H_2 suboptimal leader-follower consensus control of multi-agent systems

Yuan Gao, Junjie Jiao, Sandra Hirche

Chair of Information-oriented Control (ITR),
Technical University of Munich (TUM), Germany

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Motivation

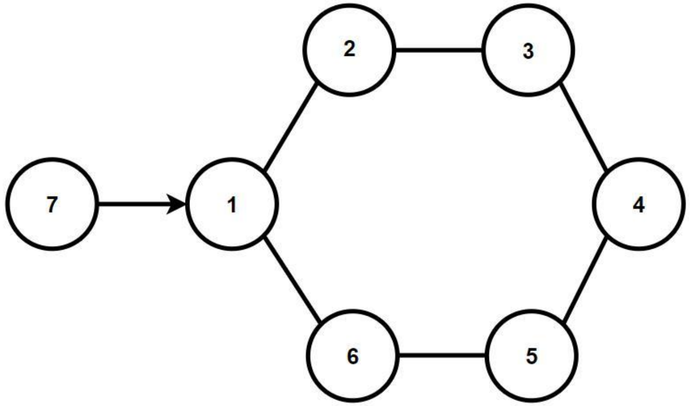
Agriculture monitoring

Cooperative manipulation

UAVs, robot arms → agents → multi-agent system

In practice, external disturbances exist, due to e.g., strong wind, measurement noise in sensors, etc.
→ Distributed control protocols with performance guarantees

Leader-follower System



Follower:

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + Ed_i(t)\,,\\ y_i(t) &= C_1x_i(t) + D_1d_i(t)\,,\\ z_i(t) &= C_2x_i(t) + D_2u_i(t)\,, \quad i = 1,\dots,N-1\end{aligned}$$

Leader:

$$\begin{aligned}\dot{x}_N(t) &= Ax_N(t)\,,\\ y_N(t) &= C_1x_N(t)\,,\\ z_N(t) &= C_2x_N(t)\,.\end{aligned}$$

Assumption:

(A, B) stabilizable and (C_1, A) detectable.

Goal

Design distributed protocols that achieve leader-follower consensus and guarantee certain performance.

Related Work

Leader-follower Consensus

- Second-order agent dynamics [Hong+ 2008]
- General linear agent dynamics [Ni+ 2010] [Li+ 2017]

Consensus with Guarantees

- H_∞ performance: state feedback [Zhang+ 2017] [Pirani+ 2019]
- H_2 performance: state/output feedback [Li+ 2011] [Jiao+ 2020]

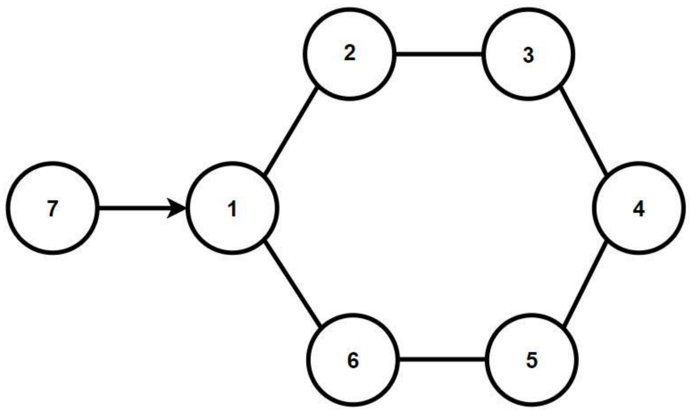
Problem

Distributed H_2 control of leader-follower systems using dynamic output feedback, in particular, extending results in [Jiao+ 2020].

Distributed Protocol

Distributed protocol:

$$\dot{w}_i = (A - GC_1)w_i + \sum_{j=1}^{N} a_{ij} \left[BF(w_i - w_j) + G(y_i - y_j) \right],$$
$$u_i = Fw_i, \quad i = 1, \ldots, N-1, \quad w_N = 0$$



Assumption:

- Leader $\xrightarrow{\text{outputs}}$ some followers
- Followers $\xleftrightarrow{\text{outputs}}$ followers: connected, simple, and undirected.

Performance Outputs

Performance outputs:

$$\epsilon_i = z_i - z_N, \quad i = 1, \dots, N-1$$

Error states and outputs:

$$e_i = x_i - x_N,$$
$$\xi_i = y_i - y_N, \quad i = 1, \dots, N-1$$

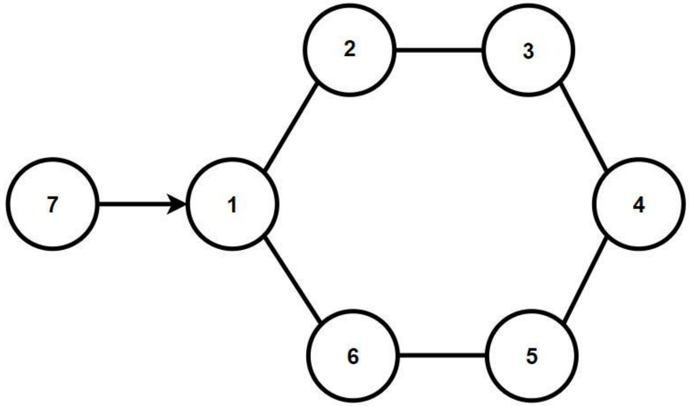
Denote

$$\boldsymbol{e} = \left[e_1^\top, \dots, e_{N-1}^\top\right]^\top, \quad \boldsymbol{\epsilon} = \left[\epsilon_1^\top, \dots, \epsilon_{N-1}^\top\right]^\top, \quad \boldsymbol{\xi} = \left[\xi_1^\top, \dots, \xi_{N-1}^\top\right]^\top,$$
$$\boldsymbol{u} = \left[u_1^\top, \dots, u_{N-1}^\top\right]^\top, \quad \boldsymbol{d} = \left[d_1^\top, \dots, d_{N-1}^\top\right]^\top, \quad \boldsymbol{w} = \left[w_1^\top, \dots, w_{N-1}^\top\right]^\top$$

Compact Form

Global error system:

$$\begin{aligned}
\dot{e} &= (I_{N-1} \otimes A)e + (I_{N-1} \otimes B)\boldsymbol{u} + (I_{N-1} \otimes E)\boldsymbol{d},\\
\xi &= (I_{N-1} \otimes C_1)e + (I_{N-1} \otimes D_1)\boldsymbol{d},\\
\epsilon &= (I_{N-1} \otimes C_2)e + (I_{N-1} \otimes D_2)\boldsymbol{u}.
\end{aligned}$$



Distributed protocol:

$$\begin{aligned}
\dot{\boldsymbol{w}} &= (I_{N-1} \otimes (A - GC_1))\boldsymbol{w} + (L_1 \otimes BF)\boldsymbol{w} + (L_1 \otimes G)\boldsymbol{\xi},\\
\boldsymbol{u} &= (I_{N-1} \otimes F)\boldsymbol{w}.
\end{aligned}$$

Laplacian matrix:

$$L = \begin{bmatrix} L_1 & L_2 \\ 0_{1\times(N-1)} & 0 \end{bmatrix}, \quad L_1 \in \mathbb{R}^{(N-1)\times(N-1)}, \quad L_2 \in \mathbb{R}^{(N-1)\times 1}$$

Problem

Controlled error system:

$$\begin{bmatrix}\dot{e}\\ \dot{w}\end{bmatrix} = \underbrace{\begin{bmatrix} I_{N-1}\otimes A & I_{N-1}\otimes BF \\ L_1 \otimes GC_1 & I_{N-1}\otimes (A - GC_1) + L_1 \otimes BF \end{bmatrix}}_{A_o} \begin{bmatrix} e \\ w \end{bmatrix} + \underbrace{\begin{bmatrix} I_{N-1}\otimes E \\ L_1 \otimes GD_1 \end{bmatrix}}_{E_o} d,$$

$$\epsilon = \underbrace{\begin{bmatrix} I_{N-1}\otimes C_2 & I_{N-1}\otimes D_2 F\end{bmatrix}}_{C_o} \begin{bmatrix} e \\ w \end{bmatrix}.$$

Impulse response matrix:

$$T_{F,G}(t) = C_o e^{A_o t} E_o$$

H_2 cost functional:

$$J(F,G) := \int_0^\infty \mathrm{tr}\left[T_{F,G}{}^\top(t) T_{F,G}(t)\right] dt$$

Problem

Let $\gamma > 0$. Design local gains F and G such that the protocol achieves

- leader-follower consensus $(x_i - x_N \to 0)$
- $J(F,G) < \gamma$.

Main Results

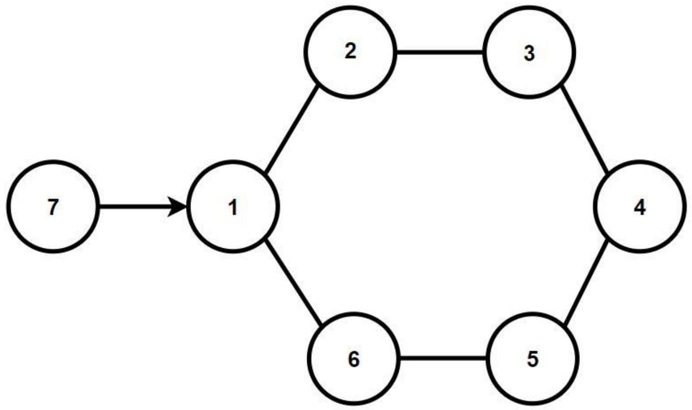
Theorem

- Let $\gamma > 0$.
- Assume $D_1 E^\top = 0,\ D_2^\top C_2 = 0,\ D_1 D_1^\top = I_r,\ D_2^\top D_2 = I_m.$
- Compute $Q > 0, \quad AQ + QA^\top - QC_1^\top C_1 Q + EE^\top < 0.$
- If $0 < c < \frac{2}{\lambda_1 + \lambda_{N-1}}$: compute $P > 0, \quad A^\top P + PA + (c^2\lambda_1^2 - 2c\lambda_1)PBB^\top P + C_2{}^\top C_2 < 0.$
- If $\frac{2}{\lambda_1 + \lambda_{N-1}} \leq c < \frac{2}{\lambda_{N-1}}$: compute $P > 0, \quad A^\top P + PA + (c^2\lambda_{N-1}^2 - 2c\lambda_{N-1})PBB^\top P + C_2{}^\top C_2 < 0.$
- If – in addition – we have $\mathrm{tr}\left(C_1 QPQC_1^\top\right) + \mathrm{tr}\left(C_2 QC_2^\top\right) < \frac{\gamma}{N-1}.$

The protocol with $F := -cB^\top P,\ G := QC_1{}^\top$ achieves

  - leader-follower consensus,
  - and $J(F, G) < \gamma.$

Simulation



Communication graph

- System matrices:

$$A = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.6 & 0 \\ 1 & 0 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 1.2 \\ 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

- (A, B) stabilizable (C_1, A) detectable.
- $D_1 E^\top = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad D_2^\top C_2 = \begin{bmatrix} 0 & 0 \end{bmatrix},$
  $D_1 D_1^\top = 1, \quad D_2^\top D_2 = 1.$
- Let $\gamma = 92$.

Simulation

Let $c = \dfrac{2}{\lambda_{N-1}}$ and compute $\quad P > 0, \quad Q > 0$

$$A^\top P + PA + (c^2\lambda_6^2 - 2c\lambda_6)PBB^\top P + C_2{}^\top C_2 + \delta I_2 = 0, \quad \delta = 0.001$$

$$AQ + QA^\top - QC_1^\top C_1 Q + EE^\top + \eta I_2 = 0, \quad \eta = 0.001$$

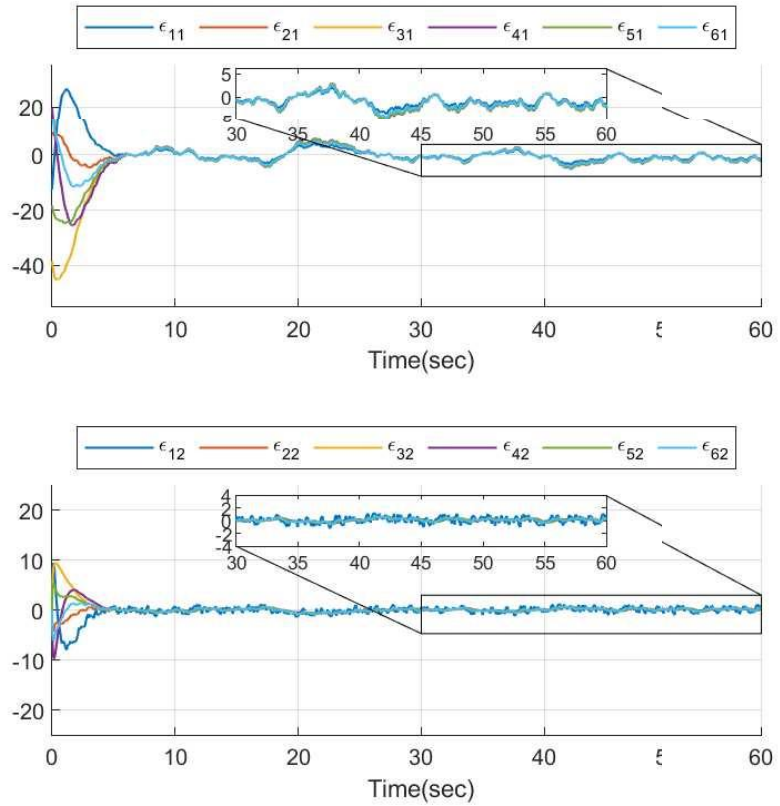
Subsequently, we compute

$$F = (1.3414, -4.5669), \quad G = (1.0407, 1.2213)^\top$$

We also compute

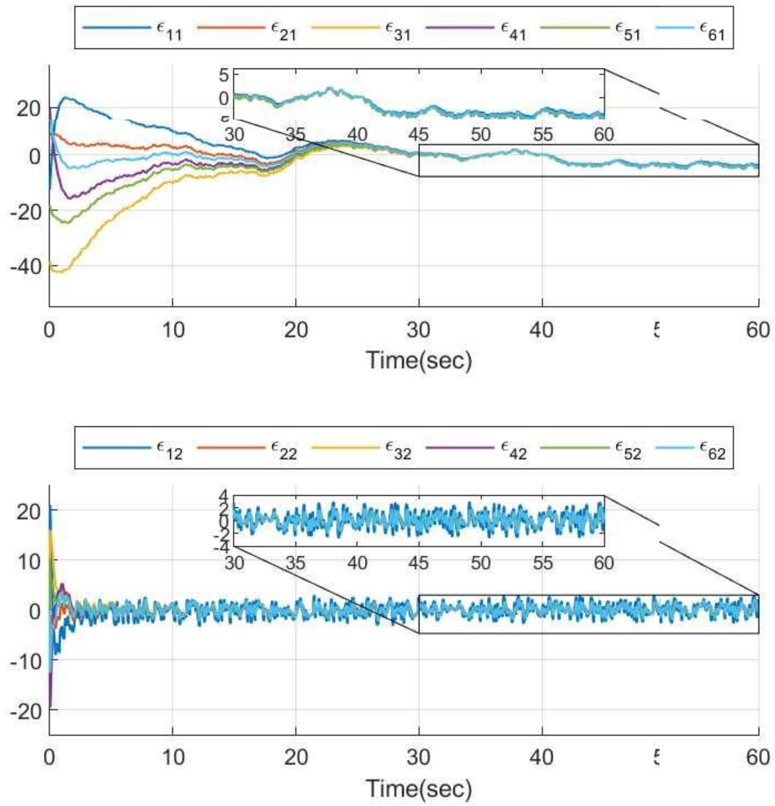
$$6(\mathrm{tr}(C_1 QPQC_1{}^\top) + \mathrm{tr}(C_2 QC_2{}^\top)) = 91.0974 \ < \ \gamma = 92.$$

Simulation



$||T_{F,G}||^2_{H_2} = 76.37 < \gamma = 92$

Our protocol: error outputs with disturbances



$||T'||_{H_2} = 547.9$

Protocol in [Li+ 2017]: error outputs with disturbances

Summary

In this work, we have

- formulated an H_2 suboptimal leader-follower consensus control problem.
- provided a design method for obtaining one such protocol.
- used a simulation example to illustrate the performance of our protocol.

Extensions:

- heterogeneous multi-agent systems
- containment control (multiple leaders) [Gao+ 2023]

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