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Homework Assignment 3

H 3.1 (Variational Methods) 10 Points

Instead of using the grey value constancy assumption, let us assume that the y-derivative of the image f remains constant over time, i.e.

$$f_y(x+u, y+v, t+1) = f_y(x, y, t) \tag{1}$$

(a) Linearise the constancy assumption (1) w.r.t the flow functions u and v.

(b) Write down an energy functional similar to Horn and Schunck based on the linearised constancy assumption from (a).

(c) Compute the Euler-Lagrange equations for this energy functional.

(d) Discretise the Euler-Lagrange equations from (c).

(e) Starting from the discrete equations computed in (d), derive an iterative scheme that computes the minimiser.

H 3.2 (Stereo) 6 Points

Consider a camera with focal distance 2. Its image coordinate system is orthogonal with square pixels of size 1. The principal point in this coordinate system is located in $(2,3)^\top$. Furthermore, the position of the world coordinate system relative to the camera coordinate system is given by a rotation around the z-axis by an angle of 90° and a translation by the vector $(5,0,-1)^\top$.

(a) Compute the intrinsic matrix A_{int} (including the focal length f).

(b) Compute the extrinsic matrix A_{ext}.

(c) Compute the full projection matrix P.

P 3.3 (Horn and Schunck)

Please download the required file `cv19_ex03.tgz` from ILIAS. To unpack the data, use `tar xvfz cv19_ex03.tgz` .

In the routine `flow`, supplement the missing code such that it computes one Jacobi iteration step. Make sure the image boundaries are treated correctly. To compile the program, type

```
gcc -O3 -o hsTemplate hsTemplate.c -lm
```

The filling-in effect that is characteristic for variational methods can be studied with the image pair `pig1.pgm`, `pig2.pgm`. To this end, investigate the result for different numbers of iterations. What is a good value for the regularisation parameter α in this case?

Submission:

The theoretical problem(s) have to be submitted in handwritten form before the next tutorial (December 06th).

Deadline for Submission is: Friday, December 06th, 9:45 am (before the tutorial)

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Classroom Assignment 3

C 3.1 (Eigenvalue Analysis)

Let $\mathbf{J} \in \mathbb{R}^{n \times n}$ be a symmetric $(n \times n)$ matrix with real components. We consider its corresponding quadratic form given by

$$E : \mathbb{R}^n \longrightarrow \mathbb{R}, \qquad E(\mathbf{v}) = \mathbf{v}^\top \mathbf{J} \mathbf{v}$$

Show that among all vectors $\mathbf{v} \in \mathbb{R}^n$ with $|\mathbf{v}| = 1$, the function value $E(\mathbf{v})$ is minimal for the eigenvector of $\mathbf{J}$ corresponding to its smallest eigenvalue. What can we say about E if $\mathbf{J}$ is positive definite?