

5 – Continuous Probability Distributions and Central Limit Theorem

1. Suppose X is a normally distributed random variable with $\mu = 16.2$ and $\sigma^2 = 1.3225$. Find the probability that
 - a. X is greater than 16.8
 - b. X is less than 14.9
 - c. X is between 13.6 and 18.8
 - d. X is between 16.5 and 16.7
2. Human body temperatures are normally distributed with a mean of 98.20°F and a standard deviation of 0.62°F. A temperature greater than 100.6°F is a fever. What proportion of people have fevers?
3. Men's heights are normally distributed with a mean of 69.0 inches and a standard deviation of 2.8 inches. Women's heights are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. The Boeing 757 airplane's doors are 72 inches from top to bottom.
 - a. What proportion of men can fit through the door without bending?
 - b. What proportion of women can fit through the door without bending?
4. Birth weights in Norway are normally distributed with a mean of 3570g and a standard deviation of 500g.
 - a. If a hospital requires special treatment for babies that weigh less than 2700g, what percentage of newborn babies require special treatment?
 - b. If the hospital requires special treatment for the smallest 3% of babies, what birth weight separates babies that require special treatment from those that don't?
5. In a certain antipsychotic medication (tablet form) the mass of the active ingredient is normally distributed with mean 51 mg and standard deviation 2.5 mg.
 - a. If the rated content of the active ingredient in the tablets is 50 mg, then what percentage of these tablets will have less than the rated amount of active ingredient?
 - b. Suppose a patient receives 10 such tablets. What is the probability that at least one of the tablets will have less than the rated amount of the active ingredient? [*Hint: normal + binomial*]
 - c. If the acceptable limits of the amount of active ingredient are 47 mg to 55 mg, then what percentage of tablets will lie outside the acceptable limits?
 - d. It turns out that a simple adjustment on the machine used to manufacture the tablets allows one to change the mean content without changing the standard deviation. To what level (in mg) should the mean be raised in order that only 1% of the tablets lie below the lower acceptable limit? At this setting for the mean, what percentage of tablets lie above the upper acceptable limit?

- e. With some effort it is possible to reduce the standard deviation. With the mean set at 51 mg, to what value must the standard deviation be reduced in order that only 5% of all tablets will have a mass of active ingredient which is outside the acceptable limits quoted above?
6. Specifications for a mechanical product require metal washers with an inside diameter of 0.300 ± 0.002 cm. If the inside diameters of the washers supplied by a given manufacture are normal distributed with $\mu = 0.301$ cm and $\sigma = 0.001$ cm, then what percentage of these washers will meet specifications?
7. The length of a structural component of a device is an approximately normally distributed random variable with a standard deviation of 0.90 mm. The fabricating machine can be adjusted to achieve any desired mean value. What must the mean value be so that 90% of the components have a length of 12.10 mm or greater?
8. The amount of time T that a surveillance camera will run without having to be reset is a continuous random variable that is exponentially distributed with a mean of 50 days. Find the probability that such a camera will
- have to be reset in less than 25 days (i.e., $T < 25$)
 - last at least 65 days
 - last between 60 and 80 days
9. The waiting time T between arrivals at a passport office is modelled by an exponential random variable with a rate of 0.2 per minute. Find the probability that T is:
- less than 5 minutes
 - between 7 and 9 minutes
 - more than 10 minutes
10. An air-actuated electric switch has an exponential life distribution with mean of 1000 hours.
- What proportion of the devices last at least 1150 hours?
 - Find the median lifetime of the devices.
11. An integrated circuit chip has an exponential failure rate of 0.048 per thousand hours. What is the probability that it will operate satisfactorily for at least 15000 hours?
12. The elevator in a women's gym is limited to 10 passengers. Women's weights are approximately normally distributed with mean 154 lbs and standard deviation of 33 lbs.
- If 10 women are randomly selected, find the probability that their total weight will not exceed the maximum capacity of 1750 lb.
 - If we want a 99.99% probability that the elevator will not be overloaded whenever 10 women are randomly selected as passengers, what should be the maximum allowable weight?

13. The new Lucky Lady Casino wants to increase revenue by providing buses that can transport gamblers from other cities. Research shows that these gamblers tend to be older, they tend to play slot machines only, and they have losses with a mean of \$82 and a standard deviation of \$60. The buses carry 40 gamblers per trip. The bus costs \$200 to operate, and the casino gives each bus passenger \$50 worth of vouchers. The casino needs to recover its costs in order to make a profit. Find the probability that if a bus is filled with 40 passengers, the casino makes a profit.
14. A gym has 210 members. The weights of members have a distribution that is approximately normal with a mean of 163 lbs and a standard deviation of 32 lbs. The design for a new building includes an elevator with a capacity limited to 12 passengers.
- If the elevator is designed to safely carry a load up to 2100 lbs, what is the probability that 12 randomly-selected gym members will exceed the limit?
 - What is the maximum number of passengers that should be allowed if we want a 99.9% chance that the elevator is not overloaded when it is filled with gym members?

Answers:

- 1a. 0.301
- 1b. 0.129
- 1c. 0.976
- 1d. 0.0652
- 2. 0.005%
- 3a. 85.77%
- 3b. 99.96%
- 4a. 4.09%
- 4b. 2630g
- 5a. 0.3446
- 5b. 0.9854
- 5c. 10.96%
- 5d. 52.82; 19.12%
- 5e. 2.041 mg
- 6. 0.8400
- 7. 13.25mm
- 8a. 0.3935
- 8b. 0.2725
- 8c. 0.09930
- 9a. 0.6321
- 9b. 0.08130
- 9c. 0.1353
- 10a. 0.3166
- 10b. 693.1h
- 11. 0.4868
- 12a. 97.78%
- 12b. 1927 lbs
- 13. 99.8%
- 14a. 0.0918
- 14b. 10 passengers