Automatic Signal Design for Quantum Illumination

Yuan Liu
3030079603
Department of Computer Science
The University of Hong Kong
yuan59@connect.hku.hk

Fei Meng
3030049359
Department of Computer Science
The University of Hong Kong
mengf@connect.hku.hk

Abstract

Quantum illumination is an emerging technique in detecting the presence of an object with low reflectivity and highly noisy background. An important problem in quantum illumination is to design the optimal detecting signal that is optimal to the detection. However, this problem is hard to solve analytically for the general situation. Therefore, we seek to find an machine learning algorithm that can help us to design the optimal detecting signal automatically.

1 Introduction

Quantum illumination [7] is a novel technique in detecting whether a distant object is present or not, subjecting to highly noisy background and low reflectivity. For conventional illumination [7], a signal is sent to the region where the object might be present. If the object is absent, then only the environmental noise is received. However, if the object is present, then the signal is reflected and mixed with the environmental noise. To detect the presence or absence of the object is therefore a hypothesis testing problem, where one needs to differentiate the environmental noise and the mixture of the reflected signal and the environmental noise.

Quantum mechanics provides more accurate detection in the above scenario, since light signals are composed by billion's of photons which follow the quantum mechanical law [9]. In the quantum mechanical region, we can even consider the illumination problem using only a few photons. In particular, if only one photon is considered, then it is called one-shot illumination [11].

What's new in this quantum mechanical region? A novel resource in this quantum region is called quantum entanglement [5], which is a non-classical correlation that attributes to many quantum advantages, such as speedup in quantum computation, super-dense coding, unconditional secure quantum cryptography, etc [9]. When applied to quantum metrology [2], it is expected to increase the detection precision. This is basically the motivation to consider quantum illumination.

In quantum illumination, the signal sent is entangled with an idler system kept by the detector [11]. If the object is absent, then the signal is lost, and what's left is the composite state of the partial state of the idler system and the detected noise state. If the object is present, then the received state is a mixture of the entangled signal state and the environmental noise. If we know the quantum channel which described how the signal state is updated after the detection, and we know which signal state is sent, then by Helstrom theorem [4], we know what is the optimal measurement that optimally differentiate them. However, the major problem is that we usually do not know as a prior what is the optimal entangled signal state. What's worse, we do not even know the exact quantum channel that models the update of the signal state from its emission to its reflection and detection. This makes it impossible to design the optimal signal state analytically.

However, machine learning algorithm [8] provides new insights in designing such an optimal entangled signal state, even when the intermediate quantum channel is fully unknown. In this course

COMP 9501 Machine Learning Course Project (CS 2021), Hong Kong.

project, we design an method to use machine learning algorithm to find the optimal signal state of the quantum illumination problem.

In the rest of this report, we will first review some related works and then move on to the discussion of basic notations, concepts used in quantum illumination. After establishing the preliminary concepts and tools, we then introduce how we tackle this problem in Methodology section. Then we summarize our experiments results in Experiments section and conclude our project in the Conclusion section.

2 Related Works

There have been some research on using machine learning algorithm in radar signal processing [6], where DCNN networks are used to process the reflected signal. However, there is little work on the design of such signal. This is because classically, such signal deign is not as important as the processing of the reflected signal. But this is different in quantum regime, where the fine control of the quantum parameters is very effective. This inspired researchers to investigate how can we use machine learning algorithms to control a quantum system [10], or to prepare a quantum state [1]. In this project, we proposed to optimize the quantum signal used in the quantum illumination problem using machine learning algorithms.

As a baseline, we refer to the theoretical work by Yung et al [11], where the exact performance and the optimal signal are analytically derived for a special type of quantum channels. In their work, the environmental noise ρ_E can be arbitrary. And they assume that the signal undergoes an evolution that is simple: if the object is absent, then the signal is totally lost, corresponding to the channel

$$\mathcal{E}_0(\rho) = \mathrm{tr}[\rho]\rho_E \ . \tag{1}$$

If the object is present, then it undergoes a simple reflection

$$\mathcal{E}_1(\rho) = \eta\,\mathrm{tr}[\rho] + (1-\eta)\rho_E \ . \tag{2}$$

Under this assumption, they derived the optimal signal state for detection. The entanglement spectrum of the optimal signal state is inversely proportional to the noise spectrum of the environmental noise, and is irrelevant to the reflectivity η. We will use this result to check if our network runs correctly.

3 Methodology

3.1 Mathematical and Physical Preliminary.

In this subsection, we will introduce the mathematical and physical preliminary for quantum illumination. In quantum mechanics, the state of a system is described by a density matrix, which is a positive semi-definite matrix whose trace is one. In this report, we follow the convention in quantum information science and use Greek letters such as ρ or σ to denote quantum state.

Pure and mixed state. If the density matrix has eigenvalue 1 and all other values 0, then we call this quantum state a pure state. Otherwise, it is a mixed state. For pure state, we can represent it by the eigenvector whose eigenvalue is 1 of the density matrix, and denote it using braket notation. For example, if a quantum state ψ is a pure state, then it can be written as $\psi = |\psi\rangle\langle\psi|$, where $|\psi\rangle$ known as a ket, which is the eigenstate of the matrix ψ with eigenvalue 1, i.e. $\psi|\psi\rangle = |\psi\rangle$ and $\langle\psi|$ known as a bra, which is the complex conjugation of the vector $|\psi\rangle$. In this sense, there is no ambiguity between a ket $|\psi\rangle$ and a pure state ψ.

Composition of states. If we have more systems, the state of the composite system is obtained by taking tensor product of its composition. For example, if quantum system A is at quantum state ρ_A and quantum system B is at quantum state ρ_B, then the joint state of the composite system AB is $\rho_{AB} = \rho_A \otimes \rho_B$. States of this form is known as product state. Moreover, if with probability p_i, the state of system A is $\rho_{A,i}$ and the state of the system B is $\rho_{B,i}$, then the state of the composite system is $\rho_{AB} = \sum_i p_i \rho_{A,i} \otimes \rho_{B,i}$, which is a probabilistic mixture of product state. This type of state is known as separable state. Then we can discuss quantum entanglement.

Quantum entanglement. For quantum entangled states, it is defined as the quantum state that is not separable. For example, consider the quantum state of a two qubits system. The state of each qubit system can be described by a 2x2 density matrix. Using $|0\rangle$ and $|1\rangle$ as basis, any pure state $|\psi\rangle$ can be

expressed as $|\psi\rangle = a|0\rangle + b|1\rangle$, for two complex numbers a, b. Therefore, the joint pure state of the two qubit systems $|\Psi\rangle_{AB} = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$ generally cannot be written in the product form and therefore is entangled.

Evolution of states. The evolution of the state of a quantum system is described by operations called quantum channels that send quantum state to quantum state, which is modeled mathematically by completely positive and trace preserving maps (CPTP maps). By convention, we use script letters such as $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{E}$ to denote quantum channels. In particular, we use $\mathcal{I}$ to denote the identity channel, which does nothing on a quantum state, meaning $\mathcal{I}(\rho) = \rho$ for any ρ. If a quantum channel $\mathcal{A}$ acts on a quantum system A and a quantum channel B acts on the other quantum system B, then the action on the composite system AB is obtained by tensor product $\mathcal{A} \otimes \mathcal{B}$.

Measurement on quantum states. In quantum mechanics, information is obtained via quantum measurement, which is modeled by a positive operator valued measure (POVM). For example, let us consider the binary outcome measurement $\{\Pi_0, \Pi_1\}$ which is applicable in this hypothesis testing scenario. This measurement consists two positive operators Π_0 and Π_1 that satisfies the normalization condition $\Pi_0 + \Pi_1 = I$, where I is the identity operator. If you perform this measurement on a quantum state ρ, then with probability $p_0 = \mathrm{tr}[\Pi_0\rho]$ you get the outcome 0 and with probability $p_1 = \mathrm{tr}[\Pi_1\rho]$ you get the outcome 1. A simple strategy for well designed POVM in a hypothesis testing should be choose hypothesis 0 when the measurement outcome is 0 while choose hypothesis 1 otherwise.

Quantum channels for illumination. In fact, the presence and absence of the object define two quantum channels $\mathcal{E}_1$ and $\mathcal{E}_0$ respectively. Consider for example the simplest condition where the signal is completely lost and only environmental noise state ρ_E is received if the object is absent, and if the signal is partially reflected with reflectivity η if the object is present. Mathematically, we have

$$\mathcal{E}_0(\rho_A) = \rho_E \,, \tag{3}$$

where ρ_A is the signal state sent and

$$\mathcal{E}_1(\rho_A) = \eta\rho_A + (1-\eta)\rho_E \,. \tag{4}$$

In general, these channels may take more complicated form whose precise mathematical expressions are unknown. This is the basic motivation that inspire us to exploit machine learning algorithm to help us design the optimal signal.

For quantum illumination, the signal state is entangled with an ancillary system B. Then the joint action of the above two quantum channels on the signal-ancilla system with state ρ_{AB} is

$$\mathcal{E}_0 \otimes \mathcal{I}(\rho_{AB}) = \rho_E \otimes \rho_B \,, \tag{5}$$

where $\mathcal{I}$ is the identity channel and ρ_B is the reduced density state of the composite state ρ_{AB}, and

$$\mathcal{E}_1 \otimes \mathcal{I}(\rho_{AB}) = \eta\rho_{AB} + (1-\eta)\rho_E \otimes \rho_B \,. \tag{6}$$

Measurement for quantum illumination. If the quantum states to be differentiated are known, say ρ_1, ρ_0 with a prior probability p_1 and p_0, then by Helstrom theorem we know that the optimal POVM is the projector onto the positive and negative support of $p_1\rho_1 - p_0\rho_0$. However, in practice, if the quantum channel is unknown, then the quantum states received would be unknown even though we have full control in picking a signal state. In this case, we should use an algorithm to differentiate unknown state. More precisely, in this case we do not know the reflected state of the signal and the ancilla when the object is present. But we do know the received state when the object is absent; the state we have would be $\rho_E \otimes \rho_B$. If we could find that the received state is not this state, then we conclude that the object is there.

SWAP test for quantum illumination. According to the above analysis, we want to certify whether two quantum states are the same. Then there is an easier test than using POVM to do this. This is known as SWAP test, which involves another ancillary qubit. This ancilla undergoes first a Hadamard gate which transforms the input state $|0\rangle$ to the coherent superposition $|+\rangle = 1/2(|0\rangle + |1\rangle)$. After the Hadamard gate, the ancilla acts as a control bit to apply the controlled SWAP operation on the two quantum states which we want to test. Then we apply a second Hadamard gate on the control qubit, following by a measurement on the computational basis. In this way, the probability to obtain outcome 0 is $1/2(1 + \mathrm{tr}[\rho_1\rho_2])$ while the probability to obtain the outcome 1 is $1/2(1 - \mathrm{tr}[\rho_1\rho_2])$, if the two quantum state to be differentiated are ρ_1 and ρ_2. For quantum illumination, we can pick $\rho_1 = \mathcal{E}_0 \otimes \mathcal{I}(\rho_{AB})$ and $\rho_2 = \mathcal{E}_1 \otimes \mathcal{I}(\rho_{AB})$, and maximize the cost function $1/2(1 - \mathrm{tr}[\rho_1\rho_2])$ to find the optimal probe state.

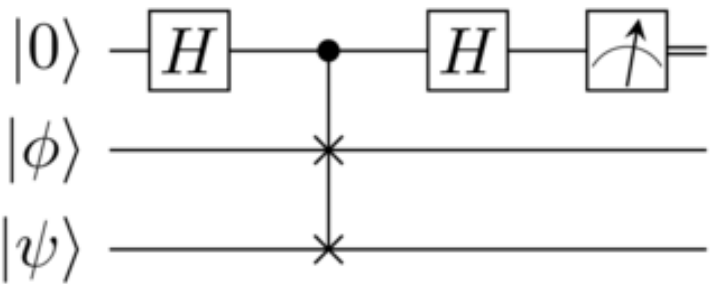


Figure 1: SWAP Test.

3.2 Machine Learning Method.

In this project, we first train a network with the environmental noise and reflectivity as inputs. When we use the POVM measurement as detection, the quantity that we want to minimize is the minimal detection error. By Helstrom theorem [4], such error is,

$$p_{err} = \frac{1}{2}(1 - ||p_0\rho_0 - (1-p_0)\rho_1||) \, , \tag{7}$$

which is called the Helstrom loss for the detection, where the prior probability p_0 is given by the detection scenario, and $||A|| = tr(\sqrt{A^\dagger A})$ is the trace norm of a matrix A.

On the other hand, for the SWAP test, what we want to minimize is the probability that get outcome 0, which means that the test cannot tell whether there is an object. In this case, the cost function is given by the probability of getting 0 in the SWAP test, which is.

$$p_{0,\mathrm{SWAP}} = \frac{1}{2}(1 + \mathrm{tr}[\rho_0 \, \rho_1]) \, . \tag{8}$$

We designed a fully connected network which takes the environmental noise and the reflectivity as input and the parameters for the optimal entangled signal state as output. The detailed setup and training procedures is reported in the Experiment section.

4 Experiments

4.1 Experiment 1

Dataset

Training Set: We generate 2000 environment states $\rho_E = \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{bmatrix}$ and reflectivity η randomly as our training set, with the ρ_E spectrum $\lambda_0 \geq \lambda_1$ and the trace norm $tr(\rho_E) = \lambda_0 + \lambda_1 = 1$ and $\eta \in [0, 1]$.

Test Set1: Fixed environment state ρ_E as a completely mixed state$\rho_E = \frac{1}{2} \cdot \mathbb{I}$, and uniformly choose 20 values from $[0, 1]$ as the reflectivity.

Test Set2: Fixed the reflectivity as 0.5, uniformly choose 20 values for λ_0 from $[0.5, 1]$ and $\lambda_1 = 1 - \lambda_0$ as the environment states ρ_E.

In the training process, for the Helstrom loss, fix the prior probability p_0 as a certain value.

Network Structure

The input of the network is the environment state ρ_E and reflectivity η. The output of the network is the spectrum of the illumination state ρ_{AB}. The network consists of 5 fully connected layers, each with 10 nodes. The three layers in the middle are all embedded with residual structure[3], which is introduced to solve the problem of vanishing gradients and exploding gradients.

For the choice of the activation function, we combine the ReLU function and the Sigmod function.Since the calculation efficiency of the ReLU function is very high, which can effectively

accelerate the speed of training convergence. However, it is a semi-linear function and suitable for deeper networks. Our network is shallow, so we use the ReLU function on the the first 4 layers and the Sigmoid function in the last layer to increase the nonlinearity of the network. The Sigmoid function is computationally expensive and converges slowly, so we only use it in the last layer.

Learning Strategy

In the training process, we use Stochastic Gradient Descent(SGD) to optimize the loss functions. The batch size is set to be 1, and we adjust the learning rate dynamically. If the learning rate is set too small, it takes a long time to converge or even stuck at a local minimal value. And if the learning rate is set too large, it may oscillate near the minimum value but cannot converge to it. Therefore, the initial learning rate in our experiment is set to be 0.01 and decayed by 2 every 5 epochs, where the total training epochs is 50.

Loss function

The loss function is exactly the error we want to minimize.

Helstrom loss:(the prior probability p_0 is a determined value, e.g. 0.5)

$$Loss_1 = \frac{1}{2}(1 - ||p_0\rho_0 - (1-p_0)\rho_1||)$$

where $||A|| = tr(\sqrt{A^\dagger A})$ denotes the trace norm of matrix A.

SWAP loss:

$$Loss_2 = \frac{1 + tr(\rho_0 \cdot \rho_1)}{2}$$

Result

Using test set 1 and Helstrom loss function, we demonstrate the test error under different reflectivity η in Figure2, which matches the theoretical research well.

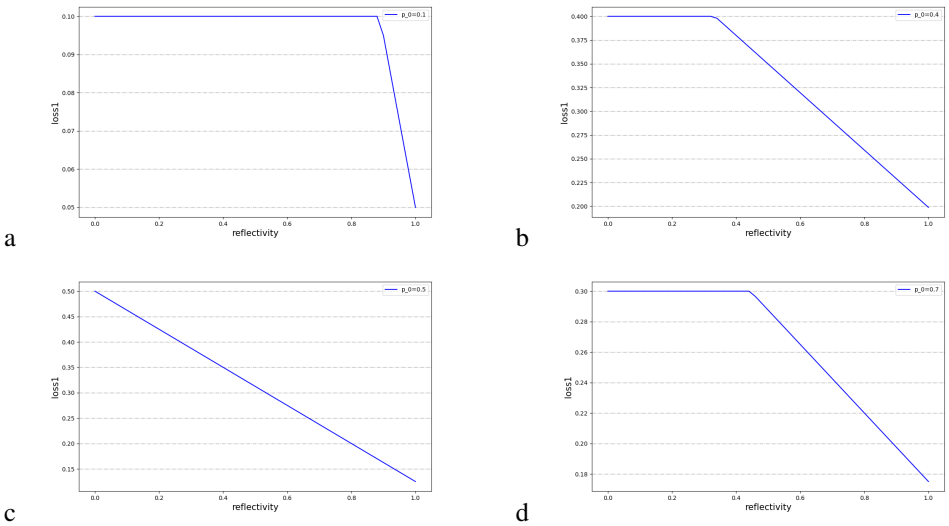
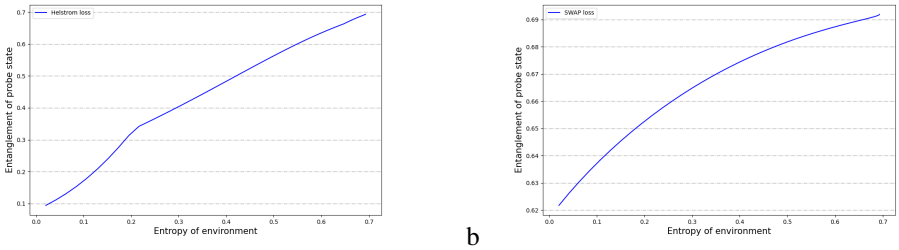


Figure 2: Helstrom loss as a function of reflectivity η in a completely mixed environment $\rho_E = \frac{1}{2} \cdot \mathbb{I}$, for different prior probability p_0. **a**: $p_0 = 0.1$. **b**: $p_0 = 0.4$. **c**: $p_0 = 0.5$. **d**: $p_0 = 0.7$.

Using test set 2 with Helstrom loss function(with prior probability $p_0 = 0.5$) and test set 2 with SWAP loss, we demonstrate the entanglement of probe state under different entropy of environment ρ_E in Figure3, which matches the theoretical research well.



a b

Figure 3: Relations between entropy of the environment and entanglement of the corresponding optimal probe states. **a**: Helstrom loss funtion. **b**: SWAP loss function.

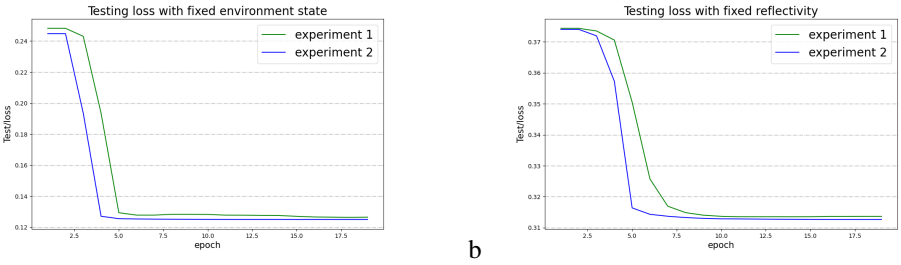
4.2 Experiment 2

According to the theoretical research and what we found in the experiments, the best probe state is independent with reflectivity when the environment is fixed with prior probability $p_0 = 0.5$. Thus, in experiment 2 we explore two more special training settings.

Training Setting 1: Fixed environment state ρ_E as a completely mixed state$\rho_E = \frac{1}{2} \cdot \mathbb{I}$, and then generate 2000 reflectivity η randomly from $[0, 1]$.

Training Setting 2: Fixed the reflectivity as 0.5, uniformly choose 2000 values for λ_0 from $[0.5, 1]$ and $\lambda_1 = 1 - \lambda_0$ as the environment states $\rho_E \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_1 \end{bmatrix}$.

In this case, as Figure4, the convergence speed of the network is accelerated and they converges to the same value.



a b

Figure 4: The convergence curve for experiment1 and experiment2. **a**: training set 1 and test set 1. **b**: training set 2 and test set 2

4.3 Experiment 3

In experiment 2, we observed that if fixed one variable in both training set and test set, we can also get the best probe state and minimal error as the experiment 1 and theoretical research. It seems our input samples are independent with each other and our network is just a black box, which tries to minimize the loss of each input simply using gradient descent. Thus, what if we just input one single sample and train the network repeatedly for enough epochs? we speculated that it could also work to use this 'training during test time' strategy. However, further experiment shows that our intuition is wrong.

As shown in Figure5, the network converges faster even than experiment 2, but it can't converge to the minimal error as experiment 1 and experiment 2.

This may because the loss functions for every input sample are non-convex. If we just repeat optimize one loss function, it's easy for it to get a local minimal value for this loss function and can't achieve the global minimal. On the other hand, if we input different samples and train them stochastic, the network could learn a common feature for them.

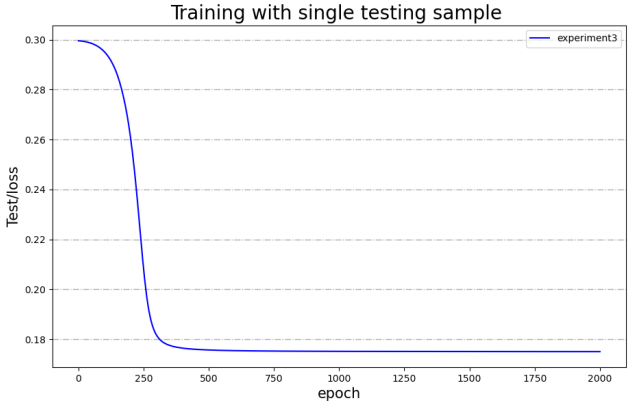


Figure 5: The convergence curve of training one sample

5 Conclusion

In this project, we design an algorithm that automatically finds the optimal entangled probe state for quantum illumination. It is shown that the network can indeed find the optimal probe state, as verified in the special case discussed in [11]. Both the error loss and the probe state is confirmed by the analytical results [11], which suggests the validity of the network.

Our project therefore confirms that it is indeed possible to design a machine learning algorithm for improving the quantum illumination technology. Future work can be done to integrate such an algorithm into the real experimental setup, which presumably can improve the performance of real application of quantum illumination.

Source code

The source code is provided on the github site: https://github.com/YuanLiu-Q/Quantum_illumination

References

[1] Marin Bukov, Alexandre GR Day, Dries Sels, Phillip Weinberg, Anatoli Polkovnikov, and Pankaj Mehta. Machine learning meets quantum state preparation. the phase diagram of quantum control. *arXiv preprint arXiv:1705.00565*, 10, 2017.

[2] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone. Quantum metrology. *Phys. Rev. Lett.*, 96(1):010401, 2006.

[3] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 770–778, 2016.

[4] Carl W. Helstrom. Quantum detection and estimation theory. *J. Stat. Phys.*, 1(2):231–252, 1969.

[5] Ryszard Horodecki, Paweł Horodecki, Michał Horodecki, and Karol Horodecki. Quantum entanglement. *Rev. Mod. Phys.*, 81(2):865, 2009.

[6] Youngwook Kim. Application of machine learning to antenna design and radar signal processing: A review. In *2018 International Symposium on Antennas and Propagation (ISAP)*, pages 1–2. IEEE, 2018.

[7] Seth Lloyd. Enhanced sensitivity of photodetection via quantum illumination. *Science*, 321(5895):1463–1465, 2008.

[8] Mehryar Mohri, Afshin Rostamizadeh, and Ameet Talwalkar. *Foundations of machine learning*. MIT press, 2018.

[9] Michael a. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information: 10th Anniversary Edition*. 2011.

[10] Murphy Yuezhen Niu, Sergio Boixo, Vadim N Smelyanskiy, and Hartmut Neven. Universal quantum control through deep reinforcement learning. *npj Quantum Information*, 5(1):1–8, 2019.

[11] Man-Hong Yung, Fei Meng, Xiao-Ming Zhang, and Ming-Jing Zhao. One-shot detection limits of quantum illumination with discrete signals. *npj Quantum Information*, 6(1):1–8, 2020.