COMP 2121 DISCRETE MATHEMATICS	# Assignment 1 Fall 2024

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Section	Total	Actual
Question 1	40	
Question 2	15	
Question 3	10	
Question 4	10	
Question 5	10	
Total	85	

✓	Instructions
☐	• Assignment must be done using Microsoft Word or an alternative word processor – **type** your work in this document. • Handwritten assignments will not be marked. • The header of every page has math templates and logic symbols that are needed. You can copy them into your text
☐	• The assignment must be done in a **group** of two students – no individual assignments will be accepted.
☐	• Just the answer will not give you credit for a problem. • When you solve a problem, you must provide necessary **explanations** – yes this means explanations in English. Normally one paragraph is sufficient, but it may take more depending on a question.
☐	• Do not evaluate final answer unless the question asks you to do that. Leave it as a **formula**, following the format in lectures and labs.
☐	• **PRINT** the completed assignment – you are handing in a paper copy.
☐	• **Due** at the beginning of the **Lecture** on **October 2, 2024**. • No late assignments will be accepted. • Electronic copies will not be accepted.

$\frac{a}{b}$ x^y $\binom{n}{k}$ ¬ ∧ ∨ → ∴

Q1) The System Administrator has set the following rules for the password:

- ✓ The password is a string made of 14 characters.
- ✓ The available characters are Hexadecimal digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.
- ✓ Repetition is allowed unless otherwise stated.

*** Do not evaluate expressions ***

a) How many passwords have exactly seven A's and at least five B's?

Explanation:

If B = 5:

C(14, 7) × C(7,5) × 14^2

If B = 6:

C(14, 7) × C(7, 6) × 14

If B = 7:

C(14, 7) × C(7, 7)

= C(14, 7)

Total: C(14, 7) × C(7,5) × 14^2 × C(14, 7) × C(7, 6) × 14 × C(14, 7)

= C(14, 7) × [C(7,5) × 14^2 + C(7, 6) × 14 + 1]

Answer:

$\boldsymbol{\binom{14}{7}[\binom{7}{5} \times 14^2 + \binom{7}{6} \times 14 + 1]}$

b) How many passwords have exactly two A's and exactly three B's, so that the three B's are sandwiched between the A's? 010**A**1 **B**8**BB** 3112**A** is an example of such a string.

Explanation:

Assume first A at i, second A at j (i > j)

∵ 3B inside need at least 4 bits,

∴ i ∈ [1,10], j ∈ [i+4,14]

∴ i has 10 – 1 + 1 = 10 possible bit, j has 14 – i – 4 +1 = 11 – i

∴ **The total possibility of A's position has**

$\sum_{i=1}^{10}(11-i)$

$S_n = \frac{n}{2} \times (a_1 + a_n),\ a_1 = 11 - 1 = 10, a_{10} = 11 - 10 = 1$

S_{10}=$\frac{10 \times (10+1)}{2}$ = **55**

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∵ The bits between i and j is

j – i – 1

also ∵ j – i ∈[4, 13]

∴ j – i – 1 ∈ [3, 12]

∴ **The total possibility of B's position has:**

C(j – i - 1, 3)

∵ j – i – 1 ∈ [3, 12]

∴ C(j – i - 1, 3)

$\frac{a}{b}$ x^y $\binom{n}{k}$ ¬ ∧ ∨ → ∴

$=\sum_{n=3}^{12} C(n,3)$
= 1 + 4 + 10 + 20 + 35 + 56 + 84 + 120 + 165 + 220
= **715**

∵ The remaining position has
14 – 2 – 3 = 9

∴ **The possibility of remaining position has:**
$\mathbf{14^9}$

∴ **The total possibility has:**
55 × **715** × $\mathbf{14^9}$

Answer:
55 × **715** × $\mathbf{14^9}$

c) How many passwords have at least one A, at least one B, at least C, and have no other characters?

Explanation:
Total possibility:
3^{14}
Only 2 of them:
3×2^{14}
Only 1 of them:
3 × 1
Total:
$\mathbf{3^{14} - 3 \times 2^{14} - 3}$

Answer:
$\mathbf{3^{14} - 3 \times 2^{14} - 3}$

d) How many passwords with exactly 3 B's, have the sum of all digits equal to 40 and have no adjacent B's? Examples of such passwords are 00230 **B**11**B** 000**B**0, **B**1101 0**B**00 00**B**04, etc.

Explanation:
∵ 3 B, total 14 bits
∴ has 14 – 3 = 11 bits to put non-B chars
∵ No adjacent B
∴ has 11 + 1 = 12 bits to put B
C(12, 3)
∵ The sum of all digits equal to 40
Also ∵ 3 × B = 33
∴ Remaining value 40 - 33 = 7
∴ available number ∈ [0, 7]
∴ Possible situations:

$\frac{a}{b}$ x^y $\binom{n}{k}$ ¬ ∧ ∨ → ∴

7 × 1 and 4 × 0:	**C(11, 7) = 330**
1 × 7 and 10 × 0:	**C(11, 1) = 11**
1 × 6, 1 × 1 and 9 × 0:	**C(11, 1) × C(10, 1) = 11 × 10 = 110**
1 × 5, 2 × 1 and 8 × 0:	**C(11, 1) × C(10, 2) = 11 × 45 = 495**
1 × 5, 1 × 2 and 9 × 0:	**C(11, 1) × C(10, 1) = 11 × 10 = 110**
1 × 4, 1 × 3 and 9 × 0:	**C(11, 1) × C(10, 1) = 11 × 10 = 110**
1 × 4, 1 × 2, 1 × 1 and 8 × 0:	**C(11, 1) × C(10, 1) × C(9, 1) = 11 × 10 × 9 = 990**
1 × 4, 3 × 1 and 7 × 0:	**C(11, 1) × C(10, 3) = 11 × 120 = 1320**
2 × 3, 1 × 1 and 8 × 0:	**C(11, 2) × C(9, 1) = 55 × 9 = 495**
1 × 3, 2 × 2 and 8 × 0:	**C(11, 1) × C(10, 2) = 11 × 45 = 495**
1 × 3, 1 × 2, 2 × 1 and 7 × 0:	**C(11, 1) * C(10, 1) × C(9, 2) = 11 × 10 × 36 = 3960**
1 × 3, 4 × 1 and 6 × 0:	**C(11, 1) × C(10, 4) = 11 × 210 = 2310**
3 × 2, 1 × 1 and 7 × 0:	**C(11, 3) × C(8, 1) = 165 × 8 = 1320**
2 × 2, 3 × 1 and 6 × 0:	**C(11, 2) × C(9, 3) = 55 × 84 = 4620**
1 × 2, 5 × 1 and 5 × 0:	**C(11, 1) × C(10, 5) = 11 × 252 = 2772**

∴ Total possibility

C(12,3)×(sum of these numbers)

Answer:

C(12,3)×(sum of these numbers)

$$\frac{a}{b} \quad x^y \quad \binom{n}{k} \quad \neg \quad \wedge \quad \vee \quad \rightarrow \quad \therefore$$

Q2) Consider the following programming segment. Your answer must rely on a combination structure. Answers that use sigma notation will not be accepted.

```
counter = 100
for i = 4 to (n+3) do {
      counter = counter + 11
      for j = i+1 to (3n+15) do {
            counter = counter + 22
            for k = j+1 to (n+8) do {
                  counter = counter + 33
            }
      }
}
// assume n ≥ 10
```

a) Determine the value of the variable counter after the segment is executed. Provide your answer as a function of n (i.e., a formula which depends on n). Make sure to explain how/why the parts of the formula relate to counting.

Explanation:

1. $(n+3)-4+1=n$
   += $11n$
2. $C(3n+15-4,2)-C(3n+15-n-3,2)\times 22$
   += $22[C(3n+12,2)-C(2n+12,2)]$
3. += $33[C(n+5,3)-C(5,3)]$

   counter = $100+11n+22[C(3n+12,2)-C(2n+12,2)]+33[C(n+5,3)-C(5,3)]$

Answer:

counter = $\mathbf{100+11n+22[C(3n+12,2)-C(2n+12,2)]+33[C(n+5,3)-C(5,3)]}$

b) Evaluate your answer in part a) for n = 50. Show the work

counter = $100+11\times 50+22[C(3\times 50+12,2)-C(2\times 50+12,2)]+33[C(50+5,3)-C(5,3)]$

$\frac{a}{b}$ x^y $\binom{n}{k}$ $\neg$ $\wedge$ $\vee$ $\rightarrow$ $\therefore$

$= 100 + 550 + 22 \times (13041 - 6216) + 33 \times (26235 - 10)$

= 100 + 550 + 150150 + 865425

= **1016225**

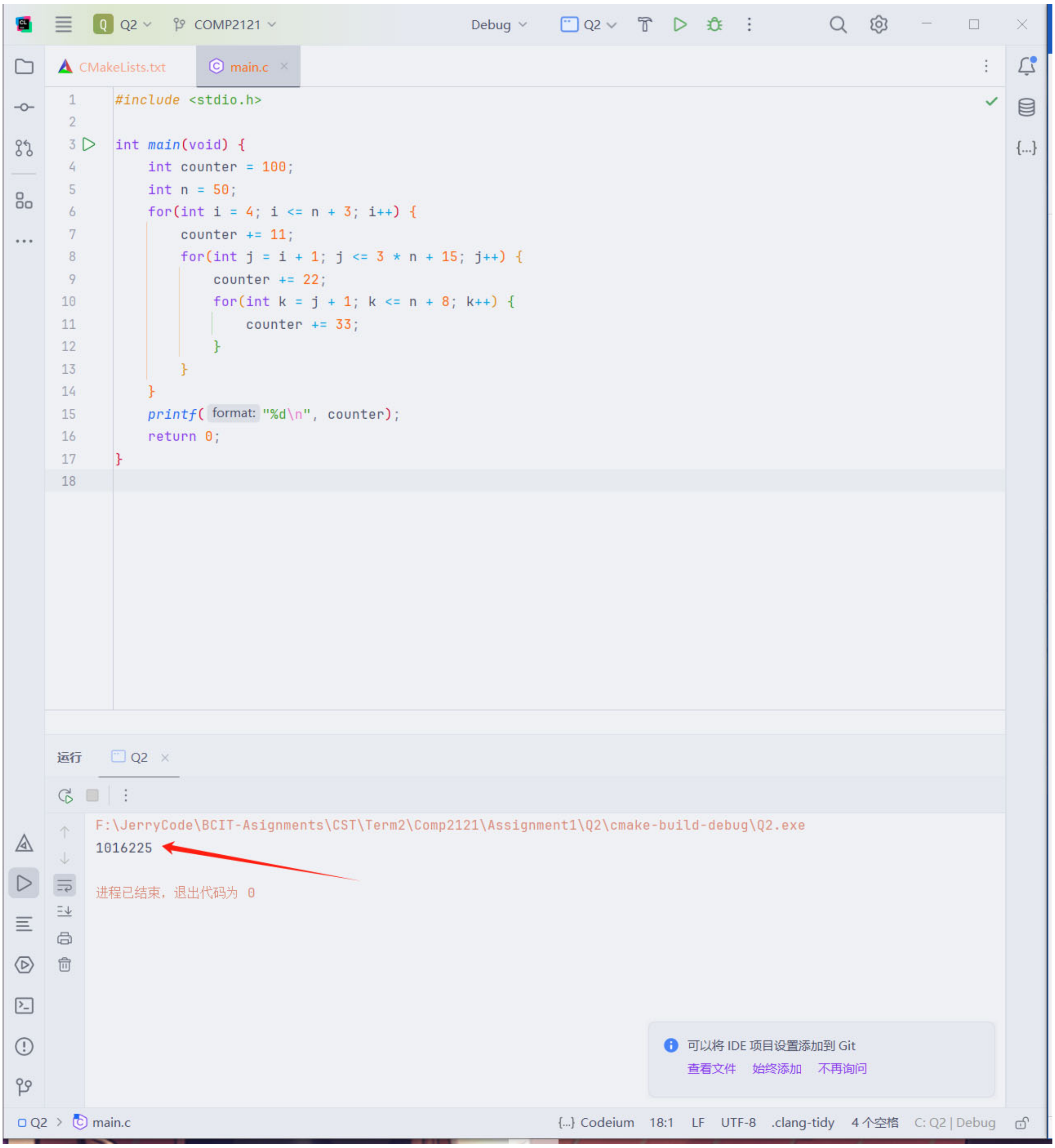
c) Check your answer in part b) by implementing the code in a programming language of your choice. Use the value n = 50 and print the variable `counter` after the code execution. You must provide two screenshots: implementation and output.

```c
#include <stdio.h>

int main(void) {
    int counter = 100;
    int n = 50;
    for(int i = 4; i <= n + 3; i++) {
        counter += 11;
        for(int j = i + 1; j <= 3 * n + 15; j++) {
            counter += 22;
            for(int k = j + 1; k <= n + 8; k++) {
                counter += 33;
            }
        }
    }
    printf( format: "%d\n", counter);
    return 0;
}

```

$\frac{a}{b}$ x^y $\binom{n}{k}$ $\neg$ $\wedge$ $\vee$ $\rightarrow$ $\therefore$



e) What do you conclude?
Nested loops are complicated; computers and programming languages are truly great inventions.

$\frac{a}{b}$ x^y $\binom{n}{k}$ ¬ ∧ ∨ → ∴

Q3) Use the truth table to show that the following argument is NOT valid.

Clearly, **1) indicate in red/bold in the table** what makes you come to that conclusion, and then **2) explain your answer below the table**

$(a \vee b) \rightarrow (a \rightarrow c)$

____

$\therefore a \vee c$

a	b	c	a∨b	(a∨b)→(a→c)	a∨c
0	0	0	0	**1**	**0**
0	0	1	0	1	1
0	1	0	1	**1**	**0**
0	1	1	1	1	1
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Explanation:

In some cases, when (a∨b)→(a→c) is 1, a∨c is 0.

$\frac{a}{b}$ x^y $\binom{n}{k}$ ¬ ∧ ∨ → ∴

Q4) Use rules of inference and <u>direct proof</u> to prove the argument is valid.

$q \vee \neg t$

$(w \vee \neg a) \rightarrow (\neg q \wedge k)$

t

$(\neg w \wedge t) \rightarrow x$

____

$\therefore x$

STEPS		REASON
1	$q \vee \neg t$	Premise
2	t	Premise
3	q	Rule of Disjunctive Syllogism(step1,3)
4	$(w \vee \neg a) \rightarrow (\neg q \wedge k)$	Premise
5	$\neg(\neg q \wedge k)$	Step 3
6	$\neg(w \vee \neg a)$	Modus Ponens
7	¬w ∧ a	DeMorgan's Law(step 6)
8	$\neg w$,a	Conjunction
9	¬w ∧ t	Conjunction(step1,6)
10	$(\neg w \wedge t) \rightarrow x$	Premise
11	x	Modus Ponens

$\frac{a}{b}$ x^y $\binom{n}{k}$ $\neg$ $\wedge$ $\vee$ $\rightarrow$ $\therefore$

Q5) Use rules of inference and proof by contradiction to prove the argument is valid.

Note: contradiction pointing to x and $\neg x$ will not be accepted. In other words, if you start with $\neg x$, then independently prove x (which is Q4), and say lines x and $\neg x$ are in contradiction – proof will not be accepted. Instead, follow instructions from the lecture/lab.

$q \vee \neg t$

$(w \vee \neg a) \rightarrow (\neg q \wedge k)$

t

$(\neg w \wedge t) \rightarrow x$

____

$\therefore x$

STEPS		REASON
1	$\neg x$	Assume
2	t	Premise
3	$(\neg w \wedge t) \rightarrow x$	Premise
4	$\neg(\neg w \wedge t)$	Modus tollens(step1,3)
5	w ∨ ¬t	DeMorgan's Law
6	w	Rule of disjunctive syllogism
7	$(w \vee \neg a) \rightarrow (\neg q \wedge k)$	Premise
8	$(w \vee \neg a)$	Step 6
9	$(\neg q \wedge k)$	Modus Ponens
10	$\neg q$	Rule of conjunction
11	$q \vee \neg t$	Premise
12	$\neg t$	Rule of disjunctive syllogism
13	x	Contradiction $\neg t$ and t