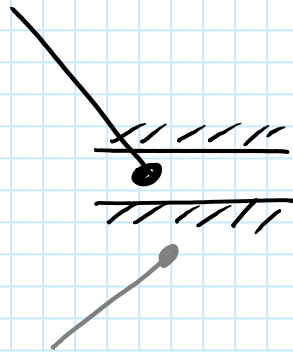
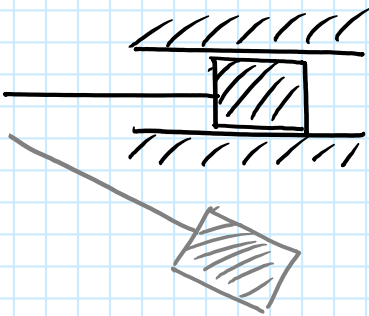
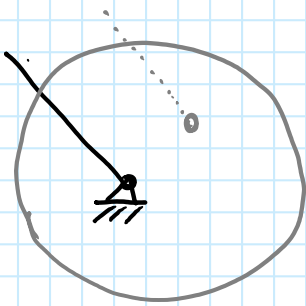
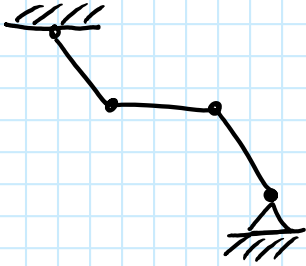


## 6. Loops & Contact

### 6.1. Loops



## 6. Loops and Contact - Page 2

a)

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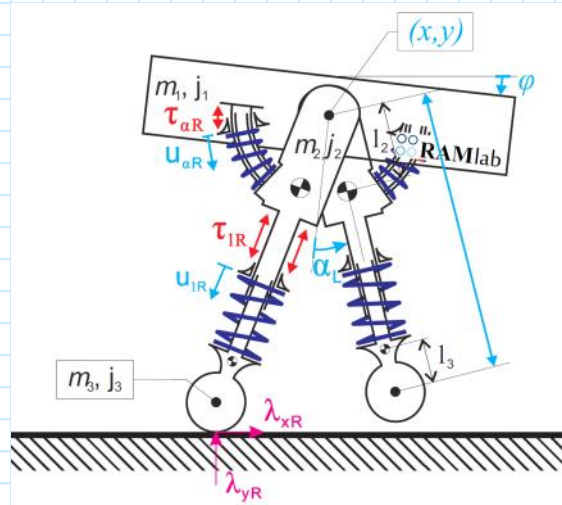
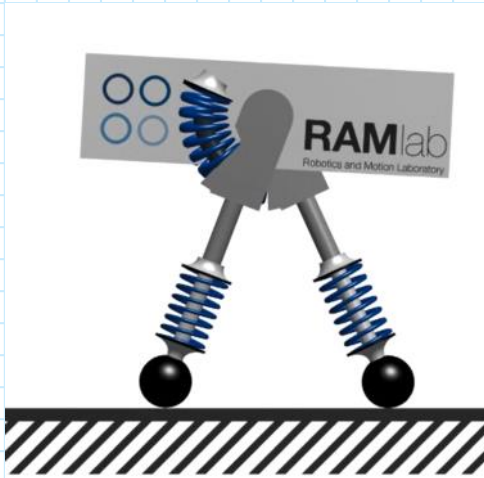
How is the constraint enforced?

## 6. Loops and Contact - Page 3

$$\begin{pmatrix} M & -J_\lambda^T \\ J_\lambda & 0 \end{pmatrix} \begin{pmatrix} \ddot{\vec{q}} \\ \vec{\lambda} \end{pmatrix} = \begin{pmatrix} \vec{f} + \vec{g} + \vec{c} \\ -\vec{\sigma}_\lambda \end{pmatrix}$$

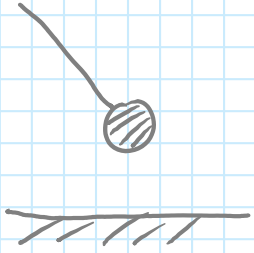
b) Potential Problems

# Example: Bipedal Robot w/ Floating Base

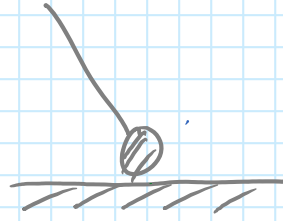


## 6.2. Unilateral Constraints & Collisions $\vec{c} = g_c(\vec{q}) \stackrel{!}{\geq} 0$

Open Contact

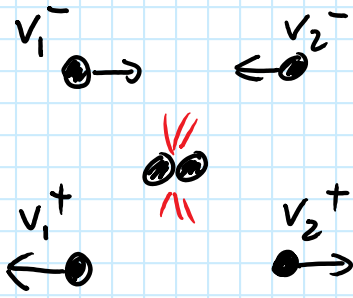


Closed Contact



$$\begin{pmatrix} M & -J_\lambda^T \\ J_\lambda & 0 \end{pmatrix} \begin{pmatrix} \ddot{\vec{q}} \\ \ddot{\lambda} \end{pmatrix} = \begin{pmatrix} \vec{f} + \vec{g} + \vec{c} \\ -\vec{\sigma}_\lambda \end{pmatrix}$$

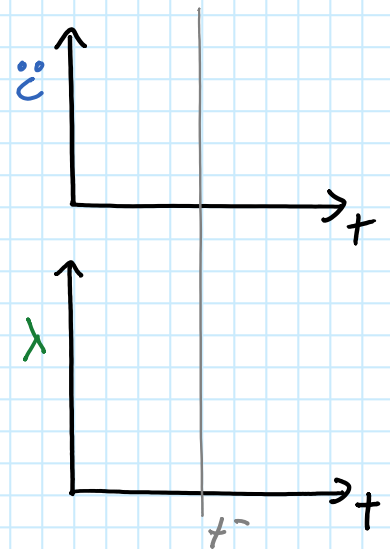
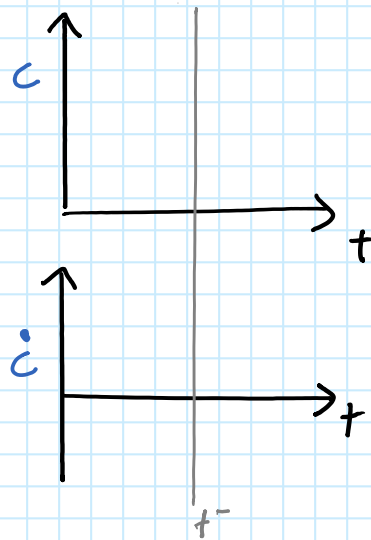
# Example: Two Particles



Notation:

"-"  $\rightarrow$  just before the impact " $\vec{v}^-$ "

"+"  $\rightarrow$  just after the impact " $\vec{v}^+$ "



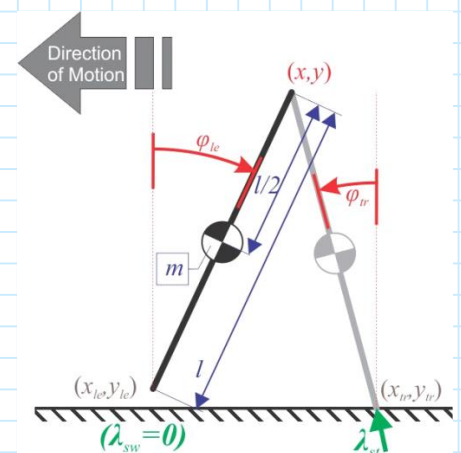
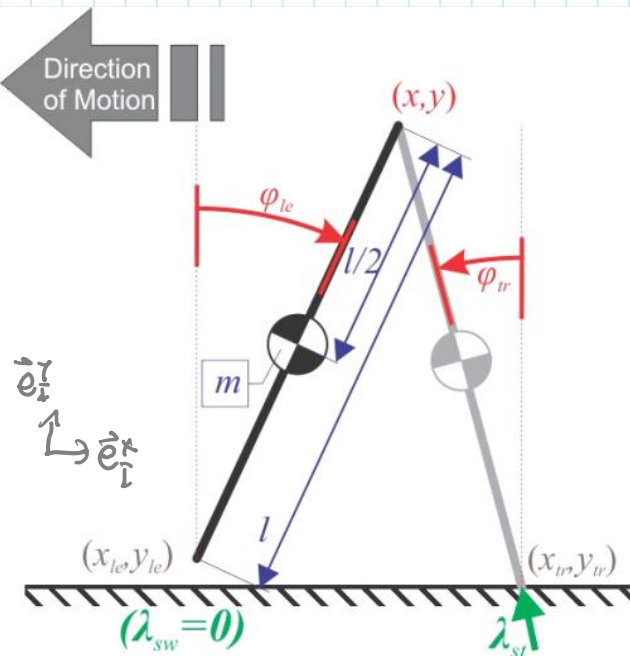
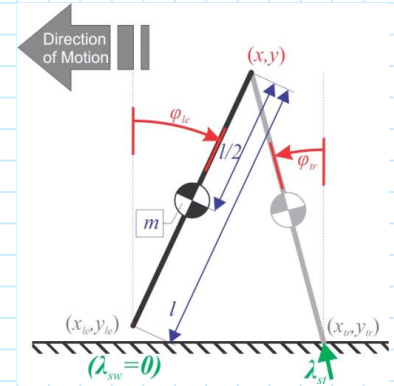
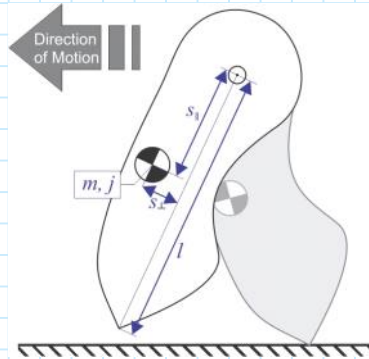
## 6. Loops and Contact - Page 7

How to generalize this?

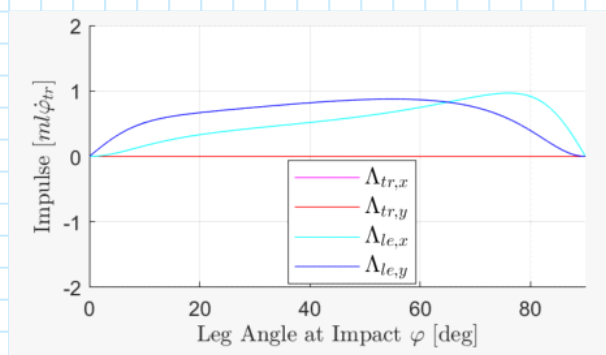
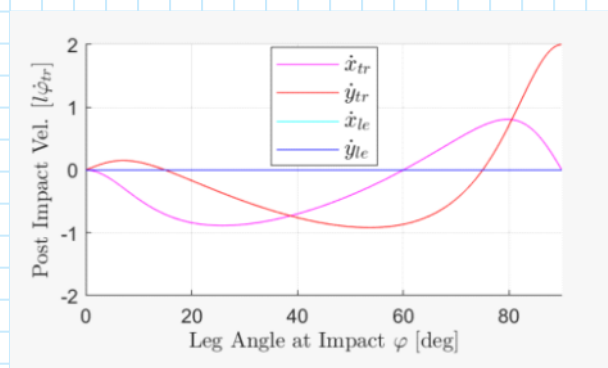
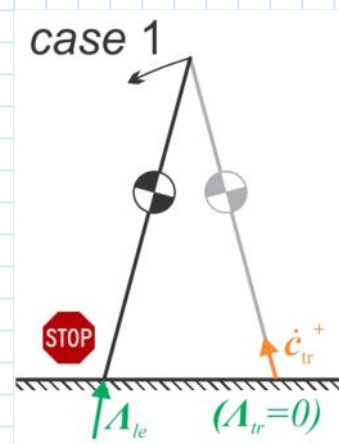
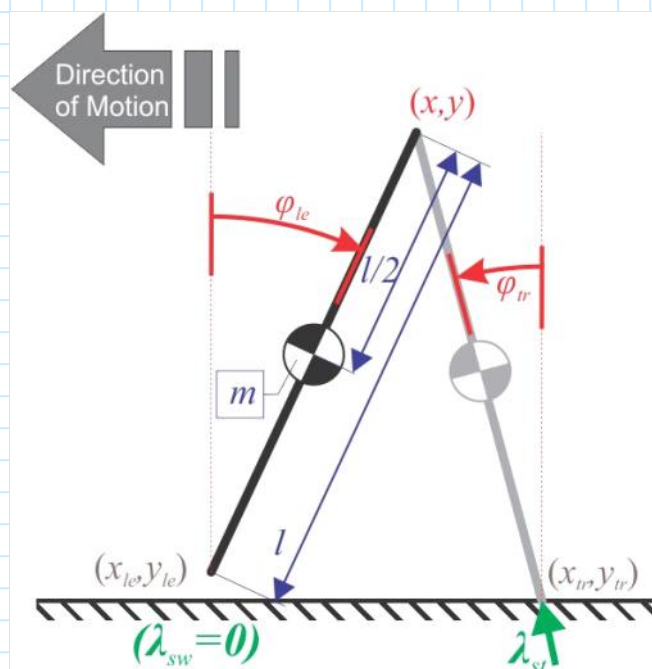
Example: two particles in 1D







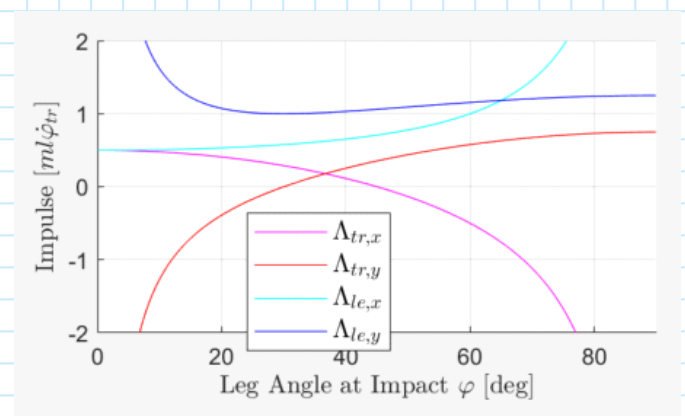
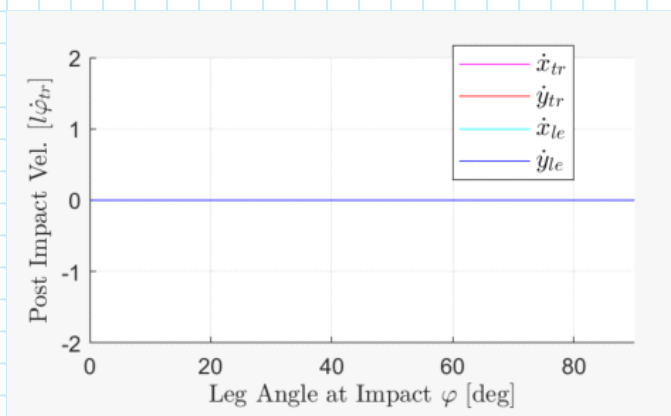
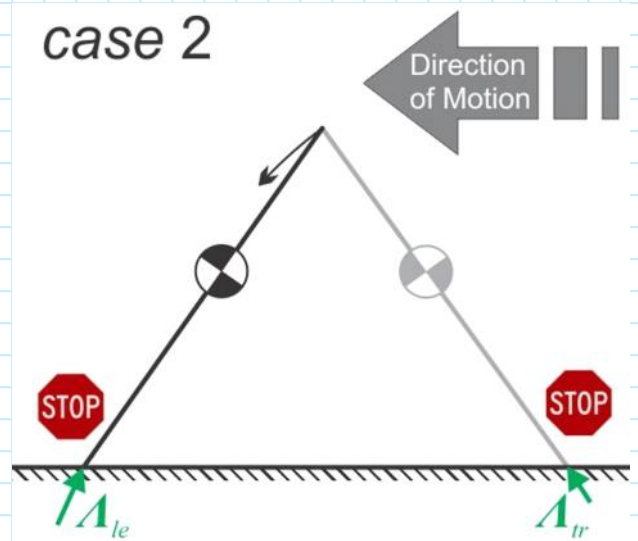
## 6. Loops and Contact - Page 11

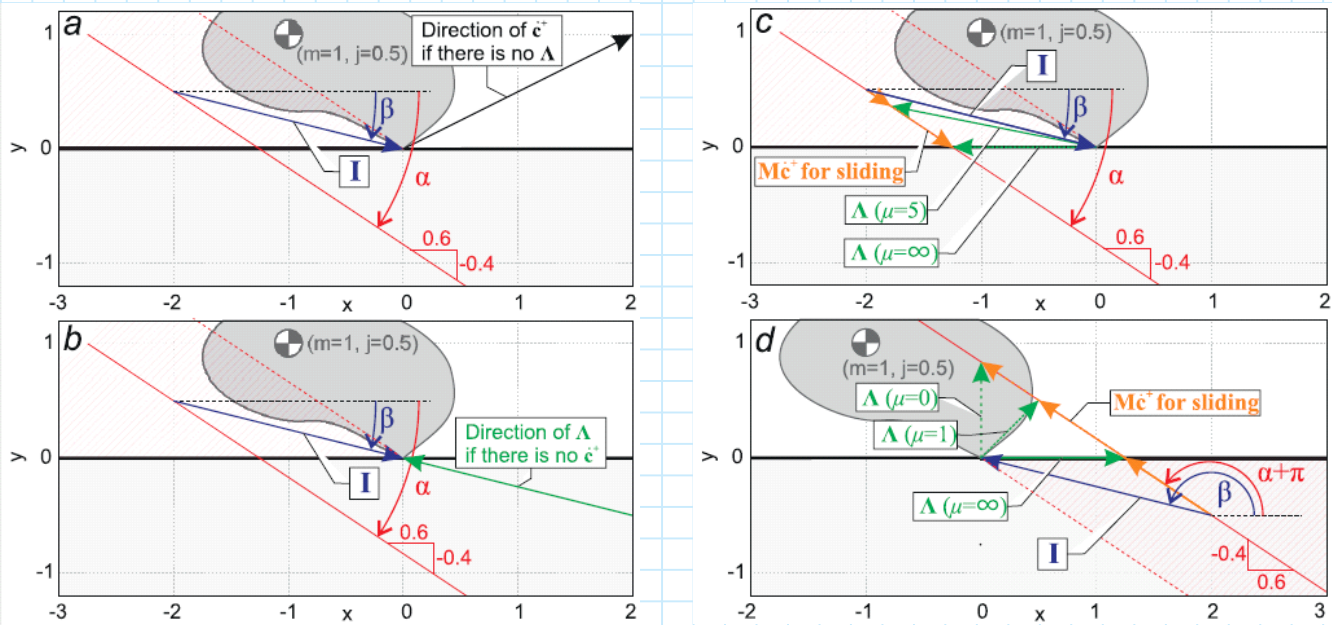


## 6. Loops and Contact - Page 12

$$\begin{bmatrix} M_T^v & M_L^v \\ M_L^T & M_L^v \end{bmatrix} \begin{bmatrix} \dot{c}_T^+ - \dot{c}_T^- \stackrel{v}{(=0)} \\ \dot{c}_L^+ \stackrel{v}{(=0)} - \dot{c}_L^- \stackrel{v}{(=0)} \end{bmatrix} = \begin{bmatrix} \vec{\Lambda}_T \\ \vec{\Lambda}_L \end{bmatrix}$$

case 2





Article

## Ambiguous collision outcomes and sliding with infinite friction in models of legged systems

C David Remy

### Abstract

This work explores the collision process at foot contact in models of legged robots. In particular, we highlight that for legged systems the widely used assumption of no sliding of the contact points can yield inconsistent outcomes. For certain contact configurations and system parameters, neither the assumption of lift-off of the trailing foot nor the assumption that it stays fixed on the ground yield valid solutions. The foot will slide, even if an infinite coefficient of friction is assumed. In a related effect, we present configurations in which the solution of the collision process is ambiguous. These behaviors are examined for the models of a minimalistic biped, a passive dynamic walker, and the five-link bipedal robot RAMone. The paper provides background for these non-intuitive behaviors and investigates the influence of system parameters onto the collision behavior of a passive dynamic walker. By studying bipedal models that range from minimalistic to physically accurate, this paper establishes a bridge between the theoretical study of unilateral contact in multibody dynamic simulations and the application of these simulation methods in the modeling of ground contact in actual legged systems.

### Keywords

Legged Robots, Mechanics, Design and Control, Biologically-Inspired Robots, Human-centered and Life-like Robotics, Dynamics, Mechanics, Design and Control, Animation and Simulation, Simulation, Interfaces and Virtual Reality, Contact modelling, manipulation

### 1. Introduction

The feet of legged systems have to alternate between aerial phases in which they are moving freely through the air and contact phases in which they are exerting forces against

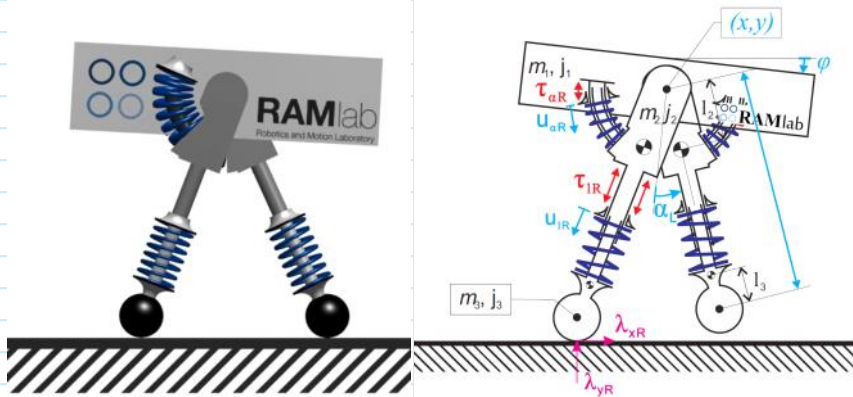
differential equations (Marhefka and Orin, 1999). For simplicity and performance, simulations of legged robots thus often employ completely rigid contact models. These rigid contact models rely on a set of unilateral contact constraints



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## 6.2 Virtual Model Control



Previously, we simulated this robot with  $\vec{\tau} = \vec{0}$

How to determine  $\vec{\tau}$ ?  
(i.e., how to do control)



⑤  $\Rightarrow$  Use contact forces  $\vec{\lambda}$  to create  $\vec{\tau}_{des}$ :

$$\vec{\tau}_{des} = \mathcal{Y}_{\lambda}^T \vec{\lambda}_{des} + S \vec{\tau}_{motor} = \begin{bmatrix} \mathcal{Y}_{\lambda}^T & S \end{bmatrix} \begin{pmatrix} \vec{\lambda}_{des} \\ \vec{\tau}_{motor} \end{pmatrix}$$

$\in \mathbb{R}^{n_q \times (n_c + n_m)}$

$\in \mathbb{R}^4 \rightarrow \text{double cont.}$   
 $\in \mathbb{R}^2 \rightarrow \text{single cont.}$   
 $\emptyset \rightarrow \text{no cont.}$



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