# Midterm Solutions V2

October 25, 2024 7:44 PM

**Name:** Answer key

**ID:** ______________________ **Set:** ______

**British Columbia Institute of Technology**

## MATH 3042 – Midterm Exam

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|---|---|
| **Program:** | Computer Systems Technology |
| **Course Name:** | Applied Probability and Statistics for CST |
| **Course Number:** | Math 3042 |
| **Date:** | October 23, 2024 |
| **Time Allotted:** | 90 min |
| **Exam Pages:** | 11 (including this page) |
| **Total Marks:** | 45 (25% weight for the course) |

## Instructions

1) Do not open the exam or write anything on these pages before you are told to begin.

2) You may use a scientific calculator with statistics functions. No other devices are allowed.

3) If your answer is a probability, round to four digits after the decimal point. Otherwise, round to three significant digits.

4) A formula sheet is provided separately. No other notes or written materials are allowed.

5) No communication of any sort is allowed with other students or any other person besides your instructor or other exam invigilator.

6) All answers are to be written clearly in this examination booklet.

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Total |
|---|---|---|---|---|---|---|---|---|---|
| /5 | /4 | /5 | /3 | /9 | /5 | /4 | /4 | /6 | /45 |

**Question 1** [5 marks]

The temperatures (in °F) of the O-rings for 18 test firings of a rocket engine were recorded in the following stem and leaf plot.

```
  The decimal point is 1 digit(s)
to the right of the |

  3 | 1
  4 | 888
  5 | 337
  6 | 1679
  7 | 0189
  8 | 034
```

a. [2] The mean value is $\bar{X} = 63.7$ °F. Determine the median and mode.

$$Q_2 = \frac{66+67}{2} = 66.5\ °F$$

$$\text{mode} = 48\ °F$$

b. [2] Which of the three measures, mean or median or mode, *best* describes the center of this data set? Explain.

The median is best since the mean is skewed by a low extreme value and the mode is biased by the repeated 48s.

c. [1] Determine the 30th percentile value of this data.

30% of 18 = 5.4 → between 5th and 6th values

$$P_{30} = \boxed{53.0\ °F}$$

**Question 2** [4 marks]

You are designing a first-person shooter (FPS) video game in which the player has the choice between several weapons. You wish to assess the popularity of the RBFG-2008 Cannon, so you study 10 experienced players and determine the percentage of time each player used that weapon while completing the first episode of the game. The data are as follows (in percent):

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 4 | 4 | 5 | 6 | 7 | 7 | 9 | 13 |

a. [2] Calculate the range and the interquartile range.

Range $= 13 - 3 = 10$

$IQR = Q_3 - Q_1 = 7 - 4 = 3$

b. [2] Does this data set contain any outliers? Answer YES or NO and support your answer.

upper fence $= Q_3 + 1.5 \times IQR = 7 + 1.5 \times 3 = 11.5$

Yes, since 13 is above the upper fence.

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**Question 3** [5 marks]

Park rangers in Yellowstone National Park have recorded measurements of the variable

$X =$ time (in minutes) between eruptions of the geyser "Old Faithful".

A sample of measured $X$ values are recorded in the table below.

| | | | | | |
|---|---|---|---|---|---|
| 76 | 80 | 84 | 50 | 93 | 55 |

a. [3] Calculate the mean and standard deviation of $X$.

$$\bar{X} = \frac{\Sigma x}{n} = \frac{438}{6} = \boxed{73.0 \text{ min}}$$

$$s = \sqrt{\frac{1}{n-1} \cdot \Sigma (x-\bar{x})^2} = \sqrt{\frac{1}{5}\left[(76-73)^2+(80-73)^2+(84-73)^2+(50-73)^2 + (93-73)^2+(55-73)^2\right]} = \boxed{16.9 \text{ min}}$$

b. [2] Determine any *unusual* values as determined by $Z$-scores. Provide enough detail to support your answer.

$$\bar{X} - 2 \cdot s = 73 - 2 \times 16.9 = 39.2$$
$$\bar{X} + 2 \cdot s = 73 + 2 \times 16.9 = 106.8$$

No values are outside this interval, so there are no unusual values.

Or, using $X_{min} = 50$, $Z = \frac{50-73}{16.9} = -1.36$

and $X_{max} = 93$, $Z = \frac{93-73}{16.9} = 1.18$

These are within $-2 \leq Z \leq 2$ so they are not unusual.

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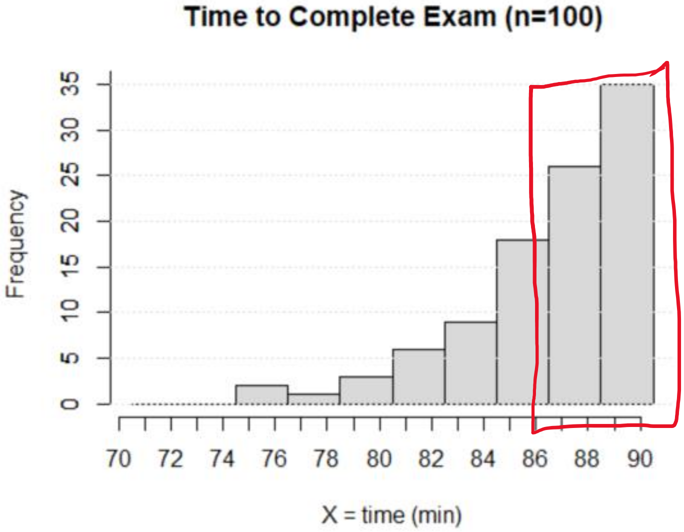
**Question 4** [3 marks]

One hundred students wrote a midterm exam.

Let $X$ = the time a student requires to complete the exam. A histogram of $X$ is shown.




35 + 26 = 61 students so median is in the class with class mark 87.5 min.

a. [1] Use the histogram to estimate the *median* value of $X$. (Aim for $\pm 1$ min accuracy.)

$Q_2 \cong 87.5$ min

b. [2] The mean time was $\bar{X} = 85.2$ min. Is the coefficient of Skewness ($Sk$) positive or negative? Explain.

$Sk$ is negative since $\bar{X} - \text{median} < 0$.

or $Sk$ is negative due to the left tail in the distribution of $X$.

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**Question 5** [9 marks]

A survey was carried out with 3320 students at a technical school. The table below contains frequencies associated with two categorical variables:

- Handedness (*Left/Right/Ambidextrous*), and
- Phone Type (*iPhone/Android/Other*)

[Some frequencies are shown as "?" for the purpose of this question. You do *not* need to fill in all these hidden frequencies.]

| Phone Type / Handedness | **iPhone** | **Android** | **Other** | Row Totals |
|---|---|---|---|---|
| **Left** | 292 | ? | ? | 498 |
| **Right** | 1676 | ? 1135 | 7 | 2818 |
| **Ambidextrous** | 0 | ? | ? | 4 |
| Column Totals | 1968 | 1340 | ? 12 | Grand Total = 3320 |

Suppose you randomly select one student from the school.

a. [1] What is the probability that the student uses an iPhone?

$1968/3320 = 0.5928$

b. [1] What is the probability that the student uses an iPhone, given that the student is Left-handed?

$292/498 = 0.5863$

c. [2] What is the probability that the student is Right-handed *or* uses an Android?

$(2818 + 1340 - 1135)/3320 = 0.9105$

**(Question 5 continued)**

d. [2] Are the categories Ambidextrous and iPhone independent? Answer YES or NO and explain.

No. $P(\text{iPhone}) = 0.5928$ but

$P(\text{iPhone} \mid \text{Ambidextrous}) = 0/4 = 0.000$

e. [3] Assume that 1% of Left-handed students and 0% of Ambidextrous students use an Other phone type. What is the probability that a randomly selected student is Left-handed, given that they use an Other phone type?

$$P(\text{Left} \mid \text{Other}) = \frac{P(\text{Left}) \cdot P(\text{Other} \mid \text{Left})}{P(\text{Left}) \cdot P(\text{Other} \mid \text{Left}) + P(\text{Right}) \cdot P(\text{Other} \mid \text{Right}) + P(\text{Ambi}) P(\text{Other} \mid \text{Ambi})}$$

$$= \frac{\frac{498}{3320} \times 0.01}{\frac{498}{3320} \times 0.01 + \frac{2818}{3320} \times \frac{7}{2818} + 0}$$

$$= \frac{498 \times 0.01}{498 \times 0.01 + 7} = \boxed{0.4157}$$

or

$$P(\text{Left} \mid \text{other}) = \frac{|\text{Left} \cap \text{Other}|}{|\text{Other}|} = \frac{1\% \text{ of } 498}{12}$$

$$= 0.4157$$

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**Question 6** [5 marks]

The rules of a gambling game are as follows:

- You roll *two* fair six-sided dice. Let $X$ = the sum of the two dice.
- If $X = 7$ then you win \$60.
- If $X = 12$ then you win \$200.
- Otherwise, you win nothing.

a. [2] If you play this game repeatedly, how much do you win per game on average?

$$\mu = \sum x \cdot P(x)$$
$$= \$60 \times \frac{6}{36} + \$200 \times \frac{1}{36} + \$0 \times \frac{29}{36} = \boxed{\$15.56}$$

b. [2] You decide to play this game over and over until you win some money (\$60 or \$200). What is the probability that you first win some money on the 10th game?

$X$ = geometric with $p = 7/36$, $q = \frac{29}{36}$

$$P(X=10) = \left(\frac{29}{36}\right)^9 \times \left(\frac{7}{36}\right) = \boxed{0.0278}$$

c. [1] On average, how many games must you play until you win some money?

$$\mu = \frac{1}{p} = \frac{36}{7} = \boxed{5.14 \text{ games}}$$

**Question 7** [4 marks]

Among a group of 75 students in the CST program, 20 have completed a Co-Op term. Suppose 4 students are randomly selected from this group (without replacement).

a. [2] What is the probability that exactly one of the selected students has completed a Co-Op term?

$N = 75$
$K = 20$
$N - k = 55$
$n = 4$
$X$ = successes in sample

$P(1) = C(20,1) \times C(55,3) / C(75,4) = \boxed{0.4317}$

b. [2] Given that the first two students selected have not completed a Co-Op term, what is the probability that the remaining two students have completed a Co-Op term?

73 students remain
20 have completed Co-Op.

Answer : $\frac{20}{73} \times \frac{19}{72} = \boxed{0.0723}$

or $P(\text{remaining two completed Co-Op} \mid \text{first two not Coop})$

$= \frac{P(\text{not first two} \cap \text{remaining two Co-op})}{P(\text{first two not Coop})}$

$= \frac{\frac{55 \times 54}{75 \times 74} \times \frac{20 \times 19}{73 \times 72}}{\frac{55 \times 54}{75 \times 74}} = 0.0723$

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**Question 8** [4 marks]

A manufacturer produces steel brackets used in EYEKEYA furniture. The quality control engineer discovers that 25% of the brackets in a very large batch were defective. These brackets have been randomly mixed with the remaining (75%) non-defective brackets.

a. [2] In a sample of 5 brackets from this batch, what is the probability that none of them are defective?

$n=5$
$p=0.25$

$$P(X=0) = C(5,0)\,p^0 q^5 = (0.75)^5 = \boxed{0.2373}$$

b. [2] In a sample of 5 brackets from this batch, what is the probability that at least three are defective?

$n=10$
$p=0.25 \quad q=0.75$

$$P(X\geq 3) = 1 - P(X\leq 2)$$
$$= 1-(0.75)^5 - 5\times 0.25\times 0.75^4 - 10\times 0.25^2\times 0.75^3$$
$$= \boxed{0.1035}$$

**Question 9** [6 marks]

When moving large amount of data, there are on average 4 random bit errors for every $10^{12}$ bits. Suppose you move a database containing $1.5 \times 10^{11}$ bits.

a. [2] What is the expected number of bit errors that occur?

$$\lambda = \frac{4}{10^{12}\text{ bits}} \times 1.5 \times 10^{11}\text{ bits} = \boxed{0.6}$$

binomial: $n = 1.5 \times 10^{11}$ $p = \frac{4}{10^{12}}$ $\rightarrow \mu = np = \boxed{0.6}$

b. [2] What is the probability that *at least one* bit error occurs?

$X$ = number of bit errors

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0.6} = \boxed{0.4512}$$

or use binomial: $1 - \left(1 - \frac{4}{10^{12}}\right)^{1.5 \times 10^{11}} \approx \boxed{0.4512}$

c. [2] What is the probability that two bit errors occur?

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^{2}}{2!} = \frac{e^{-0.6} \cdot (0.6)^2}{2!} = \boxed{0.0988}$$

or use binomial : $n = 1.5 \times 10^{11}$ trials

$p = \frac{4}{10^{12}}$ $q = 1 - \frac{4}{10^{12}}$

$$P(X=2) = C(1.5 \times 10^{11}, 2) \cdot \left(\frac{4}{10^{12}}\right)^2 \cdot \left(1 - \frac{4}{10^{12}}\right)^{1.5 \times 10^{11} - 2}$$

$\doteq \boxed{0.0988}$

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NOTE: Many calculators cannot evaluate the binomial expressions.