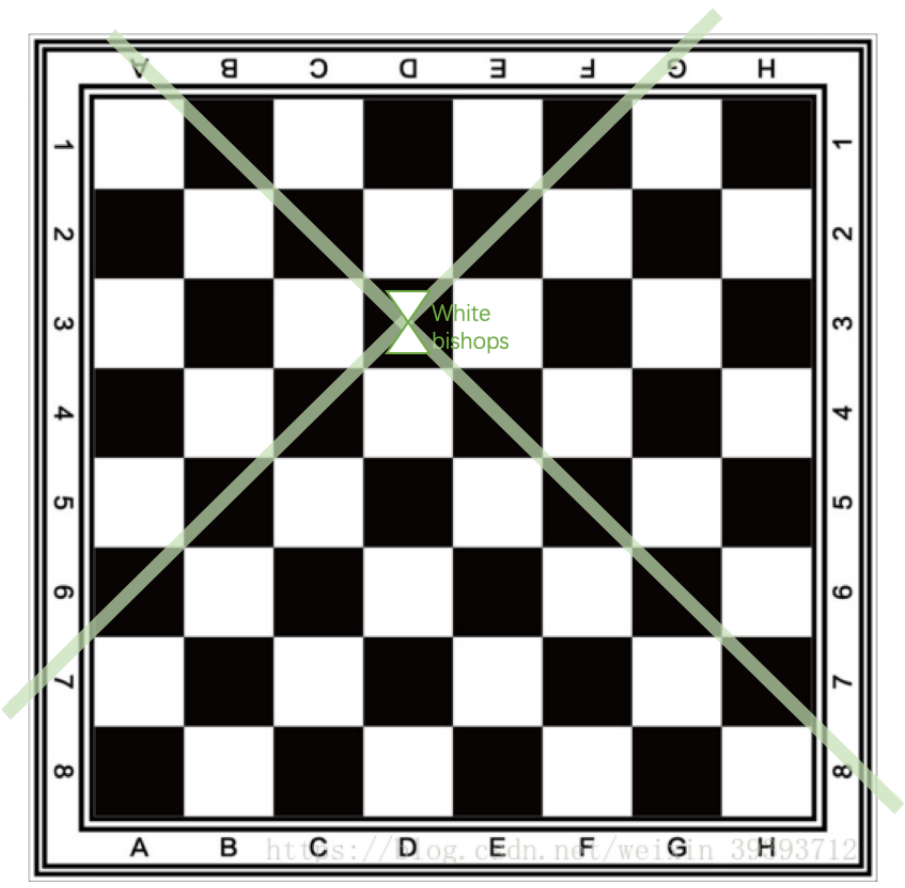
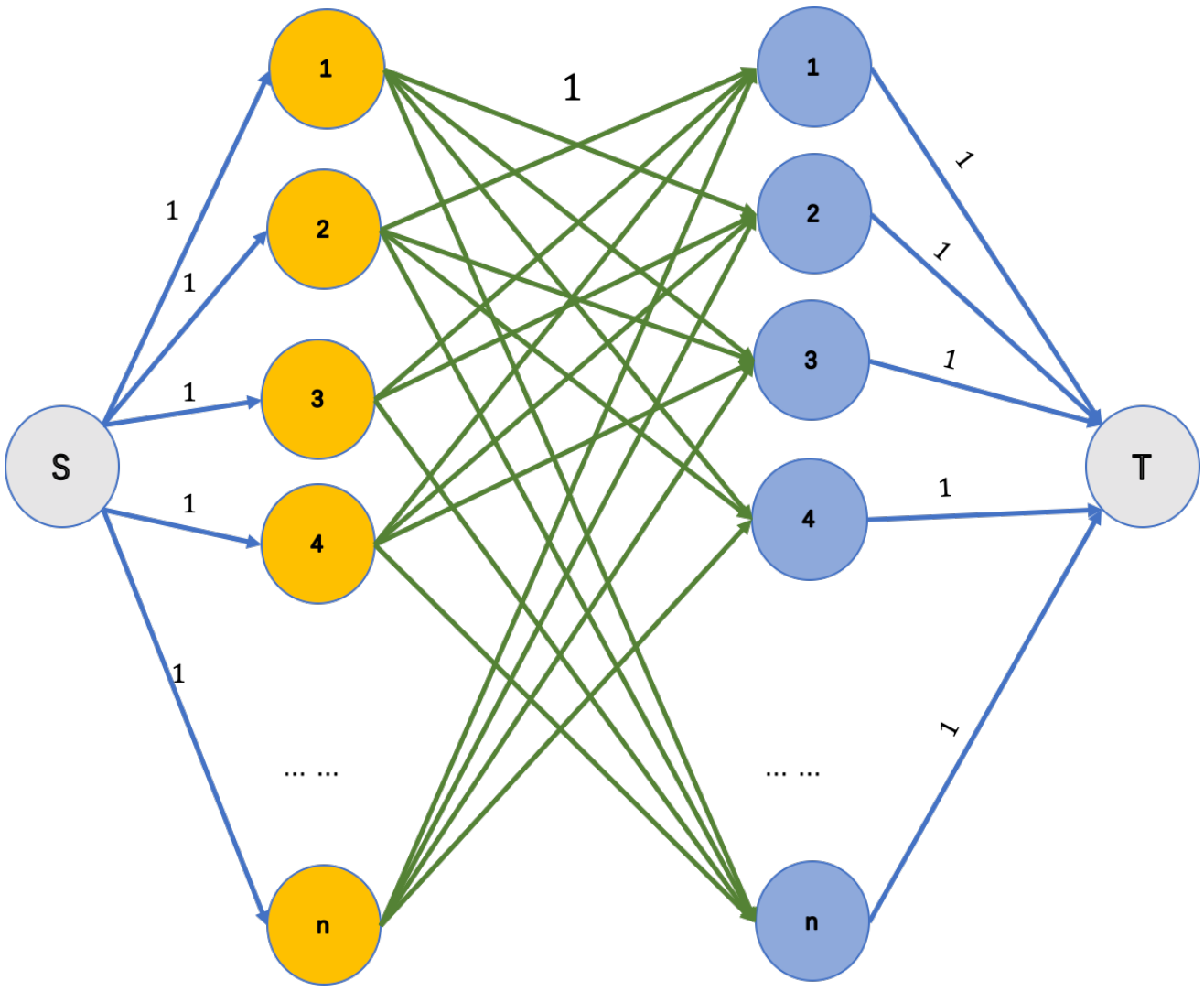
Yuan Gao z5239220 Q2

You are given a usual $n \times n$ chess board with k white bishops on the board at the given cells (a_i, b_i), $(1 \leq a_i, b_i \leq n, 1 \leq i \leq k)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the k bishops (recall that bishops go diagonally).

> **Step 1: Determine the coordinates:**
>
> > According to the question:
> > If the white bishop in the chess board, the two diagonally cannot place rooks
>
> 
>
>
> When the bishop in the place (i, j)The two diagonallies can present as:
>
> $$x' = i + j, Dx\{(1, x' - 1), (2, x' - 2), (3, x' - 3) \ldots\ldots (x' - 1, 1)\}$$
>
> If $i > j$:
>
> $$y' = i - j, Dy\{(y' + 1, 1), (y' + 2, 2), (y' + 3, 3) \ldots\ldots (n, n - y')\}$$
>
> If $i < j$:
>
> $$y' = j - i, Dy\{(1, 1 + y'), (2, 2 + y'), (3, 3 + y') \ldots\ldots (n - y', n)\}$$
>
> If the coordinates in set Dx and Dy, this place cannot put rook.
>
> **Step 2: Create the flow network:**
>
> > - Create a complete bipartite graph
> >
> > > The first part present i, $(1 \leq i \leq n)$
> > > The second part present j, $(1 \leq j \leq n)$
> >
> > - Create virtual Source(S) and sink(T)
> > - Delete the edge $edge(i, j)$ which i,j in the set of Dx, Dy.
> > - The flow of evey edge is 1
>
> This graph shows that the case of puting bishop in the coordinate (i, i)



Step 3: Calculate the largest number of black rooks:

> Use Ford-Fulkerson method count maximal flow M
> The M is the largest number of black rooks