Problem 35 (Specific Joint Functions)

(a) Rotational joint, where $\mathbf{q}=[\gamma]$

$$\mathbf{A}_{D_PD_S}(\mathbf{q})=\begin{bmatrix}\cos\mathbf{q} & -\sin\mathbf{q} & 0\\ \sin\mathbf{q} & \cos\mathbf{q} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$${}_{D_P}\mathbf{r}_{D_PD_S}(\mathbf{q})=[0\quad 0\quad 0]^T$$

$${}_{D_P}\boldsymbol{\omega}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[0\quad 0\quad \dot{\mathbf{q}}]^T$$

$${}_{D_P}\dot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[0\quad 0\quad 0]^T$$

$${}_{D_P}\dot{\boldsymbol{\omega}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[0\quad 0\quad \ddot{\mathbf{q}}]^T$$

$${}_{D_P}\ddot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[0\quad 0\quad 0]^T$$

$$R(\mathbf{q})=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

$$S(\mathbf{q})=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

(b) Translational joint, where $\mathbf{q}=[\Delta x]$

$$\mathbf{A}_{D_PD_S}(\mathbf{q})=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$${}_{D_P}\mathbf{r}_{D_PD_S}(\mathbf{q})=[\mathbf{q}\quad 0\quad 0]^T$$

$${}_{D_P}\boldsymbol{\omega}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[0\quad 0\quad 0]^T$$

$${}_{D_P}\dot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[\dot{\mathbf{q}}\quad 0\quad 0]^T$$

$${}_{D_P}\dot{\boldsymbol{\omega}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[0\quad 0\quad 0]^T$$

$${}_{D_P}\ddot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[\ddot{\mathbf{q}}\quad 0\quad 0]^T$$

$$R(\mathbf{q})=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$S(\mathbf{q})=\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

(c) 3DOF joint, where $\mathbf{q}=[\Delta x\quad \Delta y\quad \gamma]^T$

$$\mathbf{A}_{D_PD_S}(\mathbf{q})=\begin{bmatrix}\cos q_3 & -\sin q_3 & 0\\ \sin q_3 & \cos q_3 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$${}_{D_P}\mathbf{r}_{D_PD_S}(\mathbf{q})=[q_1\quad q_2\quad 0]^T$$

$${}_{D_P}\boldsymbol{\omega}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[0\quad 0\quad \dot{q}_3]^T$$

$${}_{D_P}\dot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[\dot{q}_1\quad \dot{q}_2\quad 0]^T$$

$${}_{D_P}\dot{\boldsymbol{\omega}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[0\quad 0\quad \ddot{q}_3]^T$$

$${}_{D_P}\ddot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[\ddot{q}_1\quad \ddot{q}_2\quad 0]^T$$

$$R(\mathbf{q})=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 0 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

$$S(\mathbf{q})=\begin{bmatrix}0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 1 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

(d) Rolling contact joint, where $\mathbf{q}=[\gamma]$

$$\mathbf{A}_{D_PD_S}(\mathbf{q})=\begin{bmatrix}\cos\mathbf{q} & -\sin\mathbf{q} & 0\\ \sin\mathbf{q} & \cos\mathbf{q} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$${}_{D_P}\mathbf{r}_{D_PD_S}(\mathbf{q})=[-r\mathbf{q}\quad 0\quad 0]^T$$

$${}_{D_P}\boldsymbol{\omega}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[0\quad 0\quad \dot{\mathbf{q}}]^T$$

$${}_{D_P}\dot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}})=[-r\dot{\mathbf{q}}\quad 0\quad 0]^T$$

$${}_{D_P}\dot{\boldsymbol{\omega}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[0\quad 0\quad \ddot{\mathbf{q}}]^T$$

$${}_{D_P}\ddot{\mathbf{r}}_{D_PD_S}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})=[-r\ddot{\mathbf{q}}\quad 0\quad 0]^T$$

$$R(\mathbf{q})=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

$$S(\mathbf{q})=\begin{bmatrix}0 & 0 & -r & 0 & 0\\ 0\cdots0 & 0 & 0\cdots0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$