COMP 2121
DISCRETE MATHEMATICS

Lecture 10

Sample Space:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\Pr(sum \le 9) = \frac{30}{36}$$

$$\Pr(sum \le 9) = 1 - \Pr(sum > 9) = 1 - \frac{6}{36} = \frac{30}{36}$$

<u>Relationship (Counting, Venn Diagrams)</u>

Recall that for a finite set A, |A| denotes the number of elements of A and is referred to as the *cardinality*, or *size*, of A. For example, set A = {1, 2, 5,23} has a cardinality 4, i.e. |A| = 4.

Example 1. In a class of 50 college freshmen, 30 are studying C++, 25 are studying Java, and 10 are studying both languages. Draw Venn diagram and answer how many freshmen are studying C++ or Java. How many students are not studying any programming language?

Theorem: *If A and B are finite sets, then* $|A \cup B| = |A| + |B| - |A \cap B|$. *Consequently, finite sets A and B are mutually disjoint if and only if* $|A \cup B| = |A| + |B|$.

Example 2. How many 8-bit strings start with 1011 or end with 01?

Example 3. How many 8-bit strings start with 1011 or end with 01, but not both?

Example 4. How many 8-bit strings start with 1011 but do not end with 01?

Theorem:	*If A, B, C are finite sets, then* $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Example 5. A student visits an arcade each day after school and plays one game of either Pac-Man, Tetris, or Donkey Kong. In how many ways can he play one game each day so that he plays each of the three types at least once during a given school week?

Probability

Definition: *A sample space S is the set of all possible outcomes of a random process or experiment.*

For example, when rolling a single die the sample space is S = {1, 2, 3, 4, 5, 6}. Here we feel that each of six outcomes has the *same*, or *equal*, *likelihood* of occurrence.

The following definition is given under the assumption of equal likelihood:

Definition: *Let S be a sample space. Each subset A of S (including the empty set) is called an event. Each element of S determines an outcome, so if |S| = n and a $\in$ S, A $\subseteq$ S, then:*

Pr({a}) =the probability that {a} occurs = $\frac{1}{n}$

Pr(A) = the probability that A occurs= $\frac{|A|}{|S|} = \frac{|A|}{n}$

Observation: Pr(S)=1 and $\Pr(\overline{A}) = 1 - \Pr(A)$

Example 6. A Dillon rolls a fair die, what is the probability he gets (a) an even number, (b) a number different from number 3?

Example 7. What is the probability that a randomly selected 12-bit string starts with 1011 or ends with 011?

So far we were dealing with the assumption of equal likelihood. However, that is not a general case. In general, events are not necessarily equally likely. We will restrict our discussion to independent events.

If two events are independent, then probability of $A \cap B$ is:

$$Pr(A \cap B) = Pr(A)Pr(B).$$

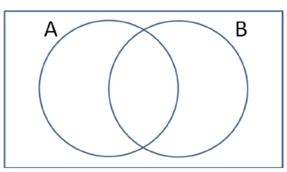
This is very intuitive, and it has analogy with the product rule. By the definition of intersection: $x \in A \cap B \Leftrightarrow x \in A \wedge x \in B$. The "key word" here is AND. We have learned that "AND" and multiplication are very much related.

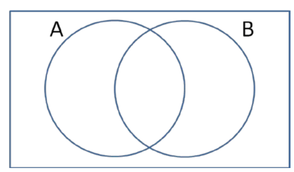
Also, the probability of union is very much related to the counting argument presented for cardinality of the union:

$$Pr\ (A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B).$$

Example 8. Printer is working 80% of time and a photocopier is working 70% of the time.

What percent of the time is at least one machine working?





What percent of the time is at least one machine down?

Example 9. In the communication network shown below, each link may be up or down. Assuming, that the nodes connecting the links are always functioning and that failures of the links are not related, what is the probability that functioning set of links is connecting node A to node C?