Problem 8 (Integrating Angular Velocities)

(a) See provided solution file.

(b) See provided solution file.

(c) Method (a) integrates the transformation matrix $\mathbf{A}_{IC}$ and numerical errors are introduced directly to this matrix. Because of that, $\mathbf{A}_{IC}$ ceases to be an orthonormal matrix that represents rotation. You can see that, for example, in the last frame of the animation where the coordinate vectors have changed in length. In contrast, method (b) integrates the Cardan angles which at any point in time define rotation. So, even though the angle values accumulate integration errors, the resulting transformation $\mathbf{A}_{IC}$ based on these angles is always an orthonormal matrix that represents rotation. This makes method (b) more suited to describe angular motions.

☠ See provided solution file. Line 57 substituted the Euler integration step with an exact solution of the differential equation.