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Determine if $f(n) = O(g(n))$ or $g(n) = O(f(n))$ or both (i.e., $f(n) = \Theta(g(n))$) or neither of the two, for the following pairs of functions

(a) $f(n) = (\log_2(n))^2$, $g(n) = \log_2\left(n^{\log_2 n}\right)^2$

(b) $f(n) = n^{10}$, $g(n) = 2^{\sqrt[10]{n}}$

(c) $f(n) = n^{1+(-1)^n}$, $g(n) = n$

a)

$g(n) = \log_2 n^{2\log_2 n} = 2(\log_2(n))^2$

$f(n) = (\log_2(n))^2$

(1) When $M = 4$, $n_0 = 1$, there are $|g(n)| \leq M|f(n)|$ for all $n \geq n_0$

Therefore, $g(n) = O(f(n))$

(2) When $c = 1$, $n_0 = 1$, there are $|f(n)| \leq M|g(n)|$ for all $n \geq n_0$

Therefore, $f(n) = O(g(n))$

Hence,

$f(n) = \Theta(g(n))$

b)

$f(n) = n^{10}, g(n) = 2^{\sqrt[10]{n}} = 2^{n^{\frac{1}{10}}}$

When $M = 1$, $n_0 = 10$, there are$|g(n)| \leq M|f(n)|$ for all $n \geq n_0$

Therefore, $g(n) = O(f(n))$

c)

(1) If n is odd,

$f(n) = 1,\ g(n) = n$

When $M = 1,\ n_0 = 2$, there are $|f(n)| \leq M|g(n)|$ for all $n \geq n_0$

Therefore, $f(n) = O(g(n))$

(2) If n is even,

$f(n) = n^{n+1},\ g(n) = n$

When $M = 1,\ n_0 = 2$, there are $|g(n)| \leq M|f(n)|$ for all $n \geq n_0$

Therefore, $g(n) = O(f(n))$