Problem 47 (Bowling Pin)

(a) The constraint violation c is the height of the lower-left corner of the pin above the ground:

$$c = y - y_{cog}\cos\varphi - x_{cog}\sin\varphi\,.$$

Hence, the constraint Jacobian is

$$\mathbf{J}_\lambda = \frac{\partial c}{\partial q} = \begin{bmatrix} 0 & 1 & y_{cog}\sin\varphi - x_{cog}\cos\varphi \end{bmatrix}.$$

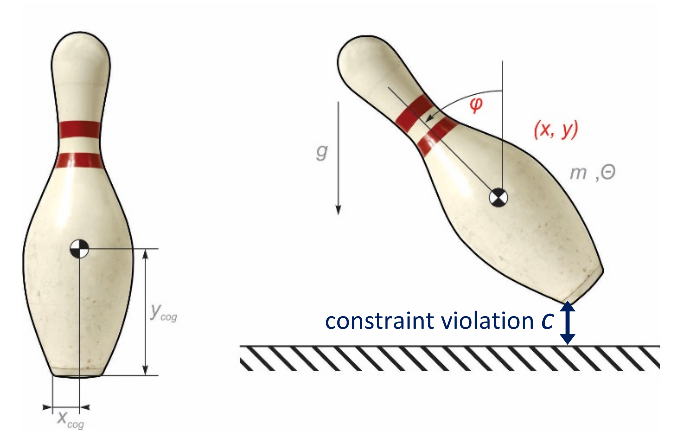

Figure 6

(b) For the specified pre-impact velocity, the bin is colliding with the ground without rotation. The collision impulse Λ is maximal when the contact point is exactly below the center of gravity of the pin – in this case, no energy is converted into rotational energy, and all translational energy lost during the impact is due to the collision impulse. This case corresponds to the angle

$$\hat{\varphi}^- = \arctan\left(\frac{x_{cog}}{y_{cog}}\right)$$

(c) Compared to the case (b) above, the pin has a non-zero pre-impact rotational energy. Hence, for some collision angles, the collision impulse may 'absorb' not only the translational energy of the pin but also some of its rotational energy. Thus, the contact point exactly below the center of gravity may not maximize the impulse in this case.

(d) The contact impulse Λ if found as

$$\Lambda = -M_\lambda \dot{c}^- = -M_\lambda \mathbf{J}_\lambda \dot{q}^- = -M_\lambda \dot{y}^-,$$

where

$$M_\lambda^{-1} = \mathbf{J}_\lambda M^{-1}\mathbf{J}_\lambda^T = \mathbf{J}_\lambda \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/\theta \end{bmatrix} \mathbf{J}_\lambda^T = ,$$

$$= \frac{1}{m} + \frac{\left(x_{cog}\cos\varphi^- - y_{cog}\sin\varphi^-\right)^2}{\theta}$$

and

$$M_\lambda = \frac{m\theta}{m\left(x_{cog}\cos\varphi^- - y_{cog}\sin\varphi^-\right)^2 + \theta}.$$

Therefore, the contact impulse is given by

$$\Lambda = -\dot{y}^- \frac{m\theta}{m\left(x_{cog}\cos\varphi^- - y_{cog}\sin\varphi^-\right)^2 + \theta}$$

and is maximal when $\left(x_{cog}\cos\varphi^- - y_{cog}\sin\varphi^-\right)^2 = 0$, such that

$$\Lambda_{\max} = -\dot{y}^{-}\frac{m\theta}{m \cdot 0 + \theta},$$
$$\Lambda_{\max} = -m\dot{y}^{-}.$$