Problem 19 (Calculating Inertia Matrices)

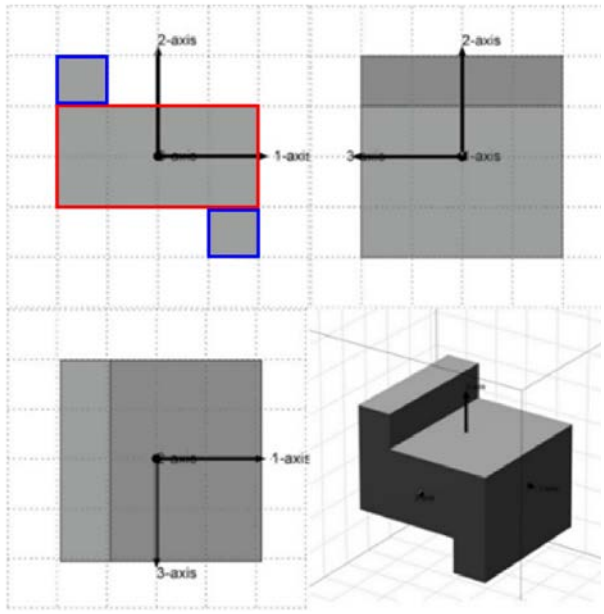

Figure 2

We will compute the inertia about the 3-axis of the given object as a sum of the inertia of the central part of it (highlighted with red in the 1- and 2-axis plane in Figure 2) and the inertias of the two parts highlighted with blue. The inertia of the red part about the 3-axis is

$$I_{red} = \frac{1}{12}\left(\frac{8}{10}m\right)\left(h_{red}^2 + w_{red}^2\right) = \frac{8}{120}m\left(16+4\right) = \frac{1}{120}160m$$

The blue parts have the same inertia that we can find using the parallel axis theorem:

$$I_{blue} = \frac{1}{12}\left(\frac{m}{10}\right)\left(h_{blue}^2 + w_{blue}^2\right) + \frac{m}{10}\left(1.5\sqrt{2}\right)^2 = \frac{1}{120}m\left(1+1\right) + \frac{9}{20}m = \frac{1}{120}56m$$

Thus, the inertia of the whole object about the 3-axis is

$$I_3 = I_{red} + 2I_{blue} = \frac{1}{120}m\left(160 + 2\cdot 56\right) = \frac{1}{120}272m$$