Problem 39 (Turbine)

(a)

$$I_1 = \begin{bmatrix} 50000 & 0 & 0 \\ 0 & 450000 & 0 \\ 0 & 0 & 450000 \end{bmatrix} kg \cdot m^2$$

(b) The rotation matrix is given by

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & -\alpha & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.001 & 0 \\ 0.001 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, the inertia matrix of the misaligned turbine is

$$I_2 = AI_1A^T \approx \begin{bmatrix} 1 & -\alpha & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 50000 & 0 & 0 \\ 0 & 450000 & 0 \\ 0 & 0 & 450000 \end{bmatrix} \begin{bmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 50000.45 & -400 & 0 \\ -400 & 450000.05 & 0 \\ 0 & 0 & 450000 \end{bmatrix} kg \cdot m^2$$

(c) The equations of motion of the turbine are given by

$$\vec{F}_1 + \vec{F}_2 = m\vec{g}$$

$$\vec{M} = I_2\dot{\vec{\omega}} + \vec{\omega} \times I_2\vec{\omega}$$

where $\vec{M}$ is the force moment about the origin and

$$\vec{\omega} = \begin{bmatrix} 1500 \\ 0 \\ 0 \end{bmatrix} rpm = \begin{bmatrix} \frac{1500}{60}2\pi \\ 0 \\ 0 \end{bmatrix} \frac{1}{s} \approx \begin{bmatrix} 50\sqrt{10} \\ 0 \\ 0 \end{bmatrix} \frac{1}{s}$$

is the angular velocity, with $\dot{\vec{\omega}} = 0$. Expanding the equations, we get

$$F_1 + F_2 = mg$$

$$-F_1\frac{d}{2} + F_2\frac{d}{2} + mg \cdot 0 = -400\left(50\sqrt{10}\right)^2 \; Nm$$

which leads to

$$F_1 + F_2 = 10^6 N$$

$$-F_1 + F_2 = -2 \cdot 10^6 N$$

and, hence, $F_1 = \begin{bmatrix} 0 & 1.5 \cdot 10^6 & 0 \end{bmatrix}^T N$, $F_2 = \begin{bmatrix} 0 & -0.5 \cdot 10^6 & 0 \end{bmatrix}^T N$.