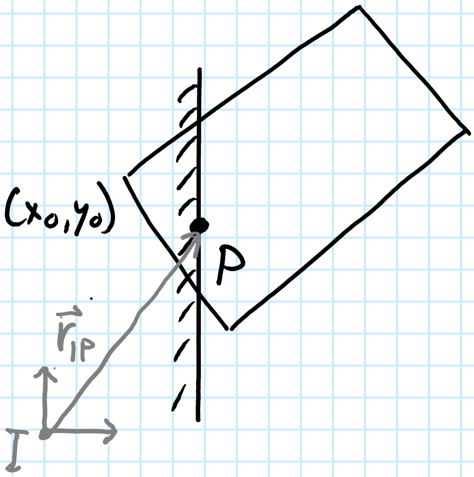
3. Constraints (Joints)

Example 1:



3.2. Constraint Derivatives

a) Velocities

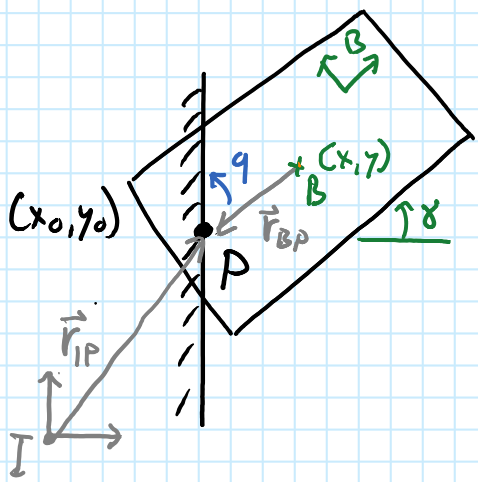
Explicit Constraints $\vec{x} = f_c(\vec{q})$

Implicit Constraints $g_c(\vec{x}) = \vec{0}$

Note: The meaning of $\vec{x}$ is not yet clearly defined. For now think of it as ${}_I\vec{x}$

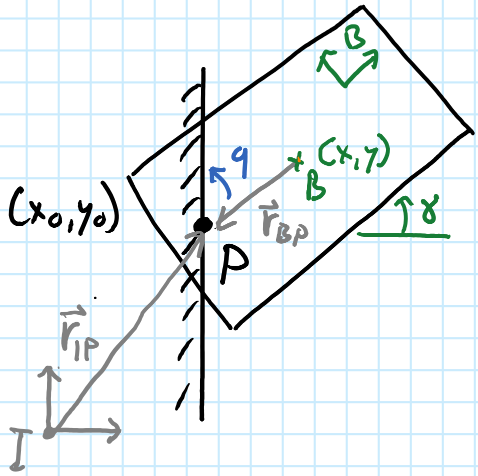
In Example 1

$$\vec{x} = \begin{pmatrix} x_0 + p_x \cos q \\ y_0 + p_y \cos q \\ q \end{pmatrix}$$

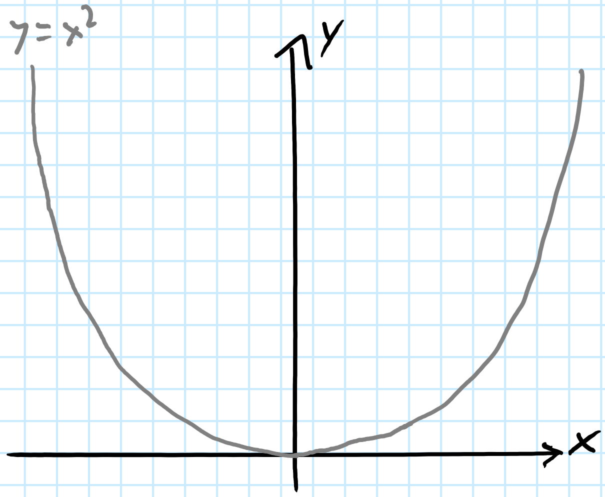


In Example 1

$$\vec{g}_c(\vec{x}) = \begin{pmatrix} x_0 - x + p_x \cos\gamma \\ y_0 - y + p_x \sin\gamma \end{pmatrix} = \vec{0}$$



Example 2: Quadratic Rollercoaster



b) Accelerations

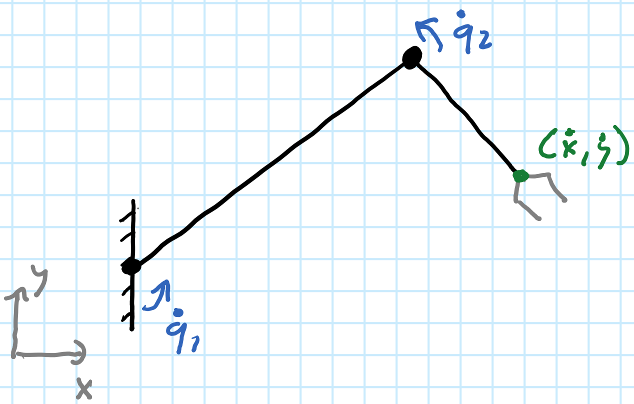
$$\dot{\vec{x}} = J_f \, \dot{\vec{q}}$$

$$\vec{0} = J_g^T \, \dot{\vec{x}}$$

c) What is the Jacobian?

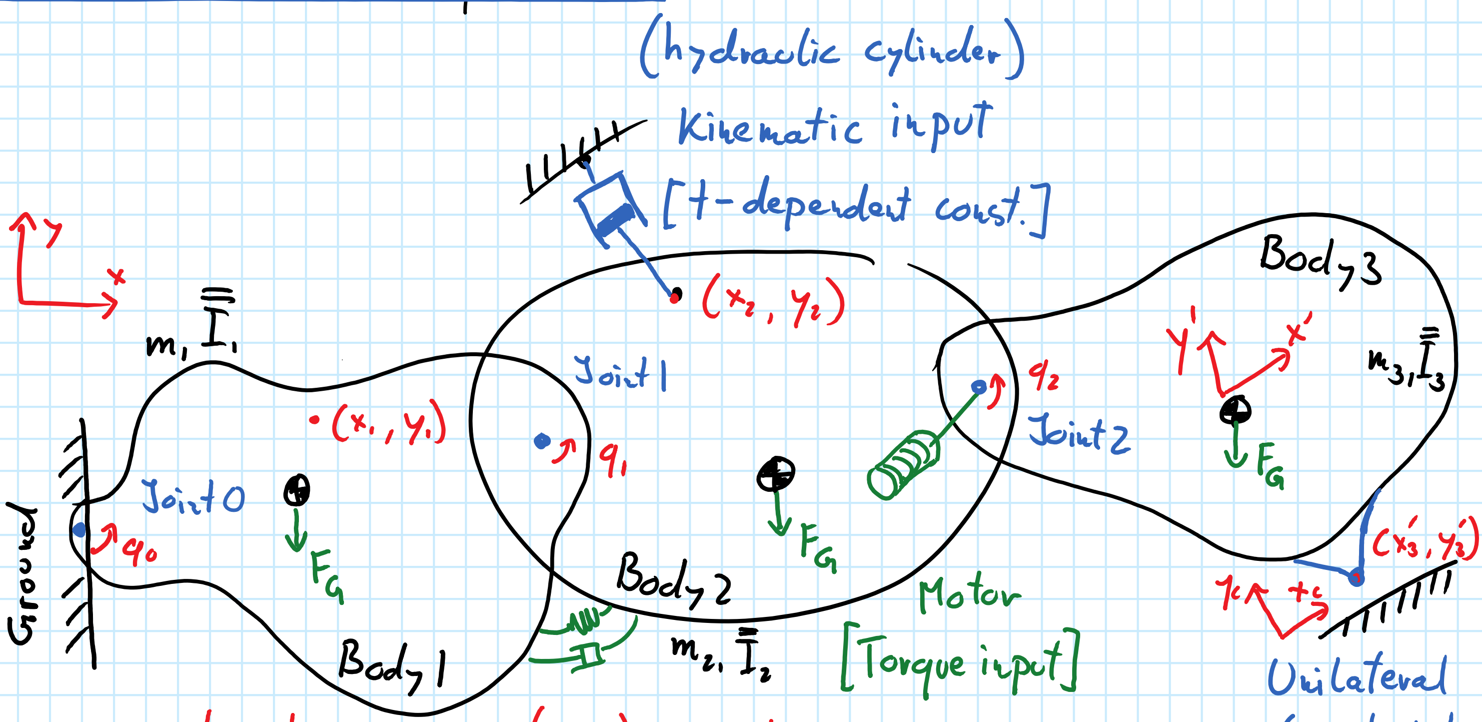
$$J_f = \frac{\partial \vec{f}_c}{\partial \vec{q}} = \begin{pmatrix} \frac{\partial \vec{f}_{c,1}}{\partial q_1} & \frac{\partial \vec{f}_{c,1}}{\partial q_2} & \\ \frac{\partial \vec{f}_{c,2}}{\partial q_1} & \frac{\partial f_{c,2}}{\partial q_2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$\Rightarrow \dot{x}_i = \frac{\partial f_{c,i}}{\partial q_1} \dot{q}_1 + \frac{\partial f_{c,i}}{\partial q_2} \dot{q}_2 + \ldots$$



$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = J_f \dot{\vec{q}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}$$

3.3. Constraint Classification



q = generalized coordinates

(x, y) = cartesian coordinates

explicit constraint $\vec{x} = \vec{f}_c(\vec{q})$

implicit constraint $\vec{g}_c(\vec{x}) = \vec{0}$

3.4. Non-Holonomic Constraints

Example: Skid-steering