COMP 2121 DISCRETE MATHEMATICS | LAB 3

1. Let $a \rightarrow (b \wedge c)$ be false. What is the truth value of the following statements:

   i) $a \wedge b \wedge c$ *ii)* $\neg a \vee (b \wedge c)$

   iii) $(b \wedge c) \rightarrow a$ *iv)* $\neg(b \wedge c) \rightarrow \neg a$

   v) $(b \wedge t) \rightarrow (a \vee r)$ *vi)* $(a \vee r) \rightarrow (b \wedge t)$

2. Construct a truth table for the statement $\neg(a \vee \neg b) \rightarrow \neg a$.

3. Negate the statement $(a \vee \neg b) \rightarrow \neg c$ and simplify the result.

4. Negate the sentence: If $a = b$ and $c + d > 50$ then this graph is bipartite.

5. Consider the following argument and provide reason for each step

$\neg b$
$(\neg a \lor c) \rightarrow t$
$\neg a \lor n$
$\neg b \rightarrow \neg t$

$\therefore n \lor k$

	STEPS	REASON
1)	$\neg b$	
2)	$\neg b \rightarrow \neg t$	
3)	$\neg t$	
4)	$(\neg a \lor c) \rightarrow t$	
5)	$\neg(\neg a \lor c)$	
6)	$a \land \neg c$	
7)	a	
8)	$\neg a \lor n$	
9)	n	
10)	$\therefore n \lor k$	

6. Use the rules of inference to show that the following arguments are valid. Provide the rule for each step.

		STEPS	REASON
a)	$\neg t$ $\neg s$ $a \to t$ ——— $\therefore \neg(a \vee s)$		

		STEPS	REASON
b)	$x \to (y \to z)$ x $\neg y \to \neg x$ ——— $\therefore z$		

		STEPS	REASON
c)	$\neg s$ $p \to (q \to r)$ $t \to q$ $p \vee s$ ——— $\therefore \neg r \to \neg t$		

STEPS REASON

d) $a \wedge b$
$\neg x$
$a \rightarrow (r \wedge b)$
$(r \vee n) \rightarrow (x \vee p)$
___
$\therefore p$

7. Use <u>contradiction</u> to prove the following argument:

STEPS REASON

$p \rightarrow q$
$(q \wedge r) \rightarrow s$
r
___
$\therefore p \rightarrow s$

If all of the problems can't be done in the lab period, students can work on them on their own.

No Name identity: $p \to q \Leftrightarrow \neg p \vee q$

Contrapositive: $p \to q \Leftrightarrow \neg q \to \neg p$

	Rule of Inference	Related Logical Implication	Name of Rule
1)	$\begin{array}{l} p \\ \underline{p \to q} \\ \therefore q \end{array}$	$[p \wedge (p \to q)] \to q$	Rule of Detachment (Modus Ponens)
2)	$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Law of the Syllogism
3)	$\begin{array}{l} p \to q \\ \underline{\neg q} \\ \therefore \neg p \end{array}$	$[(p \to q) \wedge \neg q] \to \neg p$	Modus Tollens
4)	$\begin{array}{l} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$		Rule of Conjunction
5)	$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array}$	$[(p \vee q) \wedge \neg p] \to q$	Rule of Disjunctive Syllogism
6)	$\begin{array}{l} \underline{\neg p \to F_0} \\ \therefore p \end{array}$	$(\neg p \to F_0) \to p$	Rule of Contradiction
7)	$\begin{array}{l} \underline{p \wedge q} \\ \therefore p \end{array}$	$(p \wedge q) \to p$	Rule of Conjunctive Simplification
8)	$\begin{array}{l} \underline{p} \\ \therefore p \vee q \end{array}$	$p \to p \vee q$	Rule of Disjunctive Amplification
9)	$\begin{array}{l} p \wedge q \\ \underline{p \to (q \to r)} \\ \therefore r \end{array}$	$[(p \wedge q) \wedge [p \to (q \to r)]] \to r$	Rule of Conditional Proof
10)	$\begin{array}{l} p \to r \\ \underline{q \to r} \\ \therefore (p \vee q) \to r \end{array}$	$[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$	Rule for Proof by Cases
11)	$\begin{array}{l} p \to q \\ r \to s \\ \underline{p \vee r} \\ \therefore q \vee s \end{array}$	$[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$	Rule of the Constructive Dilemma
12)	$\begin{array}{l} p \to q \\ r \to s \\ \underline{\neg q \vee \neg s} \\ \therefore \neg p \vee \neg r \end{array}$	$[(p \to q) \wedge (r \to s) \wedge (\neg q \vee \neg s)] \to (\neg p \vee \neg r)$	Rule of the Destructive Dilemma