Probabilistic Machine Learning

Tutorial 1

Start: 10.15

Keep your solution open in the background

Filip de Roos

27 April 2020

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Faculty of Science
Department of Computer Science
Chair for the Methods of Machine Learning

Goals for tutorial/exercise sheet

Week 2

Discuss

- Tutorial practicalities
- Basic probability theory
- Deep learning has made other ML obsolete(?)

Exercise sheet

- EXAMple question
- Theory question
  - Solution
  - Interpretation/Visualization
- Practical: Deep learning for Regression

Tutorials over Zoom

and beyond

Communication (in a controlled manner)

- How to talk: Participants -> raise hand -> unmuted
- Feedback: Participants -> yes/no
- (Annotation): Raise hand -> Request to draw
- Chat: Ask questions / communicate

One more thing:

- Utilize the forum on Ilias for assistance

The rules of the game:

1. Sum Rule:

$$P(A) = P(A, B) + P(A, \neg B) \tag{S}$$

2. Product Rule:

$$P(A, B) = P(A \mid B)P(B) = P(B \mid A)P(A) \tag{P}$$

3. Bayes' Rule

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)} = \frac{P(A, B)}{P(A, B) + P(A, \neg B)} \tag{B}$$

Theory question

Example $P(B \mid A) \geq P(B)$

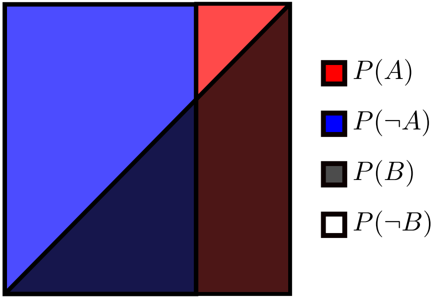

Figure: Graphical depiction of $P(B \mid A) \geq P(B)$

	$\neg A$	A	$\sum \rightarrow$
$\neg B$	8/18	1/18	1/2
B	4/18	5/18	**1/2**
$\sum \downarrow$	2/3	1/3	1

$$P(B \mid A) = \frac{P(B, A)}{P(A)} = \frac{5/18}{1/3} = \mathbf{5/6}$$

Theory question

Story for interpretation

$P(B \mid A) \geq P(B)$

- B: Student passes the course
- A: Student did all homework
- $P(B \mid A) \geq P(B)$: Student is **more** likely to pass the course if the student has **done** the homework

(S) + (P) + (B) + Assumption

Theory question

(a): $P(B \mid A) \geq P(B) \Rightarrow P(B \mid \neg A) \leq p(B)$

$P(B \mid \neg A) \leq p(B)$

Student is **less** likely to pass the course if the student did **not** do the homework

$$\begin{aligned} P(B \mid \neg A) &= \frac{P(\neg A, B)}{P(\neg A)} \\ &= \frac{P(B) - P(A, B)}{1 - P(A)} \\ &= P(B)\left(\frac{1 - P(A \mid B)}{1 - P(A)}\right) \\ &\leq P(B) \end{aligned}$$

$$\begin{aligned} P(A \mid B) = \frac{P(B \mid A)}{P(B)} P(A) &\geq P(A) \\ 1 - \underbrace{\frac{P(B \mid A)}{P(B)} P(A)}_{P(A|B)} &\leq 1 - P(A) \end{aligned}$$

Theory question

(b): $P(B \mid A) \geq P(B) \Rightarrow P(A \mid B) \geq p(A)$

$P(A \mid B) \geq p(A)$

Student is **more** likely to have done the homework if the student **passed** the course

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \geq \frac{P(B)}{P(B)}P(A) = P(A)$$

Theory question

(c): $P(B \mid A) \geq P(B) \Rightarrow P(A \mid \neg B) \leq P(A)$

$P(A \mid \neg B) \leq P(A)$

Student is **less** likely to have done the homework if the student did **not** pass the course
It is more likely that the student did not do the homework if he/she failed the course

$$
\begin{aligned}
P(A \mid \neg B) &= \frac{P(\neg B, A)}{P(\neg B)} \\
&= \frac{P(A) - P(B, A)}{P(\neg B)} \\
&= \frac{(1 - P(B \mid A))}{(1 - P(B))} P(A) \\
&\leq P(A)
\end{aligned}
$$

$$
\begin{aligned}
P(B \mid A) &\geq P(B) \\
1 - P(B \mid A) &\leq 1 - P(B)
\end{aligned}
$$

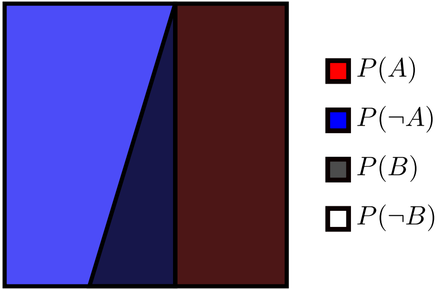

Figure: Example of $P(B \mid A) = 1$

	$\neg A$	A	$\sum \rightarrow$
$\neg B$	3/6	0	1/2
B	1/6	2/6	**1/2**
$\sum \downarrow$	2/3	1/3	1

$$P(B \mid A) = \frac{P(B, A)}{P(A)} = \frac{2/6}{1/3} = \mathbf{1}$$

Theory question

Narrative

$P(B \mid A) = 1$

- ~~B: Student passes the course~~
- ~~A: Student did the homework~~
- ~~$P(B \mid A) = 1$: Student **passes** the course if the student has **done** the homework~~

$P(B \mid A) = 1$

- B: Student has at least 1/3 of homework sufficient
- A: Student passes the course
- $P(B \mid A) = 1$: If the student passed the course he/she had at least 1/3 of the homework marked sufficient (with certainty)

Theory question

(d): $P(B \mid A) = 1 \Rightarrow P(\neg A \mid \neg B) = 1$

$P(\neg A \mid \neg B) = 1$

If the student did not get at least 1/3 sufficient, then the student did not pass the course (with certainty)

$$
\begin{aligned}
P(\neg A \mid \neg B) &= \frac{P(\neg A, \neg B)}{P(\neg B)} \\
&= \frac{P(\neg A, \neg B)}{P(A, \neg B) + P(\neg A, \neg B)} \\
&= \frac{P(\neg A, \neg B)}{\underbrace{P(\neg B \mid A)}_{0} P(A) + P(\neg A, \neg B)} \\
&= \frac{P(\neg A, \neg B)}{P(\neg A, \neg B)} = 1
\end{aligned}
$$

$$
\begin{aligned}
P(A) &= P(A, B) + P(A, \neg B) \\
&= \underbrace{(P(B \mid A) - P(\neg B \mid A))}_{=1} P(A) \\
&\Rightarrow P(\neg B \mid A) = 0
\end{aligned}
$$

Theory question

(e): $P(B \mid A) = 1 \Rightarrow P(B \mid \neg A) \leq P(B)$

$P(B \mid \neg A) \leq p(B)$

It is less likely that the student got 1/3 of homework sufficient if the student did not pass the course

from 2.a we have:

$$\begin{aligned} P(B \mid \neg A) &= \frac{P(B, \neg A)}{P(\neg A)} = ... \\ &= P(B)\left(\frac{1 - P(A \mid B)}{1 - P(A)}\right) \\ &= P(B)\left(\frac{1 - \frac{1}{P(B)}P(A)}{1 - P(A)}\right) \\ &\leq P(B) \end{aligned}$$

$$\begin{aligned} P(A \mid B) = \frac{P(B \mid A)}{P(B)}P(A) &\geq P(A) \\ \frac{1}{P(B)}P(A) &\geq P(A) \\ 1 - \underbrace{\frac{1}{P(B)}P(A)}_{P(A|B)} &\leq 1 - P(A) \end{aligned}$$

Theory question

(f): $P(B \mid A) = 1 \Rightarrow P(A \mid B) \geq P(A)$

$P(A \mid B) \geq P(A)$

It is more likely that the student passes the course if he/she got 1/3 of homework marked sufficient

$$\begin{aligned} P(A \mid B) &= \frac{P(B, A)}{P(B)} \\ &= \underbrace{\frac{P(B \mid A)}{P(B)}}_{\geq 1} P(A) \quad \text{in (b)} \\ &= \underbrace{\frac{1}{P(B)}}_{\geq 1} P(A) \\ &\geq P(A) \end{aligned}$$

EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Deep Learning