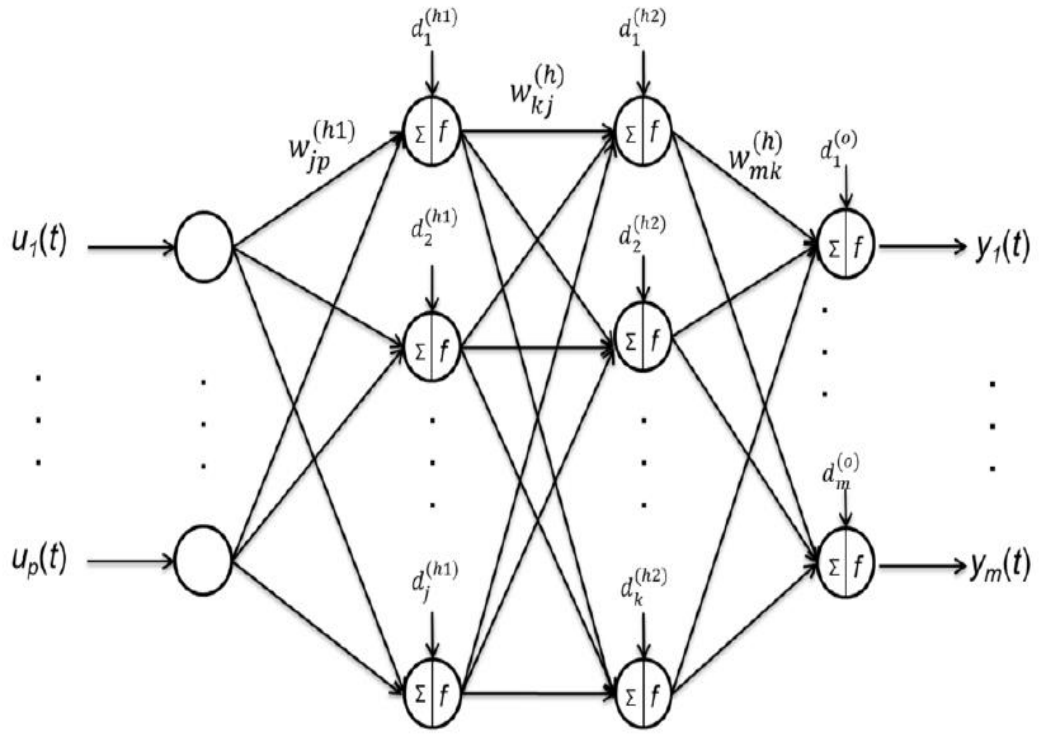


Fig. 1. Structure of three-layer feedforward neural network.

$$f^{h1}(v_j) = \text{sigmoid}(v_j) \quad f^{h2}(v_k) = \tanh(v_k) \quad y_m = f^{out}(v_m) = v_m$$

(1) Update rule for the weights of the output neurons:

$$w_{mk}(t+1) = w_{mk}(t) + \Delta w_{mk}(t)$$

$$= w_{mk}(t) - \eta \frac{\partial E_m(t)}{\partial w_{mk}(t)}$$

$$= w_{mk}(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial w_{mk}(t)}$$

$$= w_{mk}(t) - \eta e_m(t)(-1) f^{out'}\big(v_m(t)\big) f^{h2}\big(v_k(t)\big)$$

$$= w_{mk}(t) - \eta e_m(t)(-1)(1) f^{h2}\big(v_k(t)\big)$$

$$= w_{mk}(t) - \eta \big(d_m(t) - y_m(t)\big)(-1)(1) f^{h2}\big(v_k(t)\big)$$

$$= w_{mk}(t) + \eta \big(d_m(t) - y_m(t)\big) \frac{e^{v_k} - e^{-v_k}}{e^{v_k} + e^{-v_k}}$$

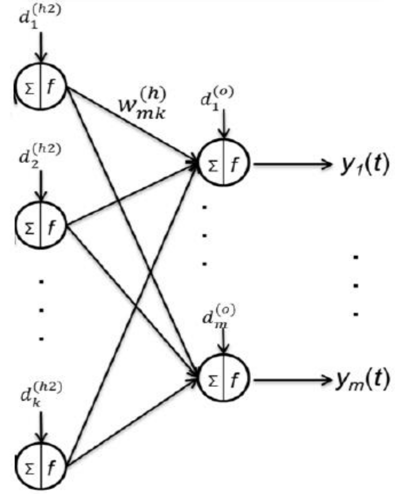
Let: bias=b
design=d

$$E_m(t) = \frac{1}{2}\left(\sum e_m(t)^2\right)$$

$$e_m(t) = d_m(t) - y_m(t)$$

$$y_m = \boldsymbol{f^{out}}(v_m) = v_m$$

$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$



(2) Update rule for the biases of the output neurons:

$$b_m(t+1) = b_m(t) + \Delta b_m(t) \quad = b_m(t) - \eta \frac{\partial E_m(t)}{\partial b_m(t)}$$

$$= b_m(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial b_m(t)}$$

$$= b_m(t) - \eta e_m(t)(-1) f^{out'}\big(v_m(t)\big)(1)$$

$$= b_m(t) - \eta e_m(t)(-1)(1)(1)$$

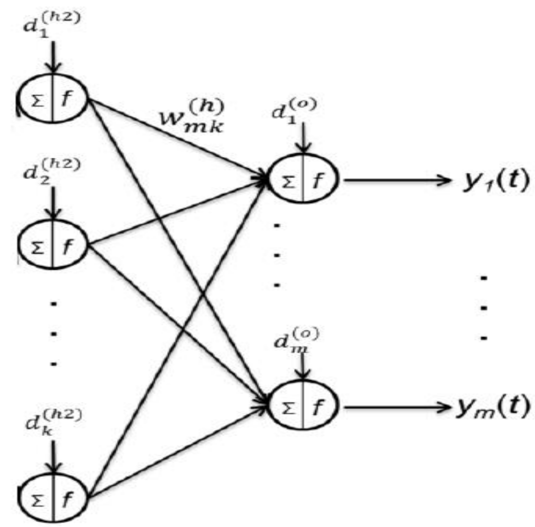
$$= b_m(t) + \eta\big(d_m(t) - y_m(t)\big)$$

$$E_m(t) = \frac{1}{2}\left(\sum e_m(t)^2\right)$$

$$e_m(t) = d_j(t) - y_j(t)$$

$$y_m = f^{out}(v_m) = v_m$$

$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$



(3) Update rule for the weights of the 2nd hidden neurons:

$$w_{kj}(t+1) = w_{kj}(t) + \Delta w_{kj}(t)$$

$$= w_{kj}(t) - \eta \frac{\partial E_m(t)}{\partial w_{kj}(t)}$$

$$= w_{kj}(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial f^{h2}(v_k(t))} \frac{\partial f^{h2}(v_k(t))}{\partial v_k(t)} \frac{\partial v_k(t)}{\partial w_{kj}(t)}$$

$$= w_{kj}(t) - \eta \sum_m \{e_m(t)(-1) f^{out'}(v_m(t)) w_{mk}(t) f^{h2'}(v_k(t)) f^{h1}(v_j(t))\}$$

$$= w_{kj}(t) - \eta \sum_m \{e_m(t)(-1)(1) w_{mk}(t)(1 - (f^{h2}(v_k(t))^2) f^{h1}(v_j(t))\}$$

$$= w_{kj}(t) + \eta \sum_m \{(d_m(t) - y_m(t)) w_{mk}(t)[1 - (tanh v_k(t))^2] \frac{1}{1+e^{-v_j}}\}$$

$$E_m(t) = \frac{1}{2}(\sum e_m(t)^2)$$

$$e_m(t) = d_m(t) - y_m(t)$$

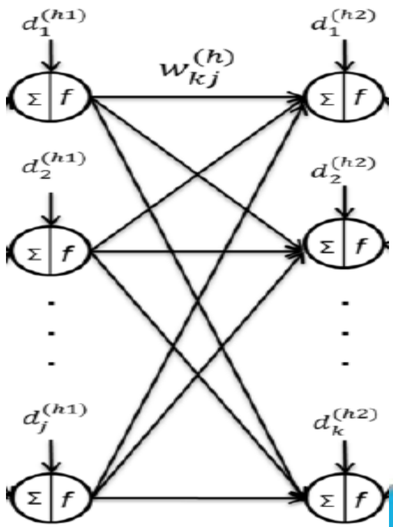
$$y_m = f^{out}(v_m) = v_m$$

$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$

$$f^{h2}(v_k) = \frac{e^{v_k} - e^{-v_k}}{e^{v_k} + e^{-v_k}}$$

$$v_k = \sum_k w_{kj} f^{h1}(v_j(t)) + b$$

$$f^{h1}(v_j) = \frac{1}{1+e^{-v_j}}$$



(4) Update rule for the biases of the 2nd hidden neurons:

$$b_k(t+1) = b_k(t) + \Delta b_k t \quad = b_k(t) - \eta \frac{\partial E_m(t)}{\partial b_k(t)}$$

$$= b_k(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial f^{h2}(v_k(t))} \frac{f^{h2}(v_k(t))}{\partial v_k(t)} \frac{\partial v_k(t)}{\partial b_k(t)}$$

$$= b_k(t) - \eta \sum_m \{e_m(t)(-1) f^{out'}(v_m(t)) w_{mk}(t) f^{h2'}(v_k(t)) 1\}$$

$$= b_k(t) - \eta \sum_m \{e_m(t)(-1) f^{out'}(v_m(t)) w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right]\}$$

$$= b_k(t) - \eta \sum_m \{(d_m(t) - y_m(t))(-1) 1 w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right]\}$$

$$= b_k(t) + \eta \sum_m \{(d_m(t) - y_m(t)) w_{mk}(t) [(1 - (tanh v_k(t))^2)]\}$$

$$E_m(t) = \frac{1}{2}\left(\sum e_m(t)^2\right)$$

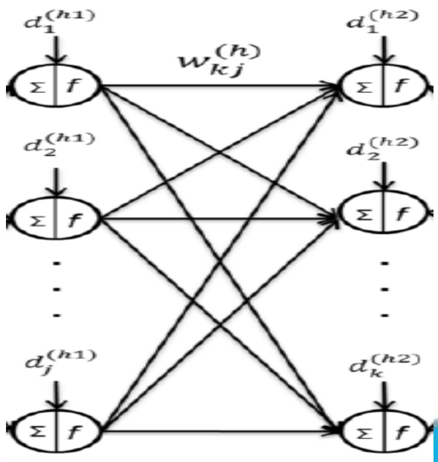
$$e_m(t) = d_m(t) - y_m(t)$$

$$y_m = f^{out}(v_m) = v_m$$

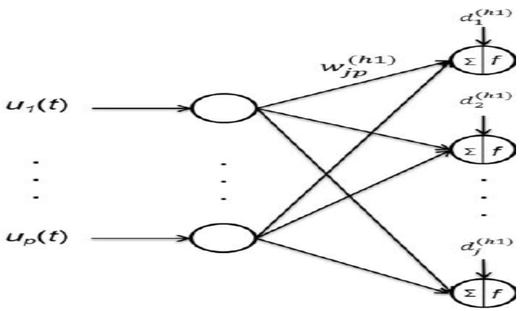
$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$

$$f^{h2}(v_k) = \frac{e^{v_k} - e^{-v_k}}{e^{v_k} + e^{-v_k}}$$

$$v_k = \sum_k w_{kj} f^{h1}(v_j) + b_k$$



(5) Update rule for the weights of the 1st hidden neurons:



$$E_m(t) = \frac{1}{2}\left(\sum e_m(t)^2\right)$$

$$e_m(t) = d_m(t) - y_m(t)$$

$$y_m = f^{out}(v_m) = v_m$$

$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$

$$f^{h2}(v_k) = \frac{e^{v_k} - e^{-v_k}}{e^{v_k} + e^{-v_k}}$$

$$v_k = \sum_k w_{kj} f^{h1}(v_j) + b_k$$

$$f^{h1}(v_j) = \frac{1}{1 + e^{-v_j}}$$

$$v_j = \sum_j w_{jp} x_p + b_j$$

$$w_{jp}(t+1) = w_{jp}(t) + \Delta w_{jp}(t)$$

$$= w_{jp}(t) - \eta \frac{\partial E_m(t)}{\partial w_{jp}(t)}$$

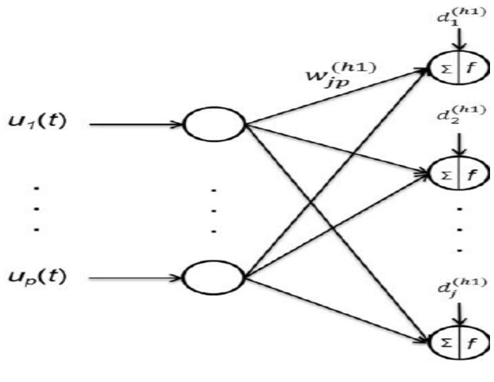
$$= w_{jp}(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial f^{h2}(v_k(t))} \frac{\partial f^{h2}(v_k(t))}{\partial v_k(t)} \frac{\partial v_k(t)}{\partial f^{h1}(v_j(t))} \frac{\partial f^{h1}(v_j(t))}{\partial v_j(t)} \frac{\partial v_j(t)}{\partial w_{jp}(t)}$$

$$= w_{jp}(t) - \eta \sum_k \sum_m \{e_m(t)(-1) f^{out'}(v_m(t)) w_{mk}(t) f^{h2'}(v_k(t)) w_{kj}(t)\, f^{h1'}(v_j(t))\, u_p(t)\}$$

$$= w_{jp}(t) - \eta \sum_k \sum_m \{e_m(t)(-1)(1) w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right] w_{kj}(t) \left[f^{h1}(v_j(t))\left(1 - f^{h1}(v_j(t))\right)\right] u_p(t)\}$$

$$= w_{jp}(t) + \eta \sum_k \sum_m \{(d_m(t) - y_m(t)) w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right] w_{kj}(t) \left[f^{h1}(v_j(t))\left(1 - f^{h1}(v_j(t))\right)\right] u_p(t)\}$$

(6) Update rule for the biases of the 1st hidden neurons:



$$E_m(t) = \frac{1}{2}\left(\sum e_m(t)^2\right)$$

$$e_m(t) = d_m(t) - y_m(t)$$

$$y_m = f^{out}(v_m) = v_m$$

$$v_m = \sum_m w_{mk} f^{h2}(v_k) + b_m$$

$$f^{h2}(v_k) = \frac{e^{v_k} - e^{-v_k}}{e^{v_k} + e^{-v_k}}$$

$$v_k = \sum_k w_{kj} f^{h1}(v_j) + b_k$$

$$f^{h1}(v_j) = \frac{1}{1 + e^{-v_j}}$$

$$v_j = \sum_j w_{jp} u_p + b_j$$

$$b_j(t+1) = b_j(t) + \Delta b_j(t) \quad = b_j(t) - \eta \frac{\partial E_m(t)}{\partial b_j(t)}$$

$$= b_j(t) - \eta \frac{\partial E_m(t)}{\partial e_m(t)} \frac{\partial e_m(t)}{\partial y_m(t)} \frac{\partial y_m(t)}{\partial v_m(t)} \frac{\partial v_m(t)}{\partial f^{h2}(v_k(t))} \frac{\partial f^{h2}(v_k(t))}{\partial v_k(t)} \frac{\partial v_k(t)}{\partial f^{h1}(v_j(t))} \frac{\partial f^{h1}(v_j(t))}{\partial v_j(t)} \frac{\partial v_j(t)}{\partial b_j(t)}$$

$$= b_j(t) - \eta \sum_k \sum_m \{e_m(t)(-1) f^{out\prime}(v_m(t)) w_{mk}(t) f^{h2\prime}(v_k(t)) w_{kj}(t)\, f^{h1\prime}(v_j(t))\,(1)\}$$

$$= b_j(t) - \eta \sum_k \sum_m \{e_m(t)(-1)(1) w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right] w_{kj}(t) \left[f^{h1}(v_j(t))\left(1 - f^{h1}(v_j(t))\right)\right](1)\}$$

$$= b_j(t) + \eta \sum_k \sum_m \{(d_m(t) - y_m(t)) w_{mk}(t) \left[\left(1 - f^{h2}(v_k(t))^2\right)\right] w_{kj}(t) \left[f^{h1}(v_j(t))\left(1 - f^{h1}(v_j(t))\right)\right](1)$$