HA04

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4.1

$$E(u,v) = \int_\Omega (f_x u + f_y v + f_t)^2 + \alpha \cdot \psi(|\nabla u|^2 + |\nabla v|^2) \, dx\,dy$$

where $E(u,v)$ is the flow-driven isotropic energy function with linearized constancy assumption.

ⓐ Solution to the Euler-Lagrange Equation:

$$\begin{cases} E(u,v) = \int_\Omega F(x,y,u,v,u_x,u_y,v_x,v_y)\,dx\,dy \\ \min_{u,v} E(u,v) \rightarrow \begin{matrix} F_u - \partial_x F_{u_x} - \partial_y F_{u_y} = 0 \\ F_v - \partial_x F_{v_x} - \partial_y F_{v_y} = 0 \end{matrix} \Rightarrow \begin{cases} F_u - \operatorname{div}\left(\begin{bmatrix} F_{u_x} \\ F_{u_y} \end{bmatrix}\right) = 0 \\ F_v - \operatorname{div}\left(\begin{bmatrix} F_{v_x} \\ F_{v_y} \end{bmatrix}\right) = 0 \end{cases} \\ \text{with } n^T \begin{pmatrix} F_{u_x} \\ F_{u_y} \end{pmatrix} = 0, \; n^T \begin{pmatrix} F_{v_x} \\ F_{v_y} \end{pmatrix} = 0 \quad \text{(boundary conditions)} \end{cases}$$

Applying to our equations:

$$F_u = 2 \cdot (f_x u + f_y v + f_t) \cdot f_x$$
$$F_v = 2 \cdot (f_x u + f_y v + f_t) \cdot f_y$$
$$F_{u_x} = \alpha \cdot \psi'(|\nabla u|^2 + |\nabla v|^2) \cdot 2 \cdot u_x$$
$$F_{v_x} = \alpha \cdot \psi'(|\nabla u|^2 + |\nabla v|^2) \cdot 2 \cdot v_x$$
$$F_{u_y} = \alpha \cdot \psi'(|\nabla u|^2 + |\nabla v|^2) \cdot 2 \cdot u_y$$
$$F_{v_y} = \alpha \cdot \psi(|\nabla u|^2 + |\nabla v|^2) \cdot 2 \cdot v_y$$

Finally:

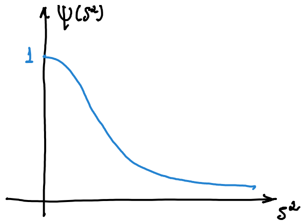
$$\begin{cases} (f_x u + f_y v + f_t) \cdot f_x - \alpha \cdot \operatorname{div}\left(\psi'(|\nabla u|^2 + |\nabla v|^2) \cdot \nabla u\right) = 0 & \text{(I-a)} \\ (f_x u + f_y v + f_t) \cdot f_y - \alpha \cdot \operatorname{div}\left(\psi'(|\nabla u|^2 + |\nabla v|^2) \cdot \nabla v\right) = 0 & \text{(I-b)} \end{cases}$$

ⓑ If $\psi(s^2) = \lambda^2 \log\left(1 + \frac{s^2}{\lambda^2}\right)$

then $\frac{\partial \psi(s^2)}{\partial s^2} = \psi'(s^2) = \lambda^2 \cdot \frac{1}{1 + s^2/\lambda^2} \cdot \frac{1}{\lambda^2}$

$$\Rightarrow \boxed{\psi'(s^2) = \frac{1}{1 + s^2/\lambda^2}}$$

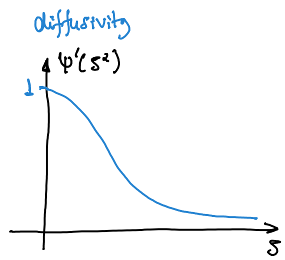
ⓒ The term $\operatorname{div}(\psi'(|\nabla u|^2 + |\nabla v|^2) \cdot \nabla u)$ is represents a nonlinear diffusion. The term $\psi'(s^2)$ is the diffusivity factor, which decreases the difusion when s^2 increases.

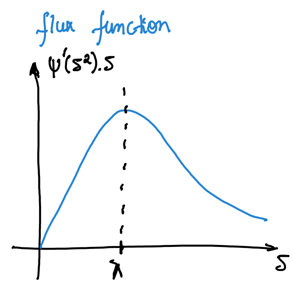


(d) The nonlinear diffusion term in the E-L equation (I-a) is:

- $\operatorname{div}\left(\psi'(|\nabla u|^2+|\nabla v|^2)\cdot\nabla u\right)$

which can be analysed in terms of diffusivity and flux functions:





- As a main effect, we have forward diffusion when $\frac{\partial}{\partial u}\left(\psi'(|\nabla u|^2+|\nabla v|^2)\cdot\nabla u\right) > 0$ and backward diffusion (edge enhancing) when $\frac{\partial}{\partial u}\left(\psi'(|\nabla u|^2+|\nabla v|^2)\cdot\nabla u\right) < 0$

  We increase diffusion when edges are not present (∇u or $\nabla v \approx 0$) and we enhance edges when edges are present (high values of ∇u or ∇v)

- Concerning the optical flow computation, we allow discontinuities of flow when we have edges (since there is less smoothness penalization on those areas, since s^2 is high).

- The optical flow image will look less blurry due to the nonlinear diffusion and present high flow discontinuity on object boundaries, which is desirable, since objects are usually moving differently from the background or other objects

- One drawback is that now we have to solve a set of nonlinear equations instead of linear

(e) Lets consider now a robustness in the data term:

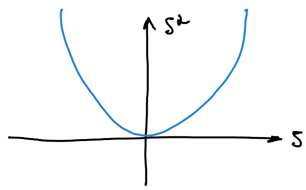
$$E(u,v) = \int_\Omega \psi\left((f_x u + f_y v + f_t)^2\right) + \alpha\left(|\nabla u|^2 + |\nabla v|^2\right) dx\,dy$$

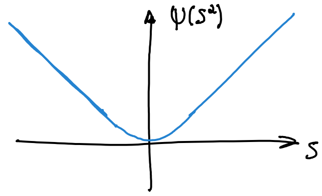
$$\begin{cases} F_u = \psi'\left((f_x u + f_y v + f_t)^2\right)\cdot 2(f_x u + f_y v + f_t)\cdot f_x \\ F_v = \psi'\left((f_x u + f_y v + f_t)^2\right)\cdot 2(f_x u + f_y v + f_t)\cdot f_y \\ F_{u_x} = 2\cdot\alpha\cdot u_x \end{cases}$$

$$\begin{cases} F_{uy} = 2\alpha\, u_y \\ F_{vx} = 2\alpha\, v_x \\ F_{vy} = 2\alpha\, v_y \end{cases}$$

$$\Rightarrow \begin{cases} \psi'\left((f_x u + f_y v + f_t)^2\right)(f_x u + f_y v + f_t)\cdot f_x + \Delta u = 0 \\ \psi'\left((f_x u + f_y v + f_t)^2\right)\cdot 2(f_x u + f_y v + f_t)\cdot f_y + \Delta v = 0 \end{cases}$$

f) The main effect is that we penalize less outliers (the pixels that have a big error in the constancy assumption $(f_x u + f_y v + f_t)^2 = s^2$)





, since s^2 grows quadratically and $\psi(s^2)$ grows almost linear for values far from the origin.

There are many impacts of using Robust data terms:

Advantage:

- Improve results w.r.t. outliers and noise (noise may increase s^2 such that unreliable locations have too much weight and flow is never driven to those areas)

Disadvantage:

- Computationally expensive (we now have a system of nonlinear equations)