due on Monday, 11 May 2020, 10am sharp

Probabilistic Machine Learning

Exercise Sheet #3

1. **EXAMple: Automated Trading** You have been hired to run a (terribly simplistic) hedge fund. You have a total amount of B to spend on two kinds of stock packets: oil futures and the stock of airlines. Over the past years, average returns on oil futures were 5% with a standard deviation of 8%, while airline stock has risen by 2% on average, with a standard deviation of 4%. But oil futures and airline stock are anticorrelated: If one rises, the other tends to fall. Their covariance is -4.5%. In other words, the joint distribution of their returns r_{oil} and r_{air} is approximately Gaussian,

$$p(r_{\text{oil}}, r_{\text{air}}) = \mathcal{N}\left[\begin{pmatrix} r_{\text{oil}} \\ r_{\text{air}} \end{pmatrix}; \begin{pmatrix} 0.05 \\ 0.02 \end{pmatrix}, \begin{pmatrix} 0.080 & -0.045 \\ -0.045 & 0.040 \end{pmatrix}\right] \tag{1}$$

   You can distribute your total budget B into oil (x) and airlines ($B - x$) into a position with return $R = x r_{\text{oil}} + (B - x) r_{\text{air}}$

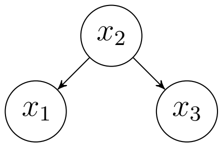
   (a) What is the marginal predictive distribution $p(R)$ under this model?

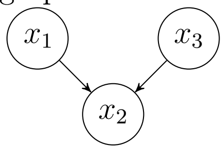
   (b) How would you choose the split x to maximize expected return $\mathbb{E}[R]$?

   (c) How would you choose x to minimize variance $\mathbb{E}[R^2] - \mathbb{E}[R]^2$?

   (d) For $B = 1$, which split x maximizes expected return if the expected variance of the portfolio is to be below 3%?

2. **Theory Question.** Consider the following two directed graphical models

   $\mathcal{H}_1$: 

   $\mathcal{H}_2$: 

   and the four matrices

$$A := \begin{bmatrix} 9 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 9 \end{bmatrix} \quad B := \begin{bmatrix} 8 & -3 & 1 \\ -3 & 9 & -3 \\ 1 & -3 & 8 \end{bmatrix} \quad C := \begin{bmatrix} 9 & 3 & 0 \\ 3 & 9 & 3 \\ 0 & 3 & 9 \end{bmatrix} \quad D := \begin{bmatrix} 9 & -3 & 0 \\ -3 & 10 & -3 \\ 0 & -3 & 9 \end{bmatrix}$$

   Assuming the variables x_1, x_2, x_3 have a joint Gaussian distribution $p(x_1, x_2, x_3) = \mathcal{N}(\boldsymbol{x}; \mu, \Sigma)$

   (a) which of the four matrices *could* be their *inverse* covariance (aka. precision) matrix Σ^{-1},

      i. if their joint distribution is described by $\mathcal{H}_1$?

      ii. if their joint distribution is described by $\mathcal{H}_2$?

   Now let the three variables y_1, y_2, y_3 have covariance matrix $\Sigma_{(3)}$ and inverse Covariance matrix $\Sigma_{(3)}^{-1}$

$$\Sigma_{(3)} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} \qquad \Sigma_{(3)}^{-1} = \begin{bmatrix} 1.5 & -1 & 0.5 \\ -1 & 2 & -1 \\ 0.5 & -1 & 1.5 \end{bmatrix}$$

   Consider just the bivariate subset y_1, y_2. Which of the following two statements about *their* covariance matrix $\Sigma_{(2)}$ and inverse covariance matrix $\Sigma_{(2)}^{-1}$ are true?

$$\text{(a) } \Sigma_{(2)} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \qquad \text{(b) } \Sigma_{(2)}^{-1} = \begin{bmatrix} 1.5 & -1 \\ -1 & 2 \end{bmatrix}$$

3. **Practical Question** This week you will complete the mini-project to build an autonomous agent that can play the pen-and-paper game *Battleships*. For more, refer to `Exercise_03.ipynb` which you find in the zipped folder `Exercise_03_battleships.zip`.