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Optimum Design

Lecture 9
Implementation of Lagrange Multiplier

Spring 2016

Lagrange Multiplier Theorem

Consider the problem:

minimizing $f(\mathbf{x})$

subject to $h_i(\mathbf{x}) = 0$, $i = 1$ to p.

Let $\mathbf{x}^*$ be a regular point that is a local minimum for the problem.

There exist Lagrange multipliers v_j^*, $j = 1$ to p such that

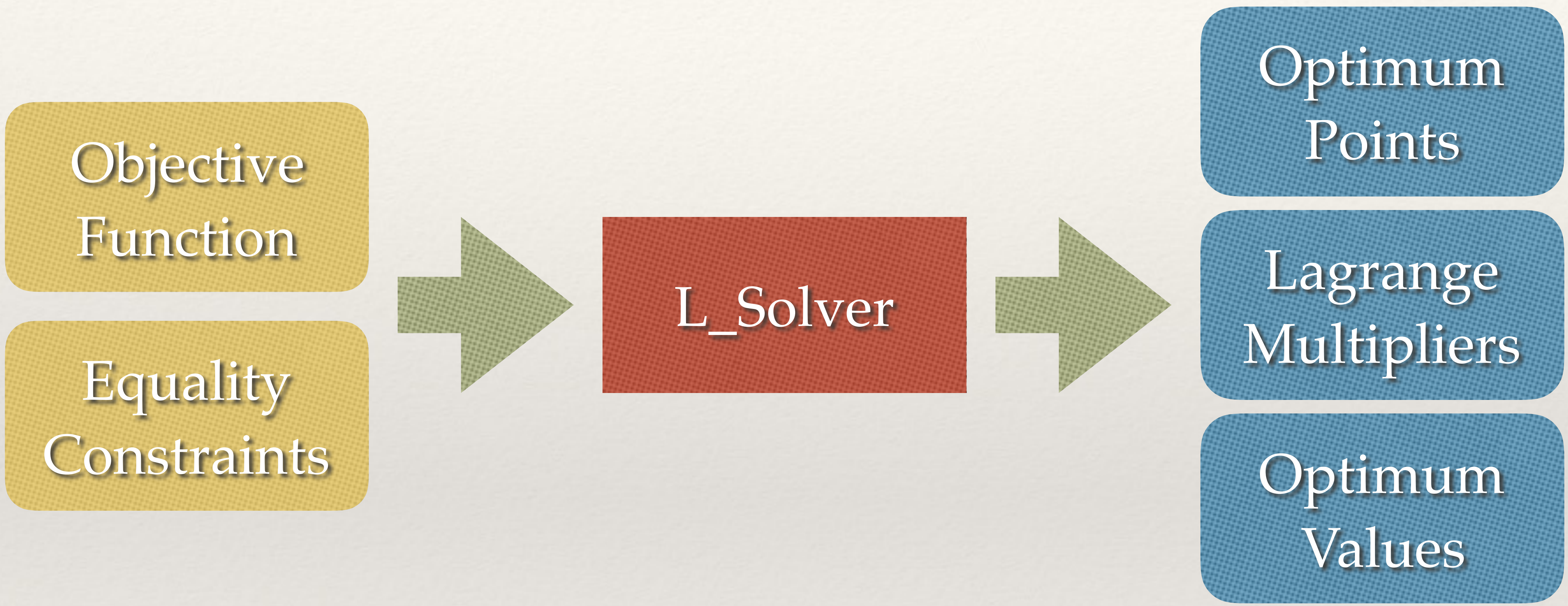
$$\frac{\partial f(\mathbf{x}^*)}{\partial x_i} + \sum_{j=1}^{p} v_j^* \frac{\partial h_j(\mathbf{x}^*)}{\partial x_i} = 0; \quad i = 1 \quad \text{to} \quad n$$

Lagrange function $L(\mathbf{x}, \mathbf{v}) = f(\mathbf{x}) + \sum_{j=1}^{p} v_j h_j(\mathbf{x}) = f(\mathbf{x}) + \mathbf{v}^T \mathbf{h}(\mathbf{x})$

Explanation of Homework 2

- This homework is designed to help you:
  - Understand and implement the Lagrange Multiplier Theorem.
  - Enhance your MATLAB programming skill to conduct optimization studies and to present your results.
- What are we going to do?
  - Write a general solver for up to 2nd-order equality constrained problem.

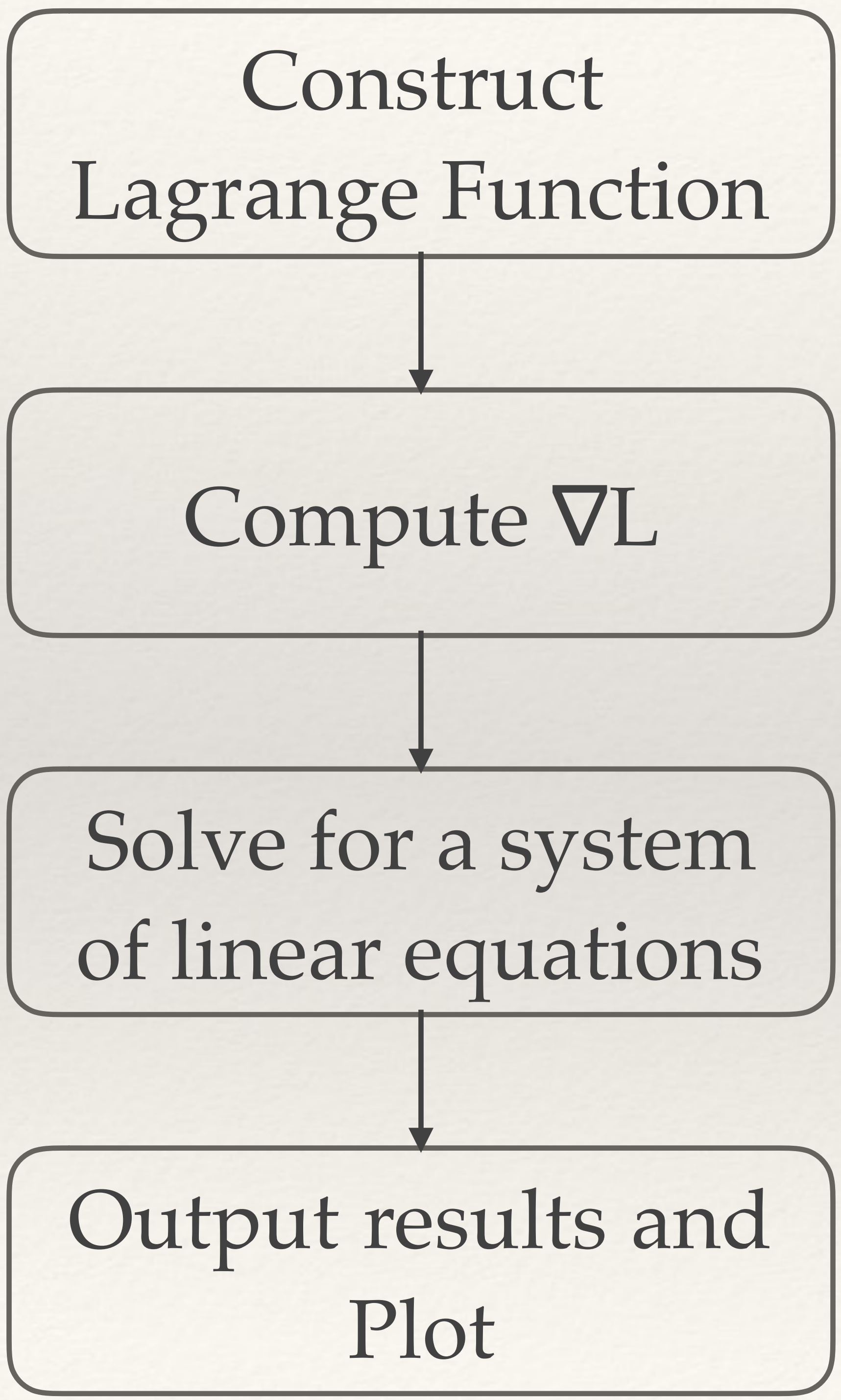
Task 1: Implement L Solver



About L_Solver

- function [sol] = l_solver(f,h,nvar,ncos)
  - f: objective function
  - h: vector of equality constraints
  - nvar: number of design variables
  - ncos: number of constraints

Flowchart for Your Program



$$L(\mathbf{x}, \mathbf{v}) = f(\mathbf{x}) + \mathbf{v}^T \mathbf{h}(\mathbf{x})$$

$$\nabla L = \frac{\partial L}{\partial x_i}$$

$$\nabla L = 0$$

Note:
These are all vector operations

Useful MATLAB Functions

- Creating symbolic variables and functions

  http://www.mathworks.com/help/symbolic/syms.html

- Differentiation:

  http://www.mathworks.com/help/symbolic/differentiation.html

- Solve System of Linear Equations:

  http://www.mathworks.com/help/symbolic/solve-a-system-of-linear-equations.html

- Coefficients of polynomial:

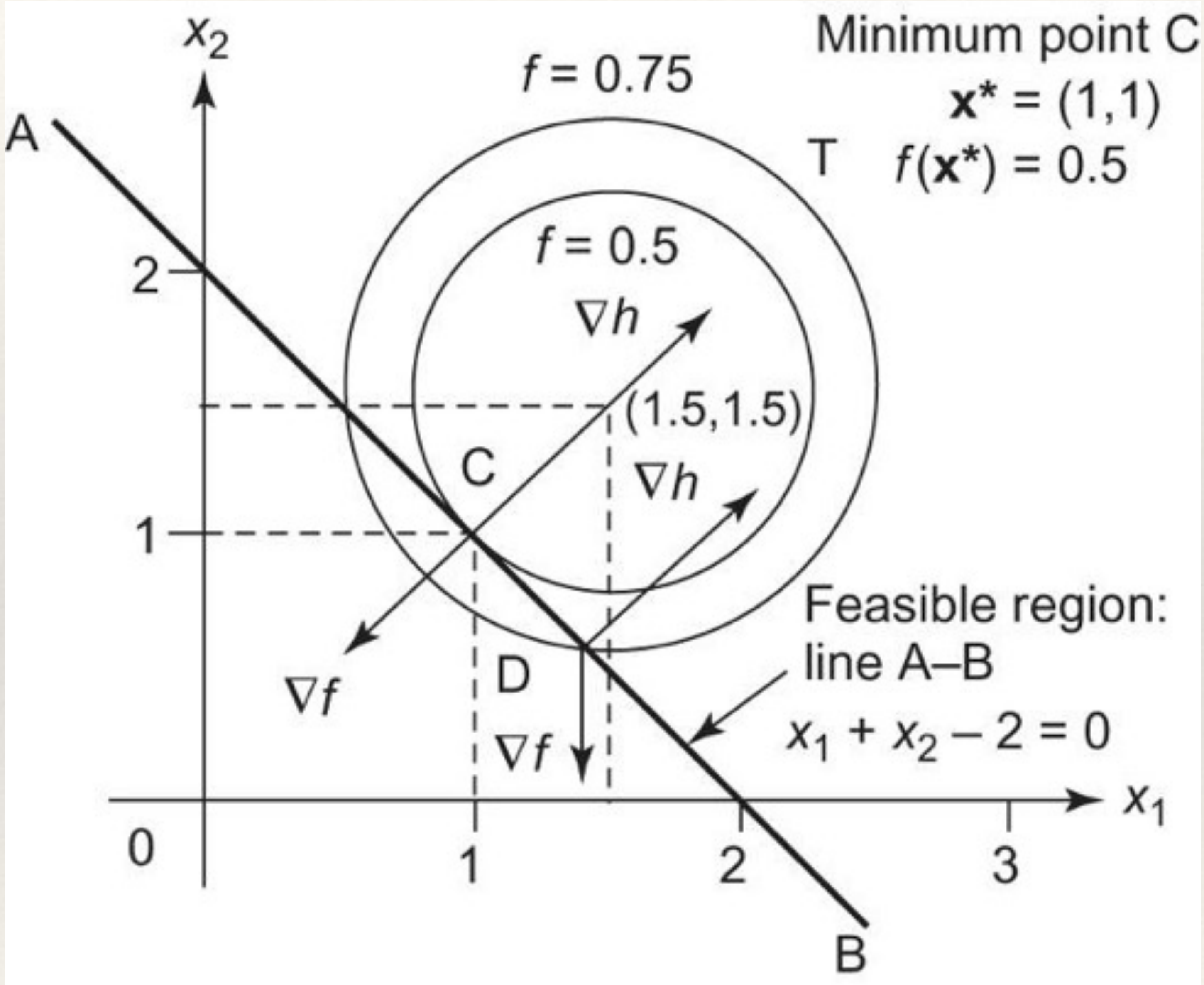
  http://www.mathworks.com/help/symbolic/coeffs.html

Demo

$$\text{min.}\quad f(\mathbf{x}) = (x_1 - 1.5)^2 + (x_2 - 1.5)^2$$

$$h(\mathbf{x}) \equiv x_1 + x_2 - 2 = 0$$

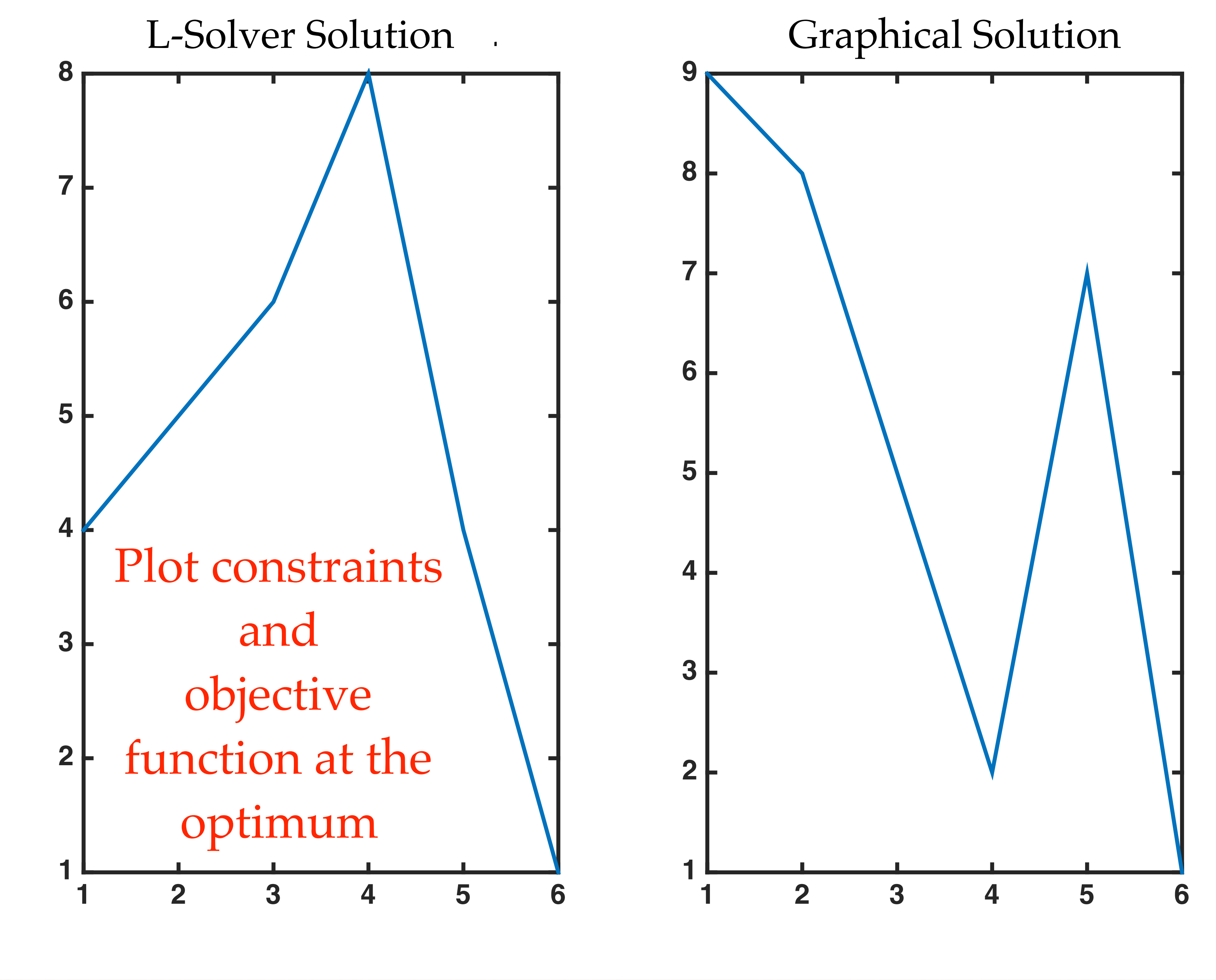
- syms
- diff()
- solve()



Validation

- A test_func.m is available on Moodle for you to validate your L solver
- There are four types of problems that your L Solver should be able to solve:
  1. Two-variable non-constrained problem
  2. Two-variable equality-constrained problem
  3. Three-variable non-constrained problem
  4. Three-variable constrained problem

Task 2: Solve a Design Problem (TBD)



Use subplot (1,2,i)

http://www.mathworks.com/help/matlab/ref/subplot.html

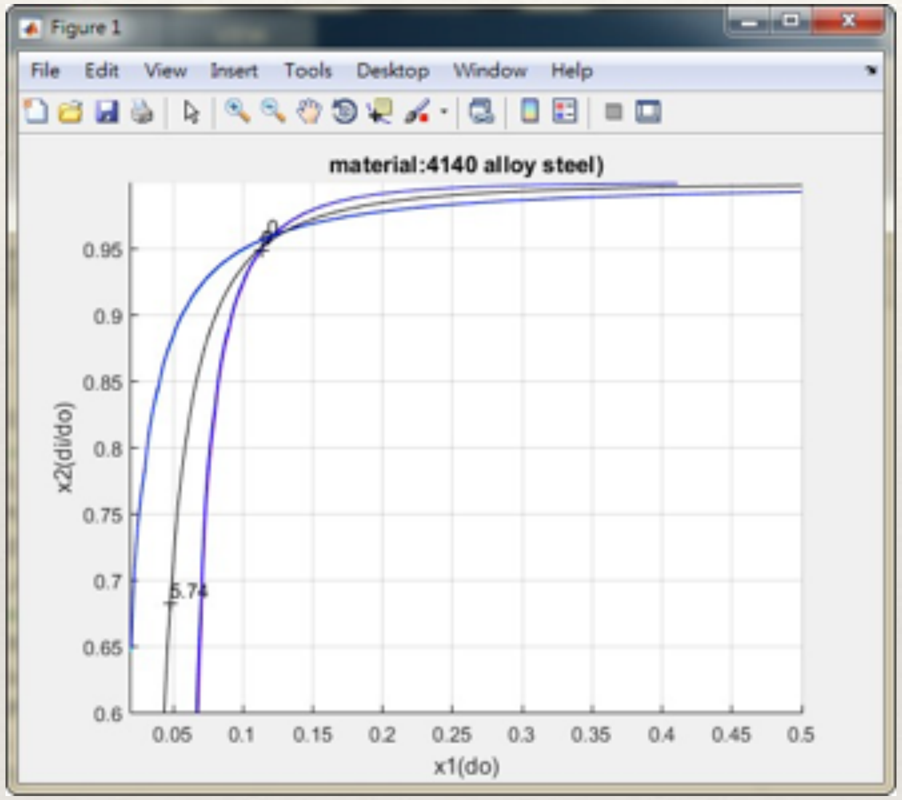
Deliverables

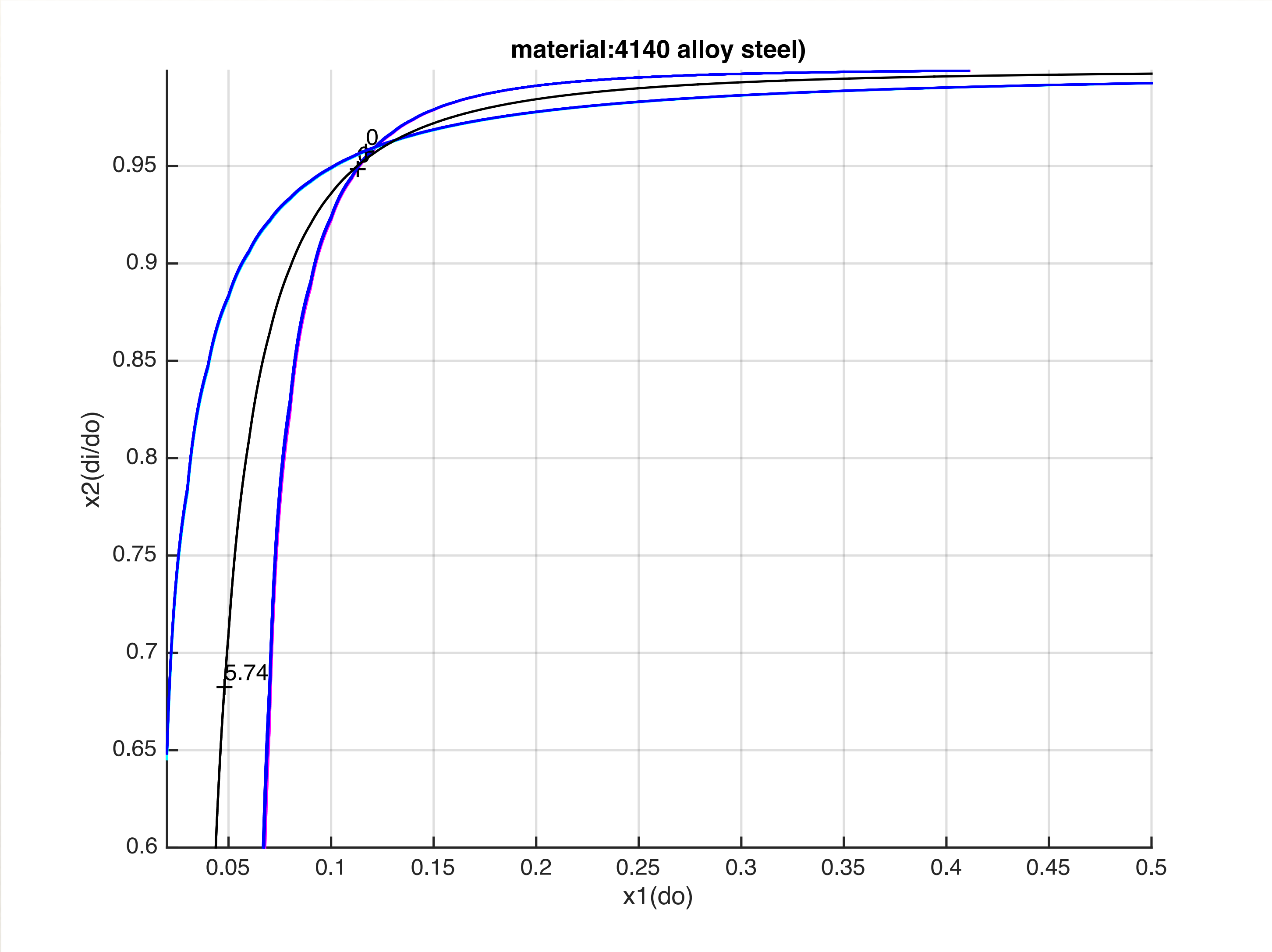
- A short memo
  - Briefly describe the tasks and your solution
  - Compare two methods & their results
- MATLAB m-files
  - l_solver.m (function file)
  - prob.m (script file)
  - Your m-file will be first tested by the provided test_func.m
  - Your m-file will be next tested by your prob.m

Comments on Homework 1

- Do not zip your files
- Basic grammar requirement #1: spell check (red underlines)
- Basic grammar requirement #2: Look at grammar suggestions (green underlines)
- Figure quality

Use Export Tool





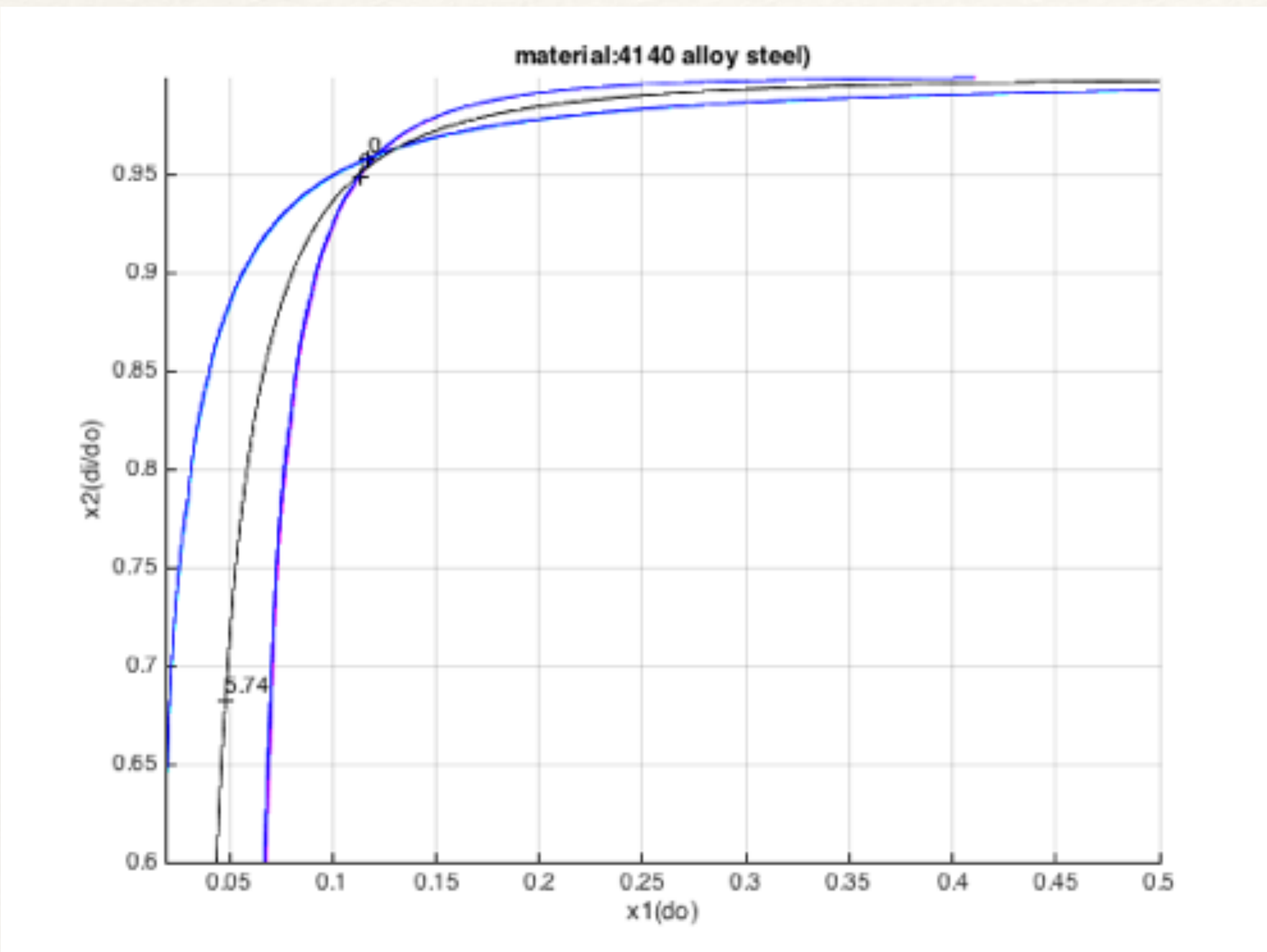
Windows screenshot:

1. unnecessary stuff
2. Poor quality

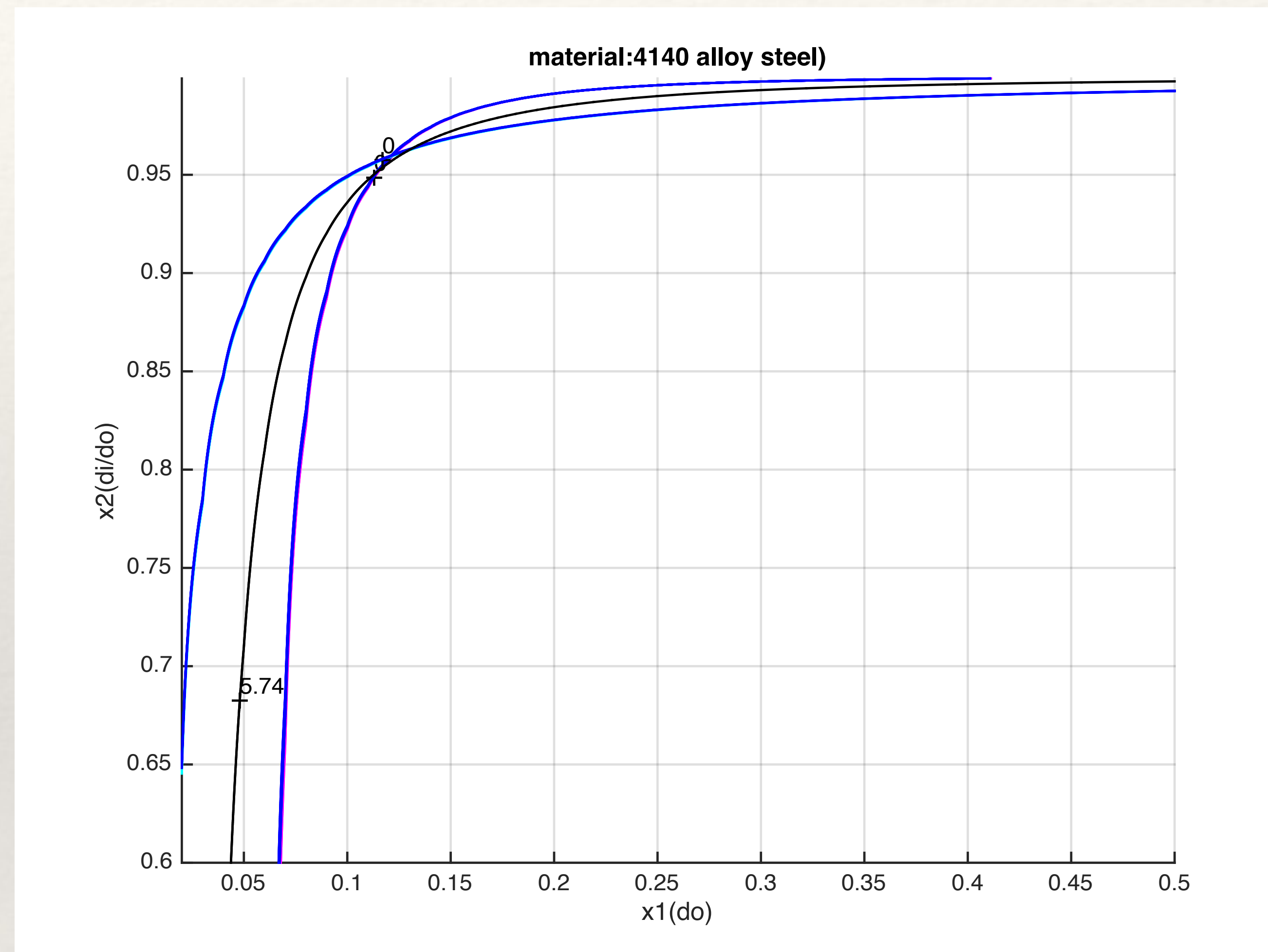
Use MATLAB export tool:

1. Pure
2. High quality

Pixel Graph vs. Vector Graph



Export as PNG



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