COMP 2121
DISCRETE MATHEMATICS

Lecture 4

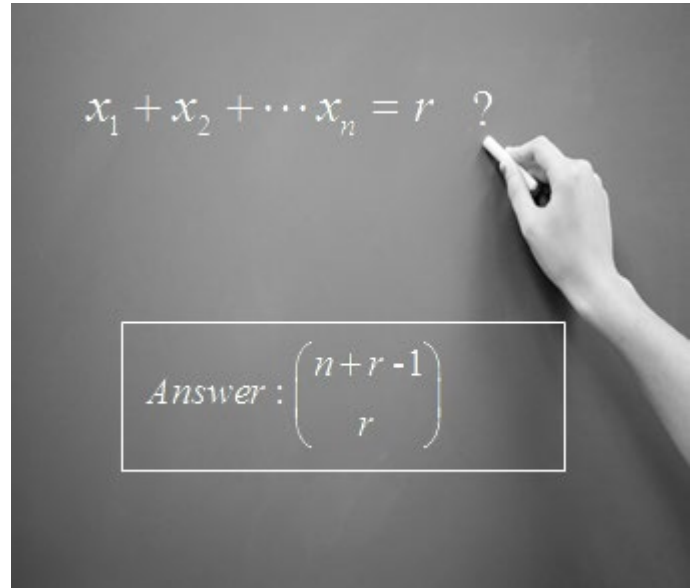$$x_1 + x_2 + \cdots x_n = r \quad ?$$

$$Answer : \binom{n+r-1}{r}$$

Combinations with Repetition Allowed

Example 1. (Review problem): Consider the following program segments. What value does a variable `counter` have after the following code execution?

```
counter = 15
for i = 1 to 5
     for j = 1 to 5
         counter = counter + 8
     next j
     counter = counter + 9
next i
```

```
counter = 15
for i = 1 to 5
     for j = i+1 to 5
         counter = counter + 8
     next j
     counter = counter + 9
next i
```

```
counter = 15
for i = 1 to 6
     for j = i+1 to 10
         counter = counter + 8
     next j
     counter = counter + 9
next i
```

Example 2. In how many ways can one distribute 12 identical blue marbles among three math instructors: Andrew, David and Scott?

Theorem: *When we wish to select, with repetition, r of n distinct objects, we find that we are considering all arrangements of r 'x's and $n-1$ |'s and thus their number is:*
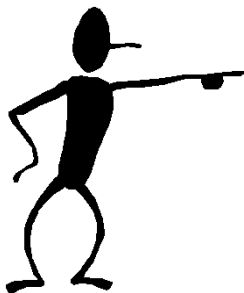
$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

Example 3. How many arrangements of the letters in CANADIAN have at least two letters between the N's? (Similar to example from Lecture 3)

Example 4. Consider the following program segments. How many times is the print statement executed?

```
for i = 1 to 5
    for j = i to 5
        print i, j
    next j
next i
```

```
for i = 1 to 5
    for j = 1 to i
        print i, j
    next j
next i
```

Summary:

⚖	Order matters	Order does not matter
Repetition is allowed	n^k	$\binom{n+k-1}{k}$
Repetition is Not allowed	$P(n,k) = \dfrac{n!}{(n-k)!}$	$\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$