Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Department Intelligent Systems
University of Stuttgart

Mathematics for Computer Vision – Training Sheet

Problem 0.1 (Eigenvalues and Eigenvectors)

Determine the Eigenvalues and Eigenvectors of the following 2×2 matrices and state if the matrices are negative (semi)-definite, positive (semi-)definite or indefinite:

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} , \qquad A_2 = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix} , \qquad A_3 = \begin{pmatrix} 1 & -5 \\ -5 & 1 \end{pmatrix} .$$

Problem 0.2 (Complex Numbers)

Perform the following operations with the complex numbers $a = 1 + 2i$ and $b = 1 - i$:

- addition
- subtraction
- multiplication
- division

Problem 0.3 (Transformation in Polar Coordinates)

Given the complex numbers $a = 1 + 2i$ and $b = 1 - i$ from Problem 2:

- What is the norm (magnitude) of a and b?
- Having the norm of a and b you just need to compute the corresponding angles $\arg(a)$ and $\arg(b)$ to rewrite them in polar coordinates.
- Write down a and b in polar coordinates.

What are the complex-valued equivalents of the following numbers in polar coordinates

$$\begin{aligned} c_1 &= e^{\pi i} , \\ c_2 &= 4 \cdot a \cdot e^{\frac{\pi}{4} i} , \\ c_3 &= 1 \cdot e^{2\pi i + 1} . \end{aligned}$$

Problem 0.4 (Partial Derivatives)

Compute the partial derivatives of the following functions. How does the gradient look like for each of the functions?

$$\begin{aligned} f_1(x,y) &= \sin(xy) + y^4 \,, \\ f_2(x,y) &= e^{xy^4} + xy \,, \\ f_3(x,y) &= a \cdot \ln(x) \cdot e^{xy} \,, \\ f_4(x,y,z) &= a \cdot xyz + b \cdot xz + c \cdot z \,. \end{aligned}$$

Problem 0.5 (Mathematical Operators)

Compute the divergence of:

$$\mathbf{g}(x,y) = \begin{pmatrix} a \cdot x^3 \\ y \cdot \cos x \end{pmatrix} \,.$$

Compute the Laplacian of:

$$h(x,y,z) = \sin(xy) + y^4 + z^2 \,.$$

Guidelines for the Tutorials

- **This training sheet** should recall some mathematical preliminaries and is **not** graded. If you have problems to solve it, contact your tutor as soon as possible.

- The following exercise sheets contain **three types of problems:**
- **Homework Assignments (H)** have to be submitted **at the beginning** of the next tutorial. They will be corrected and given back to you in the tutorial after the next.
- **Programming Assignments (P)** are intended to help you to further familiarize with the contents of the lecture. They **don't** have to be submitted and will not be graded, but example solutions will be provided online later on.
- **Classroom Assignments (C)** are intended to be solved **within** the next tutorial. They **don't** have to be submitted and will not be graded, but your tutor will help you to complete them correctly.

- In order to gain **admission to the exam**, you have to achieve **50% of the total points** from the homework **(H)** assignments within the semester.
- **Regular attendance** of the tutorials is not mandatory, but highly recommended as preparation for the exam.