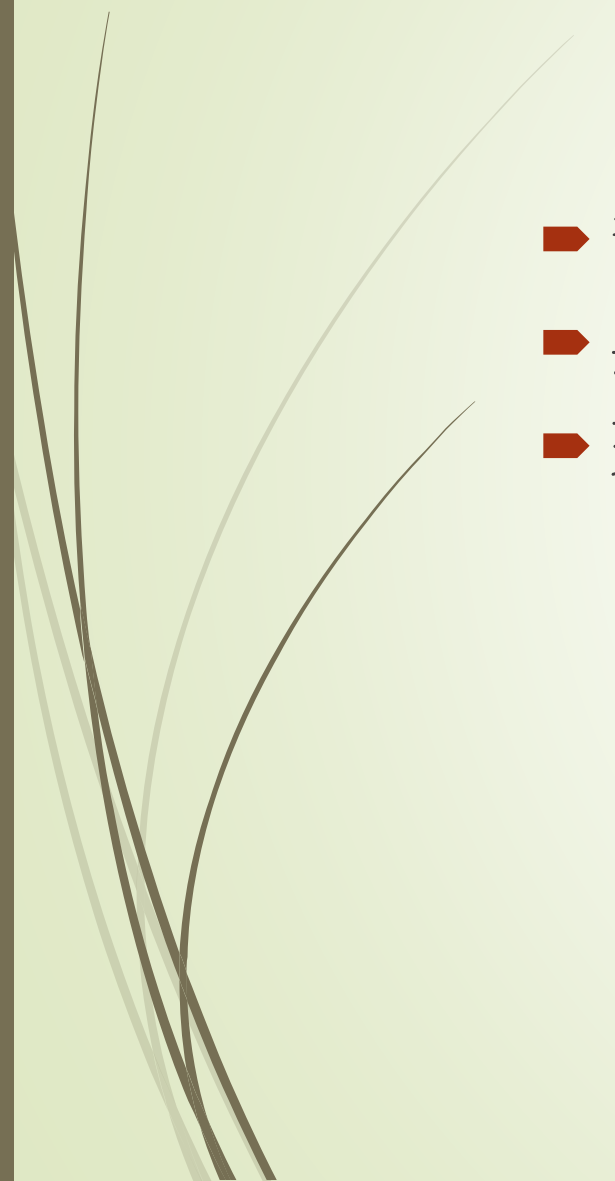


# Anomalous Hall Effect

彭泽龙 2017.12.15



# outline

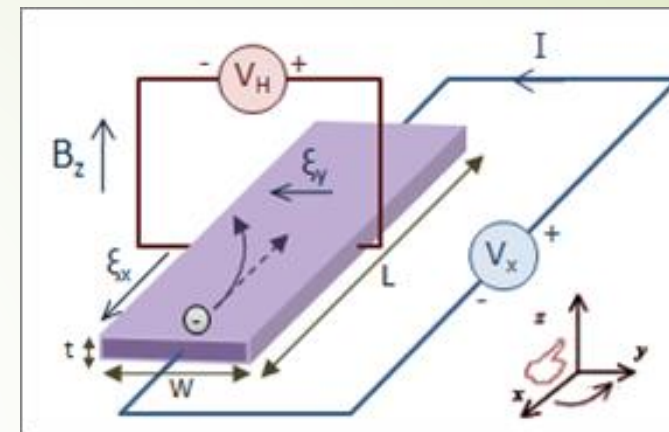
- 背景
  - 实验
  - 理论
- 

## 背景

1879年，霍尔效应

1881年，反常霍尔效应

1980年，量子霍尔效应



非磁性导体中， $\rho_{xy}$ 随外场线性增加

铁磁体中，弱场时增加很快，之后趋于饱和。

$$\rho_{xy} = R_0 H_z + R_s M_z,$$

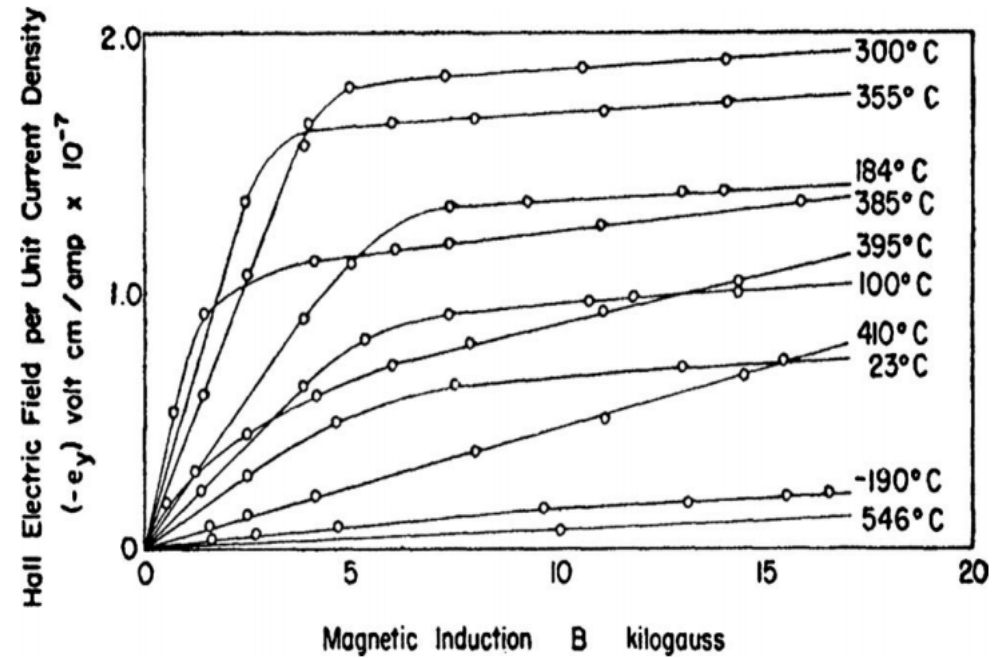


FIG. 1. The Hall effect in Ni (data from [Smith, 1910](#)). From [Pugh and Rostoker, 1953](#).

# AHE机制

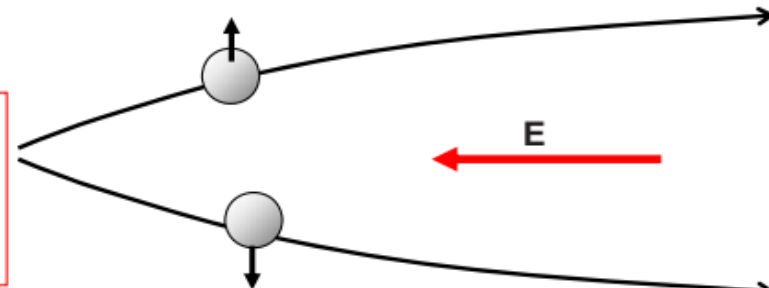
- Intrinsic
- Skew scattering
- Side jump

## a) Intrinsic deflection

Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.

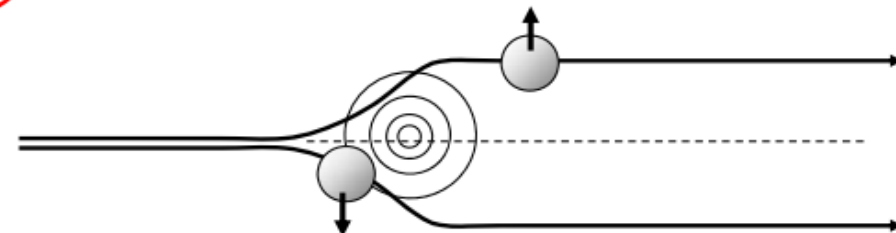
$$\frac{d\langle\vec{r}\rangle}{dt} = \frac{\partial E}{\hbar\partial\vec{k}} + \frac{e}{\hbar} \vec{E} \times \vec{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature



## b) Side jump

The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity. The time-integrated velocity deflection is the side jump.



## c) Skew scattering

Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.

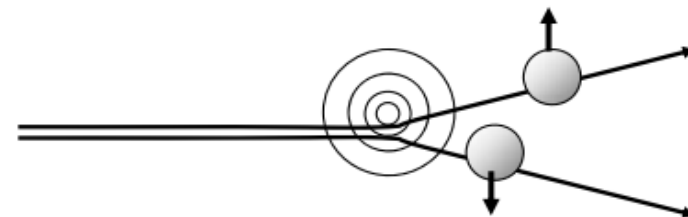


FIG. 3. (Color online) Illustration of the three main mechanisms that can give rise to an AHE. In any real material all of these mechanisms act to influence electron motion.

# 实验

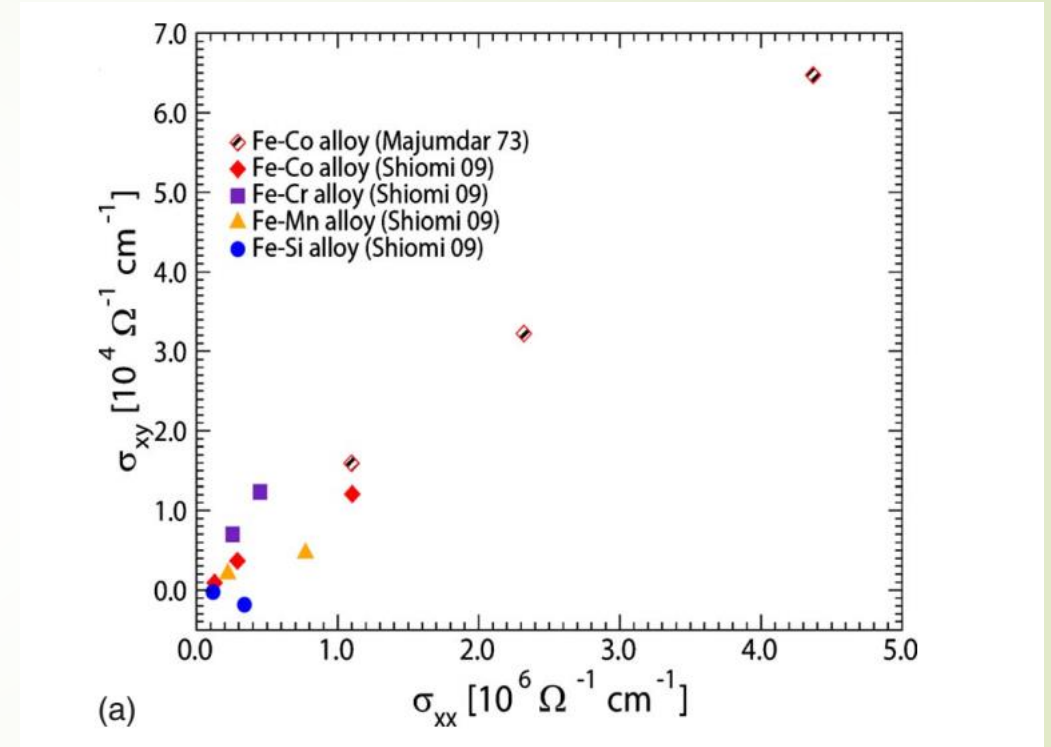
- High conductance,  $10^6 \text{ } (\Omega\text{cm})^{-1}$ , skew  
散射占主导
- Good-metal,  $10^4 \sim 10^6$ , 对xx电导不敏感
- Bad-metal,



# High conductance regime

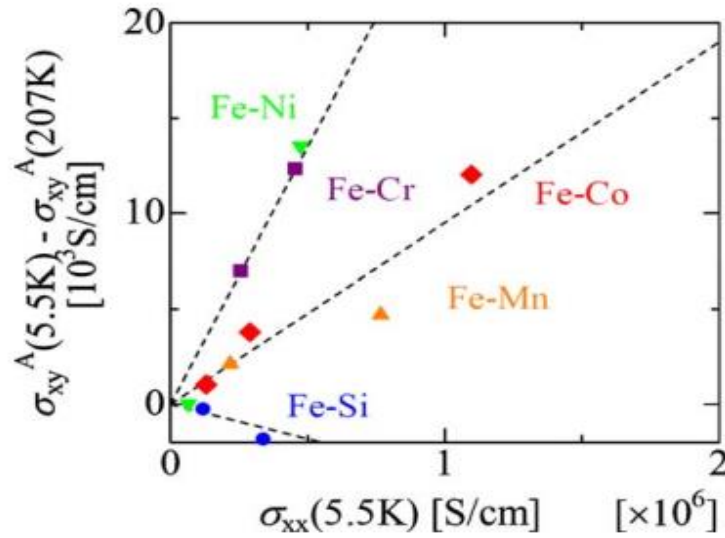
为了使磁化达到饱和状态，必须用强外场，这导致正常霍尔效应也很大，AHE不容易观察到。

这种线性变化趋势为skew散射提供了证据。



Majumdar and Berger, 1973 and Shiomi *et al.*, 2009

# High conductance regime



Shiomi *et al.*, 2009

高温部分被认为是其他机制

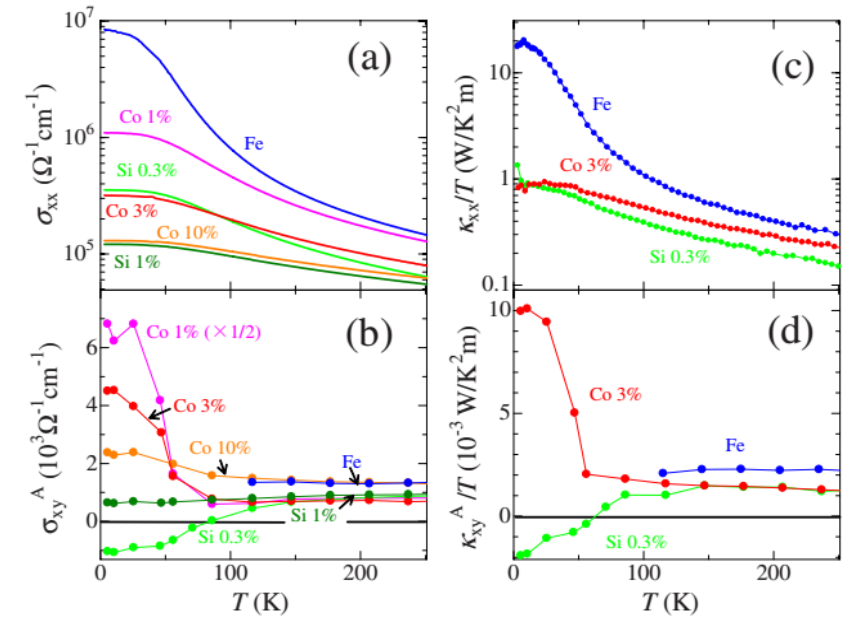


FIG. 1. (Color online) Longitudinal and anomalous Hall conductivities of charge and heat. Temperature ( $T$ ) profiles of (a) the electrical conductivity ( $\sigma_{xx}$ ), (b) the anomalous part of electrical Hall conductivity ( $\sigma_{xy}^A$ ), (c) the thermal conductivity ( $\kappa_{xx}$ ) divided by  $T$ , and (d) the anomalous part of thermal Hall conductivity ( $\kappa_{xy}^A$ ) divided by  $T$ .

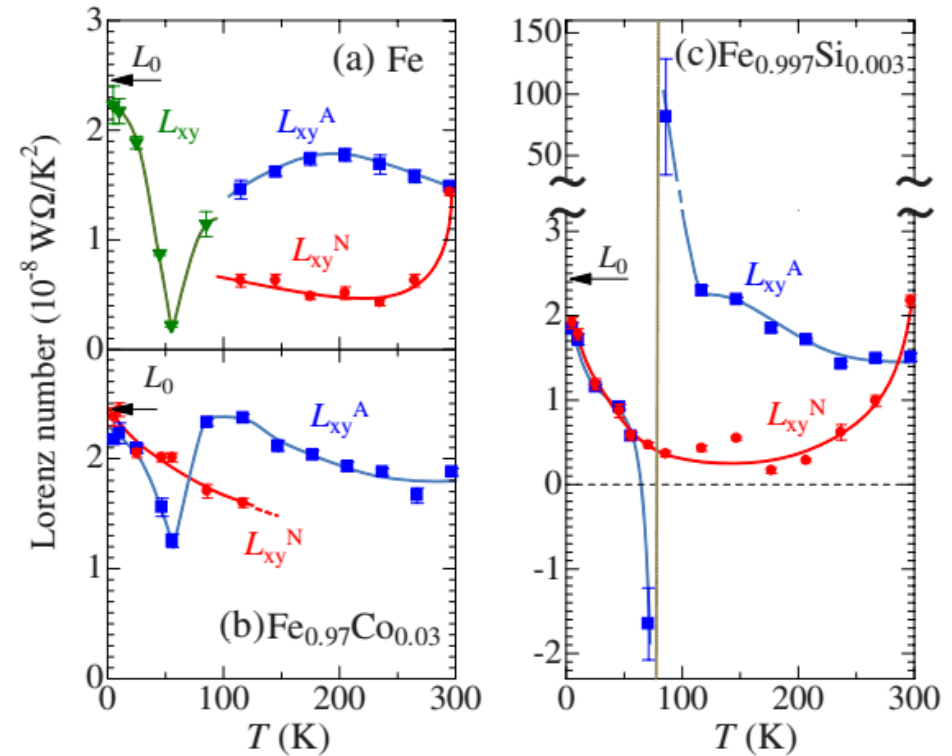


# High conductance regime

非弹性散射效应影响NH，也影响低温的AH (<100K)， >150K时几乎不影响AHC，是由于Berry Phase AHE无耗散性质

Lorentz number的改变标志着非弹性散射

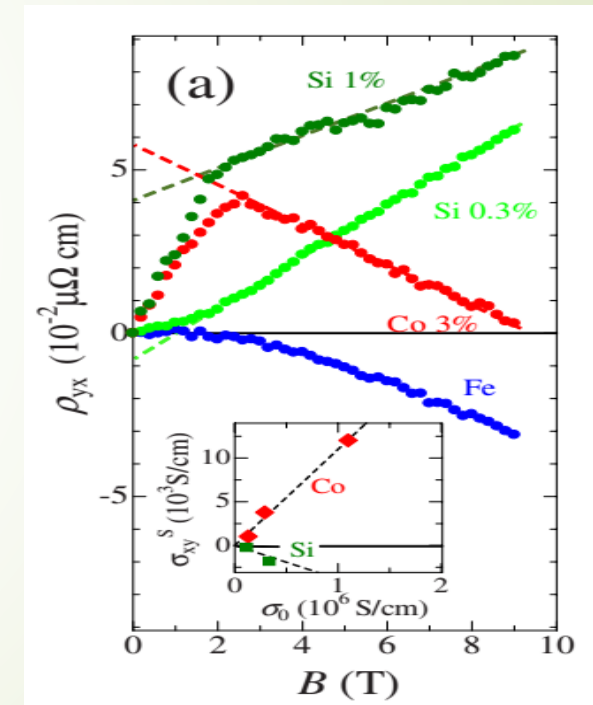
$$L = K / \sigma T$$



Shiomi et al., 2009

# High conductance regime

- 随着杂质数量增加， $\sigma$ 迅速减小，intrinsic AHE开始主导



Shiomi *et al.*, 2009

# Good metal

$\sigma_{xy}$ 与 $\sigma_{xx}$ 无关，以不依赖散射的机制为主，intrinsic和side jump

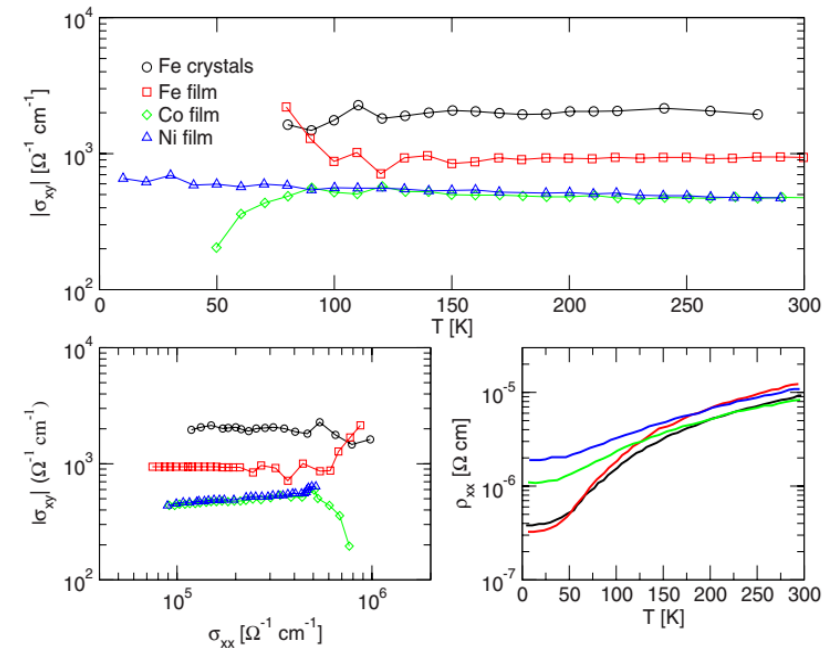


FIG. 5. (Color online) Measurements of the Hall conductivity and resistivity in single-crystal Fe and in thin foils of Fe, Co, and Ni. The top and lower right panels show the  $T$  dependence of  $\sigma_{xy}$  and  $\rho_{xx}$ , respectively. The lower left panel plots  $|\sigma_{xy}|$  vs  $\sigma_{xx}$ . From [Miyasato et al., 2007](#).

Miyasato et al., 2007

# Bad metal

改变薄膜厚度改变来调节电阻  
localization

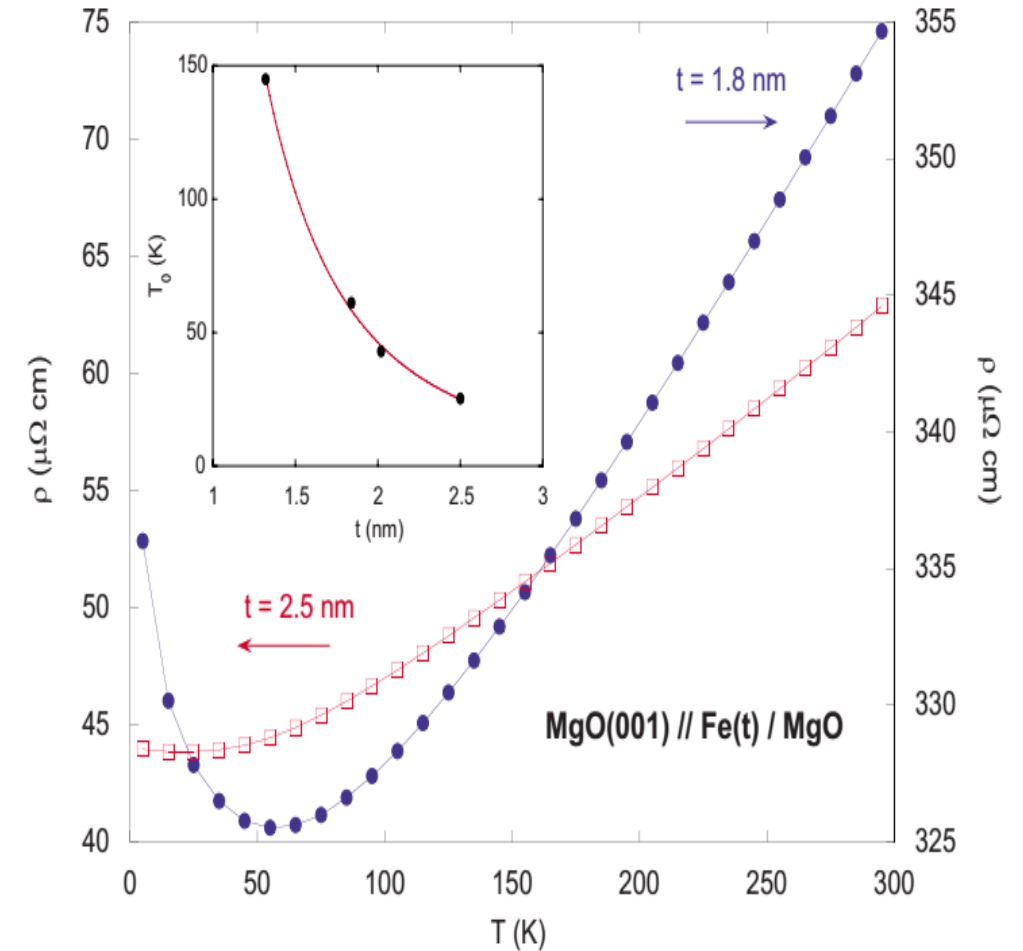
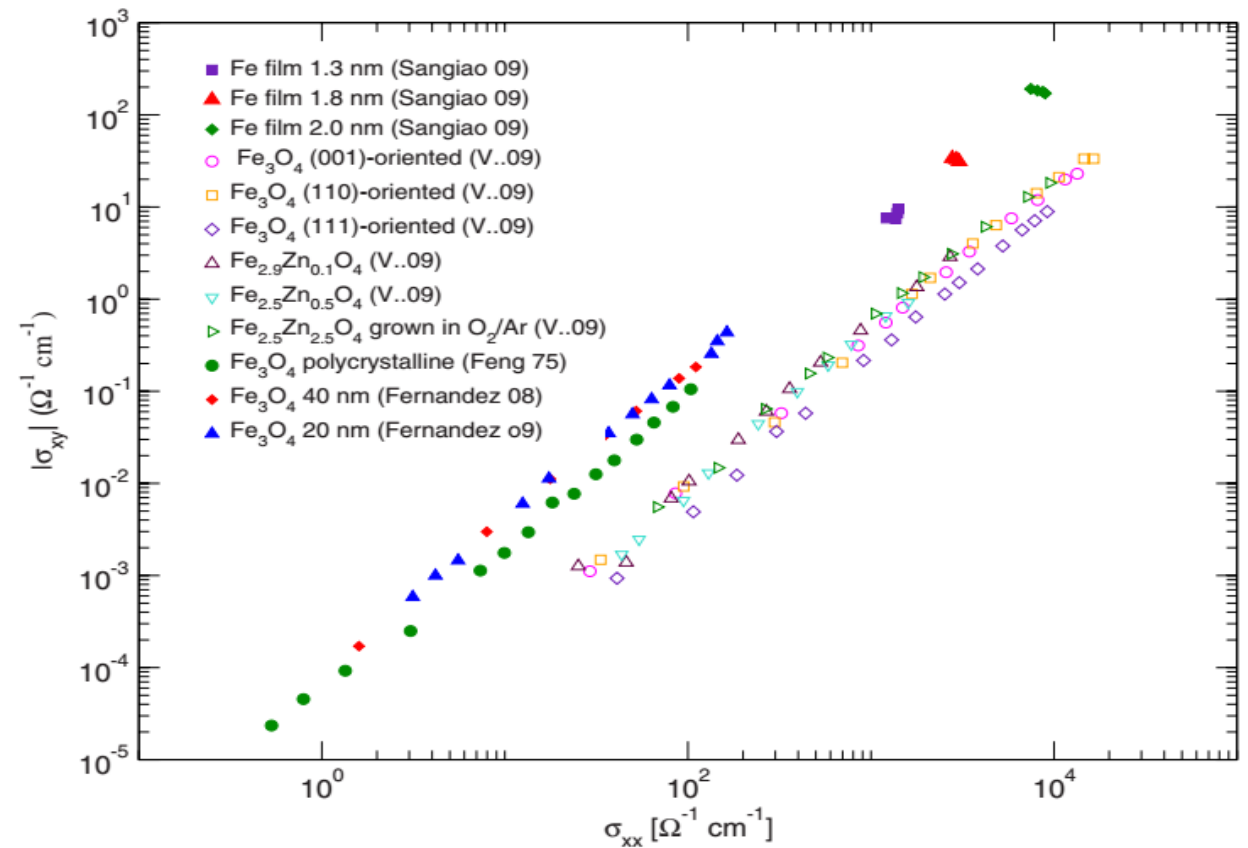



FIG. 7. (Color online) The  $T$  dependence of  $\rho$  in epitaxial thin-film  $\text{MgO}(001)/\text{Fe}(t)/\text{MgO}$  with  $t=1.8$  and  $2.5 \text{ nm}$ . The inset shows how  $T_0$ , the temperature of the resistivity minimum, varies with  $t$ . From [Sangiao \*et al.\*, 2009](#).

# Bad metal

$\alpha \sim 1.6-1.8$



Nagaosa N, Sinova J, Onoda S, et al. Anomalous Hall effect[J]. Reviews of Modern Physics, 2009, 82(2):1539-1592.



# 对称性分析

AHE的出现是破坏时间反演对称性的结果





# Intrinsic

- **Karplus-Luttinger** 理论——忽略所有晶格无序，考虑自旋轨道耦合，电子获得垂直于外场的反常速度；带间矩阵元对反常霍尔流有重要贡献

$$H_T = H_0 + H_{\text{SOI}} + H_E,$$

$$H_{\text{SOI}} = \frac{\hbar e}{2m^2 c^2} (\mathbf{s} \times \nabla V) \cdot \mathbf{p},$$

$$\bar{v}_a = -ieE_b \sum_{n,k} \rho'_0(E_{nk}^p) J_a^{nn}(\mathbf{k}),$$

$$R_s \cong \frac{2e^2 H_{\text{SO}}}{m\Delta^2} \delta \left\langle \frac{m}{m^*} \right\rangle \rho^2,$$

# intrinsic

- ➡ Kubo公式
- ➡ 半经典Boltzmann 理论

$$\sigma_{ij}^{AH-int} = -\varepsilon_{ij\ell} \frac{e^2}{\hbar} \sum_n \int \frac{d\mathbf{k}}{(2\pi)^d} f(\varepsilon_n(\mathbf{k})) b_n^\ell(\mathbf{k}),$$

$$\mathbf{a}_n(\mathbf{k}) = i \langle n, \mathbf{k} | \nabla_{\mathbf{k}} | n, \mathbf{k} \rangle$$

$$\mathbf{b}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{a}_n(\mathbf{k}).$$

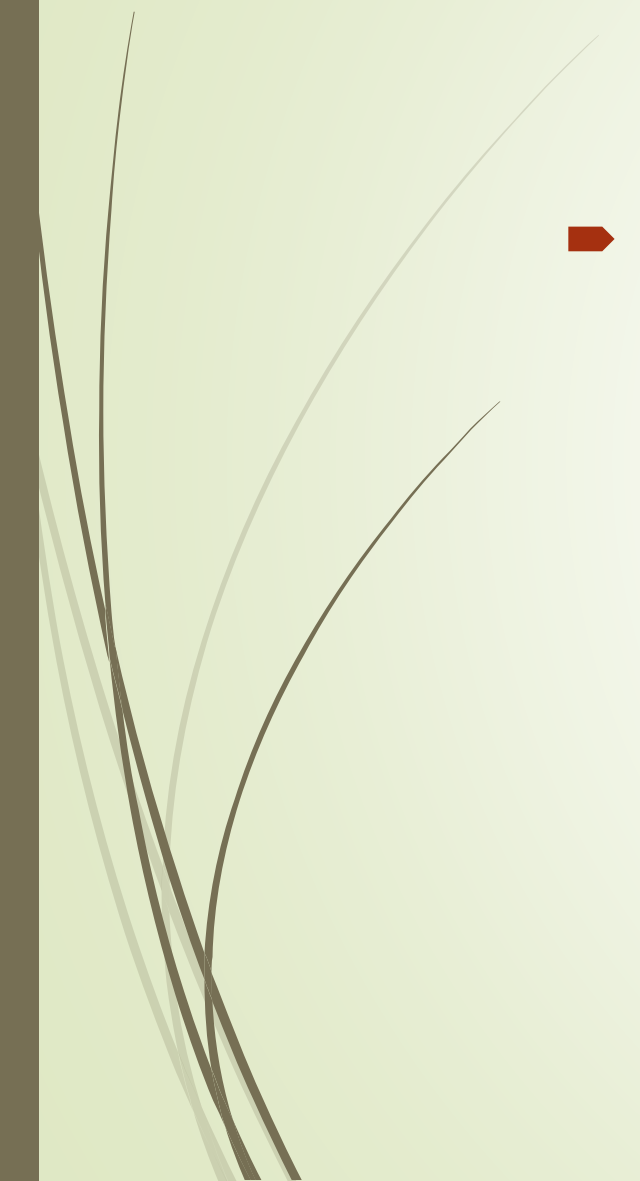

$$\dot{\mathbf{r}}_c = \frac{\partial E_n(\mathbf{k}_c)}{\partial \mathbf{k}_c} - \hbar \dot{\mathbf{k}}_c \times \mathbf{b}_n(\mathbf{k}_c)$$

# Screw scattering

- ➡ Smit提出，自旋轨道耦合，载流子经过杂质散射后，携带垂直于入射动量和磁化方向的动量。

$$\langle \mathbf{k}' s' | V | \mathbf{k}, s \rangle = \tilde{V}_{\mathbf{k}, \mathbf{k}'} \left( \delta_{s, s'} + \frac{i\hbar^2}{4m^2 c^2} (\langle s' | \boldsymbol{\sigma} | s \rangle \times \mathbf{k}') \cdot \mathbf{k} \right).$$

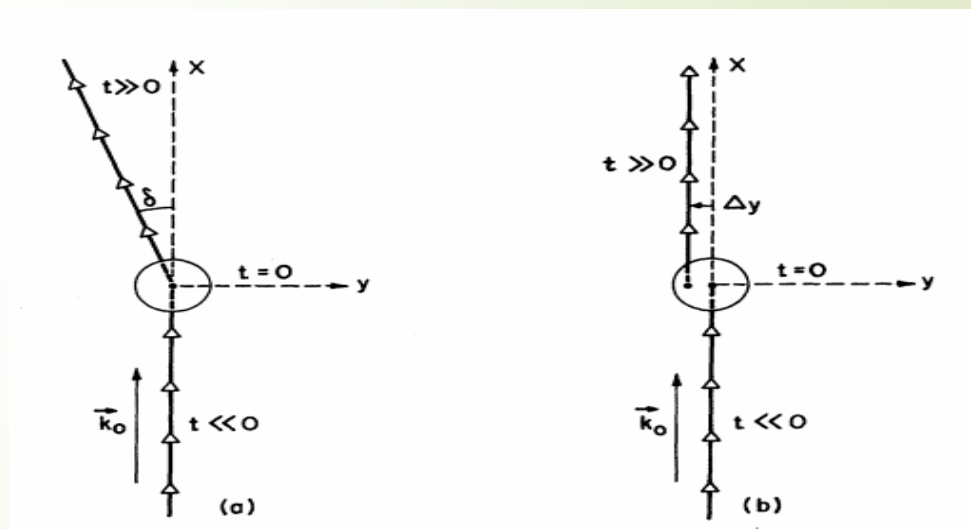
$$W_{\mathbf{k}\mathbf{k}'}^A = -\tau_A^{-1} \mathbf{k} \times \mathbf{k}' \cdot \mathbf{M}_s.$$



## ► Kondo理论——sd相互作用

# Side jump

- Berger
- 可以类比KL反常速度机制，看作粒子受到杂质电场的影响
- 实验研究中容易产生困扰



Berger L. Side-Jump Mechanism for the Hall Effect of Ferromagnets[J]. Physical Review B Condensed Matter, 1970, 2(11):4559-4566.

# Linear Response

➡ 半经典

$$\hbar \dot{\mathbf{k}}_c = -e\mathbf{E},$$

$$\dot{\mathbf{r}}_c = \frac{\partial E_n(\mathbf{k}_c)}{\partial \mathbf{k}_c} - \hbar \dot{\mathbf{k}}_c \times \mathbf{b}_n(\mathbf{k}_c),$$






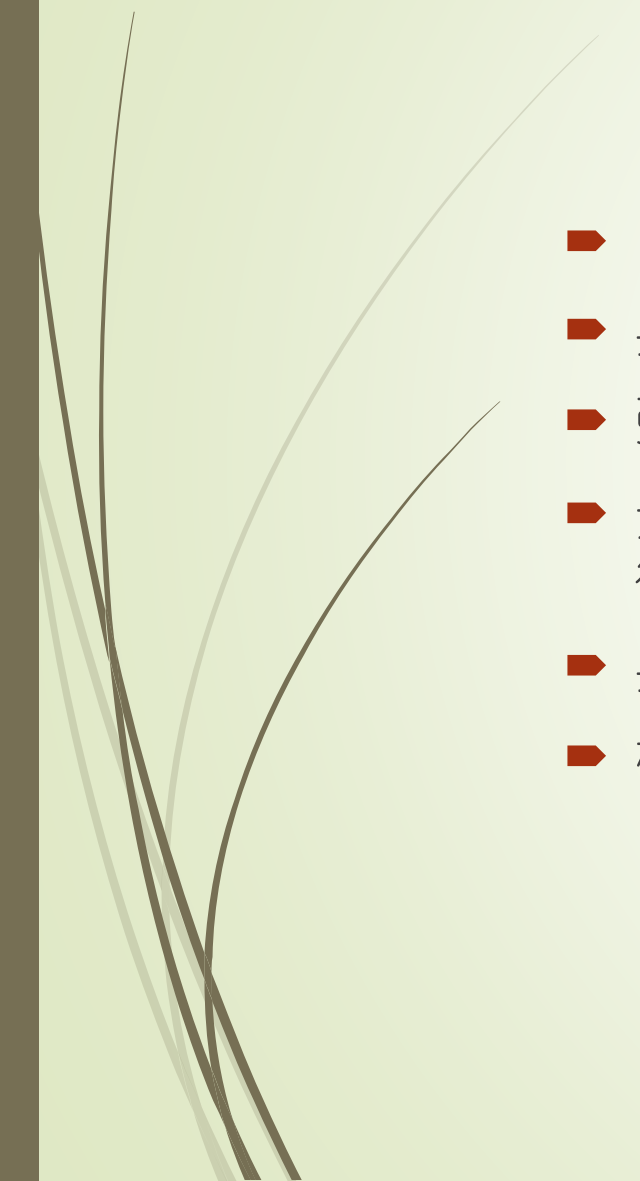
▸ Kubo formula





➡ Keldysh



- 
- 
- $\sigma_{xy}$  不依赖于  $\sigma_{xx}$  时，通常理解为 intrinsic AHE 的贡献
  - 实验材料分为三大类，高电导类，好金属类，坏金属类
  - 强自旋轨道耦合的材料适合研究本征 AHE 机制
  - 通过第一性原理计算贝里曲率，确认了费米面附近的能带交叉会导致大的本征 AHE
  - 实空间贝里曲率导致的 AHE 有所进展
  - 格林函数处理输运现象的理论已经建立

# Reference

- Nagaosa N, Sinova J, Onoda S, et al. Anomalous Hall effect[J]. Reviews of Modern Physics, 2009, 82(2):1539-1592.
- Shiomi Y, Onose Y, Tokura Y. Extrinsic anomalous Hall effect in charge and heat transport in pure iron, Fe<sub>0.997</sub>Si<sub>0.003</sub>, and Fe<sub>0.97</sub>Co<sub>0.03</sub>[J]. Phys.rev.b, 2009, 79(10):-.
- Berger L. Side-Jump Mechanism for the Hall Effect of Ferromagnets[J]. Physical Review B Condensed Matter, 1970, 2(11):4559-4566.



*Thank you!*