

# COMP 2121

## DISCRETE MATHEMATICS

### Quiz 1

Name:

Student ID:

Set: A B C D (circle one)

- Time allowed: 20 min.

1. A typical PIN (personal identification number) is a sequence of 12 letters chosen from the 26 letters in the alphabet.

- Repetition is allowed unless otherwise stated.

(a) [1 mark] How many PINs have at least one letter B?

$\xleftarrow{26^{12}}$   

No Bs	at least one B
-------	----------------

 $\xleftarrow{25^{12}}$

$26^{12} - 25^{12}$

(b) [1 mark] How many PINs have exactly **three** letters B and exactly **two** letters C?

$\frac{12!}{3!2!7!} \left\{ \begin{array}{l} \text{BBBCC} \text{ --- } 24^7 \\ \text{---C---C---B---BB---} 24^7 \\ \text{---CC---B---B---B} 24^7 \end{array} \right.$

$\frac{12!}{3!2!7!} (24)^7$

OR use combinations  
 $\binom{12}{3} \binom{9}{2} \cdot (24)^7$

select pos. for 3 B's, select pos. for 2 C's, the rest

Same

(c) [1 mark] How many PINs with exactly **two** X's, exactly **one** Y, and exactly **one** Z do not have them adjacent in any way?

any arrangement of 8 digits (no x, y, z)  $\rightarrow$  W A A C A B W T  $23^8$

select any 4 positions  $\rightarrow$  W A A C A B W T  $\binom{9}{4}$

arrange x, y, z over 4 spots  $\rightarrow$  W A A C A B W T  $\frac{4!}{2!}$

$\binom{9}{4} \cdot (23)^8 \cdot \frac{4!}{2!}$

2. [2 marks] What value does a variable *counter* have after the following code execution?

Do not simplify the expression.

```
counter = 100
```

```
for i = 1 to n {
```

```
    counter = 7
```

```
}
```

```
for j = 1 to 4n {
```

```
    for k = j + 1 to 4n + 15 {
```

```
        counter = counter + 8
```

```
    }
```

```
    counter = counter + 9
```

```
}
```

```
// assume  $n \geq 1$ 
```

← reset to 7

gap: pairs never made  
 $4n+1, 4n+2, \dots, 4n+15$

$$\text{counter} = 7 + \left[ \binom{4n+15}{2} - \binom{15}{2} \right] (8) + (4n)(9)$$