logarithms
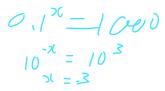
1. Without using your calculators (except to check your answers), determine or deduce the following logarithms. State the exponential relations which suggest or confirm your answers in each case.

(a) $\log_{0.1} 1000$

$$0.1^x = 1000$$
$$10^{-x} = 10^3$$
$$x = -3$$

(b) $\log_8 32$

$$8^x = 32$$
$$(2^3)^x = 2^5$$
$$3x = 5 \qquad x = \dfrac{5}{3}$$

(c) $\log_8 16$

$$8^x = 16$$
$$2^{3x} = 2^4 \qquad x = \dfrac{4}{3}$$

(d) $\log_{16} 8$

$$16^x = 8$$
$$2^{4x} = 2^3$$
$$x = \dfrac{3}{4}$$

2. Using only the information: $\log_b 2 = 0.3445$ $\log_b 5 = 0.8000$ $\log_b 7 = 0.9672$ to four decimal places, compute each of the following logarithms. You may use the basic properties of logarithms here, but you may not use a built-in logarithmic function key on your calculator. Show your work. Do not attempt to determine the value of the base b first.

$$\log_b ab = \log_b a + \log_b b$$

(a) $\log_b 35$

$$= \log_b 5 + \log_b 7 = 1.7672$$

(b) $\log_b 49$

$$= 2\log_b 7 = 1.9344$$

(c) $\log_b \left(\sqrt{10} \right)$

$$= \log_b 10^{\frac{1}{2}} = \dfrac{1}{2}\log_b 10 = \dfrac{1}{2}(1.1445) = 0.57225$$

(d) $\log_b 3.5$

$$= \log_b \left(7 \times \dfrac{1}{2} \right) = \log_b 7 - \log_b 2 = 0.6227$$

(e) $\log_b (10^{5.12})$

$$= 5.12 \log_b 10 = 5.12 (1.1445) = 5.85984$$

3. Use the properties of logarithms to simplify each of the following expressions. Your final answer in each case should be a single logarithm of an expression which is itself in simplest form:

(a) $\log_3(xy) - \log_3(xy^2) + 2\log_3 x$

$= \log_3 \frac{x\,y}{x\,y^2} \cdot x^2$

$= \log_3 \frac{x^2}{y}$

(b) $\log_{13}(16x^5) - 2\log_{13}(4x^2)$

$= \log_{13} \frac{16x^5}{16x^4} = \log_{13} x$

4. Solve the following exponential equations:

(a) $e^{4t+5} = 4872.6$

$\ln 4872.6 = 4t + 5$

(b) $8 + 3^{4x} = 17$

(c) $9\left(5^{3x-7}\right) = 4\left(3^{5x+2}\right)$

(d) $\dfrac{24}{3 - e^{2x}} = 107$

5. Use logarithms and floor or ceiling function to answer: How many bits are required to represent number N as unsigned integer?

Use the following for the two questions below

Exponential growth or decay function:
$Y = Y_0 e^{rt}$, where y is the amount of material at any time t, y_0 is the initial amount of material (when t = 0), and r is a positive number giving the fractional growth rate per unit time.

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6. Cesium-137 is a radioactive byproduct of nuclear reactors such as the one that experienced partial meltdown in Chernobyl (then in the USSR, but now in Ukraine) on April 26, 1986. Given that the half-life of Cesium-137 is 30.07 years and it decays exponentially over time,

(a) compute what fraction (or percent) of the Cesium-137 released at that time has since decayed away. You may assume that the elapsed time since the release of this material is 31.92 years for the purpose of this calculation.

(b) How many <u>more</u> years will it take for 95% of the Cesium-137 released in that event to have decayed away.