Week 1 - Lecture

August 31, 2025 3:07 PM

MATH 3042 Lecture Notes Fall 2025

1 – Descriptive Statistics: Visualizing Data

Some History

The subject of *Statistics* began in the 18th century when governments (i.e., states) in Europe began collecting data about their citizens. A modern definition is:

> Statistics is *the branch of mathematics that deals with data.*

The purpose of statistics is to *make sense* of data to support decision-making.

Ronald Fisher (1890-1962) developed foundational concepts and methods of statistical inference in connection with genetics and agricultural data.

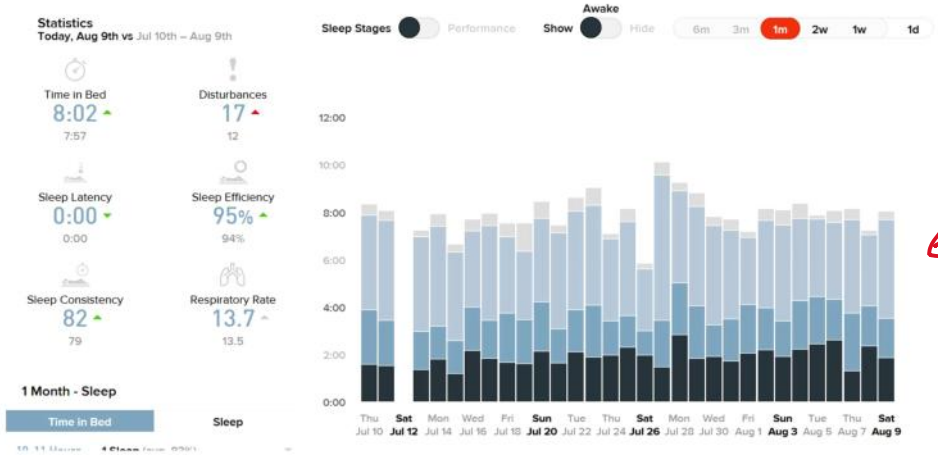
Ronald Fisher, British Statistician

Why Statistics in CST?

The core concepts of statistics and probability are used throughout *data science* (including *machine learning*, *signal processing*, *data mining*) as well as in business, politics, medicine, and in many other areas.

Also, data analytics might be a core feature of your software! For instance, the Whoop app evaluates your sleep and physical recovery by comparing it to your past data.

based on my past data



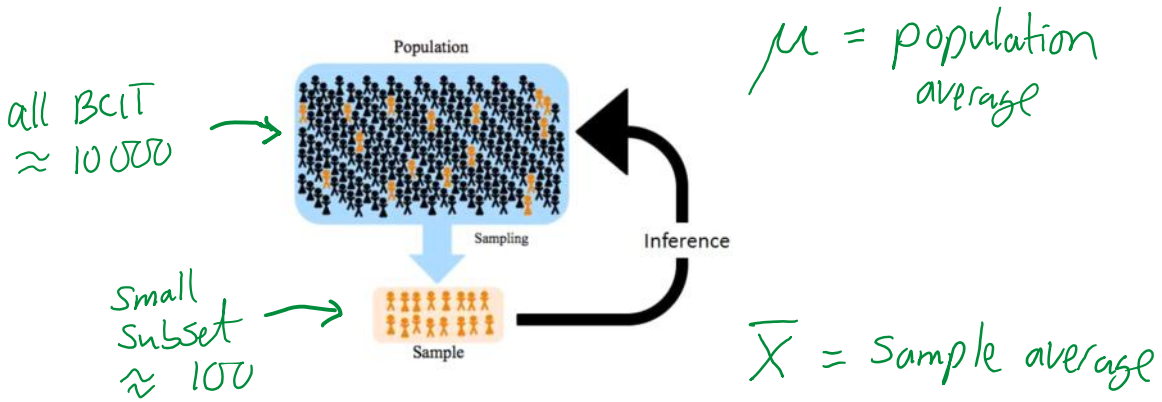
Statistical visualizations

1

Parts of Statistics

Descriptive Statistics provides methods for organizing and summarizing large data sets.

Inferential Statistics provides methods for drawing conclusions about a *population* based on data from a *sample*.



Probability Theory is the mathematical link between descriptive and inferential statistics.

Example (Student Survey) Given the *data frame* below (first 10 rows shown), we might try to answer questions like:

- Do most students exercise?
- Are students who smoke more or less likely to exercise?
- What is the typical age of a student?

Each column is a Variable

	Sex	Wr.Hnd	NW.Hnd	W.Hnd	Fold	Pulse	Clap	Exer	Smoke	Height	M.I	Age
1	Female	18.5	18.0	Right	R on L	92	Left	Some	Never	173.00	Metric	18.250
2	Male	19.5	20.5	Left	R on L	104	Left	None	Regul	177.80	Imperial	17.583
3	Male	18.0	13.3	Right	L on R	87	Neither	None	Occas	NA	NA	16.917
4	Male	18.8	18.9	Right	R on L	NA	Neither	None	Never	160.00	Metric	20.333
5	Male	20.0	20.0	Right	Neither	35	Right	Some	Never	165.00	Metric	23.667
6	Female	18.0	17.7	Right	L on R	64	Right	Some	Never	172.72	Imperial	21.000
7	Male	17.7	17.7	Right	L on R	83	Right	Freq	Never	182.88	Imperial	18.833
8	Female	17.0	17.3	Right	R on L	74	Right	Freq	Never	157.00	Metric	35.833
9	Male	20.0	19.5	Right	R on L	72	Right	Some	Never	175.00	Metric	19.000
10	Male	18.5	18.5	Right	R on L	90	Right	Some	Never	167.00	Metric	22.333

Each row is one individual (or unit)

1.1 Graphical Methods of Describing Data

Data can be either:

- *non-numerical* (also known as *categorical* or *qualitative*), or
  e.g., **Smoke**
- *numerical* (also known as *quantitative*).
  e.g., **Age**

e.g. Sex

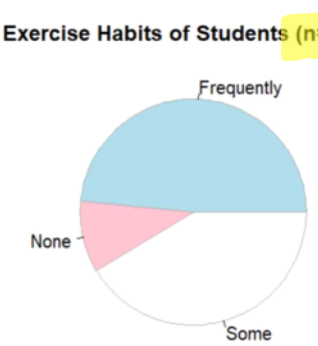
e.g. Distance

Graphs for Non-Numerical Data

Pie Chart

```
> pie(table(survey$Exer))
```

include sample size

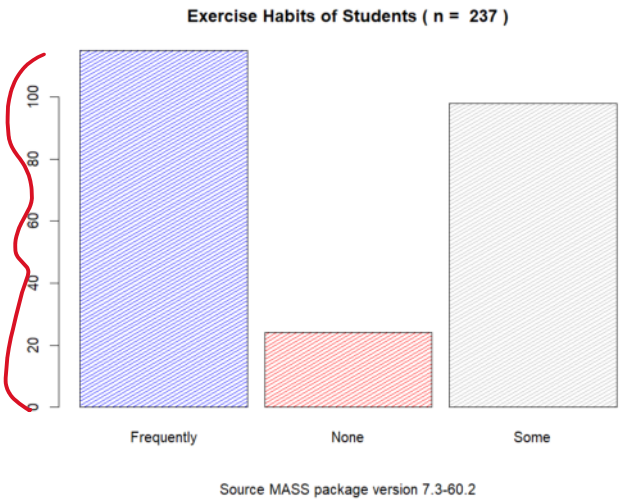


Observation: Most students exercise

Bar Chart (Column Chart)

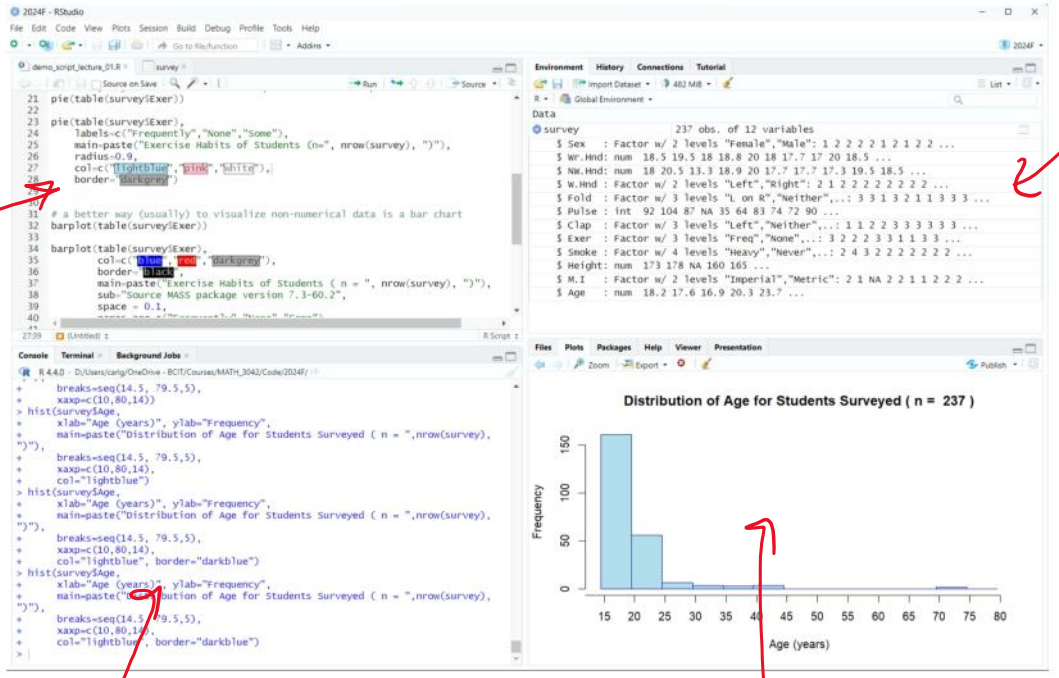
```
> barplot(table(survey$Exer))
```

we can see the absolute amounts →



based on S

Sidenote on Software We will be using the programming language R and the coding environment RStudio to create graphs and charts. R is an open-source programming language for statistical computing and graphics. The R language is widely used among statisticians and data miners for developing statistical software and data analysis.



source pane

Environment

Console Pane (interactive)

Plots, Files, etc...

Graphs for Numerical Data

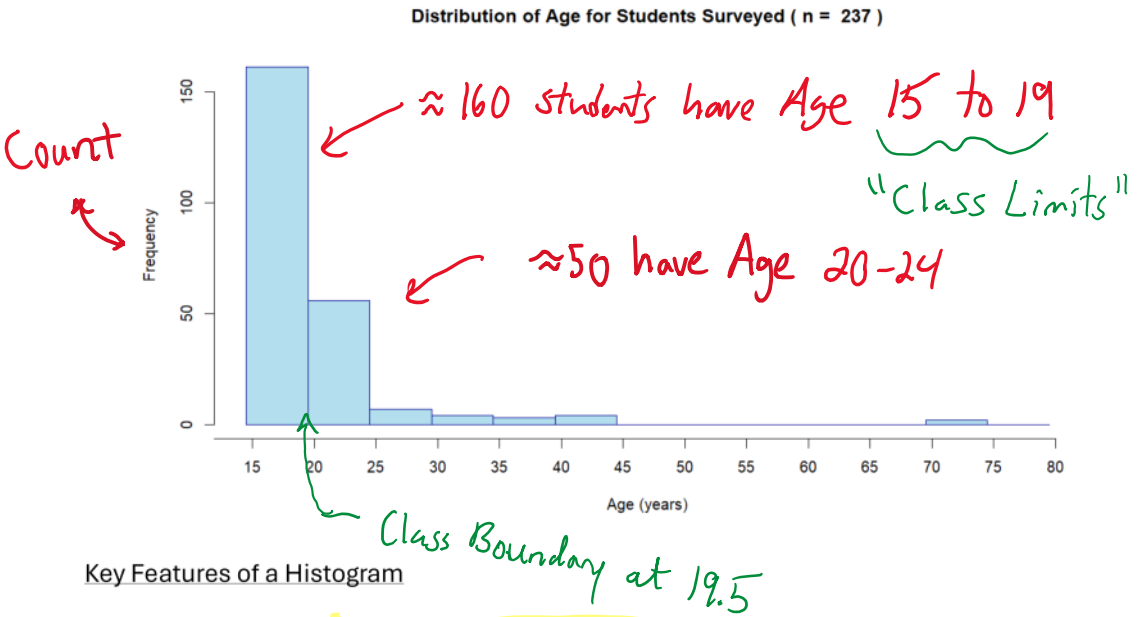
We will spend most of our time in this course dealing with *numerical* variables.

When the set of numerical values is relatively large (> 100 individuals) then it becomes impossible to understand the data without the help of graphs or numerical summaries.

Histograms, a Quick Look

Example (Student Survey) Again, we use the student survey data frame mentioned above. We can visualize the students' ages in several ways. The most important way is using a *histogram*, shown below.

```
> hist(survey$Age)
```



Key Features of a Histogram

- Variable values X go along the horizontal axis.
- The X axis is divided into *classes*.
- The *frequency* of a given class is shown as the *height* of the rectangle for that class.
- Classes have *equal width*.
- There are no *gaps* between classes.
- Each *data value* falls into a class.

[lower class limit, upper class limit]

Example (Old Faithful) Another built-in data is *faithful*, which contains data describing eruptions of the geyser called "Old Faithful" in Yellowstone, USA.

We can look at the data using the command *View(faithful)*.

	eruptions	waiting
1	3.600	79
2	1.800	54
3	3.333	74
4	2.283	62
5	4.533	85
6	2.883	55
7	4.700	88
8	3.600	85
9	1.950	51
10	4.350	85

In this data set:

- eruptions = the duration (in minutes) of observed eruptions
- waiting = the time (in minutes) in between one eruption and the next eruption

Stem-and-leaf

Let's focus on the *waiting* data. A simple (although old-fashioned) way to visualize the data is a *stem-and-leaf plot*.

```
> stem(faithful$waiting)

  The decimal point is 1 digit(s) to the right of the |

  4 | 3
  4 | 55566666777788899999
  5 | 000001111112222233333344444444
  5 | 55555566667778888899999
  6 | 00000022223334444
  6 | 555667899
  7 | 00001111123333333444444
  7 | 55555555666666666777777777777888888888888888999999999
  8 | 000000001111111111111222222222223333333333333344444444
  8 | 55555566666677888888999
  9 | 00000012334
  9 | 6
```

"stem" "leaf"

In a stem-and-leaf plot, each data value is cut into a "stem" and a "leaf". The leaf is typically the final numerical digit. The stem is everything before the leaf.

stem: 7 7|9 leaf: 9

If the data values have more than three digits of precision, it may be necessary to round each value to three digits first. For the *eruptions* data, a typical data value is cut like this:

2.283 → round → 2.28

stem: 22 → leaf: 8

```
> stem(faithful$eruptions)

 The decimal point is 1 digit(s) to the left of the |

 16 | 070355555588
 18 | 000022233333335577777777888822335777888
 20 | 00002223378800035778
 22 | 0002335578023578
 24 | 00228
 26 | 23
 28 | 080
 30 | 7
 32 | 2337
 34 | 250077
 36 | 0000823577
 38 | 2333335582225577
 40 | 000000335778888800223355557777 8
 42 | 033355557788002333335555777 78
 44 | 022223355577800000000233333577 8888
 46 | 000023335770000002357 8
 48 | 00000022335800333
 50 | 0370
```

1.98

4.58

What can we conclude about the duration of *eruptions* at Old Faithful?

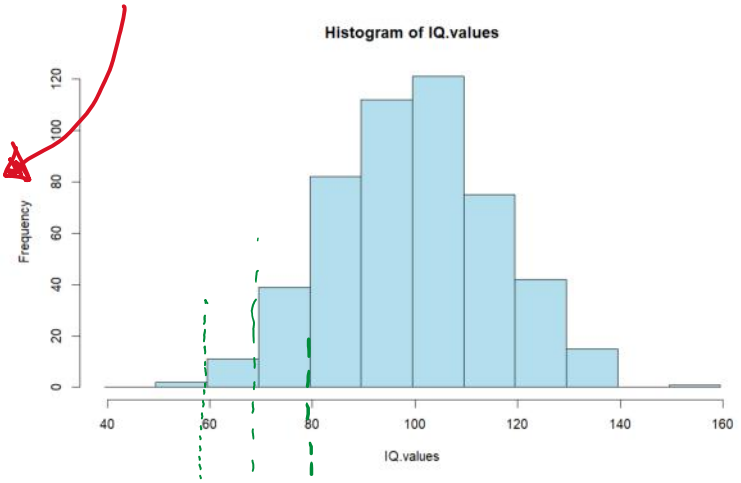
Eruptions are typically around 4.6 or 2.0 seconds

Frequency Distributions and Histograms

A stem-and-leaf plot is a crude form of a *histogram*, which we will now cover in more detail.

A histogram is always derived from a *frequency distribution* based on a choice of *classes* for the variable in question.

Lower	Upper	Frequency
50	59	1
60	69	10
70	79	40
⋮		⋮
150	159	1



59.5 69.5 79.5

class boundaries

Example (Old Faithful) One possible choice of classes for the *eruptions* data and the corresponding frequencies is given by the *frequency distribution* shown in the table below.

The first class contains all values from 1.600 to 1.799

1st rectangle

Lower Class Limit	Upper Class Limit	Frequency
1.600	1.799	12
1.800	1.999	39
2.000	2.199	20
2.200	2.399	18
2.400	2.599	3
2.600	2.799	3
2.800	2.999	2
3.000	3.199	1
3.200	3.399	4
3.400	3.599	6
3.600	3.799	10
3.800	3.999	16
4.000	4.199	31
4.200	4.399	29
4.400	4.599	35
4.600	4.799	28
4.800	4.999	11
5.000	5.199	4

The *class width* is:

$= 1.800 - 1.600$

$= 0.200$

(NOT 0.199)

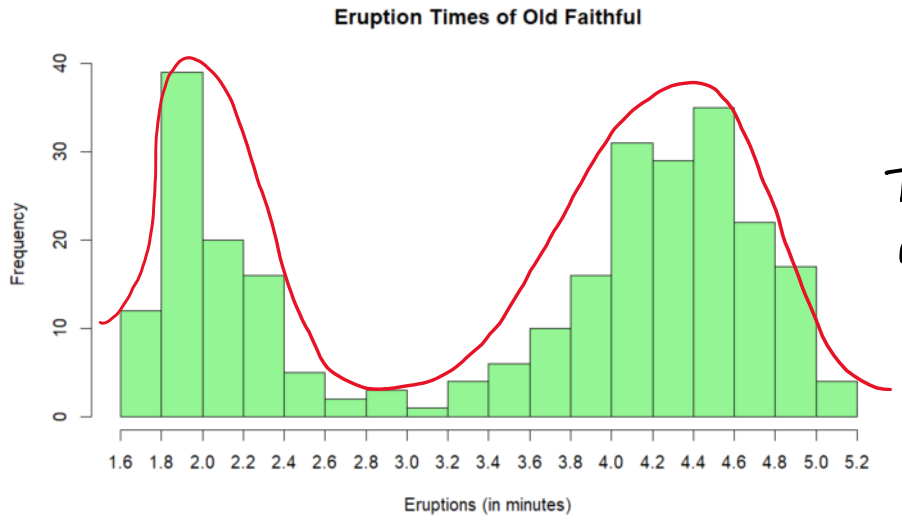
1st class boundary

$= 1.7995$

A *histogram* is a direct visual representation of the frequency distribution.

```
> hist(faithful$eruptions)
```

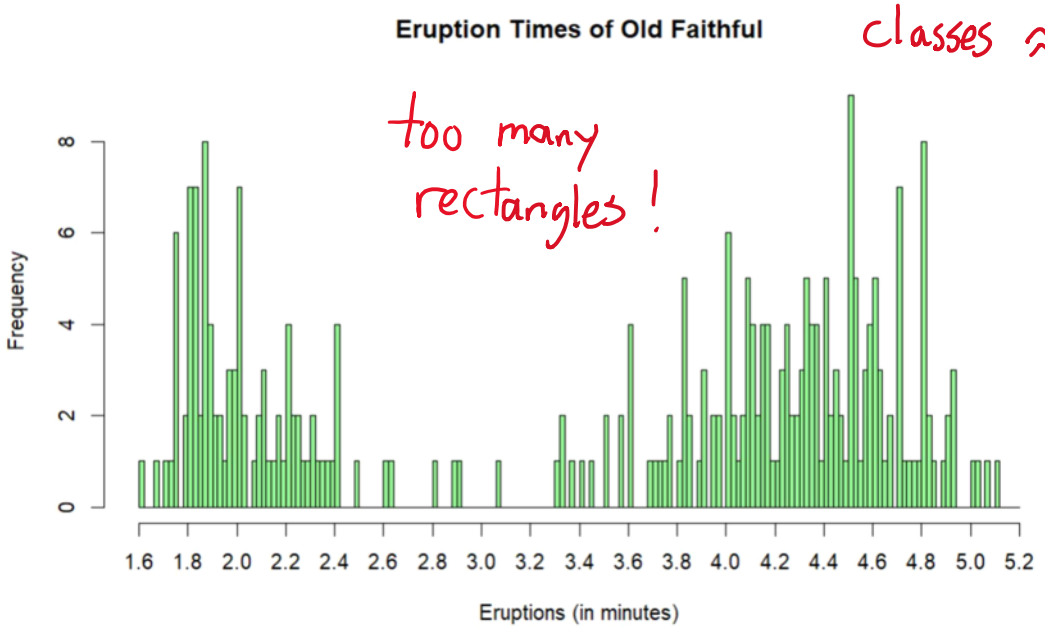
→ uses default class limits



two peaks are apparent

Using too many classes is bad. For instance, here is the same data using a class width of 0.020 (instead of 0.200).

ideal number of classes $\approx \sqrt{n}$



Cumulative Relative Frequency Distributions and Ogives

The *cumulative frequency* of a class is the number of individuals that fall into any class up to and including that class.

The *cumulative relative frequency* distribution is the *fraction* of individuals that fall into any class up to and including that class.

Lower Class Limit	Upper Class Limit	Frequency	Cumul Freq	Cumul Rel Freq
1.6	1.799	12	12	12/272 = 0.0441
1.8	1.999	39	51	51/272 = 0.1875
2.0	2.199	20	71 = 12+39+20	
... and so on ...				
5.0	5.199	4	272	272/272 = 1.0000

(complete table shown on next page)

Lower Class Limit	Upper Class Limit	Frequency	Cumul Freq	Cumul Rel Freq
	1.599		0	0.000
1.6	1.799	12	12	0.044
1.8	1.999	39	51	0.188
2.0	2.199	20	71	0.261
2.2	2.399	18	89	0.327
2.4	2.599	3	92	0.338
2.6	2.799	3	95	0.349
2.8	2.999	2	97	0.357
3.0	3.199	1	98	0.360
3.2	3.399	4	102	0.375
3.4	3.599	6	108	0.397
3.6	3.799	10	118	0.434
3.8	3.999	16	134	0.493
4.0	4.199	31	165	0.607
4.2	4.399	29	194	0.713
4.4	4.599	35	229	0.842
4.6	4.799	28	257	0.945
4.8	4.999	11	268	0.985
5.0	5.199	4	272	1.000
		Total = 272		

How many eruptions had a duration of less than 4.0 minutes?

134 eruptions or 49.3% of all

What fraction of eruptions had a duration of less than 4.2 minutes?

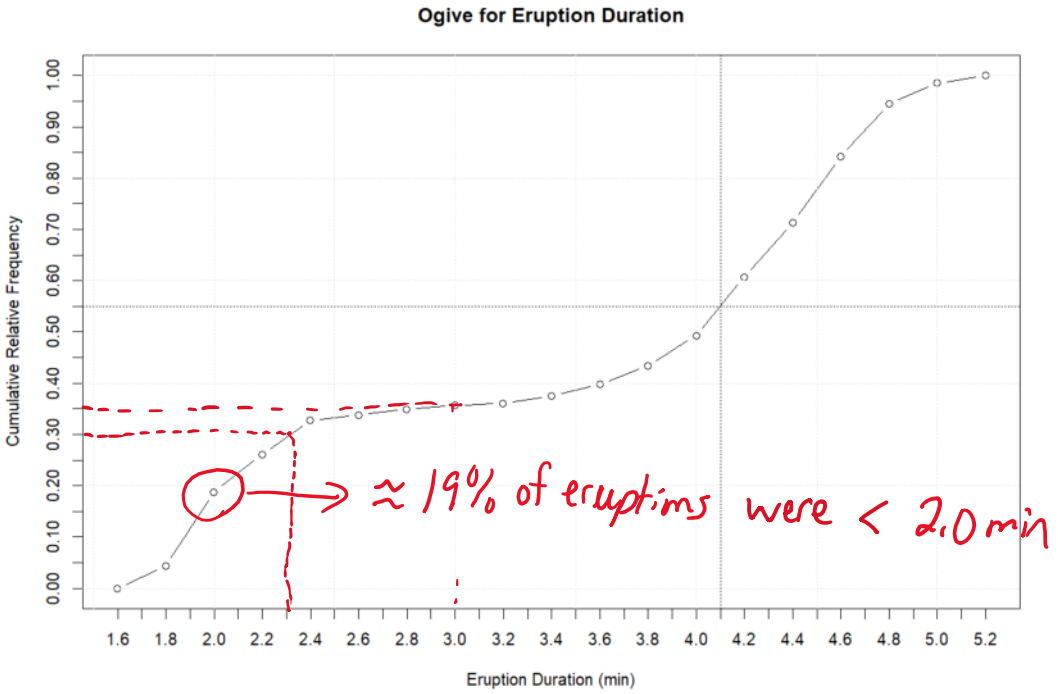
$\frac{165}{272} = 0.607$

What fraction of eruptions had a duration of less than 4.1 minutes?

at least 49.3%, less than 60.7%

An *ogive* ("oh-jive") shows the *cumulative relative frequencies* plotted against the *upper class limits*.

(We insert one additional point at the beginning of the curve to indicate the 0.0 point of cumulative relative frequency.)



What percentage of eruptions have a duration below 3.0 minutes?

≈ 35%

What percentage of eruptions have a duration below 4.1 minutes?

≈ 55%

What value of eruption duration has 30% of eruptions below and 70% above?

$P_{30} \approx 2.3$

↑ 30th percentile of eruptions

11

Scatter Plot

When we measure two or more *numerical* variables for each individual/unit, then it is possible to create a *scatter plot*.

Supposing the two variables are called X and Y, then we simply plot one point for each individual/unit using the specific X and Y values as the coordinates.

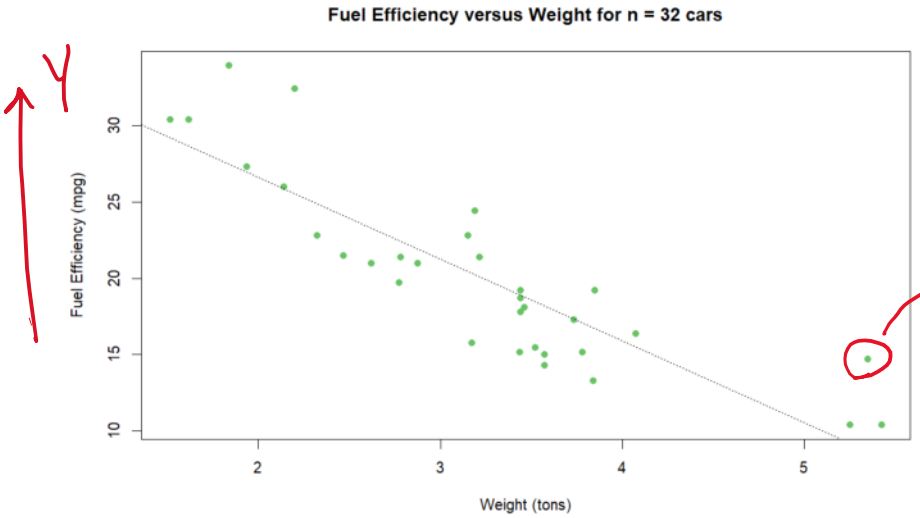
A scatter plot helps us to quickly identify whether the variables X and Y are related.

Example (Fuel Efficiency vs Car Weight) Another built-in dataset called *mtcars* provides data for several variables for different makes of cars. If we let $Y = mpg$ (meaning fuel efficiency) and $X = wt$ (meaning weight), then the scatter plot shows the relation between fuel efficiency and car weight.

mtcars

Filter

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear
Mazda RX4	21.0	6	160.0	110	3.90	2.620	16.46	0	1	4
Mazda RX4 Wag	21.0	6	160.0	110	3.90	2.875	17.02	0	1	4
Datsun 710	22.8	4	108.0	93	3.85	2.320	18.61	1	1	4
Hornet 4 Drive	21.4	6	258.0	110	3.08	3.215	19.44	1	0	3
Hornet Sportabout	18.7	8	360.0	175	3.15	3.440	17.02	0	0	3



Y

X

individual with
$X = 5.2$
$Y = 14.0$

We can see as weight increases the fuel efficiency tends to decrease.