

3 – Probability - Practice Problems

1. In an engineering statistics course, the instructor awarded 30 As, 40 Bs, 35 Cs, 15 Ds and 5 Fs.
 - a. What is the probability of getting an A?
 - b. What is the probability of getting a C or better?
2. The probability that a reckless driver will be fined, get his license revoked or both are, respectively 0.88, 0.60 and 0.55. What is the probability that this driver will be fined or get his license revoked?
3. A carton of 24 light bulbs includes 3 that are defective. If two of the bulbs are chosen at random, what are the probabilities that
 - a. neither bulb is defective
 - b. exactly one of the bulbs is defective
 - c. both bulbs are defective.
4. In the quality control of the production of glass blocks, we find the probability that a glass block is cracked is 0.004, that it has air bubbles is 0.006, and that it is discoloured is 0.009. We also know the appearance of cracks, air bubbles and discolouration are independent. What is the probability that an inspector will find a block that:
 - a. is both cracked and has air bubbles?
 - b. is either cracked or has air bubbles?
 - c. is cracked, has air bubbles, and is discoloured?
 - d. is cracked or has air bubbles or is discoloured (“or” being “exclusive or”)?
5. A machine produces components for cellular phones. At any given time, the machine is in exactly one of three states:
 - Operational
 - Out of control
 - DownExperience shows that the probability that the machine will be out of control at any moment is 0.02 and that probability that the machine is down is 0.015.
 - a. The machine can be used to produce a single item unless the machine is down. What is the probability that the machine can produce a single item?
 - b. A repair person is called when the machine is not operational. What is the probability that a repair person is called?
6. A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is 0.95; by device B, 0.98; and by both devices, 0.94. If smoke is present, find the probability that the smoke will be detected by device A or device B.

7. The probability that a certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for 1 year is 0.99. What is the probability that a new component will last 1 year? (i.e., The component must first not fail immediately *and* then last the year.)
8. A fire detection device uses three temperature-sensitive cells acting independently of one another in such a manner that any one or more can activate the alarm. Each cell has a probability of 0.8 of activating the alarm when the temperature reaches 45°C. Find the probability that the alarm will function when the temperature reaches 45°C.
9. Assume that a certain batch of 200 castings contains 5 defectives. Calculate the probability that
- a single randomly selected casting will be defective
 - of three castings selected, all will be defective
 - of three castings selected, exactly one will be defective
 - of two castings selected, at least one will be defective
10. A labour management organization wants to study the problem of workers displaced by automation within the industrial engineering field. Case reports for 100 workers whose loss of job is directly attributable to technological advances are selected within the industry. For each worker selected, it is determined whether he or she was given another job within the same company, found a job with another company in industrial engineering, found a job in a new field, or has been unemployed for longer than 6 months. In addition, the union status (union or non-union) of each worker is recorded, with the results shown in the table.

Job status Union status	Same Company	New Company (same field)	New Field	Unemployed
Union	41	12	4	1
Non-Union	16	9	11	6

Suppose a worker is randomly selected from those surveyed.

- What is the probability that a selected worker is a union member?
- What is the probability that a selected worker is a union member and was given a job with the same company? What is the probability that a selected worker is a non-union member and was given a job in the same company?
- If the selected worker found a job with a new company in the same field, what is the probability that the worker is a union member?
- If the worker is not a union member, what is the probability that the worker has been unemployed for longer than 6 months?

11. **(Past Test Question)** A storage bin contains 200 injection molded parts from 4 cavities. One of the 4 cavities has a problem which causes all the parts it produces to be defective. If two of the 200 parts are randomly selected from the bin, what is the probability that:

- a. both are from the faulty cavity (i.e. both defective)
- b. only one is from the faulty cavity

12. **(Past Test Question)** Four machines, a drill, a lathe, a grinder and a miller operate independently of each other. Their utilizations currently are:

Drill 52 %	Miller 68 %
Lathe 60 %	Grinder 85 %

- a. What is the chance of all of the machines being idle? (i.e. not in use)
- b. What is the probability of either the Drill or the Lathe (or both) being utilized?
- c. What is the probability that at least two of the machines will be utilized?

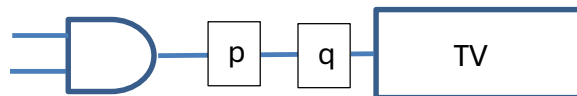
13. It is required that commercial aircraft have two independent radios. Assume that for a typical flight, the probability of radio failure is 0.002. What is the probability that both radios will fail on a particular flight?

14. When testing for current in a cable with five colour-coded wires, an electrician used a meter to test two wires at a time. How many different tests are required for every possible pairing of two wires?

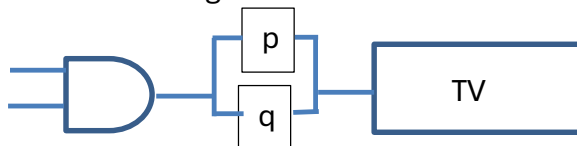
15. A lot consists of 10 articles. 6 of the articles have defects and the rest are good. Two articles are selected at random. Determine the probability that both have defects.

16. Two surge protectors p and q are used to protect an expensive television. If there is a surge in the voltage, the surge protector reduces it to a safe level. Assume that each surge protector has a 99% chance of working properly when a voltage surge occurs.

- a. If two surge protectors are arranged in series, what is the probability that a voltage surge will not damage the television?



- b. If two surge protectors are arranged in parallel, what is the probability that a voltage surge will not damage the television?



17. A manufacturer of mining safety equipment plans to ship 15 gas detectors to a customer, and as a precaution, orders that a sample of 3 of the detectors be inspected and checked. All 3 were found to be satisfactory and so the entire 15 detectors were shipped to the customer. The customer immediately put all 15 detectors to use and discovered that 2 of them were actually defective. What is the probability that the 3 detectors checked will include no defective units, when in fact 2 of the 15 detectors are defective?
18. An online service randomly generates 8-letter passwords. What is the probability that such a password consists of 8 *different* letters (assume 26 letters in alphabet)?
19. A quality control engineer inspects a random sample of 3 batteries from each lot of 24 car batteries ready to be shipped. If such a lot contains 6 batteries with slight defects, what are the probabilities that the inspector's sample will contain none of the batteries with defects?
20. An expert shot hits a target 95% of the time. What is the probability that the expert will miss the target for the first time on the 15th shot?
21. Ten horses are in a race. If you randomly guess the first, second, and third place finishers, what is the probability you were correct?
22. Suppose you have not attended classes or done any homework for a course in which you are to write a ten-question multiple choice test where each question has four choices. Therefore, you have to guess on every question and have a $1/4$ chance of getting each question correct.
- What is the probability you get none of the answers correct?
 - What is the probability you get all of the questions correct?
23. When testing blood samples for HIV, the procedure can be made more efficient and less expensive by combining samples. If samples from three people are combined and test negative, no further testing is needed. If the combined sample tests positive, three individual tests are done. The probability of an at-risk person being HIV-positive is 0.1. What is the probability that a combined sample from three at-risk people tests positive?
24. In Mathtown, there are two brands of bicycles: Binomial Bikes and Calculus Cycles. 80% the town's cyclists ride Binomial Bikes and 20% ride Calculus Cycles. Furthermore, 5% of people who ride Binomial Bikes and 8% of people who ride Calculus Cycles bring their bikes into the local shop for service. What is the probability that a randomly-chosen bike that has been brought in for service is a Binomial Bike?

25. If a person has recently been sick with COVID-19, their blood contains antibodies that make the person immune to contracting the virus again. Serology testing can be used to determine the presence of antibodies. This has the potential to be very useful, since many people who contract COVID-19 have no symptoms.

The testing method, of course, is not perfect. A particular serology test has a sensitivity of 90%; that is, 90% of people who have recently been sick with COVID-19 “test positive” (i.e., the test detects the presence of antibodies). The specificity of the test is 92%; that is, 92% of people who have not recently been sick with COVID-19 “test negative” (i.e., the test does not detect the presence of antibodies). It is known that 5% of people have been sick with COVID-19.

- a. Suppose a person tests positive for COVID-19 antibodies. What is the probability the person has recently been sick with COVID-19?
- b. Suppose a person tests negative for COVID-19 antibodies. What is the probability that the person has not recently been sick with COVID-19?

Hint: Define events

A = person tests positive for COVID-19 antibodies

B = person's blood actually contains COVID-19 antibodies

Answers

- 1a. 0.2400
- 1b. 0.8400
- 2. 0.9300
- 3a. 0.7609
- 3b. 0.2283
- 3c. 0.0109
- 4a. 0.000024
- 4b. 0.009976
- 4c. 2.16×10^{-7}
- 4d. 0.0188
- 5a. 0.985
- 5b. 0.035
- 6. 0.9900
- 7. 0.8910
- 8. 0.9920
- 9a. 0.025
- 9b. 7.61×10^{-6}
- 9c. 7.20×10^{-2}
- 9d. 0.0495
- 10a. 0.58
- 10b. 0.41 and 0.16
- 10c. 0.5714
- 10d. 0.1429
- 11a. 0.06156
- 11b. 0.3769
- 12a. 0.009216
- 12b. 0.808
- 12c. 0.895168
- 13. 0.000004
- 14. 10
- 15. 0.3333
- 16a. 0.9999
- 16b. 0.9801
- 17. 0.6286
- 18. 0.3016
- 19. 0.4032
- 20. 0.02438
- 21. 0.001389
- 22a. 0.05631
- 22b. 9.536E-7
- 23. 0.271
- 24. 0.7143
- 25a. 0.3719
- 25b. 0.9943