Problem 10 (Understanding Rotation Matrices)

(a) Matrix (4) has an element of 1.354. The associated column/row is hence longer than 1, and the matrix therefore cannot be orthonormal.

(b) Matrices (1) and (4) are symmetric. Since in rotation matrices, the inverse is equal to the transpose, these matrices would have to be their own inverse. This is obviously not true (it only holds for the identity) and, hence, they do not represent rotation. Matrix (2) and (3) are equal apart from the signs in the second column. One of these must hence be a left-handed coordinate system. One could check this by visualizing the basis vectors, or realize that for matrix (3), each term in the determinant formula is negative which leads to a negative determinant of the matrix; hence, the matrix is not orthonormal and cannot represent rotation. Thus, matrix (2) remains as the correct answer.

(c) Using the provided pictures and the grid in them, we can approximate the basis vectors of the frame \`C' in terms of the basis vectors of the frame \`I', i.e. in coordinates of the frame \`I'. As such, the 1-axis vector of the frame \`C' is about $[0.15, -0.1, 0.95]^T$, the 2-axis vector is about $[-0.6, 0.9, 0.2]^T$, and the 3-axis vector is about $[-0.9, 0.6, 0.1]^T$. Therefore, the approximate transformation matrix is

$$\begin{bmatrix} 0.15 & -0.6 & -0.9 \\ -0.1 & 0.9 & -0.6 \\ 0.95 & 0.2 & 0.1 \end{bmatrix}$$