Problem 46 (Robotic Arm. 3 – Event Detection)

(a), (b), (c) See the provided solution file. In the lines 29 and 30, you will need to set the parameters *'P_Baumgarte'* and *'D_Baumgarte'* to, respectively, 25 and 10 to run the problem with Baumgarte stabilization, and to 0 and 0 to run it without stabilization.

☠ Let P be anchorPoint, E be endeffector, and l be wireLength. We write the constraint violation as

$$c = -\left\| {}_I\vec{r}_{IP} - {}_I\vec{r}_{IE} \right\| + l \,.$$

Let $\vec{r} = {}_I\vec{r}_{IP} - {}_I\vec{r}_{IE}$ and $\vec{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^T$. Then we get

$$c = -\left\|\vec{r}\right\| + l = -\sqrt{r_x^2 + r_y^2 + r_z^2} + l \,,$$

$$\dot{c} = -\frac{r_x \dot{r}_x + r_y \dot{r}_y + r_z \dot{r}_z}{\sqrt{r_x^2 + r_y^2 + r_z^2}} + l = -\frac{\vec{r} \cdot \dot{\vec{r}}}{\sqrt{r_x^2 + r_y^2 + r_z^2}} =$$

$$= -\overrightarrow{dir} \cdot {}_I\vec{v}_{IE} = -\overrightarrow{dir}^T A_{IE}\, {}_E J_E^S \dot{\vec{q}}$$

Hence, the constraint Jacobian

$$J_\lambda = -\overrightarrow{dir}^T A_{IE}\, {}_E J_E^S \,.$$

Differentiating $\dot{c}$ once again, we find

$$\ddot{c} = -\frac{d}{dt}\left(\frac{\vec{r} \cdot \dot{\vec{r}}}{\sqrt{r_x^2 + r_y^2 + r_z^2}} \right) = -\left[\frac{\vec{r} \cdot \ddot{\vec{r}}}{\sqrt{r_x^2 + r_y^2 + r_z^2}} + \frac{\left\|\vec{r} \times \dot{\vec{r}}\right\|^2}{\left(r_x^2 + r_y^2 + r_z^2\right)^{3/2}} \right] =$$

$$= -\left[\overline{dir}^T A_{IE}\, {}_E\vec{a}_E + \frac{\left\| \overline{dir} \times \left(A_{IE}\, {}_E\vec{v}_E \right) \right\|^2}{\sqrt{r_x^2 + r_y^2 + r_z^2}} \right] = -\left[\overline{dir}^T A_{IE}\, {}_E\vec{a}_E + \frac{\left\| \overline{dir} \times \left(A_{IE}\, {}_E\vec{v}_E \right) \right\|^2}{l} \right]$$

Therefore, the bias acceleration is

$$\vec{\sigma}_\lambda = -\overline{dir}^T A_{IE}\, {}_E\vec{a}_E - \frac{\left\| \overline{dir} \times \left(A_{IE}\, {}_E\vec{v}_E \right) \right\|^2}{l} \,.$$