7 - Conf Intervals for p

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Confidence Intervals for p ← population proportion

Example Suppose you are trying to determine p, the proportion of students at BCIT who use an iPhone. You randomly select $n = 50$ students and determine that $x = 34$ use an iPhone.

a. What is the best point estimate of p, the population proportion of iPhone users? $\wedge$ = estimator

Sample proportion $\hat{p} = \frac{x}{n} = 34/50 = 0.68$

Our best available point estimate of p is 0.68.

b. What is the 95% confidence interval for p?

Formula: $\hat{p} - Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

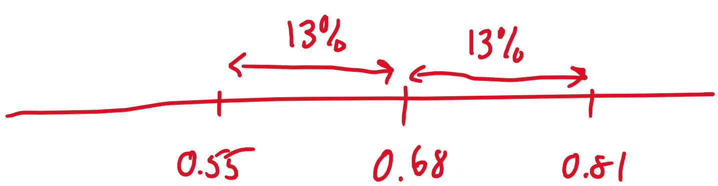
$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{0.68 \times 0.32}{50}} = 1.96\sqrt{0.004352}$

$= 0.1293 = 0.13$

lower limit $= \hat{p} - E = 0.68 - 0.13 = 0.55$

upper limit $= \hat{p} + E = 0.68 + 0.13 = 0.81$

Conclusion: We are 95% confident that the proportion of iPhone users among BCIT students is between 0.55 and 0.81.



To reduce E we need a larger n.

Why Does This Work?

assumption

Suppose the true proportion of iPhone users at BCIT is $p = 0.62$. (But suppose also that this information is *hidden* from us.) We randomly select $n = 50$ and determine $X =$ the number of iPhone users in the sample. Then:

success = iPhone
failure = not iPhone

- X is a binomial variable with
  - success $p = 0.62$ and
  - failure $q = 0.38$

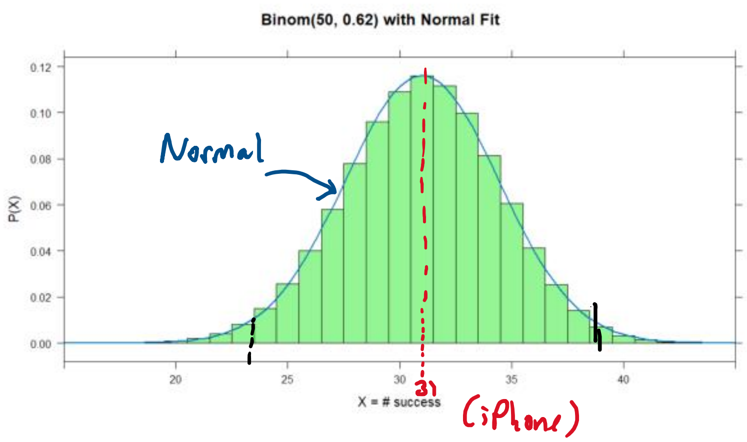
- The mean and standard deviation of X are:
  - $\mu = np =$ $50 \times 0.62 = 31$
  - $\sigma = \sqrt{npq} =$ $\sqrt{50 \times 0.62 \times 0.38} = 3.432$

- As a consequence of the Central Limit Theorem, the variable X is approximately *normally distributed* since:

$$np = 31 \geq 5$$

$$nq = 19 \geq 5$$

$\left.\begin{matrix} np \geq 5 \\ nq \geq 5 \end{matrix}\right\}$ Conditions for Normality



If X follows a normal distribution, then we know that there is a 95% probability that X has a Z-score between -1.96 and $+1.96$.

In other words:

$$-1.96 < \frac{X - np}{\sqrt{npq}} < 1.96 \quad \text{[with a 95\% probability]}$$

$$(*) \quad -1.96\sqrt{npq} < X - np < 1.96\sqrt{npq}$$

$$np - 1.96\sqrt{npq} < X < np + 1.96\sqrt{npq}$$

$$50\times 0.62 - 1.96\sqrt{50\times 0.62\times 0.38} < X < 50\times 0.62 + 1.96\sqrt{50\times 0.62\times 0.38}$$

$$31 - 6.727 < X < 31 + 6.727$$

$$24.273 < X < 37.727$$

We know 95% prob. that $\boxed{24.273 < X < 37.727}$

"usual values of X"

Back to (*)
Now solve for p

$$-1.96\sqrt{npq} < X - np < 1.96\sqrt{npq}$$

$$-X - 1.96\sqrt{npq} < -np < -X + 1.96\sqrt{npq}$$

$$\frac{-X}{-n} - \frac{1.96\sqrt{npq}}{-n} > p > \frac{-X}{-n} + \frac{1.96\sqrt{npq}}{-n}$$

$$\frac{X}{n} - \frac{1.96\sqrt{npq}}{n} < p < \frac{X}{n} + \frac{1.96\sqrt{npq}}{n}$$

$$\hat{p} - 1.96\sqrt{\frac{\cancel{n}pq}{n^{\cancel{2}}}} < p < \hat{p} + 1.96\sqrt{\frac{\cancel{n}pq}{n^{\cancel{2}}}}$$

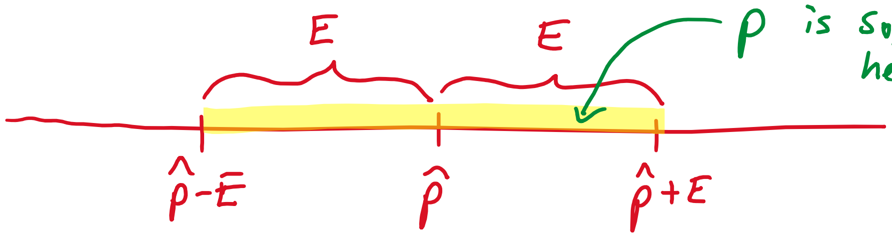
use $p \approx \hat{p}$, $q \approx \hat{q}$

$$\hat{p} - 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$Z_{\alpha/2} = 1.96$
for $\alpha = 0.05$
$1-\alpha = 0.95$

or

$$\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$



p is somewhere in here [95% probability]