Problem 13 (Twists and Screw Axis)

First, we write the velocity equation for the point *T* as

$$ {}_B\mathbf{v}_T - {}_B\mathbf{v}_B = {}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{r}_{BT} $$

Take the cross product with ${}_B\vec{\boldsymbol{\Omega}}_B$ of both sides of the equation and use the triple product formula:

$$ \begin{aligned} {}_B\vec{\boldsymbol{\Omega}}_B \times ({}_B\mathbf{v}_T - {}_B\mathbf{v}_B) &= {}_B\vec{\boldsymbol{\Omega}}_B \times ({}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{r}_{BT}) \\ &= {}_B\vec{\boldsymbol{\Omega}}_B({}_B\vec{\boldsymbol{\Omega}}_B \cdot {}_B\mathbf{r}_{BT}) - {}_B\mathbf{r}_{BT}({}_B\vec{\boldsymbol{\Omega}}_B \cdot {}_B\vec{\boldsymbol{\Omega}}_B) \\ &= {}_B\vec{\boldsymbol{\Omega}}_B({}_B\vec{\boldsymbol{\Omega}}_B \cdot {}_B\mathbf{r}_{BT}) - {}_B\mathbf{r}_{BT} \| {}_B\vec{\boldsymbol{\Omega}}_B \|^2 \end{aligned} $$

The velocity of the point *T* has to be along the axis of rotation. Therefore, ${}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{v}_T = 0$ and we now have

$$ -{}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{v}_B = {}_B\vec{\boldsymbol{\Omega}}_B({}_B\vec{\boldsymbol{\Omega}}_B \cdot {}_B\mathbf{r}_{BT}) - {}_B\mathbf{r}_{BT} \| {}_B\vec{\boldsymbol{\Omega}}_B \|^2 $$

Next, note that we can choose any point *T* along the axis of rotation. Let's choose *T* that is closest to the point *B* so that ${}_B\vec{\boldsymbol{\Omega}}_B \cdot {}_B\mathbf{r}_{BT} = 0$ and the above equation simplifies to

$$ {}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{v}_B = {}_B\mathbf{r}_{BT} \| {}_B\vec{\boldsymbol{\Omega}}_B \|^2 $$

Now we can write the location of the point *T* relative to *B*:

$$ {}_B\mathbf{r}_{BT} = \frac{{}_B\vec{\boldsymbol{\Omega}}_B \times {}_B\mathbf{v}_B}{\| {}_B\vec{\boldsymbol{\Omega}}_B \|^2} = \frac{{}_B\tilde{\boldsymbol{\Omega}}_B \cdot {}_B\mathbf{v}_B}{\| {}_B\vec{\boldsymbol{\Omega}}_B \|^2} $$