

1. Determine the truth value of the following statements (T or F) and justify your answer.
(hint: you can use counterexample to disprove the statement)

The universe for x is numbers: 8, 6, 4, 2 and 0.

The universe for y is numbers: -7, -5, -3, and -1.

a) $\exists x \exists y$ such that $-30 < x \cdot y < -5$

b) $\forall x \forall y$ such that $x \cdot y < x + y$

c) $\forall x \exists y$ such that $x + y = 1$

d) $\forall y \exists x$ such that $x + y = 1$

e) $\exists x \forall y$ such that $x \cdot y > -10$

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SELECT DISTINCT s.name
FROM student s
WHERE FORALL
  (SELECT *
   FROM requirement r
   WHERE r.major='CS'
   AND EXISTS
     (SELECT *
      FROM enrollment e
      WHERE e.student_id=s.student_id
      AND e.course_id=r.course_id));
```



2. Determine the truth value of the following statements (T or F) and justify your answer.

Sets A, B and C are defined as follows:

$$A = \{1, 2, 3\} \quad B = \{-1, -2, -3\} \quad C = \{-1, -2, -4, -9\}.$$

a) $\forall x \in A, \forall y \in B, \text{ then } x \cdot y \in C$

b) $\forall x \in A, \exists y \in B, \text{ then } x \cdot y \in C$

c) $\exists y \in B, \forall x \in A, \text{ then } x \cdot y \in C$

d) $\exists z \in C, \forall y \in B \text{ such that } z \cdot y \notin A$

e) $\exists u \in A, \exists v \in A \text{ such that } u \cdot v \in C$

3. Determine the truth value of the following statements (T or F) and justify your answer.

The universe for x is $\{-4, -2, -1\}$.

The universe for y is $\{-4, -2, 4, 6\}$

(a) $\forall x, \forall y, 0 < |x - y| < 6$

(b) $\exists x, \exists y, 0 < |x - y| < 6$

(c) $\forall x, \exists y, 0 < |x - y| < 6$

(d) $\exists x, \forall y, 0 < |x - y| < 6$

4. Determine the following:

a) $X \cup Y \cup Z$

b) $Y \cap Z$

c) $Y - (X \cup Z)$

d) $\overline{(X - Z)} \Delta Y$

$$X = \{1, 2, 3, 4\}$$

$$Y = \{1, 3, 7\}$$

$$Z = \{4, 6\} \text{ and}$$

$$\text{universe } U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

Determine the truth value of the following statements (T or F) and justify your answer.

e) $\exists p \in U, \forall q \in X \Delta Y, \text{ then } 1 \leq |p - q| \leq 3$

$$X = \{1, 2, 3, 4\}$$

$$Y = \{1, 3, 7\}$$

$$Z = \{4, 6\} \text{ and}$$

$$\text{universe } U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

f) $\forall t \in \overline{X \cup Y}, \exists w \in X - Z, \text{ then } t - w \in \overline{X \cap Y \Delta Z}$

g) $\exists a \in X - Z, \forall b \in \overline{X \cap Y \Delta Z}, \text{ then } -a + b \notin \overline{X \cap Y \Delta Z}$

