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Given positive integers M and n compute M^n using only $O(\log n)$ many multiplications.

We can take the divide and conquer method to solve it.

For instance,

$$M^{75} \text{ can be divided to } M^{75} = M \cdot M^2 \cdot M^8 \cdot M^{64} = M^{2^0} \cdot M^{2^1} \cdot M^{2^3} \cdot M^{2^6}$$

At the same time,

The binary of 75 is 1001011

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	0	1	1

Therefore,

We can calculate M^n by binary number

$$M^n = M^{2^{i_0}} \cdot M^{2^{i_1}} \cdot M^{2^{i_3}} \dots M^{2^{i_k}}$$

There is C++ code:

```
1. long long myPow(int M, int n){
2.     /*set result */
3.     long long res = 1;
4.     while(n > 0){
5.         /*Bit operation, if the last bit is 1
6.         do the res *= M*/
7.         if((n & 1)){
8.             res *= M;
9.         }
10.        M *= M;
11.        /*right shifted one time*/
12.        n = n >> 1;
13.    }
14.    return res;
15. }
```

Because M stored the last product in a while loop, the time complexity is $O(\log n)$