HA06

Mittwoch, 29. Januar 2020 13:56

6.1 $P(x|\omega_1) = \mathcal{N}(0,1)$

$p(x|\omega_2) = \mathcal{N}(0,\sqrt{2})$

a) if $p(\omega_1) = p(\omega_2)$ then we can use LRT

Assume also symmetric losses $\lambda_{11} = \lambda_{22}$ and $\lambda_{21} = \lambda_{12}$

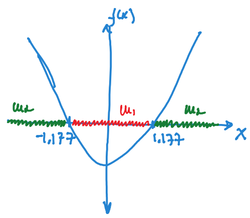
$$\Rightarrow \Lambda_{12}(x) = \frac{p(x|\omega_1)}{p(x|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} 1$$

$$\Rightarrow \Lambda_{12}(x) = \sqrt{2}\cdot \exp\left(-\frac{1}{4}x^2\right) \underset{\omega_2}{\overset{\omega_1}{\gtrless}} 1$$

$$\Rightarrow \ln\sqrt{2} - \frac{1}{4}x^2 \underset{\omega_2}{\overset{\omega_1}{\gtrless}} 0$$

$$\Rightarrow x^2 - 4\ln\sqrt{2} \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 0$$

$$\Rightarrow \boxed{|x| \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 1{,}177}$$



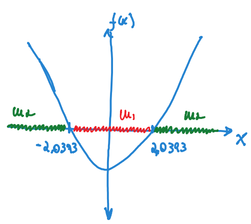
b) $\Lambda_{12}(x) = \frac{p(x|\omega_1)}{p(x|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}$ with $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 1$, $\lambda_{21} = 2$

$$\Rightarrow \sqrt{2}\cdot \exp\left(-\frac{1}{4}x^2\right) \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{1}{2}$$

$$\Rightarrow -\frac{1}{4}x^2 \underset{\omega_2}{\overset{\omega_1}{\gtrless}} -\ln(2\sqrt{2})$$

$$\Rightarrow x^2 - 4\cdot\ln(2\sqrt{2}) \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 0$$

$$\Rightarrow \boxed{|x| \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 2{,}0393}$$



Since we have given less penalty to misclassification of class 1, the range of x classified as ω_1 has increased

c) $\Lambda_{12}(x) = \frac{p(x|\omega_1)}{p(x|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}$ with $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 1$, $\lambda_{21} = 0$

$$\Rightarrow \sqrt{2}\exp\left(-\frac{1}{4}x^2\right) \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{1}{0} = \infty$$

it follows then that for all finite x

$$\sqrt{2}\exp\left(-\frac{1}{4}x^2\right) < \infty \quad \Rightarrow \quad \boxed{|x| \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \infty}$$

therefore, we always choose class ω_2
which make sense, because we do not penalize
misclassifications of ω_2, but only ω_1

(d) LRT for ω_1 and ω_3

$$\Rightarrow \Lambda_{1,3}(x) = \frac{p(x|\omega_1)}{p(x|\omega_3)} = \exp(-4x+4) \underset{\omega_3}{\overset{\omega_1}{\gtrless}} 1$$

$$\Rightarrow \boxed{x \underset{\omega_3}{\overset{\omega_1}{\lessgtr}} 1}$$

(d) Lets define the last decision rule between 2, 3

$$\Lambda_{2,3}(x) = \frac{p(x/\omega_2)}{p(x/\omega_3)} = \frac{1}{\sqrt{2}} \cdot \exp\left(\frac{x^2}{4} - 2x + 2\right) \underset{\omega_3}{\overset{\omega_2}{\gtrless}} 1$$

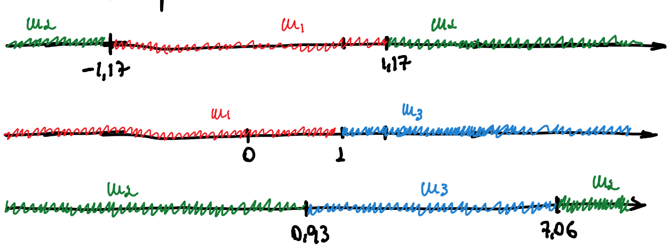
$$\Rightarrow \boxed{\begin{array}{l} \omega_3 : \ 0{,}9363 < x < 7{,}0637 \\ \omega_2 : \ x < 0{,}9363 \ , \ x > 7{,}0637 \end{array}}$$

Together with the two other classifiers:

$$\boxed{|x| \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 1{,}177}$$

$$\boxed{x \underset{\omega_3}{\overset{\omega_1}{\lessgtr}} 1}$$

interval representation:



Resulting classifier: (class with more votes wins)