Prof. Dr.-Ing. A. Bruhn
Computer Vision and Intelligent Systems Group
Institute for Visualization and Interactive Systems
University of Stuttgart

Assignment 2

Theoretical Exercise 2.1 (Sub-Pixel Refinement)

Let the three cost values (matching scores) c_0, c_{-1} and c_{+1} for the best match and its left and right neighbour be given. Considering the SSD model, we aim to fit a parabola $f(x) = ax^2 + bx + c$ through the three points $(0, c_0)$, $(-1, c_{-1})$ and $(+1, c_{+1})$.
Set up a linear equation system in the unknowns a, b, c and show that its solution yields the parabola

$$f(x) = \frac{c_{+1} - 2c_0 + c_{-1}}{2}\, x^2 + \frac{c_{+1} - c_{-1}}{2}\, x + c_0$$

Theoretical Exercise 2.2 (Taylor Linearisation)

Recall that the continuous grey value constancy assumption $f(x+u, y+v, t+1) - f(x, y, t) = 0$ leads after linearisation to the BCCE given by $f_x u + f_y v + f_t = 0$.
Show that the linearisation is correct, i.e. perform a first-order Taylor expansion of the expression $f(x+u, y+v, t+1)$ around $(x, y, t)^\top$ and plug the result into the grey value constancy assumption.
Reminder: For $\mathbf{x}, \mathbf{a} \in \mathbb{R}^n$, the m-th order Taylor expansion of a function $f(\mathbf{x})$ around $\mathbf{a}$ is given by

$$f(\mathbf{x}) \approx \sum_{k=0}^{m} \frac{1}{k!} \left[(\mathbf{x} - \mathbf{a})^\top \nabla_n \right]^k f(\mathbf{a})$$

with $\nabla_n f := (f_{x_1}, .., f_{x_n})^\top$ and $\mathbf{x} = (x_1, .., x_n)^\top$.

Theoretical Exercise 2.3 (Minimization of the Lucas/Kanade Method)

We have seen that minimising the local energy corresponding to the Lucas/Kanade model requires to solve a linear system of equations in the form $A\,\mathbf{x} = \mathbf{b}$, where

$$A := \begin{pmatrix} a & b \\ b & c \end{pmatrix} \in \mathbb{R}^{2\times 2},$$

with $\mathbf{x} := (x_1, x_2)^\top := (u, v)^\top \in \mathbb{R}^2$ and $\mathbf{b} := (d, e)^\top \in \mathbb{R}^2$.
Derive closed-form solutions for the unknowns u and v, i.e. come up with formulae how to compute u and v from a, b, c, d and e.

Hint: You may want to use Cramer's rule. It states that the unknowns x_i $(i = 1, 2)$ of above equation system can be computed as

$$x_i = \frac{\det\left(A_{i\to\mathbf{b}}\right)}{\det(A)}\,,$$

where det denotes the determinant and the matrix $A_{i\to\mathbf{b}}$ is obtained by replacing the i-th column of A by the right hand side vector $\mathbf{b}$.