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You are in an orchard which has a quadratic shape of size $4n$ by $4n$ with equally spaced trees. You purchased apples from n^2 trees which also form a square, but the owner is allowing to choose such a square anywhere in the orchard. You have a map with the number of apples on each tree. Your task is to choose such a square which contains the largest total number of apples and which runs in time $O(n^2)$. Note that the brute force algorithm would run in time $\Theta(n^2)$. (20 points)

Let's assume:

Orchard					Sum square						
	0	1	2	3	4		0	1	2	3	4
0	1	3	2	1	5	0	1	4	6	7	12
1	2	3	4	3	5	1	3	9	15	19	29
2	3	4	5	7	8	2	6	16	27	38	56
3	2	3	4	3	5	3	8	21	36	50	73
4	0	4	5	7	8	4	8	25	45	66	97

When $n = 2$

According to this graph:

The value of the element in the lower right corner $Or[i][j]$

$$Or[i][j] := Or[i][j] + Or[i - 1][j] + Or[i][j - 1] - Or[i - 1][j - 1]$$

Through this formula, the left side Sum square can be created.

The sum of n^2 square:

$$Sum = Or[i][j] - Or[i - n][j] - Or[i][j - n] + Or[i - n][j - n]$$

Therefore, we only need to traverse once to find the maximum value

The Time complexity is $O(n^2)$

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1.  /*the Two-dimensional vector represent orchard,
2.    n as the length of purchased square
3.    ii as the maximum i index
4.    jj as the maximum j index */
5.  int getMaxApple(vector<vector<int>> & orchard, int n, int & ii, int & jj){
6.      int max = -1;
7.      int tmpMax = 0;
8.      for(int i = 1; i < 4 * n; ++i){
9.          orchard[i][0] += orchard[i - 1][0];
10.         orchard[0][i] += orchard[0][i - 1];
11.     }
12.     for(int i = 1; i < 4 * n; ++i){
13.         for(int j = 1; j < 4 * n; ++j){
14.             /*The value of the element in the lower right corner*/
15.             orchard[i][j] += (orchard[i - 1][j] + orchard[i][j - 1]
16.                             - orchard[i - 1][j - 1]);
17.             if(i >= n && j >= n){
18.                 /*get the sum of square*/
19.                 int tmpMax = orchard[i][j] - orchard[i - n][j]
20.                             - orchard[i][j - n] + orchard[i - n][j - n];
21.                 if(tmpMax > max){
22.                     max = tmpMax;
23.                     ii = i;
24.                     jj = j;
25.                 }
26.             }
27.         }
28.     }
29.     return max;
```