Problem 41 (Understanding Dynamics)

(a) In Newton-Euler equations of motion, the angular and force moments are taken about either the center-of-gravity of the body or a stationary point. Therefore, the statements (2) and (3) are not correct.
In the statement (1), the moments are about the center-of-gravity, but because they are expressed in a moving frame, the extra term ${}_C\vec{\Omega}_C \times {}_C\vec{L}_G$ is missing on the left-hand side.
Thus, only the statement (4) is true.

(b) The statement (2) is not correct: instead of the Jacobians ${}_B\mathbf{J}_P^{S/R}$, it should use the Jacobians ${}_B\mathbf{J}_B^{S/R}$, as in the statement (1).

(c) Imagine the motion of the right ball that is rotation strictly around the φ axis with a fixed angle γ. In the configuration in the figure, the ball would move into the plane of the figure, along the 3-axis. This motion can be excited only by a torque around the axis of φ (a force on the ball along the 3-axis) and with no torque (no acceleration) around the axis of γ. Hence, the diagonal elements of the mass matrix have to be zero and the answers (2) and (4) are not correct.
In the described rotation, the arm of the force on the ball is the distance from the ball to the 2-axis (the axis of φ) and is equal to $\frac{l}{2}$. The corresponding torque is then given by $m\left(\frac{l}{2}\right)^2 \ddot{\varphi}$. Therefore, the element M_{11} of the mass matrix has to be $m\left(\frac{l}{2}\right)^2$. Similarly, for rotation around the axis of γ, the moment arm is l, the torque is $ml^2\ddot{\gamma}$, and the corresponding element M_{22} of the mass matrix is ml^2. Thus, the answer (3) is correct.