

## 4 - Discrete Probability Distributions – Practice Problems

1. Engineers are trying to determine where to locate a mine. At one site, 70% of the ore samples are below the minimum size needed for studying. If we randomly select 4 ore samples from all the available samples at this site, what is the probability that:
  - a. all four samples will be undersized?
  - b. none of the samples will be undersized?
  - c. Three or fewer of the samples will be undersized?
2. Based on health data, there is a 0.9989 chance that a randomly-selected 30-year-old male will live through the year and a 0.9971 that a randomly-selected 50-year-old female will live through the year.
  - a. A life insurance company charges a 30-year-old male \$180 for insuring his life for the year. If he dies, the policy pays out \$100,000 as a benefit. What is the expected cost of the insurance policy?
  - b. How much should the company charge a 50-year-old female on a \$75,000 life insurance policy in order for it to have the same expected cost as the 30-year-old male's \$100,000 policy?
3. Suppose you have not attended classes or done any homework for a course in which you are to write a ten-question multiple choice test where each question has four choices. You have to guess on every question and have a  $1/4$  chance of getting each question correct.
  - a. What is the probability you get none of the answers correct?
  - b. Suppose you need a mark of 8/10 or better in order to pass. What is the probability you will pass the test?
4. An airline has a policy of overbooking flights. The random variable  $X$  gives the probability that a flight cannot be boarded because there are more passengers than seats. Its probability distribution is given in the table on the right. What is the expected number of passengers who will not be boarded?

$X$	$P(X)$
0	0.067
1	0.141
2	0.274
3	0.331
4	0.187
5. A student sends job applications one at a time until he gets an interview. Each employer has a 15% chance of granting the student an interview.
  - a. How many applications should the student expect to send out?
  - b. What is the probability the student's fifth application is the one that results in an interview?
  - c. What is the probability the student will have to send out three or more applications?

6. In one year, there were 116 homicide deaths in Richmond, Virginia. For a randomly-selected day, find the probability that the number of homicide deaths was:
- 0
  - 1
  - 2
  - 3
  - 4

(The actual data was as follows: 268 days (73.4% of days) with no homicides; 79 days (21.6%) with 1 homicide; 1 day with 3 homicides (0.294%); no days with more than 3 homicides.)

7. A Company produces a wood product that is shipped in lots of 20. To ensure quality 5 of every 20 products produced is examined for a minimum strength. Suppose that this lot has 4 pieces that do not meet the minimum strength requirement.
- What is the probability that the inspector will find exactly one part that does not meet the minimum strength requirement?
  - What is the probability that the inspector will find at most one part that does not meet the minimum strength requirement?
  - What is the probability that the inspector will find at least one part that does not meet the minimum strength requirement?
8. A quality control engineer inspects a random sample of 3 batteries from lots of 24 car batteries that are ready to be shipped. If such a lot contains 6 batteries with defects, what are the probabilities that the inspector's sample will contain:
- None of the batteries with defects.
  - Only one of the batteries with defects
  - At least two of the batteries with defects.
9. What is the probability that a tax auditor will catch at least 2 income tax returns with illegitimate deductions, if she randomly selects 6 returns from among the 18 returns on her desk, of which 8 actually contain illegitimate deductions?
10. A newborn baby is considered to have a low birth weight if it weighs less than 2.5kg. Dutchess County, New York has been experiencing a mean of 210 low birth weight babies born every year.
- Find the probability that on a given day, there is more than one baby born with low birth weight.
  - Would it be unusual to have two babies with low birth weight born in a day?

11. If you buy a ticket on the Lotto 6/49, what are your chances of winning \$10? To win \$10 you must match 3 of the 6 (distinct) numbers drawn.
12. Radioactive atoms are unstable because they have too much energy. When they release their extra energy, they are said to decay. When studying cesium-137, a nuclear engineer found that over 365 days, 1,000,000 radioactive atoms decayed to 977,287 radioactive atoms. Find the probability that on a given day, 50 radioactive atoms decayed.
13. Consider customers arriving at a cafeteria at an average rate of 0.3 per minute.
- Find the probability that exactly 2 customers arrive in a 10 minute span.
  - Find the probability that 2 or more customers arrive in a 10 minute span.
  - Find the probability that exactly one customer arrives in a 5 minute span and one customer arrives in the next 5 minute span.
14. A lot contains 50 items, 6 of which are defective.
- What is the probability that a random sample of 5 items from the lot will contain no defective items?
  - What is the probability that a random sample of 5 items will contain not more than one defective?
  - What is the probability that a random sample of 5 items will contain more than two defectives?
15. A manufacturer of mining safety equipment plans to ship 15 gas detectors to a customer, and as a precaution, orders that a sample of 3 of the detectors be inspected and checked. All 3 were found to be satisfactory and so the entire 15 detectors were shipped to the customer. The customer immediately put all 15 detectors to use, and discovered that 2 of them were actually defective. What is the probability that the 3 detectors checked will include no defective units, when in fact 2 of the 15 detectors are defective?

## Answers

- 1a. 0.2401
- 1b. 0.0081
- 1c. 0.7599
- 2a. -\$70
- 2b. \$287.50
- 3a. 0.05631
- 3b.  $4.158 \times 10^{-4}$
- 4. 2.43
- 5a. 6.667
- 5b. 0.0783
- 5c. 0.7225
- 6a. 0.728
- 6b. 0.231
- 6c. 0.0368
- 6d. 0.00389
- 6e. 0.000309
- 7a. 0.4696
- 7b. 0.7513
- 7c. 0.7183
- 8a. 0.4032
- 8b. 0.4536
- 8c. 0.1432
- 9. 0.8801
- 10a. 0.113
- 10b. no ( $|z| < 2$ )
- 11. 0.01765 or 1 in 56.6 chance
- 12. 0.0155
- 13a. 0.2240
- 13b. 0.8009
- 13c. 0.1120
- 14a. 0.5126
- 14b. 0.8970
- 14c. 0.0093
- 15. 0.6286