Yuan Gao z5239220 Q4

(a) Compute the convolution $\langle 1,\underbrace{0,0,\ldots,0}_{k},1\rangle * \langle 1,\underbrace{0,0,\ldots,0}_{k},1\rangle$

(b) Compute the DFT of the sequence $\langle 1,\underbrace{0,0,\ldots,0}_{k},1\rangle$

a)

convolution can be treated as a multiplication:

$$
\begin{array}{r}
1\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1 \\
1\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1 \\
\hline
1\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1 \\
1\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1\phantom{\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1} \\
\hline
1\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,2\,\underbrace{0\,0\ \ldots\ldots 0\,0}_{k}\,1
\end{array}
$$

Therefore, the convolution of $\langle 1,\underbrace{0,0,\ldots,0}_{k},1\rangle * \langle 1,\underbrace{0,0,\ldots,0}_{k},1\rangle$ is $\langle 1,\underbrace{0,0,\ldots,0}_{k},2,\underbrace{0,0,\ldots,0}_{k},1\rangle$

b)

$a = \langle 1,\underbrace{0,0.\ldots,0}_{k},1\rangle$

$$y_k = A[x] = \sum_{n=0}^{k+1} e^{-j\frac{2\pi}{N}xn} a[n] \quad 0 \le x \le k+1$$

$x = 0\ A[0] = 1 + 1 = 2$

$x = 1\ A[1] = 1 + e^{-j\frac{2\pi}{N}(k+1)}$

$x = 2\ A[2] = 1 + e^{-j\frac{2\pi}{N}(k+1)\cdot 2}$

$x = 3\ A[3] = 1 + e^{-j\frac{2\pi}{N}(k+1)\cdot 3}$

$\cdots\ \cdots$

$x = k+1\ A[k+1] = 1 + e^{-j\frac{2\pi}{N}(k+1)^2}$