#### 1. Fundamentals - Page 1

# 1. Fundamentals (Recap)

# 1.1 Representing Physics

a) Scalar Quantities "s"

4) represented by a (real) number and a unit

eg. 
$$m[kg]$$
,  $d[m]$ ,  $t[s]$ ,  $k[m]$ ,  $b[m]$ ,  $E[S]$ ,  $P[W]$ 

b) Vector Quantities "v", "v", "v"

L) represented by numerical vectors / coordinates + unit

e.g., 
$$r [m]$$
,  $v [s]$ ,  $a [s^2]$ ,

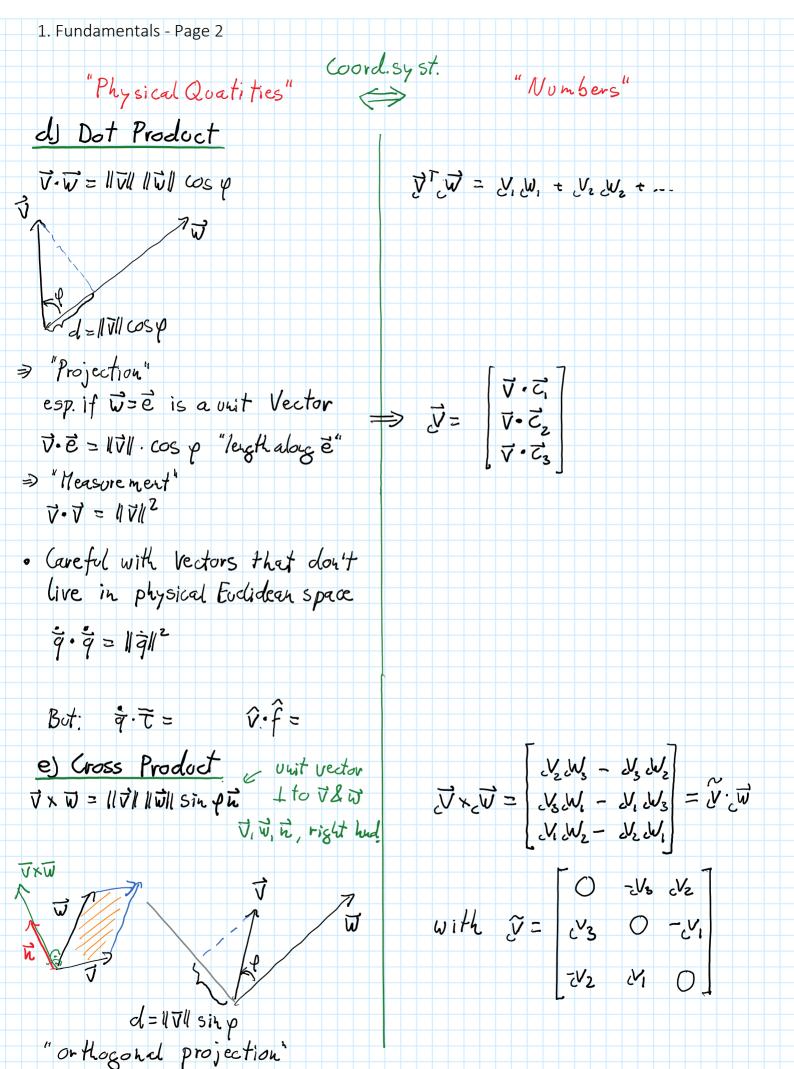
e.g., 
$$\vec{r}$$
 [m],  $\vec{v}$  [s],  $\vec{a}$  [sz],  $\vec{w}$  [rad],  $\vec{v}$  [rad],  $\vec{r}$  [N],  $\vec{p}$  [Nm]

- · These are physical quantities that can be measured.
- · A physical vektor (quantity) i can be expressed as a numerical vector vertor of 3 coordinates in a given coordinate system C.

() Coordinate System "C"

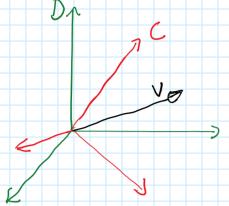
"A set of independent mutually or thogonal unit vectors ?;"

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$



# 1.2 Coordinate Transformations

two coordinate systems C&D



$$\vec{V} = \begin{bmatrix} \vec{V} \cdot \vec{d}_1 \\ \vec{V} \cdot \vec{d}_2 \\ \vec{V} \cdot \vec{d}_3 \end{bmatrix}$$

$$|\vec{v}| \Rightarrow |\vec{v}| = A_{DC} |\vec{v}|$$

$$= \frac{1}{2} c \vec{v} = \begin{bmatrix} \vec{d}_1 \cdot \vec{c}_1 & \vec{d}_2 \cdot \vec{c}_3 \\ \vec{d}_1 \cdot \vec{c}_2 & \vec{d}_2 \cdot \vec{c}_3 \end{bmatrix}$$

$$d_3 \cdot C_1$$
 $d_3 \cdot C_2$ 
 $d_4 V_2$ 

$$\Rightarrow c\vec{v} = A_{cp} \vec{v}_{p}$$

Since 
$$\vec{c}_i \cdot \vec{d}_j = \vec{d}_j \cdot \vec{c}_i \Rightarrow A_{DC} = A_{CD} = A_{CD} \Rightarrow A_{CD} = 1$$

$$A_{DC} = A_{CD}^{T} = A_{CD}^{-1}$$

for 
$$\vec{c}_{1} = \vec{0}$$

$$\vec{C}_{i} = A_{DC} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$

for 
$$\vec{c}_{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \implies \vec{c}_{i} = A_{DC} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = "first column of Aoc"$$

Similar for Gia

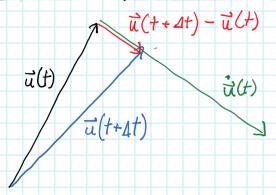
⇒ The columns of ADC are the basis vectors in coordinates of D:

$$A_{DC} = \begin{bmatrix} \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{bmatrix}$$

# 1.3 Derivatives of Vectors

$$\vec{u}(t) \rightarrow \frac{d\vec{u}}{dt} - \lim_{\Delta t \to 0} \frac{\vec{u}(t+\Delta t) - \vec{u}(t)}{\Delta t} = \vec{u}(t)$$

### a) For Physical Vectors (ū)



in an inertial frame of reference

### b) For Coordinates (ii)



$$\vec{u} = (\vec{u}) - (\widetilde{\omega}_{1c})\vec{u} = (\vec{u}) - (\widetilde{\omega}_{1c})\vec{u}$$
with  $\widetilde{\omega}_{1c} = A_{c1} A_{1c}$ 

Wic means: angular velocity of Cagainst 1 expressed in C.

$$\widetilde{\omega}_{ic} = A_{ic} c \widetilde{\omega}_{ic}$$

$$\widetilde{\omega}_{ic} = A_{ic} c \widetilde{\omega}_{ic} A_{ci}$$

## 1.4 Rigid Body Motion

must preserve distances ?
 → describe via orthonormal transformations

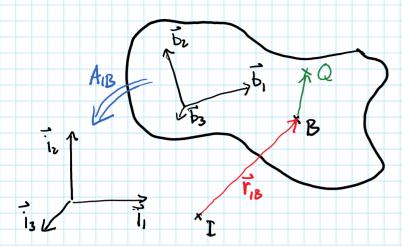
$$d = \|\vec{u}\| = \sqrt{\vec{u}} \cdot \vec{u} = const$$

$$d = (\vec{u} \cdot \vec{u})^{T} (A_{CB} g \vec{u}) = g \vec{u}^{T} A_{CB} A_{BC} c \vec{u} = g \vec{u}^{T} g \vec{u}$$

must preserve handedness (no inversion / reflection)
 → det (ABC) = +1

Body-fixed cosys B
[given by rotation AIB]
Body-fixed point B
[given by translation FIB]

## a) Motion of Points on Body



Q is body-fixed

position:

velocity:

acceleration: Ba = BaB + ( To FBQ)=

$$\vec{a}_{\alpha} = \vec{a}_{\beta} + (\vec{\beta} \hat{\vec{\Omega}}_{\beta} + \vec{\beta} \hat{\vec{\Omega}}_{\beta}^{2})_{\beta} \vec{r}_{\beta} q$$

rigid body motion defined by

$$\cdot \vec{a}_{B}$$
,  $\hat{\vec{\Omega}}_{B}$   $(\vec{\vec{l}}_{B})$ 

$$\vec{B} \vec{r}_{1Q} = \vec{B} \vec{r}_{1B} + \vec{B} \vec{r}_{BQ}$$

$$\vec{B} \vec{v}_{Q} = \vec{B} \vec{v}_{B} + \vec{B} \vec{L}_{B} \vec{B} \vec{r}_{BQ}$$

$$\vec{B} \vec{a}_{Q} = \vec{B} \vec{a}_{B} + (\vec{B} \vec{L}_{B} + \vec{B} \vec{L}_{B}) \vec{b} \vec{r}_{BQ}$$

