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You have to produce and deliver N chemicals. You need to deliver W_i kilograms of chemical C_i. Production of each chemical takes one day, and your factory can produce only one chemical at any time. However, all of the chemicals evaporate, and at the end of each day you lose p percent of the amount you had at the end of the previous day, so you need to produce more than what you need to deliver. Schedule the production of the chemicals so that the total extra weight of all chemicals needed to produce to compensate for the evaporation loss is as small as possible.

According to the problem,

The lose of per day is $lose = W_i - W_i(1-p)^{n-i} = W_i(1-(1-p)^{n-i})$

Hence,

N chemiacal in N days the total lose is

$$\begin{aligned} total\ lose &= W_1(1-(1-p)^{n-1}) + W_2(1-(1-p)^{n-2}) + W_3(1-(1-p)^{n-3}) + \cdots \\ &\quad + W_n(1-(1-p)^0) \\ &= \sum_{i=1}^{n} W_i\left(1-(1-p)^{n-i}\right) \end{aligned}$$

Therefore,

We just sort the N chemicals as increase sequence. Make lighter W earlier and heavier W later

Greedy proof:

Let us see what happens if we swap to W_i and W_{i+1}.

$$total\ lose = W_1(1-(1-p)^{n-1}) + W_2(1-(1-p)^{n-2}) + \cdots + W_i\left(1-(1-p)^{n-i}\right)$$

$$+ W_{i+1}\left(1-(1-p)^{n-(i+1)}\right) + \cdots + W_n(1-(1-p)^0)$$

$$total\ lose' = W_1(1-(1-p)^{n-1}) + W_2(1-(1-p)^{n-2}) + \cdots + W_{i+1}\left(1-(1-p)^{n-i}\right)$$

$$+ W_i\left(1-(1-p)^{n-(i+1)}\right) + \cdots + W_n(1-(1-p)^0)$$

$$Total\ lose - Total\ lose'$$

$$= W_i\left(1-(1-p)^{n-i}\right) + W_{i+1}\left(1-(1-p)^{n-(i+1)}\right)$$

$$-W_{i+1}\left(1-(1-p)^{n-i}\right) - W_i\left(1-(1-p)^{n-(i+1)}\right)$$

$$= W_i \cdot p(1-p)^{n-i-1} - W_{i+1} \cdot p(1-p)^{n-i-1}$$

Because $W_{i+1} > W_i$

$Total\ lose - Total\ lose' < 0$

$Total\ lose < Total\ lose'$

Therefore, the optimal solution is sorting the N chemicals as increase sequence.