Problem 18 (Acceleration of a Point on a Rigid Body)

The acceleration of the point *Q* is given by

$$
\begin{aligned}
{}_B\mathbf{a}_Q &= {}_B\mathbf{a}_B + {}_B\dot{\mathbf{\Omega}}_B \times {}_B\mathbf{r}_{BQ} + {}_B\mathbf{\Omega}_B \times \left({}_B\mathbf{\Omega}_B \times {}_B\mathbf{r}_{BQ} \right) = \\
&= \begin{bmatrix} a_x - \dot{\omega} r_y - \omega^2 r_x \\ a_y + \dot{\omega} r_x - w^2 r_y \\ 0 \end{bmatrix}
\end{aligned}
$$

We want to find a point with zero acceleration. Therefore,

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_x - \dot{\omega} r_y - \omega^2 r_x \\ a_y + \dot{\omega} r_x - w^2 r_y \\ 0 \end{bmatrix}
$$

Solving these equations for ω and $\dot{\omega}$ gives

$$
\omega^2 = \frac{a_x r_x + a_y r_y}{r_x^2 + r_y^2}
$$

$$
\dot{\omega} = \frac{a_x r_y - a_y r_x}{r_x^2 + r_y^2}
$$