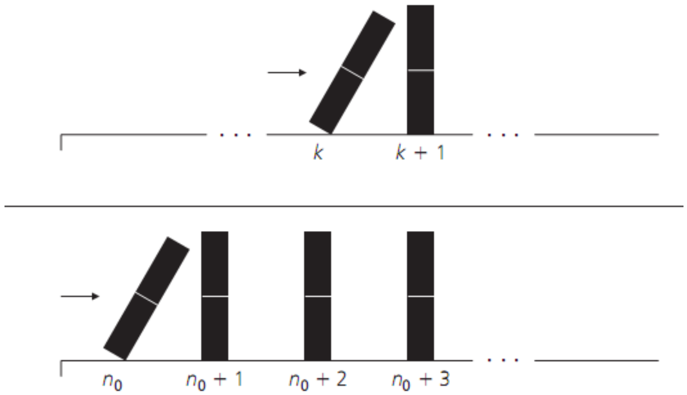
COMP 2121
DISCRETE MATHEMATICS

Lecture 13



<u>Mathematical Induction</u>

Mathematical induction is one of the more recently developed techniques of proof in the history of mathematics (used by Francesco Maurolico – 1575, Pierre de Fermat and Blaise Pascal and defined by Augustus De Morgan in 1883).

In natural sciences, deduction and induction are presented as alternative modes of thought: *deduction* being to infer a conclusion from general principles, *induction* being to enunciate a general principle after observing it to hold in a large number of specific instances.

Mathematical induction is a mathematical method to prove that a certain principle holds in processes that occur repeatedly and according to definite patterns.

> PRINCIPLE OF MATHEMATICAL INDUCTION. Let S(n) be a statement that is defined for integers *n*, and let *a* be a fixed integer. Suppose that the following two statements are true:
>
> 1. S(a) is true
> 2. For all integers k ≥ a, if S(k) is true, then S(k+1) is true.
>
> Then, the statement "*For all integers n≥ a, S(n)*" is true.

Hence, proving a statement by mathematical induction is a **two-step process**:

- ✓ <u>Step 1</u> is named the **basic step**. In step 1 we verify S(a) is true.
- ✓ <u>Step 2</u> is named the **inductive step**. First we assume that S(k) is true (we call this *inductive hypothesis*). Then we prove that S(k+1) is true.

Principle of Mathematical Induction Stated Formally is:

$$[S(a) \wedge \forall k \geq a\ (S(k) \rightarrow S(k+1))] \rightarrow \forall n \geq a\ (S(n))$$

Example 1. Sum of the first n integers.

For all $n \geq 1$, $$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

Mathematical induction can be used to prove more than sequences. Divisibility, program correctness, number of steps in algorithm can be proved using mathematical induction.

Example 2. For all integers n ≥ 1, $5^n+2^{n+1} = 3q$, for some integer q.

Note: This means that 5^n+2^{n+1} is divisible by 3.

Example 3. What does the following function return? Can you prove that by mathematical induction?

```
MysteryFunction (positive integer n)

    i = 1
    j = 4
    while i ≠ n do
        j = j + 2i + 3
        i = i + 1
    end while

    return j
```