Problem 45 (Collision of Falling Bar)

We define each of the two constraint violations as the distance from the corresponding contact point to the bottom side of the bar:

$$c = \begin{bmatrix} y\cos\varphi - x\sin\varphi - d \\ y\cos\varphi - (x-s)\sin\varphi - d \end{bmatrix}.$$

Therefore, the Jacobian is given by

$$\mathbf{J}_\lambda = \frac{\partial c}{\partial q} = \begin{bmatrix} -\sin\varphi & \cos\varphi & -x\cos\varphi - y\sin\varphi \\ -\sin\varphi & \cos\varphi & -(x-s)\cos\varphi - y\sin\varphi \end{bmatrix}.$$

At the instant of collision $q^- = \begin{bmatrix} l^- & d & 0 \end{bmatrix}^T$ and we get

$$c^- = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{J}_\lambda^- = \begin{bmatrix} 0 & 1 & -l^- \\ 0 & 1 & s-l^- \end{bmatrix}, \quad \text{and} \quad \dot{c}^- = \mathbf{J}_\lambda^- \dot{q}^- = \begin{bmatrix} \dot{y}^- - l^-\dot{\varphi}^- \\ \dot{y}^- - (l^- - s)\dot{\varphi}^- \end{bmatrix}.$$

Assuming that the contact on the left was closed before the collision, the first element of $\dot{c}^-$ has to be zero. Therefore, $\dot{y}^- = l^-\dot{\varphi}^-$ and the vector $\dot{c}^-$ becomes

$$\dot{c}^- = \begin{bmatrix} 0 \\ s\dot{\varphi}^- \end{bmatrix}.$$

Because the contact on the right was closing just before the collision, the second element of $\dot{c}^-$ has to be negative, which leads to $\dot{\varphi}^- < 0$.

The contact impulses Λ are found as $\Lambda = -M_\lambda \dot{c}$ where

$$M_\lambda^{-1} = \mathbf{J}_\lambda M^{-1} \mathbf{J}_\lambda^T = \mathbf{J}_\lambda \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/\theta \end{bmatrix} \mathbf{J}_\lambda^T.$$

At the instant of collision, we get

$$M_\lambda^{-1} = \begin{bmatrix} \dfrac{1}{m} + \dfrac{(l^-)^2}{\theta} & \dfrac{1}{m} + \dfrac{l^-(l^- - s)}{\theta} \\ \dfrac{1}{m} + \dfrac{l^-(l^- - s)}{\theta} & \dfrac{1}{m} + \dfrac{(l^- - s)^2}{\theta} \end{bmatrix}$$

and the collision impulses

$$\Lambda = -M_\lambda \dot{c}^- = \frac{\dot{\varphi}^-}{s} \begin{bmatrix} m(l^-)^2 - ml^- s + \theta \\ -\left(m(l^-)^2 + \theta\right) \end{bmatrix}.$$

The left contact opens right after the collision when the corresponding impulse Λ_1 is negative, that is

$$\Lambda_1 = \frac{m(l^-)^2 - ml^- s + \theta}{s} \dot{\varphi}^- < 0.$$

Thus, we get the condition on s from the above:

$$m(l^-)^2 - ml^- s + \theta > 0$$

$$s < l^- + \frac{\theta}{ml^-}$$