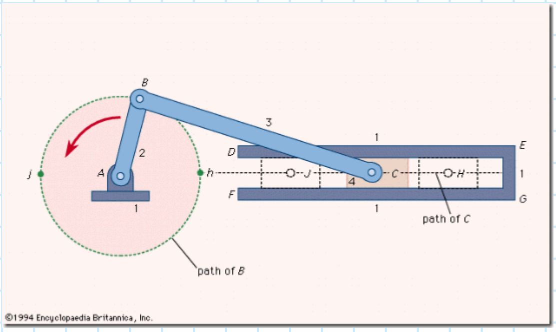
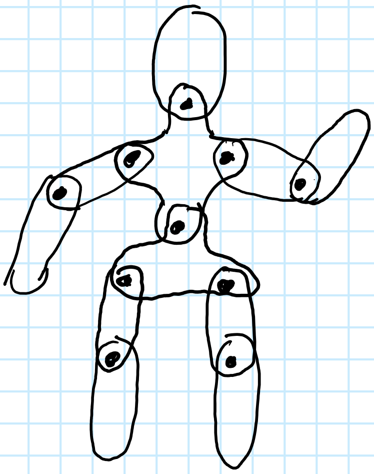
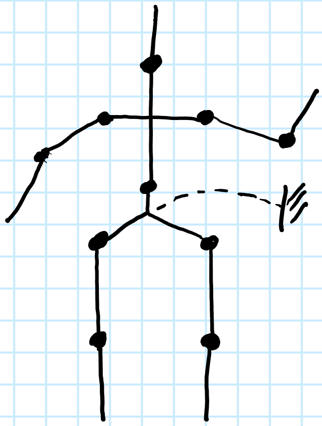
4. Recursive Kinematics



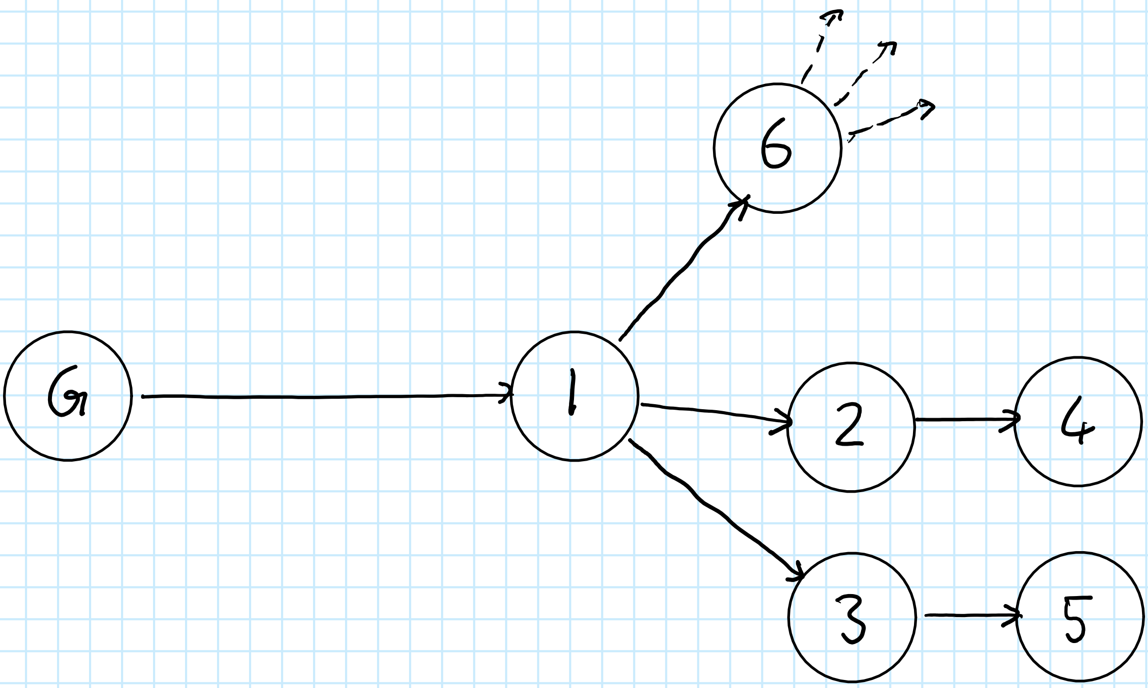
b) Representing Trees

Physical | Mass & Inertia | Linkage | Topology



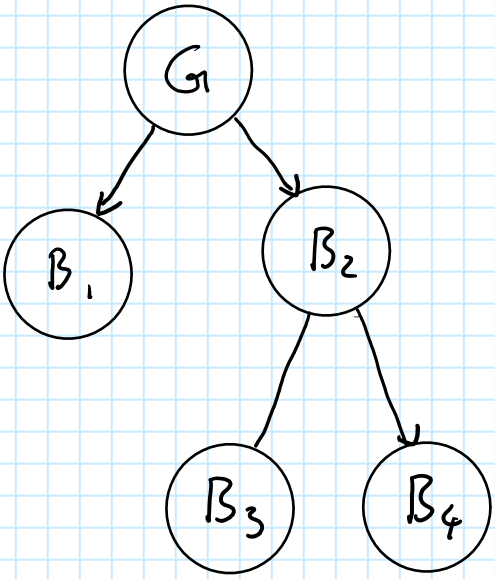


Increased abstraction



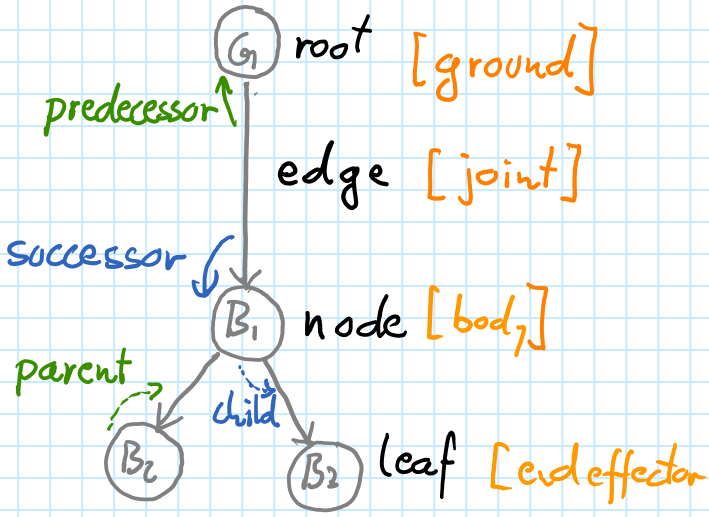
London, UK

Havanna, Cuba



4.2. Recursive Algorithms

Linked List

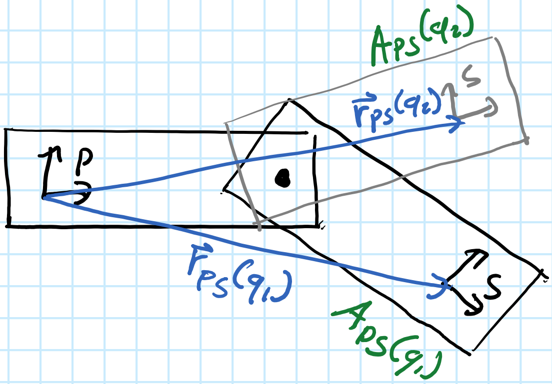


How to compute $\vec{x} = f_c(q)$ & $\dot{\vec{x}} = J\dot{\vec{q}}$?

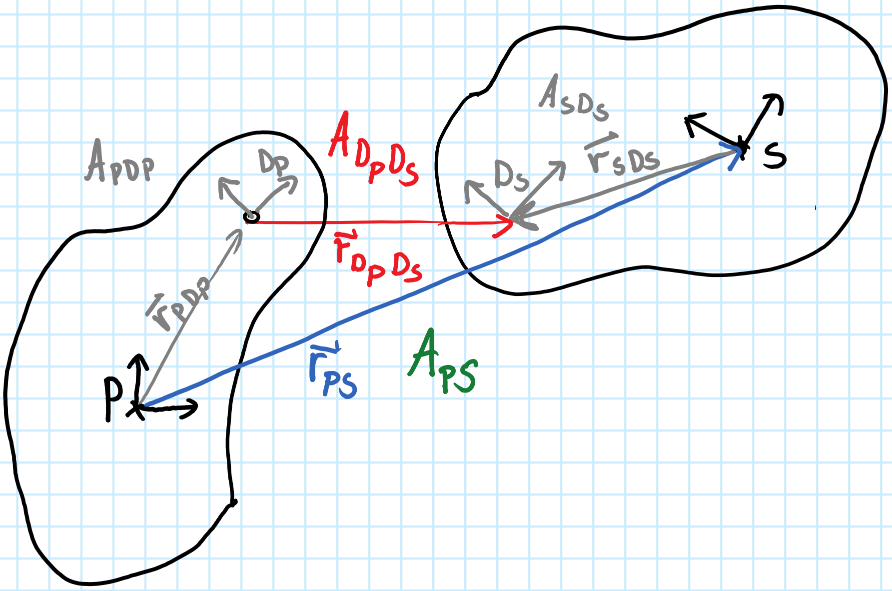
In a kinematic tree, the motion of each body depends only on all joints & bodies towards the root

"How to compute 'Δx', and what does it mean?"

Problem: Even a purely rotational joint creates translation & rotation



4. Recursive Kinematics - Page 8



Vectors

$$\vec{r}_{PS} = \vec{r}_{PD_P} + \vec{r}_{D_PD_S} - \vec{r}_{SD_S}$$

$$A_{PS} = A_{PD_P} \cdot A_{D_PD_S} \cdot A_{SD_S}^T$$

$$\Rightarrow \quad \vec{r}_{IS} = \vec{r}_{IP} + \vec{r}_{PS}$$

$$A_{IS} = A_{IP} \cdot A_{PS}$$

4. Recursive Kinematics - Page 10

4.4. Jacobians & Bias Accelerations

We can recursively compute:

A_{IS} $\quad {}_S\vec{r}_{IS}$ $\quad {}_S\vec{\Omega}_S$ $\quad {}_S\vec{v}_S$ $\quad {}_S\dot{\vec{\Omega}}_S$ $\quad {}_S\vec{a}_S$

from

A_{IP} $\quad {}_P\vec{r}_{IP}$ $\quad {}_P\vec{\Omega}_P$ $\quad {}_P\vec{v}_P$ $\quad {}_P\dot{\vec{\Omega}}_P$ $\quad {}_P\vec{a}_P$

and

$A_{D_PD_S}$ $\quad {}_{D_P}\vec{r}_{D_PD_S}$ $\quad {}_{D_P}\vec{\omega}_{D_PD_S}$ $\quad {}_{D_P}\dot{\vec{r}}_{D_PD_S}$ $\quad {}_{D_P}\dot{\vec{\omega}}_{D_PD_S}$ $\quad {}_{D_P}\ddot{\vec{r}}_{D_PD_S}$

Scleronomic

$${}_I\vec{x} = f_c(\vec{q})$$

$${}_I\dot{\vec{x}} = \frac{\partial f_c}{\partial \vec{q}}\dot{\vec{q}} = J_f \dot{\vec{q}}$$

$${}_I\ddot{\vec{x}} = J_f \ddot{\vec{q}} + \dot{J}_f \dot{\vec{q}} = \dots$$

$$= J_f(\vec{q})\ddot{\vec{q}} + {}_I\sigma(\vec{q}, \dot{\vec{q}})$$

Rheonomic

$${}_I\vec{x} = f_c(\vec{q}, t)$$

$${}_I\dot{\vec{x}} = \frac{\partial f_c}{\partial \vec{q}}\dot{\vec{q}} + \frac{\partial f_c}{\partial t}\dot{t} =$$

$$= J_f \dot{\vec{q}} + \frac{\partial f_c}{\partial t}$$

$${}_I\ddot{\vec{x}} = J_f \ddot{\vec{q}} + \dot{J}_f \dot{\vec{q}} + \frac{d}{dt}\frac{\partial f_c}{\partial t} = \dots$$

$$= J_f(\vec{q})\ddot{\vec{q}} + {}_I\sigma(\vec{q}, \dot{\vec{q}})$$

Non-holonomic

$$\left({}_I\vec{x} = f_c(\vec{q})\right)$$

$${}_I\dot{\vec{x}} = J_f \vec{\alpha}$$

$$\dot{\vec{q}} = f(\vec{\alpha})$$

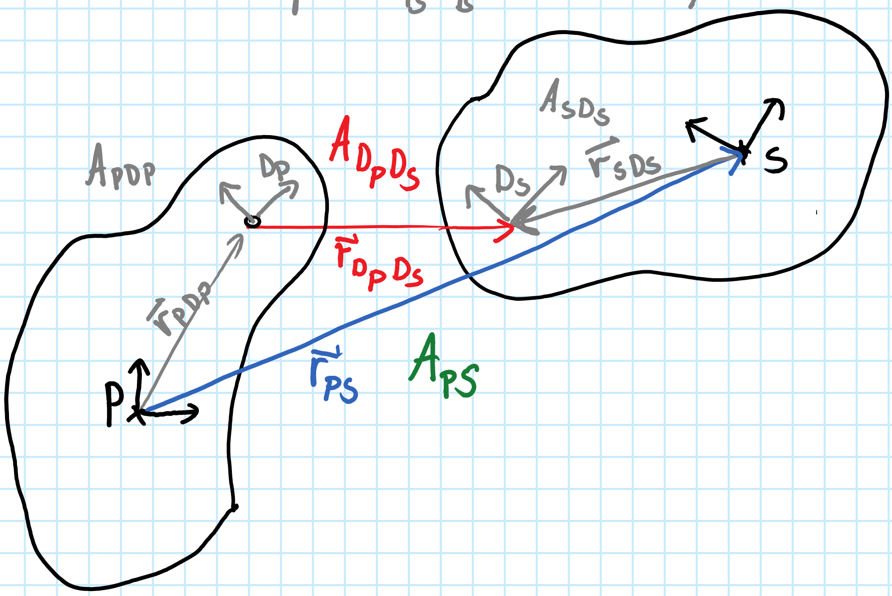
$${}_I\ddot{\vec{x}} = J_f \dot{\vec{\alpha}} + \dot{J}_f \vec{\alpha} = \dots$$

$$= J_f(\vec{q})\dot{\vec{\alpha}} + {}_I\sigma(\vec{q}, \vec{\alpha})$$

This can also be expressed for individual bodies in their coords.

a) Jacobians

Can we compute ${}_B J_B$ recursively?



Given:

$${}_P\vec{\Omega}_P = {}_P J_P^R \, \dot{\vec{q}}$$

$${}_P\vec{v}_P = {}_P J_P^S \, \dot{\vec{q}}$$

$${}_S\vec{v}_S = A_{PS}^T\left({}_P\vec{v}_P + {}_P\tilde{\Omega}_P\left({}_P\vec{r}_{PD_P} + A_{PD_P}\,{}_{D_P}\vec{r}_{D_PD_S}\right) + A_{PD_P}\,{}_{D_P}\dot{\vec{r}}_{D_PD_S}\right) - {}_S\tilde{\Omega}_S\,{}_S\vec{r}_{SD_S}$$

From:

$${}_{D_P}\vec{v}_{D_P} = A_{PD_P}^T\left({}_P\vec{v}_P + \underbrace{\cancel{{}_P\dot{\vec{r}}_{PD_P}}}_{=0} + {}_P\tilde{\Omega}_P\,{}_P\vec{r}_{PD_P}\right)$$

$${}_{D_S}\vec{v}_{D_S} = A_{D_PD_S}^T\left({}_{D_P}\vec{v}_{D_P} + {}_{D_P}\dot{\vec{r}}_{D_PD_S} + {}_{D_P}\tilde{\Omega}_{D_P}\,{}_{D_P}\vec{r}_{D_PD_S}\right)$$

$${}_S\vec{v}_S = A_{SD_S}\,{}_{D_S}\vec{v}_{D_S} - \underbrace{\cancel{{}_S\dot{\vec{r}}_{SD_S}}}_{=0} - {}_{D_S}\tilde{\Omega}_{D_S}\,{}_S\vec{r}_{SD_S}$$

b) Bias Accelerations

$$\boxed{\begin{aligned} {}_B\vec{a}_B &= {}_B J^S_B \, \ddot{\vec{q}} + {}_B\sigma^S_B \\ {}_B\dot{\vec{\Omega}}_B &= {}_B J^R_B \, \ddot{\vec{q}} + {}_B\sigma^R_B \end{aligned}}$$

4.5 Examples of Joint Functions

Type	$A_{D_pD_s}$	${}_{D_p}\vec{r}_{D_pD_s}$	${}_{D_p}\omega_{D_pD_s}$	${}_{D_p}\dot{\vec{r}}_{D_pD_s}$	${}_{D_p}\dot{\vec{\omega}}_{D_pD_s}$	${}_{D_p}\ddot{\vec{r}}_{D_pD_s}$
rotational $q = \gamma$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$
	$R \; \begin{array}{c} \gamma \\ \begin{pmatrix} 0 & 0 \\ \cdots 0 & 0 \cdots \\ 0 & 0 \end{pmatrix} \end{array}$	$S \; \begin{array}{c} \gamma \\ \begin{pmatrix} 0 & 0 \\ \cdots 0 & 0 \cdots \\ 0 & 0 \end{pmatrix} \end{array}$				
translational $q = \Delta x$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$	$\begin{pmatrix} \\ \\ \end{pmatrix}$
	$R \; \begin{array}{c} \Delta x \\ \begin{pmatrix} 0 & 0 \\ \cdots 0 & 0 \cdots \\ 0 & 0 \end{pmatrix} \end{array}$	$S \; \begin{array}{c} \Delta x \\ \begin{pmatrix} 0 & 0 \\ \cdots 0 & 0 \cdots \\ 0 & 0 \end{pmatrix} \end{array}$				

Type	$A_{D_pD_s}$	${}_{D_p}\vec{r}_{D_pD_s}$	${}_{D_p}\omega_{D_pD_s}$	${}_{D_p}\dot{\vec{r}}_{D_pD_s}$	${}_{D_p}\dot{\vec{\omega}}_{D_pD_s}$	${}_{D_p}\ddot{\vec{r}}_{D_pD_s}$
Virtual 3DOF $q=(\Delta x, \Delta y, \gamma)^T$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$
Rolling-contact $q=\gamma$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$	$\begin{pmatrix} \ \end{pmatrix}$

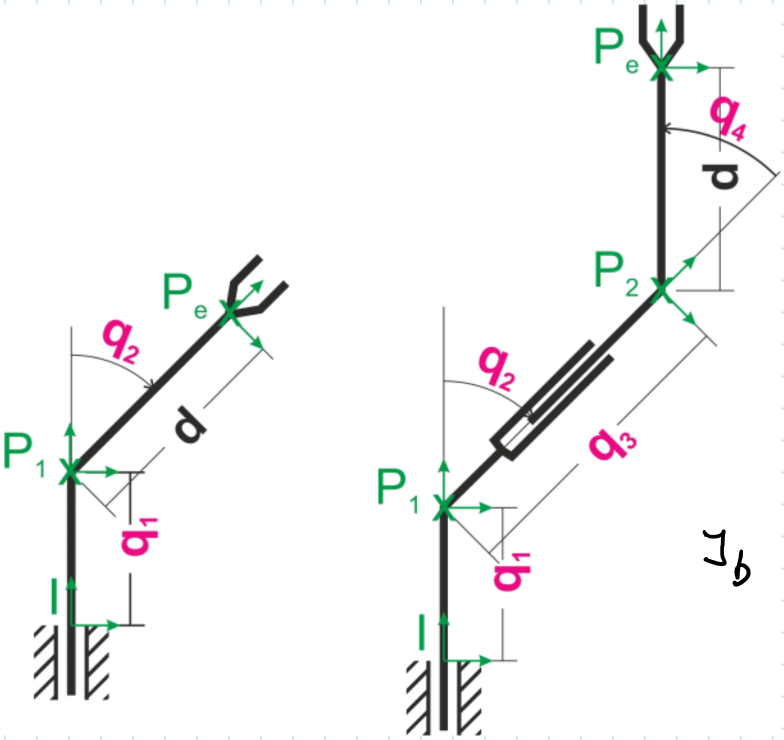
Virtual 3DOF:

$$R \quad \begin{array}{c} \begin{matrix} & \Delta x & \Delta y & \gamma & \end{matrix} \\ \begin{pmatrix} 0 & & & & 0 \\ \cdots 0 & & & & 0 \cdots \\ 0 & & & & 0 \end{pmatrix} \end{array} \qquad S \quad \begin{array}{c} \begin{matrix} & \Delta x & \Delta y & \gamma & \end{matrix} \\ \begin{pmatrix} 0 & & & & 0 \\ \cdots 0 & & & & 0 \cdots \\ 0 & & & & 0 \end{pmatrix} \end{array}$$

Rolling-contact:

$$R \quad \begin{array}{c} \begin{matrix} & \gamma & \end{matrix} \\ \begin{pmatrix} 0 & & 0 \\ \cdots 0 & & 0 \cdots \\ 0 & & 0 \end{pmatrix} \end{array} \qquad S \quad \begin{array}{c} \begin{matrix} & \dot{\gamma} & \end{matrix} \\ \begin{pmatrix} 0 & & 0 \\ \cdots 0 & & 0 \cdots \\ 0 & & 0 \end{pmatrix} \end{array}$$

(Some Notes on P21)



$$J_a = \begin{bmatrix} 0 & 0 \\ 1.0000 & 0 \\ 0 & 0 \\ 0 & -0.1061 \\ 1.0000 & 0.1061 \\ 0 & 1.0000 \end{bmatrix}$$

$$J_b \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -0.1414 & 0.7071 & 0 \\ 1.0000 & 0.1414 & 0.7071 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & -0.2914 & 0.7071 & -0.1500 \\ 1.0000 & 0.1414 & 0.7071 & 0 \\ 0 & 1.0000 & 0 & 1.0000 \end{bmatrix}$$