Team # 26718 Page 1 of 28

Team Control Number

2014 Mathematical Contest in Modeling (MCM) Summary Sheet

# The Keep-Right-Except-To-Pass Rule

#### **Abstract**

In this paper, we discuss and analyze lane changing characteristics of vehicle, lane changing motion control program and lane changing security features. Through online search resources for freeway traffic data, we establish the *overtaking models* under the keep-right rule and the keep-right rule respectively, which realize the minimum longitudinal safe distance. An application of *T-S fuzzy model in MATLAB SIMULINK* yields that the driving performance on the keep-right rule, which can effectively improve the liquidity and safety of traffic. Finally, *road intelligent system* can be carried out more safety for the overtaking operation.

First of all, applying the vehicle conversion coefficients, the number of vehicle data is converted to the equivalent standard cars on the highway, and the overtaking minimum safe distance model is built. The line chart between the accidents and equivalent standard cars is described in Figure 8. When the accident rate is lower than the equivalent standard cars, vehicle is driving safely. When the minimum safety distance of lane-changing is proportional to the limit speed, the performance of the keep-right rule is obtained. Also, the accuracy of the conclusion is verified by simulation.

Next, we apply *the VISSIM simulation software* to simulate the keep-left driving rule, and we establish the similar model. Computing the minimum safe distance for the keep-left rule, we draw a comparison chart with the minimum safety distance for the keep-right rule in Figure 13.

Finally, under the environment of SIMULINK in MATLAB, we establish *T-S fuzzy system model* and make the *intelligent control system graph* in Figure 14. Setting up the corresponding parameter, we derive that vehicle overtaking distance in intelligent control system is 13 meters, which is less than the minimum safety distance 17.7 meters in the above system. Therefore, the intelligent control system can better change vehicle overtaking the minimum safety distance, which can better ensure the safety of driving.

Team # 26718 Page 2 of 28

## **Contents**

Introduction	3
Assumptions	4
Symbols	4
Model	5
Overtaking model	5
Lane changing kinematics analysis	5
Vehicle lane change motion control solutions	6
Analysis of lane changing security features	7
Model assumptions	8
The minimum vertical safety distance model	9
Optimization Model	11
Model solution	14
Simulation and analysis of the results of real-life	15
Model test	17
Driving on the left side of building and solving of the model	18
Intelligent Transportation Systems Analysis	22
Intelligent Transportation System Definition	22
Intelligent transportation systems affected the results of the analysis before	22
Design of Intelligent Controller	23
Establish and analyze the results of the system in the Simulink environment	24
Evaluations and Solutions	25
Strengths	25
Weaknesses	26
Improvements	26
References	27

Team # 26718 Page 3 of 28

#### Introduction

The rules of worldwide vehicle traveling on the road can be divided into driving on the right and on the left, while 90 percent of the world's countries comply with the right driving rules. Right now, to develop a traveling freeway traffic rules requiring vehicles must drive on the right lane, unless the vehicle is overtaking. When overtaking, vehicle must be moved to the left lane, and return to the lane before after overtaking.

The subject asks us to solve this problem as steps as follows:

First, establish a mathematical model<sup>[1-4]</sup>, and do the performance analysis of this rule in the smooth and congestion flow of traffic from traffic flow, safety, low speed and other factors. Then, discuss whether or not this rule could effectively improve traffic flow. If could not improve, come up with a new scheme which can improve safety and traffic flow or other factors; analyze and discuss the possibility of the rule by changing the direction of travel and traffic rules could also be applied to driving on the left, or other conditions needed to added. Last, consider the same rules in the transportation control completely in intelligent systems. Think about to what extent would modify the results which be obtained before.

At the beginning, in order to analyze the performance of this rule, we divide this process into three phases, namely change lanes, beyond lanes, and and-road lanes. We take the success rate of overtake as a measure of safety performance, and the speed of safety performance also has some influence, traffic flow can be obtained from the traffic data, so we collect data, calculate traffic flow and safety performance at different stages of this rule. We also take an evaluation of the rules. Consider the following distance may affect the safety; we calculate the minimum safety distance<sup>[5-6]</sup> and ensure the safety distance to the vehicle is greater than the minimum safe distance. As an improvement of the rules, we improve the safety performance of the rule. After solving the model, we use simulation software for model simulation.

In the next step, we change direction of traffic, that is to say driving on the left<sup>[7-8]</sup>, using the data collected to calculate the minimum safety distance. We make a comparison of the minimum safety distance about driving a car on the left and right, and if the result is not very different, the driving rules can be considered that the right solution through a simple change of direction can be applied to the left with the rules, or else we make improvements to the program, making it suitable for driving left of the rules.

Finally, we reference a smart system, and create T-S fuzzy system model<sup>[9-13]</sup> for simulation of intelligent control system<sup>[14-25]</sup>. We make the results of the simulation compared with the results which without intelligent systems, and determine whether this intelligent systems change the results of the analysis before.

Team # 26718 Page 4 of 28

# **Assumptions**

In order to streamline our model, we have made several key assumptions:

- 1. The behavior of beyond one car occurs on a straight road, and the function of the car is good, it can meet traffic requirements.
- 2. Considering the vehicle which wants to go before one car needs to open the lane changing lights ahead. Therefore, setting the beyond car from opening lane changing lamps and starting beyond to changing lanes and its completed and the road course, in the original lane and passing lane beyond the car near the front and rear no vehicle traveling beyond, and its vehicles are rear-following able to maintain adequate security with its Following vehicle distance.
- 3. On the beyond road, other vehicles are doing normal road driving, after completing transcend, the car needs to return to the original lane and normally with Following vehicle.
- 4. Assume that the research on the number of lanes for bidirectional four lanes.
- 5. Assume that the vehicle in the lane changing process, transverse speed changes will not affect the change of longitudinal velocity.
- 6. Assume that the vehicle in the process of change lanes, lane changing vehicle with constant acceleration, other vehicles to keep constant motion.

# **Symbols**

Symbols	Explanation
S(0)	Initial distance between the two vehicles
S(t)	Distance between M and F cars
heta	Vehicle M formed with the horizontal line when changing lanes
W	Vehicle width
$V_{F(0)}$	Front of the vehicle lane F initial velocity
$a_{\scriptscriptstyle F}$	Front of the vehicle lane F initial velocity acceleration of the vehicle

Team # 26718 Page 5 of 28

$V_{M(0)}$	M's initial velocity
$a_{\scriptscriptstyle M}$	M's initial acceleration of the vehicle
x(t)	Longitudinal displacement
$y(t)$ , $v_y(t)$	Vehicle speed and lateral displacement
$v_x(t)$	Vehicle speed
$S_{\scriptscriptstyle M}$	The vertical distance of M car traveled
$S_{MSD}(0)$	Rear of the vehicle M and L on the target lane minimum vertical safety distance
$d_s$	Between vehicles in the same lane as its front
$d_{\scriptscriptstyle d}$	Between the front of the vehicle and its overtaking lane
$t_{r}$	Brake reaction time for the driver and co-ordination of time
$t_d$	Deceleration of growth over time
$a_{Y_0}^d$ , $a_{Y_1}^d$	Maximum deceleration of the vehicle $Y_0$ , $Y_1$

#### Model

# Overtaking model

# Lane changing kinematics analysis

Under the different traffic density condition, the transport state of the vehicle is generally not same. For example, in the low-density traffic flow state, running state of the vehicle is freedom; in the medium-density traffic flow state, the vehicle is traveling with the front vehicle, or the vehicle is overtaking state, and the tansport freedom of the vehicle is subject to the certain restrictions; in high-density state, the vehicle is always traveling with the following state, which is unable to overtake. Thus, only in the medium-density state, the possibility of changing lane exists among the vehicles. According to lane changing needs, lane changing behavior can be changed into two cases: a selective channel (discretionary lane changing) and mandatory lane change (mandatory lane changing)

The main motivates behavior change lanes selectivity is: to improve the vehicle speed, beyond the back lane after overtaking slow-moving vehicles,

Team # 26718 Page 6 of 28

and so on. In the process of selective change lanes, the driver first running speed based on the current vehicle lane driveway and objectives, to determine whether to change the lane; then observed before and after the target lane and the lane where the vehicle is running, the estimated target lane acceptable gap size, to determine whether to change lanes to meet the security requirements; If met, the vehicle change lanes the contrary, then give up the lane change.

Judgment can safely change lanes in the case, and give full consideration to vehicle safety with Following vehicle distance beyond the time, the whole process is divided into the overtaking lane changing, passing, and the road in three phases (Figure 1), and add a different lane the rate-limiting conditions, beyond the car as much as possible within a period of transcendence beyond the front of the vehicle as the goal, combined with the car following the process of moving a safe distance and overtaking cars and the process beyond the relationship between being beyond the car trip, the establishment overtaking a new model for providing vehicles and auxiliary judgment when overtaking and reduce traffic accidents caused by improper overtaking frequency of occurrence.

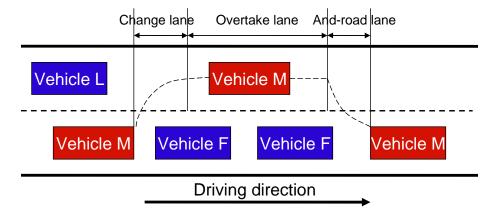


Fig.1 Three phase, of overtaking process

## Vehicle lane change motion control solutions

Throughout the process of overtaking lane changing, passing, and track three stages, will be conditions beyond the limit beyond the car when driving into internal and external too. An internal condition mainly refers to: Vehicle dimensions, maximum speed, maximum, plus or minus speed; external conditions mainly refer to: road terrain conditions, each lane speed limit, and vehicle safety with Following vehicle distance. Must meet the internal conditions and external conditions beyond the vehicle driving, otherwise, it will exist beyond the great traveling security risk.

Going beyond the vehicle in the driving, in the corresponding three stages of overtaking conditions must meet the requirements, control process is shown in Figure 2.

Team # 26718 Page 7 of 28

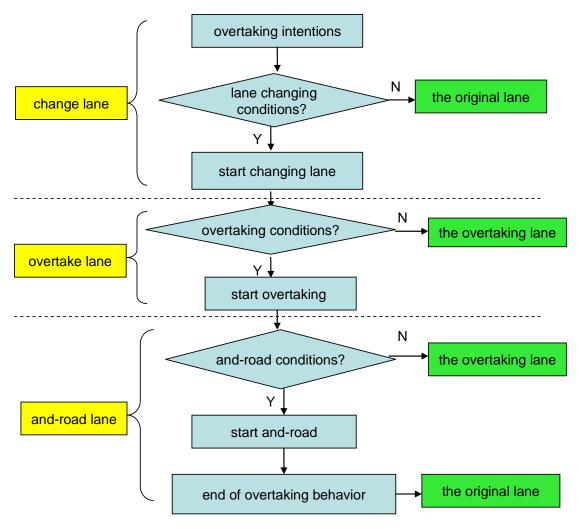


Fig.2 Control plan, of overtaking process

## Analysis of lane changing security features

In the process, change lanes, the driver is necessary to observe the state of the lane around the vehicle, but also to observe the adjacent lane on the target vehicle status, driving operation also involves controlling the angle of the vehicle longitudinal and lateral angles, so the vehicle's lane change behavior more difficult.

Vehicles traveling, it will affect the vehicle near some impact from the nearest vehicle maximum. We create a simple lane changing environment, set the vehicle (vehicle lane changing) as M cars in the same lane, which is located in front of the vehicle, respectively F cars on the target lane, the vehicle was located behind the L car this will be the car 3 car safety lane change have the most direct impact, but because of the lane between F and M cars are cars car following relations and security relations between them controlled by the F car, so M cars do not need to be considered. This article from the vehicle lane changing specific movement and safety, we study the

Team # 26718 Page 8 of 28

impact on the car for 2 cars on the road except F, draws a minimum vertical safety lane change distance.

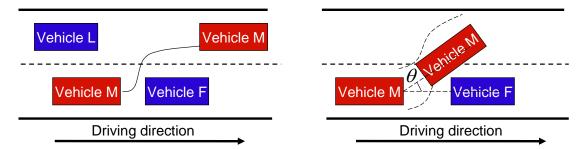


Fig. 3 Vehicle lane-changing environment model

Fig. 4 Vehicle lane-changing position

According to the vehicle shown in dashed lines in Figure 3, in the form of lane changing trajectory and collide we can see, when the M car change lanes, its main vehicle collision near the following situations: In M vehicle acceleration lane change process, and if it F longitudinal distance between the front of the vehicle does not meet the requirements of the minimum safety distance, the car will be F oblique collision; when the M car change lanes to reach the target lane, if M longitudinal distance between the vehicle and the vehicle does not meet the minimum L requires a safe distance, then L and M cars will occur laterally car scratch and rear-end collisions. So long as effective to avoid these types of accidents caused by the overtaking, you can draw a clear driving rules right performance.

# Model assumptions

Shown in Figure 4, the vehicle lane change during the positional relationship diagram, set the initial time of the vehicle lane change is  $t_0$ , the initial distance between the two vehicles is S(0), M car readies to change lanes, after a moment of t time, let S(t) is the distance between M and F cars, the angle  $\theta$  is the vehicle M formed with the horizontal line when changing lanes.

Following vehicle accordance with the minimum safety distance formula, the minimum safe distance S(t) can sure that changing lanes are available, select M car on the left front of the P point as a reference point, it can be expressed as:

$$S(t) = \int_{0}^{t} \int_{0}^{\tau} (a_F - a_M) dt d\tau + (v_{F(0)} - v_{M(0)}) t + S(0) - W \cdot \sin \theta, \tag{1}$$

where the angle  $\theta$  is the vehicle M formed with the horizontal line when

Team # 26718 Page 9 of 28

changing lanes; W is the vehicle width;  $v_{F(0)}$ ,  $a_F$ , respectively, of the front of the vehicle lane F initial velocity and acceleration of the vehicle;  $v_{M(0)}$ ,  $a_M$ , for the M's initial velocity and acceleration of the vehicle.

Therefore, to ensure that no vehicle collision occurs after time t, we must ensure that the following holds:

$$S(0) = \max \left\{ \int_{0}^{t} \int_{0}^{\tau} (a_F - a_M) dt d\tau + (v_{F(0)} - v_{M(0)}) t + S(0) - W \cdot \sin \theta \right\}, \tag{2}$$

$$\tan\left(\theta\left(t\right)\right) = \frac{\partial y(t)}{\partial x(t)} = \frac{\partial y(t)/\partial t}{\partial x(t)/\partial t} = \frac{v_x\left(t\right)}{v_y\left(t\right)},\tag{3}$$

where x(t),  $v_x(t)$  respectively denote the longitudinal displacement and the vehicle speed; y(t),  $v_y(t)$  respectively represent of the vehicle speed and lateral displacement.

#### The minimum vertical safety distance model

(1). The minimum vertical safety distances between M and L cars.

When showed in Figure 5, M vehicles change lanes, and after the collision in the form of the trajectory of the target line between L figure cars, this time may occur between the two vehicles have oblique collision and rear-end collisions.

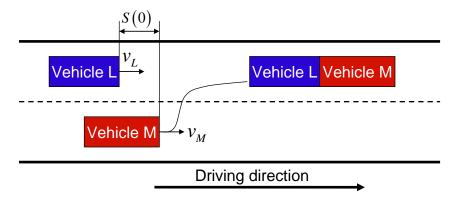


Fig.5 Position between vehicle M and L

When M car changes lanes to ensure that no collision occurs with the L car, you should meet the following conditions:

$$S(0) \ge S_L - S_M + L + W \sin \theta, \tag{4}$$

where  $t_a$  is M complete acceleration time;  $S_L$  is the time  $t_a$  period L longitudinal distance traveled by car; within hours M longitudinal distance traveled by car; L is a car length, usually taken 5 meter; W is the vehicle width,

Team # 26718 Page 10 of 28

usually take 3 meter;  $\theta$  as of the vehicle change lanes and lane M as a horizontal angle. Vehicles with lower speed than the high-speed lane change into the lane, usually expect to get a higher speed. In order to change lanes safety, vehicle acceleration lane change is completed, the speed should be consistent with the speed of the target lane, if this time the car is not on the target lane and the car collision, then the process is not behind with following vehicle collision occurs.

By the analysis of knowledge, accelerate the speed of the car  $v_L$  at the end goal should be to speed traffic lane, so the car's acceleration time is:  $t_a = (v_L - v_M)/a_M$ ,  $a_M$  is the car change lanes taken acceleration lane changing acceleration time is usually  $3 \sim 5$ s.

In the time of  $t_a$ , the vehicle M is a longitudinal distance traveled:

$$S_M = \frac{v_L^2 - v_M^2}{2a_M} = \frac{v_L + v_M}{2} \cdot \frac{v_L - v_M}{a_M} = \frac{v_L + v_M}{2} \cdot t_a. \tag{5}$$

Assume that a reasonable knowledge of the front, in addition to changing road of the vehicle M, the rest of the vehicles are kept uniform motion state, so the car on the target lane L vertical distance traveled:  $S_L = v_L t_a$ .

So the rear of the vehicle M and L on the target lane minimum vertical safety distance:

$$S_{MSD}(0) = v_L t_a - \frac{v_L + v_M}{2} \cdot t_a + L + W \sin \theta = \frac{(v_L - v_M)^2}{2a_M} + L + W \sin \theta.$$
 (6)

(2). Minimum vertical safety distance M between the vehicle and the F car.

When M car changes lanes, the line came up with the trajectory of his car between F shown in Figure 6.

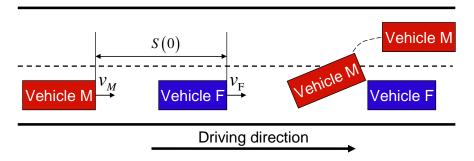


Fig.6 Position between vehicle M and F

Vehicle M collision at this time and the vehicle F is oblique collision may occur, the most likely figure is the collision point C point, if a collision does not occur at the point C, the two vehicles will no longer collide. To ensure that vehicle M and F car collision should satisfy the following conditions:

$$S(0) \ge S_M - S_F + L + W \sin \theta, \tag{7}$$

Team # 26718 Page 11 of 28

where  $S_F$  is F car within  $t_c$  hour's longitudinal distance traveled;  $S_M$  is within this time period the vehicle M vertical distance traveled.

Assume M car arrival point C at the time of  $(t_0 + t_c)$ , therefore, in this period of time, the vertical distance of M car traveled is:

$$S_M = v_M t_c + \frac{1}{2} a_M t_c^2. {8}$$

Vehicle F in front of the car M remains uniform motion, its vertical distance traveled:  $S_F = v_F t_c$ . So, the safe minimum vertical distance between vehicles M cars and F as follows:

$$S_{MSD}(0) = v_M t_c + \frac{1}{2} a_M t_c^2 - v_F t_c + L + W \sin \theta$$

$$= (v_M - v_F) t_c + \frac{1}{2} a_M t_c^2 + L + W \sin \theta.$$
(9)

## **Optimization Model**

Different lane car following a safe distance is when carried beyond the car change lanes when traveling between the original lane and passing lane in front of the vehicle and its angle of collision does not occur, the minimum safe distance from the side scraping with following vehicle.

When the car carrying transcend beyond traveling with following vehicle safe distance between vehicles in the same lane as its front:

$$d_{s} = L_{sa} + L_{ci} / 2\cos\theta - L_{ci} / 2. \tag{10}$$

Similarly, when the car carrying transcends beyond traveling with Following vehicle in a safe distance between the front of the vehicle and its overtaking lane is:

$$d_d = L_{sa} + 2(L_{ci}/2\cos\theta - L_{ci}/2), \tag{11}$$

where  $L_{sa}$  is the vehicle in three different situations safe following distance, namely with a utopia speed greater than, equal, less than the vehicle speed before.

To transcend beyond of the vehicle in a period beyond the front of the vehicle as much as the target, taking into account the relationship between the car and the vehicle is exceeded beyond the car travel between the car-driving safety distance and overtaking process, satisfy the constraints under the conditions of the given conditions beyond the vehicle to meet beyond the minimum travel speed and travel time accordingly beyond.

Constraints are three stages as follows:

Team # 26718 Page 12 of 28

(1). Changing channel conditions: beyond the car speed to meet the ultra-lane speed limit conditions; beyond the distance of the front and rear vehicles and ultra-lane vehicles meet safety requirements to prevent the vehicle side scraping; around the car with the original lane beyond the distance of the vehicle to meet the security requirements to prevent angle touch.

$$\begin{cases}
v < \overline{V_{j}}, \\
\int_{0}^{t_{c}} \left(V_{Y_{1}}^{0} + a_{Y_{1}}^{a} t\right) dt - \int_{0}^{t_{c}} \left(V_{Y_{0}}^{0} + a_{Y_{0}}^{a} t\right) dt \ge d_{s}, \\
\int_{0}^{t_{c}} \left(V_{C_{1}}^{0} + a_{C_{1}}^{a} t\right) dt - \int_{0}^{t_{c}} \left(V_{Y_{0}}^{0} + a_{Y_{0}}^{a} t\right) dt \ge d_{d},
\end{cases} (12)$$

where v is vehicle traveling beyond the speed;  $\overline{V_j}$  stands for j-lane speed limit; t represents time;  $t_c$  represents lane change time (lane change lights flashing long);  $a_{\rm Y_1}^a$  denotes in the original lane overtaking the first car of the maximum acceleration of the vehicle;  $a_{\rm Y_0}^a$  is beyond the maximum vehicle acceleration;  $V_{\rm C_1}^0$  denotes in passing lanes beyond the initial speed of the vehicle in front of the first car;  $a_{\rm C_1}^a$  represents in the ultra-lane overtaking the first car of the maximum acceleration of the vehicle.

(2). Beyond conditions: beyond the car speed to meet the ultra-lane speed limit conditions; around beyond cars and ultra-lane of the vehicle to keep a safe distance with following vehicle; beyond the period of the car change lanes in the original lane of travel is greater than the first is its transcendence travel and related security and car spacing distances with following vehicle's and I vehicle.

$$\begin{cases}
v < \overline{V_{j}}, \\
\int_{0}^{t_{c}+t_{0}} \left(V_{C_{1}}^{t_{c}} + a_{C_{1}}^{a}t\right) dt - \int_{0}^{t_{c}+t_{0}} \left(V_{Y_{0}}^{t_{c}} + a_{Y_{0}}^{a}t\right) dt \ge d_{s}, \\
\int_{0}^{t_{c}+t_{0}} \left(V_{Y_{0}}^{t_{c}} + a_{Y_{0}}^{a}t\right) dt < \max\left\{S_{Y_{i}} + (i+1)d_{s} + \sum_{i=1}^{n} L_{c_{i}}\right\},
\end{cases} (13)$$

where  $t_0$  represents beyond the stage long; i said i was beyond the original lane car beyond the first motor vehicles, and  $i = 1,2,3...; v_{c_1}^{t_c}$  said  $t_c$  moments C1 vehicle speed;  $v_{Y_0}^{t_c}$  said time  $t_c$  beyond the speed of vehicles;  $S_{Y_i}$  said first i cars were the original lane overtaking is beyond the stage at the distance traveled;  $L_{C_i}$  indicates the length of the first i motor vehicle.

Team # 26718 Page 13 of 28

(3). And-road Conditions: Conditions beyond the original lane speed limit vehicle speed to meet; around beyond cars and ultra-lane vehicles meet the security needs of the distance, angle collision prevention; around the car with the original lane beyond the distance of the vehicle to meet the security requirements, to prevent the vehicle side scratch.

$$\begin{cases}
v < \overline{V_{j}}, \\
\int_{t_{0}+t_{c}}^{t_{c}+t_{0}+t_{a}} \left(V_{C_{1}}^{t_{c}+t_{0}} + a_{C_{1}}^{a}t\right) dt - \int_{t_{0}+t_{c}}^{t_{c}+t_{0}+t_{a}} \left(V_{Y_{0}}^{t_{c}+t_{0}} + a_{Y_{0}}^{a}t\right) dt \ge d_{s}, \\
\int_{t_{c}+t_{0}+t_{a}}^{t_{c}+t_{0}+t_{a}} \left(V_{Y_{(i+1)}}^{t_{c}+t_{0}} + a_{Y_{(i+1)}}^{a}t\right) dt - \int_{t_{0}+t_{c}}^{t_{c}+t_{0}+t_{a}} \left(V_{Y_{0}}^{t_{c}+t_{0}} + a_{Y_{0}}^{a}t\right) dt \ge d_{d},
\end{cases} (14)$$

where  $t_a$  is the long and-road stage;  $V_{C_1}^{t_c+t_0}$  denotes  $(t_c+t_0)$  moments C1 vehicle speed;  $V_{Y_0}^{t_c+t_0}$  represents time  $(t_c+t_0)$  beyond the speed of vehicles;  $V_{Y_{(i+1)}}^{t_c+t_0}$  denotes the first time the original lane speed car (the first one is not beyond the car);  $a_{Y_{(i+1)}}^a$  is the original car's maximum acceleration lane section .In summary, under different conditions of speed overtaking the model can be expressed as:

$$\min Z = \max \left\{ 0, S_{Y_0} - \max \left[ S_{Y_i} + (i+1)d_s + \sum_{i=1}^n L_{C_i} \right] \right\}.$$
 (15)

$$\begin{cases} v < \overline{V_{j}}, \\ \int_{0}^{t_{c}} \left(V_{Y_{1}}^{0} - V_{Y_{0}}^{0} + \left(a_{Y_{1}}^{a} - a_{Y_{0}}^{a}\right)t\right) dt \ge d_{s}, \\ \int_{0}^{t_{c}} \left(V_{C_{1}}^{0} - V_{Y_{0}}^{0} + \left(a_{C_{1}}^{a} - a_{Y_{0}}^{a}\right)t\right) dt \ge d_{d}, \\ \int_{t_{c}}^{t_{c}+t_{0}} \left(V_{C_{1}}^{t_{c}} - V_{Y_{0}}^{t_{c}} + \left(a_{C_{1}}^{a} - a_{Y_{0}}^{a}\right)t\right) dt \ge d_{s}, \\ \int_{t_{c}}^{t_{c}+t_{0}} \left(V_{Y_{0}}^{t_{c}} + a_{Y_{0}}^{a}t\right) dt < \max\left\{S_{Y_{i}} + \left(i+1\right)d_{s} + \sum_{i=1}^{n} L_{C_{i}}\right\}, \\ \int_{t_{c}+t_{0}}^{t_{c}+t_{0}+t} \left(V_{C_{1}}^{t_{c}+t_{0}} - V_{Y_{0}}^{t_{c}+t_{0}} + \left(a_{C_{1}}^{a} - a_{Y_{0}}^{a}\right)t\right) dt \ge d_{s}, \\ \int_{t_{c}+t_{0}}^{t_{c}+t_{0}+t} \left(V_{Y_{(i+1)}}^{t_{c}+t_{0}} - V_{Y_{0}}^{t_{c}+t_{0}} + \left(a_{Y_{(i+1)}}^{a} - a_{Y_{0}}^{a}\right)t\right) dt \ge d_{d}, \\ i = 1, 2, 3, ..., j = 1, 2, 3, ..., \end{cases}$$

where  $S_{Y_0}$  denotes distance traveled beyond the car.

Team # 26718 Page 14 of 28

#### Model solution

On certain roads, from time to time by the statistics between each time period by the number of vehicles of various models of this road and through the vehicle conversion factor (see Table 1), the types of vehicles converted into equivalent standard car then aggregated results in Table 2 below.

Table 1 Various cars represent the reduction factor of model and the equivalent standard car number

The car type	The car	Midsized cars	Small bus	The big bus
Coefficient of vehicle car	1	1.5	2	3

Table 2 Every 5 minutes interval statistics on the number of vehicles

	The car	Midsized cars	Small bus	The big bus	Summary	Traffic flow	The accident probability	
10:30-10:35	12	0	1	1	17.0	204	14.3%	
10:35-10:40	3	0	1	1	8.0	96	40.0%	
10:40-10:45	10	0	2	4	26.0	312	37.5%	
10:45-10:50	12	1	0	1	16.5	198	7.1%	
10:50-10:55	8	0	1	0	10.0	120	11.1%	
10:55-11:00	3	0	1	0	5.0	60	25.0%	
11:00-11:05	9	1	0	1	13.5	162	9.1%	
11:05-11:10	10	0	3	3	25.0	300	37.5%	
11:10-11:15	5	0	1	2	13.0	156	37.5%	
11:15-11:20	5	0	0	4	17.0	204	44.4%	
11:20-11:25	3	2	1	2	14.0	168	37.5%	
11:25-11:30	5	0	2	3	18.0	216	50.0%	
11:30-11:35	9	1	1	3	21.5	258	28.6%	
11:35-11:40	9	1	0	0	10.5	126	0.0%	
11:40-11:45	10	0	1	0	12.0	144	9.1%	

From Table 2, the number of vehicles in four models can be calculated equivalent standard vehicles and traffic, according to a large proportion of the total vehicle can obtain the probability of a vehicle the size of each interval of the accident, which can determine whether the road traffic safety. The above table will be drawn into a graph (see Figure 7); we can more clearly reflect the changes in traffic flow and vehicle standards equivalent. Can be derived from the figure equivalent standard value of the car is always kept at a critical point, and a small variation of the curve and the vehicle is very close to the standard vehicle can be obtained depends on the number of equivalents of the vehicle by a small vehicle.

Team # 26718 Page 15 of 28

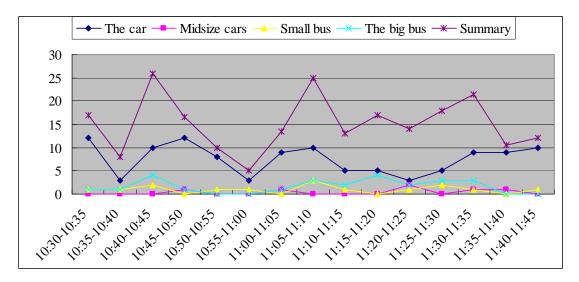


Fig.7 A highway through the number of vehicles

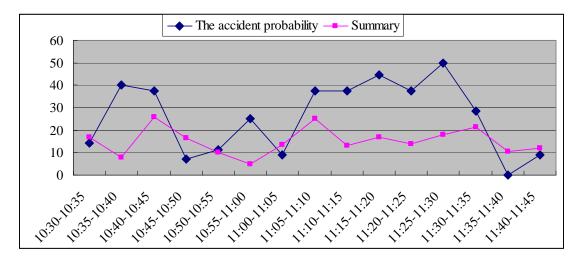


Fig.8 Accidents and standard car equivalent line chart

Figure 8 shows the comparison of accident rates and equivalent standard car, it can be seen from the figure when the accident rate is lower than the standard car equivalent of a line chart, you can consider traffic flow and traffic safety.

# Simulation and analysis of the results of real-life

The time  $t_r$  is the brake reaction time for the driver and co-ordination of time, the general value of about  $0.8 \sim 1.0$ s;  $t_d$  is the deceleration of growth over time, the general value of about  $0.1 \sim 0.2$ s;  $a_{\gamma_0}^d \cdot a_{\gamma_1}^d$  are the maximum deceleration of the vehicle, the general value of approximately  $6.86 \sim 7.84 \, m/s^2$ ; other calculation parameters as shown in Table 3, according to equation (10) to (11) for calculating the simulation, where in the length of the vehicle are taken  $4.2 \, \text{m}$ ;  $\theta$  takes  $10^\circ$ , then the vehicle or changing lanes, and

Team # 26718 Page 16 of 28

road conditions under the previous calculation results shown in the following table from the car.

Table 3 Safe distance when the vehicle lane changing

Conditions	Vehicle	Rate-limiting 60km/h		Rate-limiting 80km/h		Rate-limiting 100km/h	
$V_{Y_0}^0 < V_{Y_1}^0$	Prior to the car $V_{Y_1}^0$	50	50	60	60	80	80
	Following the car $V_{\scriptscriptstyle Y_0}^{\scriptscriptstyle 0}$	55	60	70	80	85	90
	Following distance is $d_s$ (m)	17.77	20.16	26.16	33.86	38.35	41.4
	Following distance is $d_d$ (m)	17.81	20.2	26.2	33.9	37.8	40.9
$V_{Y_0}^0=V_{Y_1}^0$	Prior to the car $V_{Y_1}^0$	50	60	70	80	90	100
	Following the car $\mathit{V}_{\mathit{Y}_{0}}^{0}$	50	60	70	80	90	100
	Following distance is $d_s$ (m)	11.3	15.9	16.06	18.3	21.5	23.8
	Following distance is $d_d$ (m)	11.34	15.94	16.1	18.34	21.7	23.94
$V_{Y_0}^0 > V_{Y_1}^0$	Prior to the car $V_{Y_1}^0$	50	50	70	70	90	90
	Following the car $\mathit{V}_{\mathit{Y}_{0}}^{0}$	40	45	50	60	70	80
	Following distance is $d_s$ (m)	7.88	10.61	6.52	12.78	5.2	15.8
	Following distance is $d_d$ (m)	7.92	10.65	6.56	12.82	5.6	15.7

Table 3 can be obtained on the analysis: the vehicle speed is greater than the current vehicle speed with following vehicle, along with the increase of the speed limit change lanes gradually increases with distance following vehicle and following vehicle Road, with gradually increasing distance, lane change distance with following vehicle and road-following and lane changing from a similar distance with following vehicle total is less than the distance and the road now; prior to analysis to vehicle speed equal to the speed obtained with a utopia vehicle speed is greater than the forward speed of the car with the same result following vehicle; current to the vehicle speed a utopia with less than when the speed change lanes with following vehicle distance and distance does not vary with road speed increases.

Team # 26718 Page 17 of 28

Table 3 derives from the consolidated results of Table 1 and 2. Figure 7 and Figure 8 are compared under the influence between traffic flow and safety. We find the relationship between them: the greater the traffic flow, the number of vehicles traveling on the highway contained when the ratio of large vehicle accident that occurred less than 10%. We consider the highway is safety. About the requirements for highway speed limit, we find that when the speed limit more stringent requirements that the speed limit is smaller, overtaking a vehicle overtaking the required safety distance is smaller, the easier it is to overtake the vehicle, the vehicle that is more secure. When the highway speed limit is too high, the greater the required safety distance, so that the vehicle overtaking is not easy to achieve, if hard to overtake, the greater the probability of traffic accidents, the more unsafe vehicles.

#### Model test

In considering only one-way dual carriageway freeway, assuming that the highway through models for small cars, medium-sized vehicles, trucks and large cars, using VISSIM software to the above rule (right road rules) and model simulation, vehicle flow animation, shown in the following figure is a screen shot:

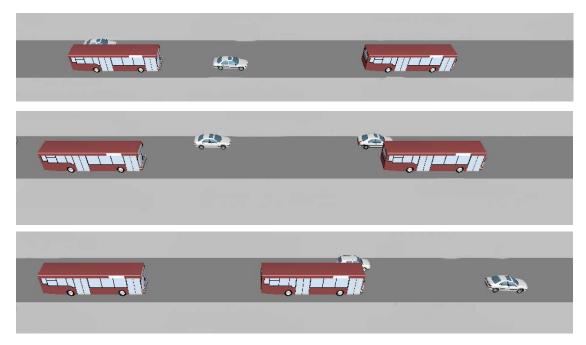


Fig.9 Traffic flow simulation software is VISSIM animation Screenshot

Screenshot from the simulation, the figure dark gray for the highway and two-lane one-way, based on the right road rules, all vehicles on the right lane. The actual situation shows that the desired speed is greater than small cars and large trucks car, and concluded that the above model: overtaking is less than expected because the actual speed of traffic speed, so small cars

Team # 26718 Page 18 of 28

overtaking at most, the figure that is small cars overtaking process screenshots.

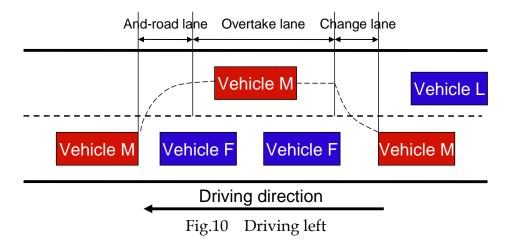
The first picture in front a small car with the car driving in the right lane, then a small car is overtaking; second graph after two small cars are overtaking, but after a small car is a continuous overtaking. Section three figures in a small car after overtaking before a successful return to the right lane, after a small car is still going to overtake.

Overtaking model, only appears when the roads open overtaking phenomenon, it is a case of road traffic simulation smooth. And because the accident occurred on the highway is a small probability event, so do not consider when doing simulation. By the simulation model, we verify the speed of the vehicle on the right road performance difference between the rules of traffic flow and safety.

The results obtain by the model and the simulation results analysis, the right road rules to improve traffic circulation, but not a good safety performance and improve results. In order to secure the performance of the traffic has increased, improving response to the rules.

Overtaking model to analyze the success is important to determine the performance of safety performance, while overtaking another success with the lateral distance between the vehicle and the vertical distance when overtaking, and therefore can add a lateral distance between the vehicle and the vertical distance to this rule, whether the vehicle can be used to constrain overtaking. Model has been obtained to prohibit overtaking a vehicle overtaking a vehicle when the vehicle in front when the minimum safe distance between vehicles as shown in Table 3, the provisions of the previous car-truck vehicle is less than the distance from the table, add this rule to the rules will improve traffic security.

# Driving on the left side of building and solving of the model



Team # 26718 Page 19 of 28

The Figure 10 shows a schematic diagram of a vehicle overtaking drive on the left, to the left and to the right to drive a car and driving a car is no essential difference, but due to the long history of evolution, different regions have developed different traffic rules appear so by today left and right traveling with the rules; traffic safety and order, different driving rules corresponding to the position of the car cabin is also different. In order to sure that the driver can see the opposite case while driving on the right side of the rule, the left cab design on the left, similarly, in driving a car on the left rule, the cab on the right design.

After checking the data shows that the main advantages of driving on the left is a human instinct to avoid its evils, because in an emergency situation, the human instinct is to turn the steering wheel to the left to protect the position of the heart. Turn left and turn right to the action than the much more quickly.

The main advantage is that drivers can drive on the right with his left hand to keep control of the steering wheel, while using the right hand to complete the shift, the operation of the instrument panel and other complex actions. This is for people who are right-handed favorable, while left-handed people is unfavorable.

Data shows: about 90% of the world's countries to implement the right line system, 10% of the national implementation of the left line system; the world's population, about 80% of the population right-handed (right-handed), only 20% of the population usual the left hand (i. e. left-handed). This is why most of the country to implement a vehicle right line system.

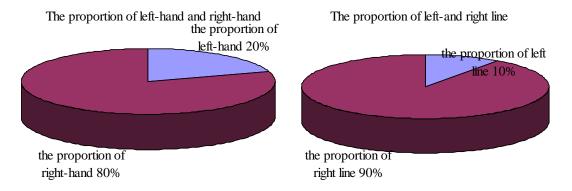


Fig. 11 Left-handed and right-handed global left-right line and pie charts

Although the vehicles drive on the left and right with no essential difference, but it is not able to rule in the right line solutions by simply changing the direction of the rules applied to the left line. We should take into account that 80% of people worldwide right-handed, so that when the right people are running 80 percent favorable, only 20% of people are negative, most people can easily manipulate the joystick on the car, in the skilled operator can reduce the response time, improved to some extent in some emergency situations when needed shift safety performance. Conversely, if driving on the left, then 80% of people worldwide are unfavorable, only 20

Team # 26718 Page 20 of 28

percent favorable, when most people are not easy to manipulate the joystick to the car, not in some emergency situations when you need to shift skilled operation, increasing the reaction time, reducing the safety performance to some extent. Considering the advantages and disadvantages of driving on the left, we can get the solution that the left safety performance is lower than the right.

In order to make right with the program rules apply to left-rule, and safe performance of the same, you can drive on the left in the rule to reduce driving speed (reducing the highway speed limit), to increase the safety distance when overtaking. Left in the car line, although the reaction time is relatively long, but if we lower the speed, in case of an emergency response will vary from reduced, thereby improving safety performance. Increase overtaking at a safe distance to avoid increasing the reaction distance is also caused by traffic accidents, thus improving safety performance.

And driving on the right road in the same situation, speed parameters and minimum safety distance drive on the left in the rule has been modified in accordance with the rules established by the simulation right in changing the direction of travel only under other conditions remain unchanged , on the left with the rules simulation, simulation results screenshot below.

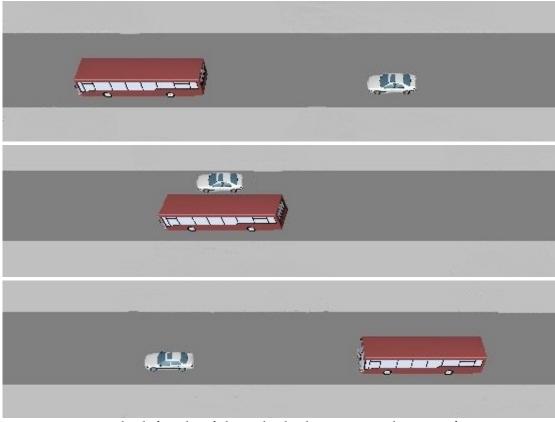


Fig. 12 The left side of the vehicle driving simulation software VISSIM flow animation Screenshot

By setting the parameters of the simulation software VISSIM export traffic data, use the same method with the rules and to the right, come Team # 26718 Page 21 of 28

forward with the vehicle speed is greater than and equal speed in both cases under the a utopia overtaking the minimum safe distance, on the right with the minimum safe distance driving rules are compared. The minimum safety distance of the overtaking left lane shows in the following table.

Conditions	Vehicles	Rate-limiting 80km/h		Rate-limiting 100km/h	
$V_{Y_0}^0 < V_{Y_1}^0$	Forward car $V_{Y_{\rm l}}^0$	60	60	80	80
	Following car $V_{Y_0}^0$	70	80	85	90
	Following distance $d_s$ (m)	26.8	36.8	39.35	43.7
	Following distance $d_d$ (m)	27.6	36.1	39.0	42.5
	Forward car $V_{Y_{\rm l}}^0$	70	80	90	100
$V_{Y_0}^0 = V_{Y_1}^0$	Following $\operatorname{car} V_{Y_0}^0$	70	80	90	100
	Following distance $d_s$ (m)	18.4	21.7	23.5	25.8
	Following distance $d_d$ (m)	18.8	21.1	23.9	25.2

Table 4 Vehicles drive on the left to change and road safety distance calculation table

From the above table can be drawn, regardless of the vehicle speed is greater than the forward speed of the car or the car - the car speed is equal to the forward speed with autopia, the car following distances are as the speed limit increases, and this conclusion by right overtaking vehicles in similar minimum safe distance.

Through a safe distance drive on the left and right driving safety distance for drawing comparison can more clearly see their similarity. Compare line chart below.

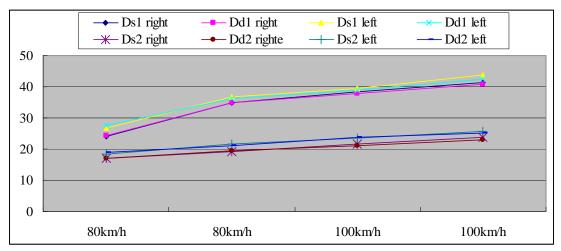


Fig. 13 Left lane to overtake a safe distance comparison chart

By comparing the figure before the vehicle speed is greater than the safe distance curve trend with four speed overtaking a utopia, draw their values although there is a gap, but the gap is small, the value is very close, and the Team # 26718 Page 22 of 28

trend is the same four curves; then by comparison with equal speed to a safe distance from the car trend curves of the four speed overtaking a utopia yield the same results. Thus it can be drawn on the left driving the vehicle in the country; we can drive on the left to solve the problem by simply changing the rules of driving directions.

## **Intelligent Transportation Systems Analysis**

#### **Intelligent Transportation System Definition**

Intelligent Transportation Systems (Intelligent Transport System, referred to as ITS) is an advanced information technology, data communications transmission technology, electronic sensor technology, electronic control technology and computer processing techniques used to effectively integrate the entire transportation management system, and to establish within a large-scale, full-functioning, real-time, accurate, efficient and integrated transport management system.

# Intelligent transportation systems affected the results of the analysis before

Rely on human judgment compliance: because people widespread luck, often choose when overtaking lane in the case of traffic congestion, leading to more severe congestion and accident, which should not have happened, and reduced road's security. In addition, because people do not have the ability to predict the future, people cannot avoid traffic congestion, also couldn't improve the traffic flow.

Intelligent Transportation Systems through relative systems to dissemination real-time traffic information to ensure people who will face the traffic environment have sufficient understanding and according to this made the right choice; through the elimination of road congestion and other traffic problems, and develop a good traffic control system to reduce the pollution of the environment; through the development of intelligent intersection and autopilot technology to improve traffic safety, reduce travel time, improve roads capacity. Michigan Department of Transportation Intelligent Transportation Systems Project Manager Greg • Kruger said: "In Detroit, if we can be of 2% to 5% of the vehicles to non-peak route guidance, you can greatly ease the traffic congestion."

Team # 26718 Page 23 of 28

## **Design of Intelligent Controller**

Takagi and Surgeon made famous T-S fuzzy system model, which antecedents is ambiguous, described the piece as a linear antecedent. T-S fuzzy model is the nature of the global nonlinear system established by more than a simple linear relationship between fuzzy partition, then the output of multiple models fuzzy reasoning and judgment, in order to represent complex nonlinear relationship. Since TS- fuzzy model can approximate arbitrary precision continuous nonlinear systems, so this more than a simple linear fuzzy inference system controller obtained by global controller, can be well controlled nonlinear systems to solve nonlinear system control issues a new way. Expression of T-S fuzzy model fuzzy implied condition statement is as follows:

IF 
$$(x_1 \ is \ \widetilde{A}_1 \ and \ x_2 \ is \ \widetilde{A}_2 \ \dots \ and \ x_i \ is \ \widetilde{A}_k)$$
  
Then  $y = p_0 + \sum_{k=1}^k p_i, x_i$ .

Which  $(x_1, x_2, ..., x_k)$  represents the antecedent of k fuzzy variables,  $(A_1, A_2, ..., A_k)$  represents a former member of the fuzzy set membership function parameter called the antecedent parameters of the piece parameter,  $(p_1, p_2, ..., p_k)$  represents after the piece parameter, y represents the fuzzy output variable. Membership function of the fuzzy subset taken as a piecewise linear convex sets composed.

If a given fuzzy input vector  $(x_1, x_2, ..., x_k)$ , the output fuzzy  $y_i (i = 1, 2, ..., n)$  weighted average calculated by the implicit rules for each of the output:

$$y = \sum_{i=1}^{n} G^{i} y^{i} / \sum_{i=1}^{n} G^{i},$$
(17)

where n is the number of fuzzy rules,  $y_i$  is calculated by the i's conclusions of the equation to get the first rule, the first rule  $G^i$  is applicable to the i's degree, calculated by the following formula:

$$G^{i} = \prod_{p=1}^{K} A_{p}^{i}(x_{p}), \tag{18}$$

where  $\Pi$  is the fuzzy operator, usually take a small operation.

For vehicle motion characteristics, dynamic target position of the vehicle x, y under the local coordinate system, and dynamic target direction of the three variables  $\theta$  as input variables of fuzzy control, the steering angle of the vehicle a as the output variable fuzzy control. According to the actual control requirements, you can roughly determine the state and control variables ranges are  $x \in [0,30]$ ,  $y \in [0,6]$ ,  $\theta \in [-0.3,0.3]$ , and  $a \in (-90,90)$  through the use of adaptive neural network parameter adjustment on T-S fuzzy membership function model obtained on the basis of fuzzy inference

Team # 26718 Page 24 of 28

system. One of the training data, check the data and test data generated by the simulation.

# Establish and analyze the results of the system in the Simulink environment

In order to ensure the correctness and effectiveness of the system, we carried out in automatic overtaking Simulink motion control experiment and analyze the experimental platform. Above, according to the design of intelligent control strategy in the Simulink environment for intelligent automatic vehicle overtaking movement modeling and simulation, system simulation structure is shown below.

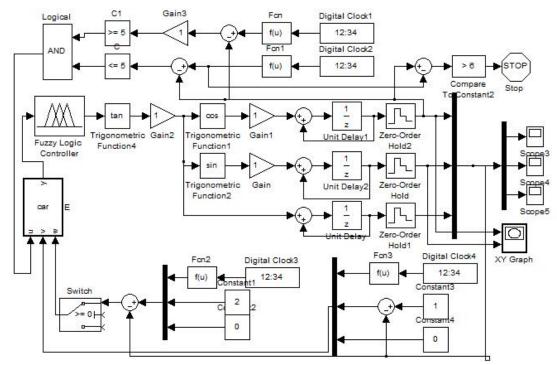
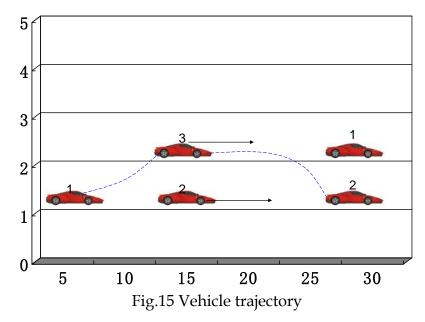


Fig.14 Control system simulation structure

In the simulation process, as compared with non-intelligent system, assuming the vehicle 1 (controlled vehicle), the initial position coordinates of the vehicle 2 and the vehicle 3 are: (2,1,0), (5,1,0) and (5,2,0); its speed were to 60km/h, 50km/h and 55km/h; wheelbase is 1m; sampling interval of 0.05s. After setting the vehicle in which the accused dynamic target position to consider the width of the body, the front and rear of the vehicle changing lanes and safe distance. Obtained by the simulation system of the vehicle's passing automatically trajectory is based on the dynamic target position shown in Figure 15.

Team # 26718 Page 25 of 28



In Figure 15: all vehicles are uniform motion, but the vehicle 1 (controlled vehicle) from the initial state (2,1,0) smoothly along a virtual route (blue line) to 60km / h constant speed tracking of vehicle 2 and the lane change, and in the state (15,2,0), the vehicle speed of a 60km / h for a uniform linear motion, a motion when the vehicle is overtaking to satisfy the condition (vehicle 1 and the vehicle 2, the vehicle 3 does not collide) , the vehicle continues along a virtual route (blue line) overtaking, then for uniform linear motion in (18.4,2,0) a safe distance from the state judgment and adjacent channel vehicle is started and that, at (25.5,1,0) state return to the original route begins uniform linear motion, eventually following vehicle the state (30,1,0), the whole process showed a smooth, natural features, more reasonably reflect the driver's driving behavior.

Compare the intelligent control system model obtained results with previous results, we can get the result that the intelligence system of the vehicle overtaking a minimum safe distance is 13 meters, another model calculated minimum safe distance is 17.77 meters. Since the 13 meters <17.77 meters, the intelligent control system is better in terms of security.

#### **Evaluations and Solutions**

# Strengths

- 1. When we create a model, we experience change lanes, beyond lanes and the and-road lanes three stages, so we have a more detailed and clearly analysis of the problem, also the results of the model is more accurate.
- 2. The model considers all kinds of vehicles that may appear on highway, we

Team # 26718 Page 26 of 28

have a very rigorous consideration about the question, so it has a wider scope application.

- 3. When establishing the minimum safety distance longitudinally model, we take all kinds of overtaking collision into consideration, so we establish a comprehensive model.
- 4. This model analyzes the traffic flow, safety, speed and other factors on the performance of the rules, obviously, the analyses are more thoroughly.
- 5. After solving the model, we exam the model with the simulation software, making the model more intuitive.

#### Weaknesses

- 1. In order to facilitate solving the model, the model is established under certain assumptions, so reducing the usefulness of the model.
- 2. The question does not give the original data, therefore, the result's deviation increased.

# **Improvements**

- 1. Models only consider the case of one-way two-lane highway, there are many highways are one-way multi-lane highway in the real, in which case the model will not be used. In order to solve the multi-lane one-way rule performance issues, we should consider the interaction of each lane to overtake a vehicle. After take this factor into account, again, modeling, analysis of traffic flow, safety, and rules limiting the impact on the performance of the model will make them more widely used in real life.
- 2. In the process of the problem analysis, you can consider the impact of individual factors on the road which may change the road safety performance and the traffic flow.

Team # 26718 Page 27 of 28

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