Problem 21 (2D Kinematics)

From simple geometry, we can compute the position and orientation of the robot at the points P_1, P_2, and P_e. For the case (a):

$$P_1\left(0 \quad q_1 \quad 0\right)$$
$$P_e\left(-d\sin q_2 \quad q_1 + d\cos q_2 \quad q_2\right)$$

And for the case (b):

$$P_1\left(0 \quad q_1 \quad 0\right)$$
$$P_2\left(-q_3\sin q_2 \quad q_1 + q_3\cos q_2 \quad q_2\right)$$
$$P_e\left(-q_3\sin q_2 - d\sin(q_2+q_4) \quad q_1 + q_3\cos q_2 + d\cos(q_2+q_4) \quad q_2+q_4\right)$$

Therefore, the Cartesian coordinate vectors are

$$\mathbf{x}_a = f_a\left(\mathbf{q}_a\right) = \begin{bmatrix} 0 \\ q_1 \\ 0 \\ -d\sin q_2 \\ q_1 + d\cos q_2 \\ q_2 \end{bmatrix}, \qquad \mathbf{x}_b = f_b\left(\mathbf{q}_b\right) = \begin{bmatrix} 0 \\ q_1 \\ 0 \\ -q_3\sin q_2 \\ q_1 + q_3\cos q_2 \\ q_2 \\ -q_3\sin q_2 - d\sin(q_2+q_4) \\ q_1 + q_3\cos q_2 + d\cos(q_2+q_4) \\ q_2+q_4 \end{bmatrix}$$

Next, taking partial derivatives of the above vectors with respect to the joint coordinates $\mathbf{q}_a$ and $\mathbf{q}_b$ respectively, we get

$$\mathbf{J}_a = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -d\cos q_2 \\ 1 & -d\sin q_2 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{J}_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -q_3\cos q_2 & -\sin q_2 & 0 \\ 1 & -q_3\sin q_2 & \cos q_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -q_3\cos q_2 - d\cos(q_2+q_4) & -\sin q_2 & -d\cos(q_2+q_4) \\ 1 & -q_3\sin q_2 - d\sin(q_2+q_4) & \cos q_2 & -d\sin(q_2+q_4) \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

For case (1), the components of $\mathbf{J}_a$ and $\mathbf{J}_b$ that correspond to translations are drawn in Figure 3.

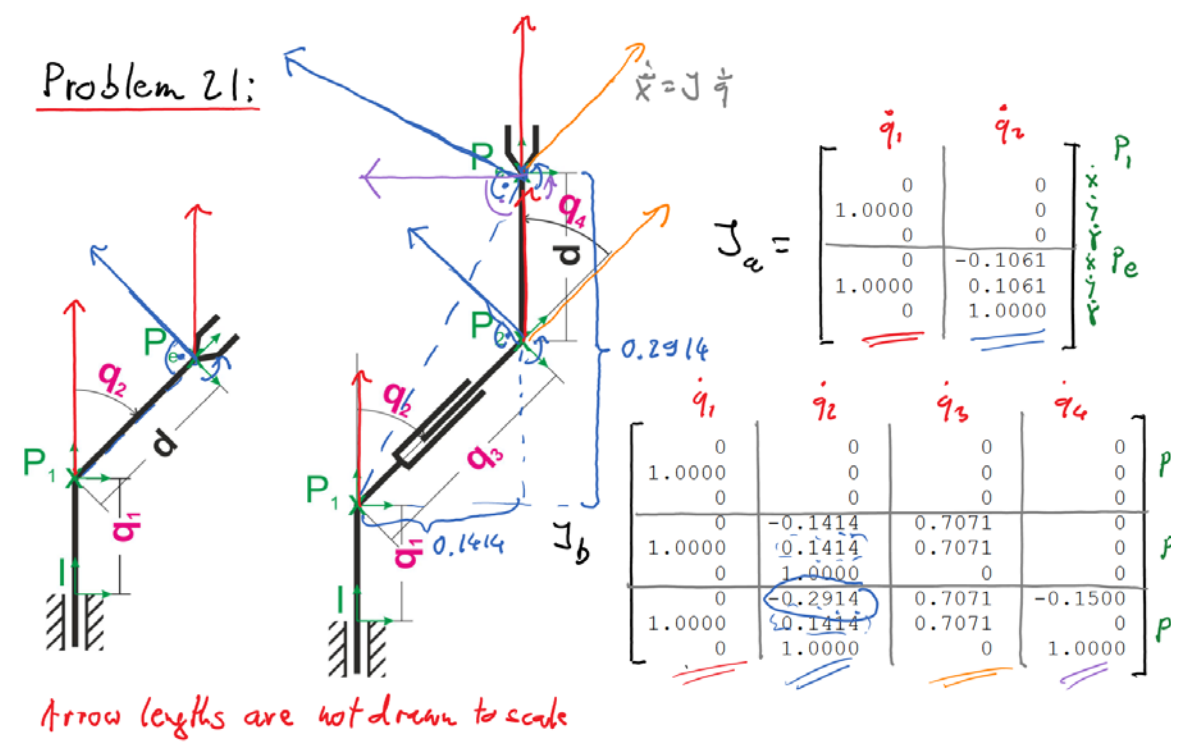
Problem 21:
$\dot{x} = J\,\dot{q}$
$J_a =$
$\dot{q}_1$ $\dot{q}_2$
0 0
1.0000 0
0 0
0 -0.1061
1.0000 0.1061
0 1.0000
P_1
P_e
J_b
$\dot{q}_1$ $\dot{q}_2$ $\dot{q}_3$ $\dot{q}_4$
0 0 0 0
1.0000 0 0 0
0 0 0 0
0 -0.1414 0.7071 0
1.0000 0.1414 0.7071 0
0 1.0000 0 0
0 -0.2914 0.7071 -0.1500
1.0000 0.1414 0.7071 0
0 1.0000 0 1.0000
0.2914
0.1414
d
Arrow lengths are not drawn to scale

Figure 3