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Assignment 4

Theoretical Exercise 4.1 (Motion Tensors)

(a) Derive the entries of the motion tensor for the assumption that the trace of the Hessian remains constant over time. Are there cases where the aperture problem does not appear?

(b) Extend the previous motion tensor to RGB colour images. Does the situation with respect to the aperture problem changes compared to the constancy assumption from (a)?

Theoretical Exercise 4.2 (Affine Horn and Schunck)

(a) In order to improve the performance of the method of Horn and Schunck, derive a variant of the original energy functional with affine parameterisation

$$\mathbf{w} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{pmatrix} .$$

Make use of the motion tensor notation. What are suitable models for the data and for the smoothness term? Be careful not to reinvent the original method in disguise!

(b) Derive the Euler–Lagrange equations that must be satisfied by each minimiser of the novel energy functional from (a). How many Euler-Lagrange equations are there in total?

Theoretical Exercise 4.3 (Photometric Invariants)

Let $(R, G, B)^\top(x, y, t)$ denote a colour image sequence, where R, G and B are the red, green and blue channel respectively. Classify the following expressions $p_1, ..., p_6$ with respect to their degree of photometric invariance w.r.t. changes of the overall intensity, shadow and shading, as well as highlights and specularities:

$$p_1 = R - 3B + G \qquad p_4 = 2\,\frac{BG+RG}{B^2-R^2} - \frac{B+R}{B-R}$$
$$p_2 = R^2 + B^2 - 2BR \qquad p_5 = (\ln B)_x + (\ln R)_x$$
$$p_3 = \frac{R-B}{R+G} \qquad p_6 = \ln B_x - \ln R_x$$