# COMP 3721
# Introduction to Data Communications

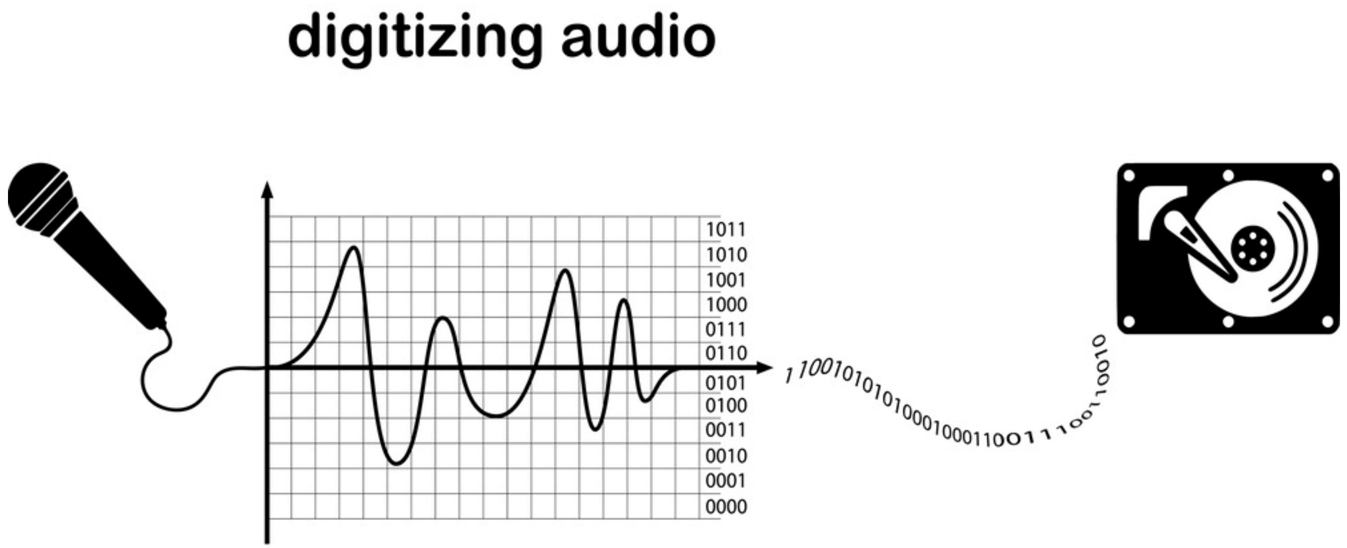
## 05a - Week 5 - Part 1

# Learning Outcomes

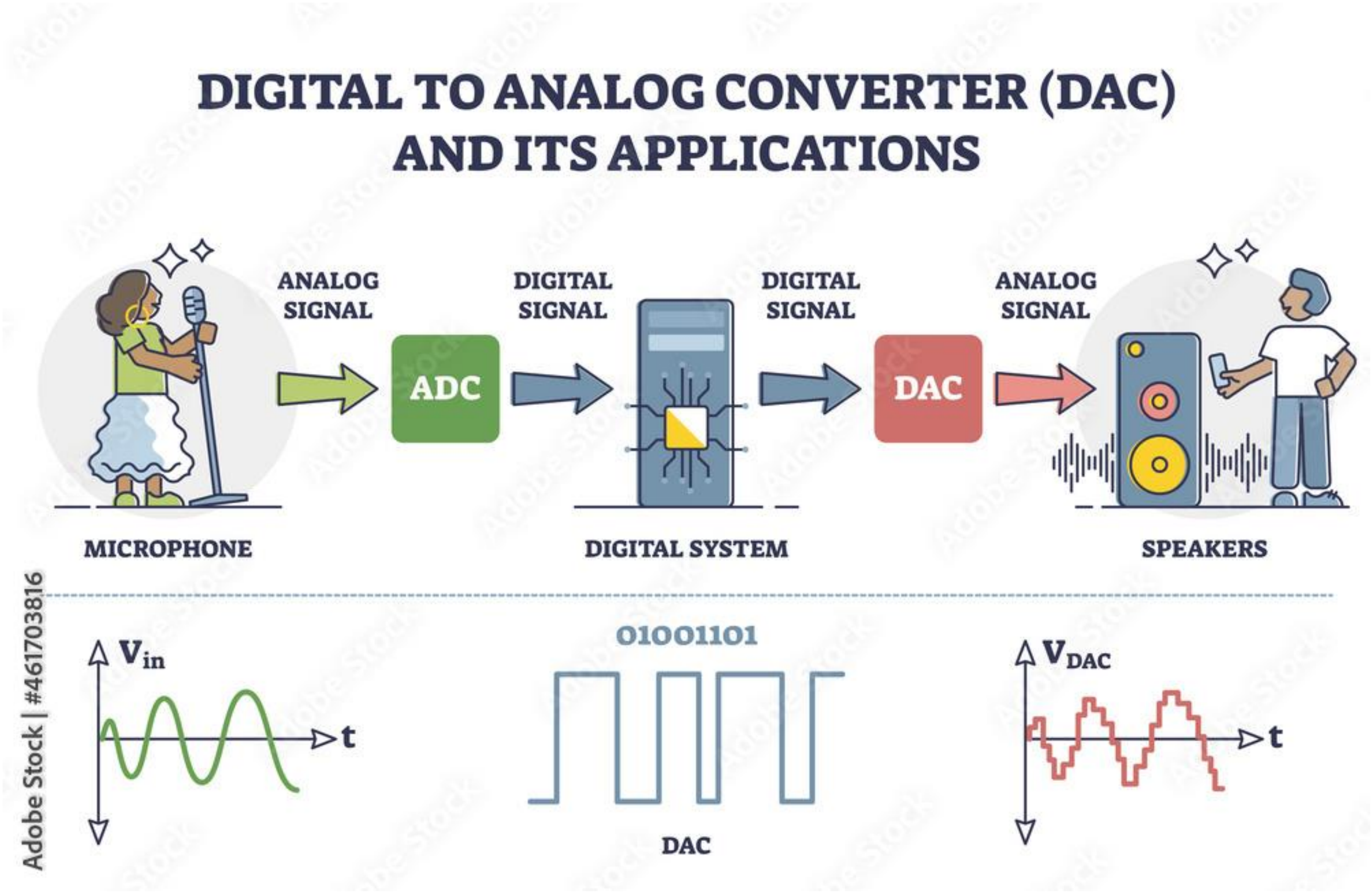
- By the end of this lecture, you will be able to:
  - Explain the Pulse Code Modulation (PCM) technique for analog-to-digital conversion.

# Introduction

- Analog to digital (A2D) conversion, why do we learn it?
- Applications in real-life:
  - The **microphone** in the recorder, samples the incoming audio signal, quantizes it into digital values, and stores them as PCM-encoded data in a file.

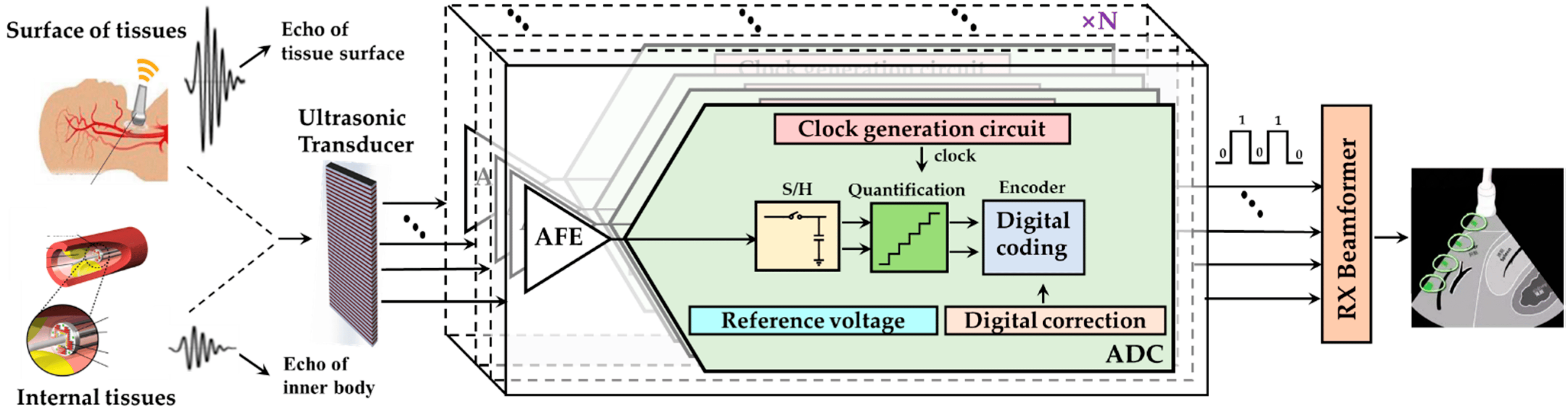


# Introduction



# Introduction

- Analog to digital (A2D) conversion, why do we learn it?
- Applications in real-life:
  - In **medical imaging**, ADC can be employed to digitize analog signals from various sensors and transducers, such as ultrasound probes or X-ray detectors. This allows for the creation of digital images and data that can be analyzed and displayed on medical equipment.



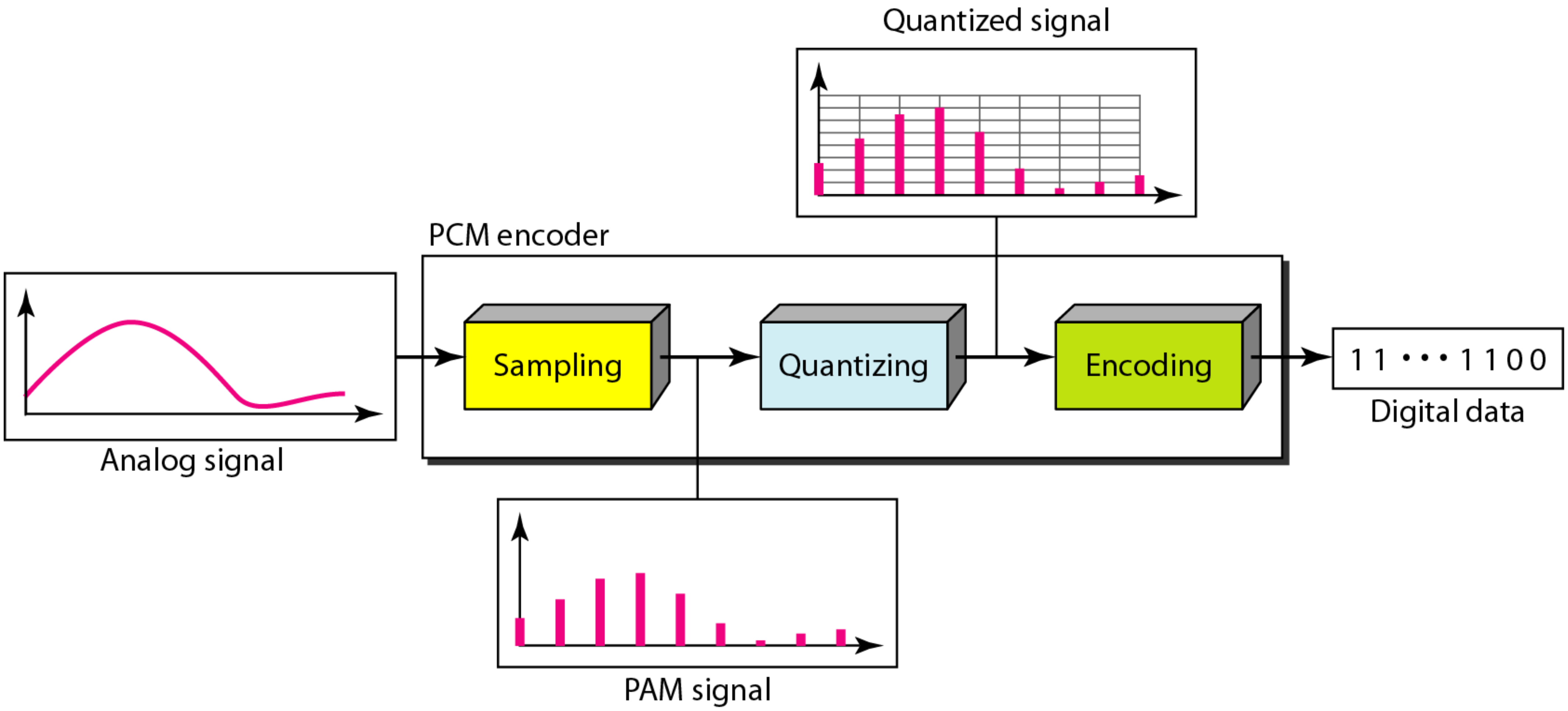
# Analog-to-Digital Conversion

- **Digitization**
  - Converting an **analog signal** to **digital data**

# Analog-to-Digital Conversion

- **Digitization**
  - Converting an **analog signal** to **digital data**
- **Pulse Code Modulation** (**PCM**)
  - The most commonly used technique for digitization.
  - A PCM encoder has **three processes**:
    1. **Sampling** of the **analog signal**
    2. **Quantizing** the **sampled signal**
    3. **Encoding** the **quantized information** as streams of bits

# PCM – Encoder


Quantized signal
PCM encoder
Sampling
Quantizing
Encoding
1 1 • • • 1 1 0 0
Digital data
Analog signal
PAM signal

# Sampling

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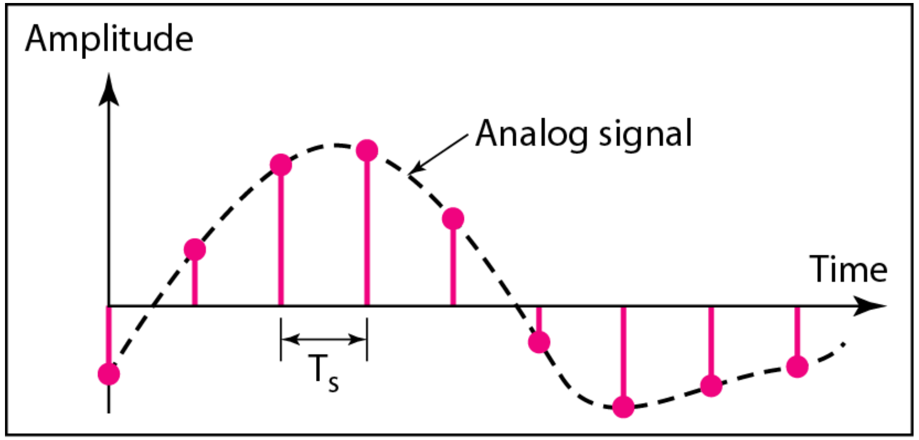
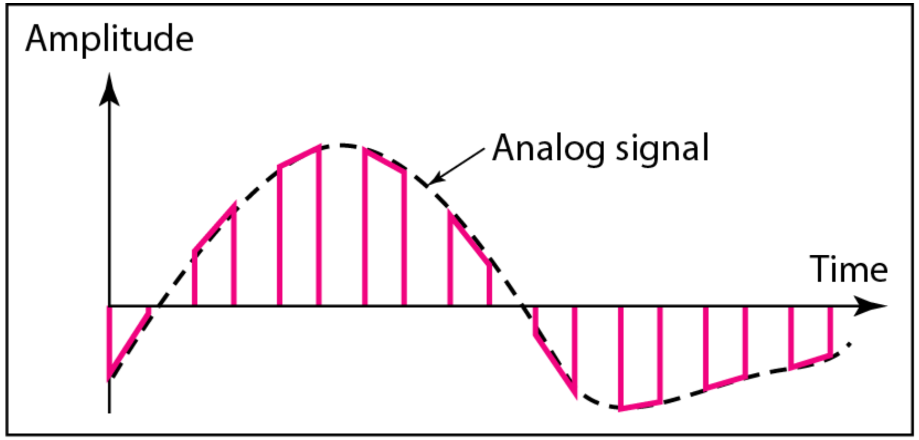
- **Sampling rate** or **sampling frequency** ($f_s$)
  - $f_s = \frac{1}{T_s}$
- A signal with an **infinite bandwidth** cannot be sampled (the signal must be bandlimited).

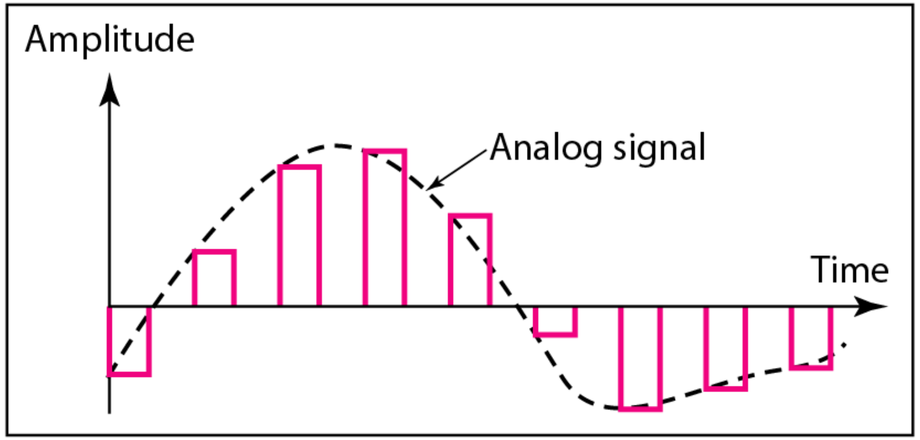
# Sampling Methods



a. Ideal sampling



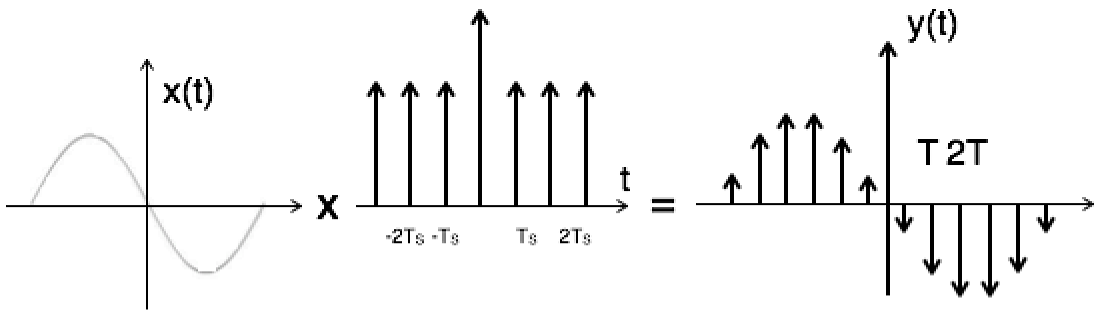
b. Natural sampling



c. Flat-top sampling (sample-and-hold)

# FYI – Ideal Sampling

- Other terminology used in the literature: **Impulse Sampling**
  - **Input signal** $x(t)$ convoluted (multiplied) with an **impulse train**
  - You **cannot** use this practically because pulse width cannot be zero and the **generation of impulse train is not possible** practically.




Train of impulse functions select sample values at regular intervals

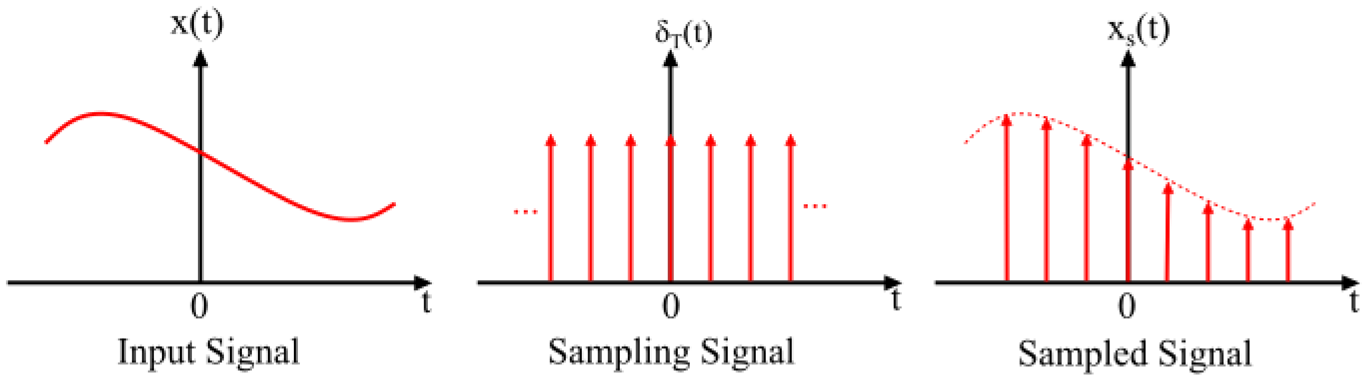
$$x_s(t) = x(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Fourier Series representation:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{jn\omega_s t}, \qquad \omega_s = \frac{2\pi}{T_s}$$

# FYI – Convolution

- **Convolution** is used to represent the process of sampling an analog signal and converting it into a discrete-time digital signal.
  - But what is convolution?
- Convolution can be thought of as a way to "**pass**" or combine two signals through each other.
  - In the time domain sampling is multiplication by an impulse train.
  - In the frequency domain sampling is convolution by an impulse train.



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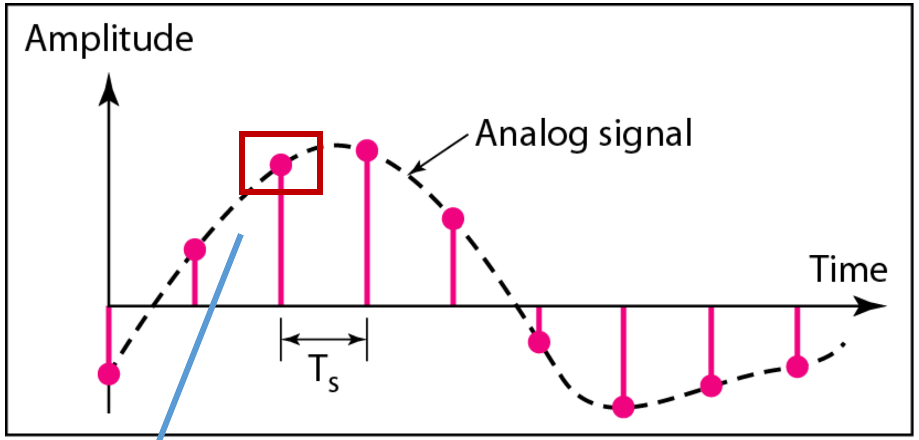
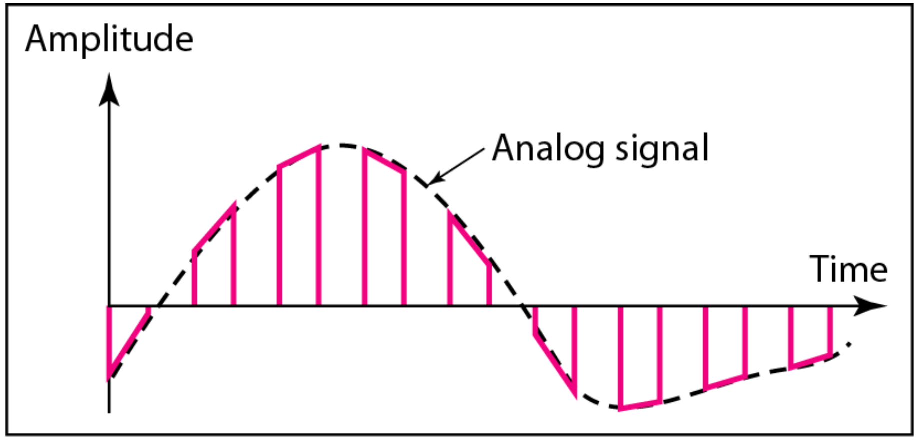


a. Ideal sampling

Cannot be easily implemented



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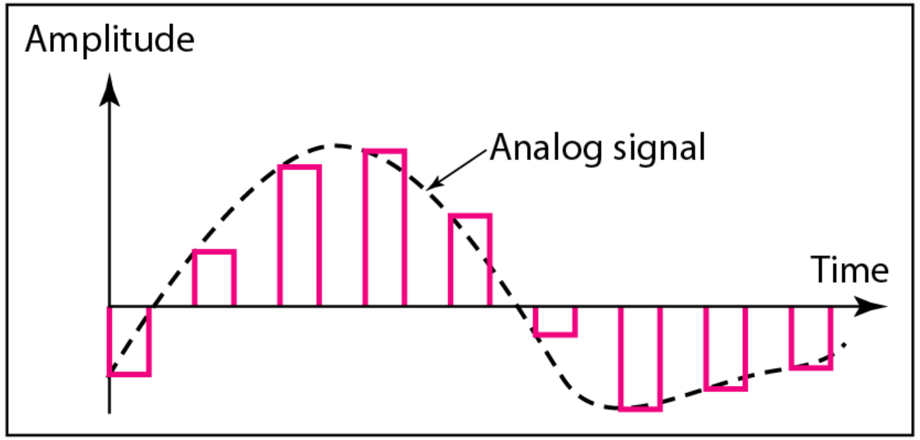

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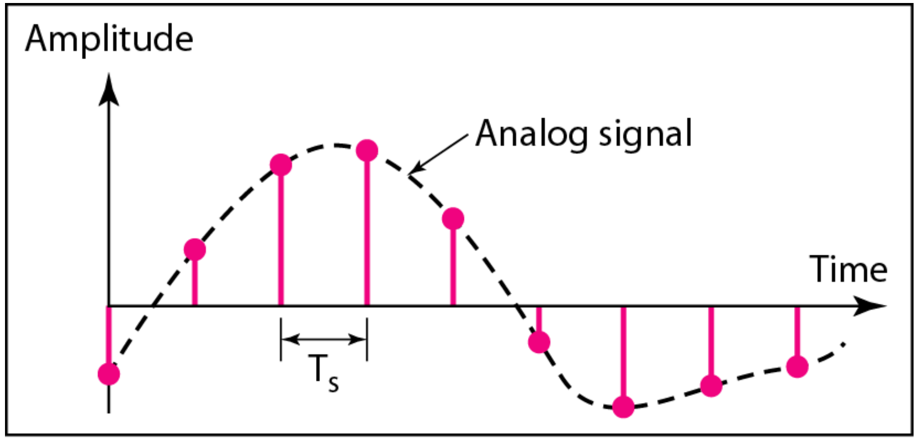


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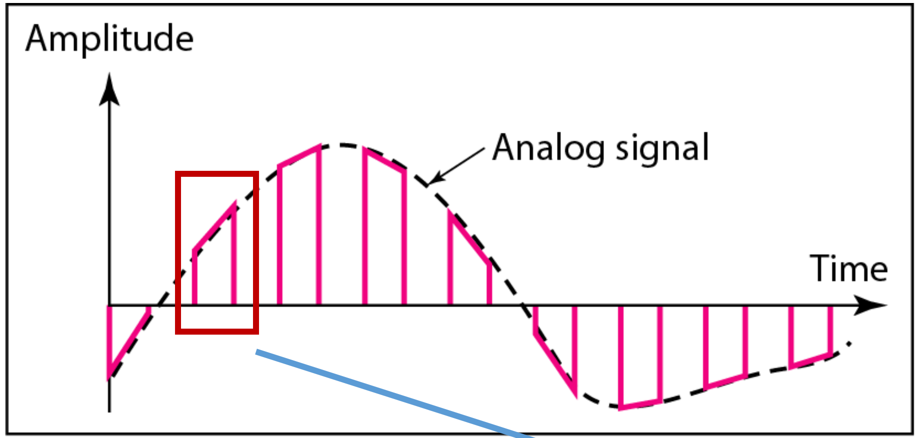


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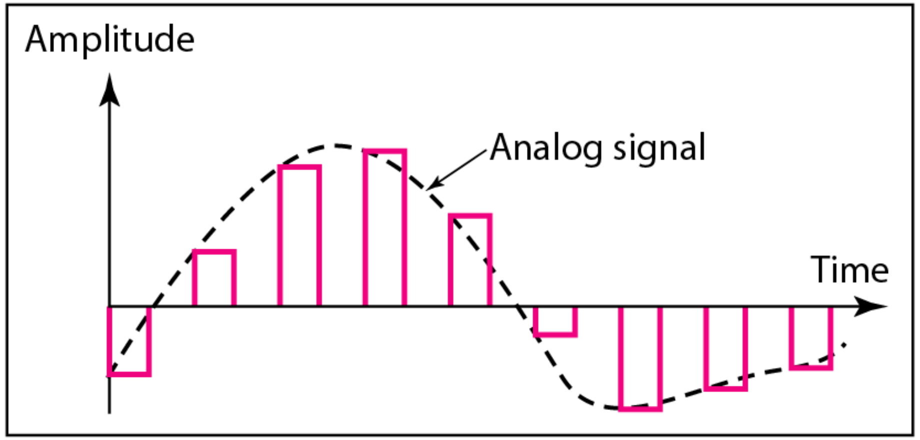
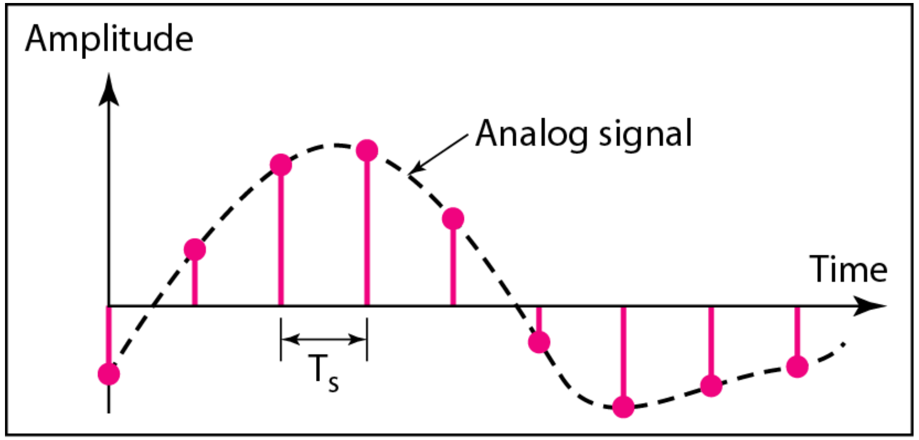



c. Flat-top sampling (sample-and-hold)

**Uses a high-speed switch**
(A circuit that can rapidly connect and disconnect a signal path)

# Sampling Methods

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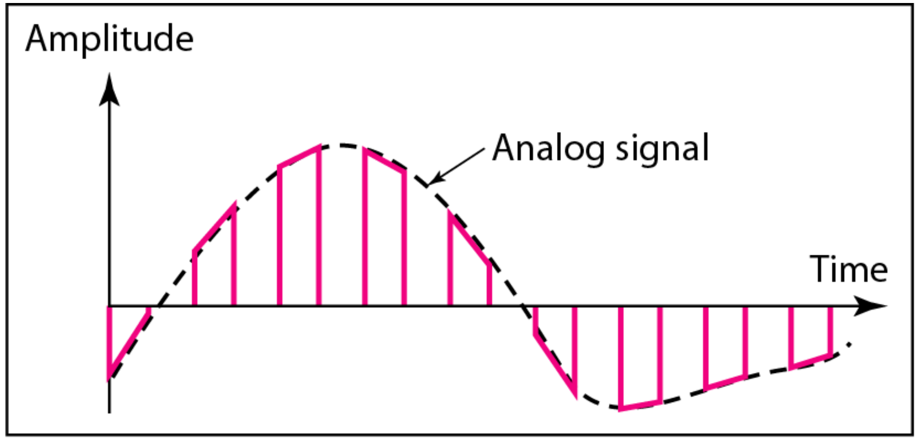


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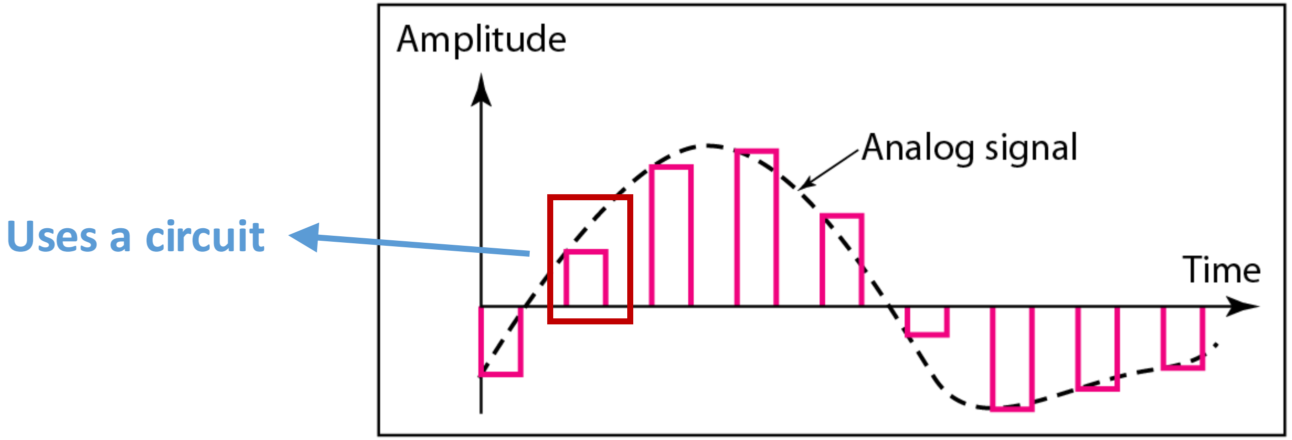


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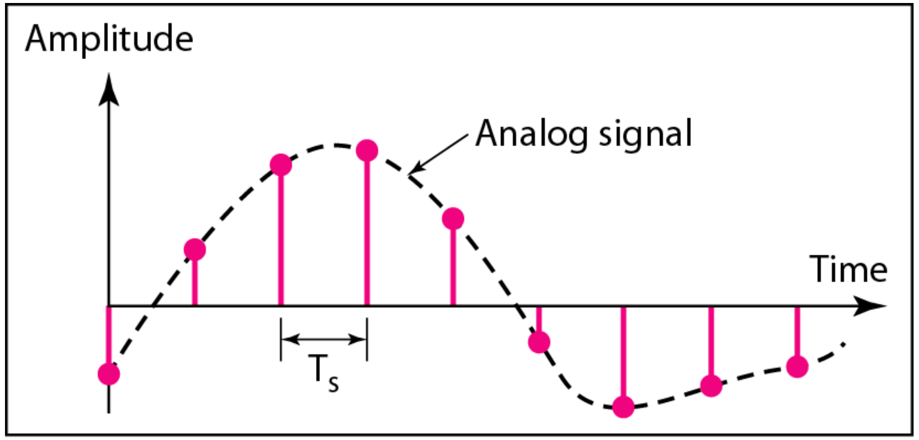

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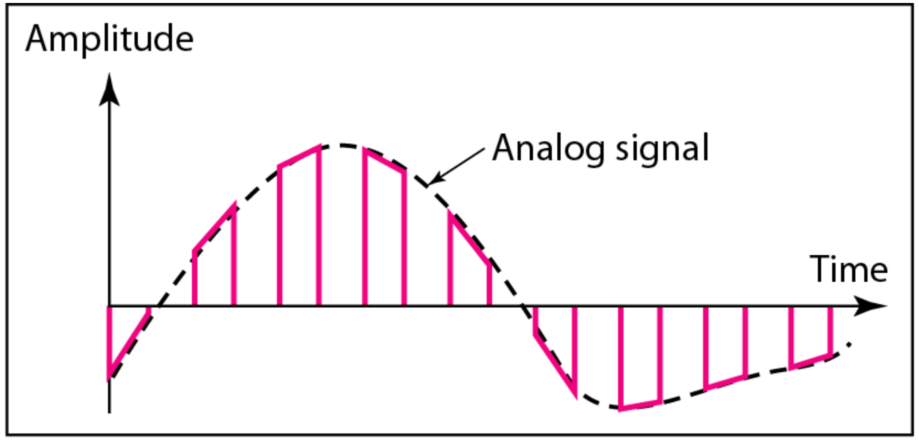
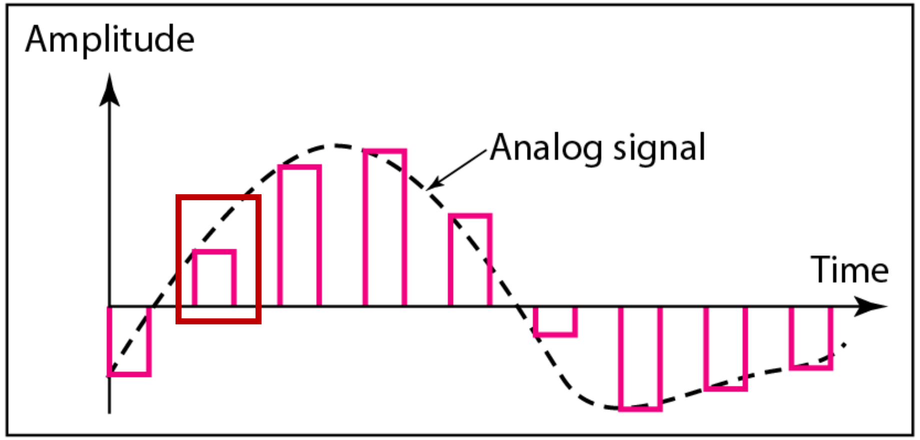


b. Natural sampling

The most commonly used method →



c. Flat-top sampling (sample-and-hold)

# Nyquist Theorem and Sampling Rate

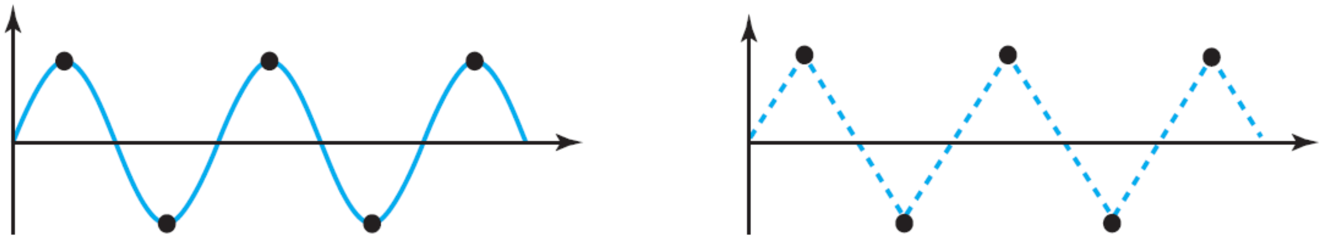
- Is there any restriction on sampling rate (sampling frequency)?

- According to the Nyquist theorem, to reproduce the original analog signal, the **sampling rate** must be **at least 2 times** the **highest frequency** contained in the signal.

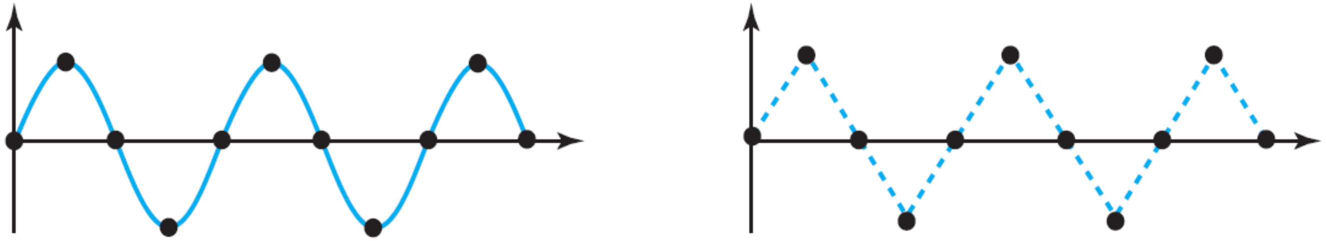
Nyquist rate → $f_N = 2f_{\max}$ (Hz)     Nyquist interval → $\frac{1}{f_N} = \frac{1}{2f_{\max}}$ (s)

- **Low-pass** analog signal → **bandwidth** = **highest frequency**
- **Bandpass** analog signal → **bandwidth** < **highest frequency**

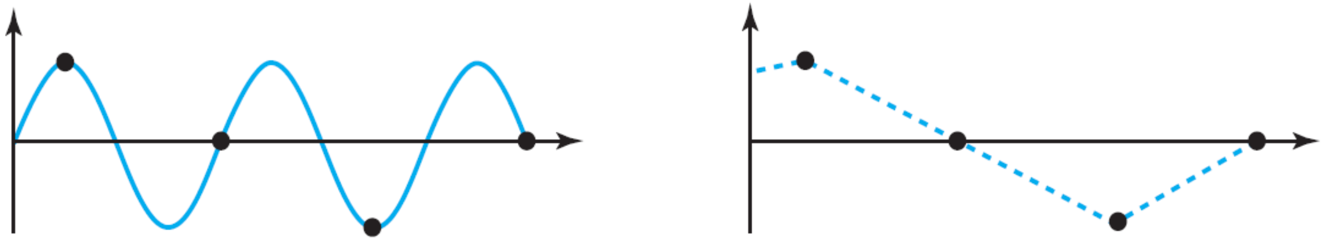
# Nyquist Theorem and Sampling Rate – Example



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$



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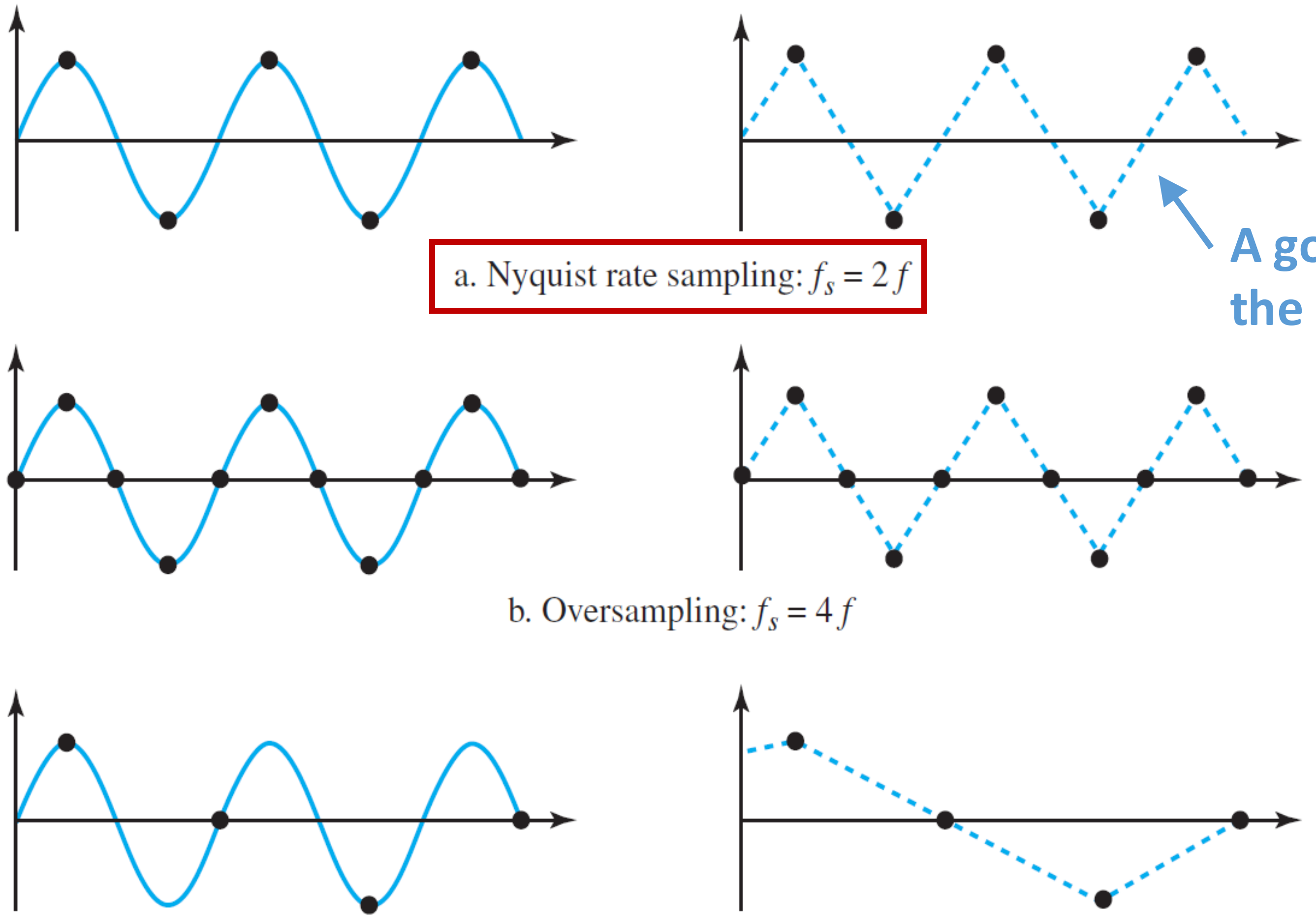

a. Nyquist rate sampling: $f_s = 2f$

A good approximation of the original sine wave

b. Oversampling: $f_s = 4f$

c. Undersampling: $f_s = f$

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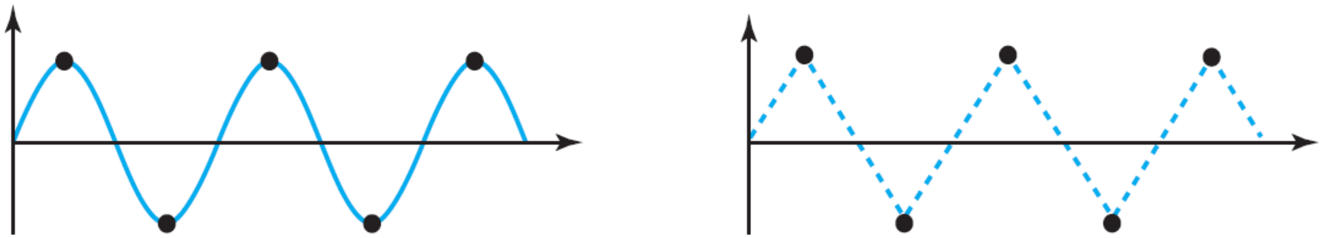

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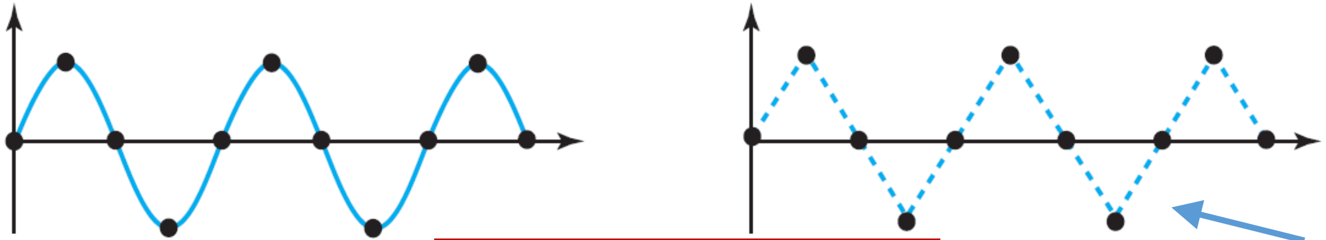
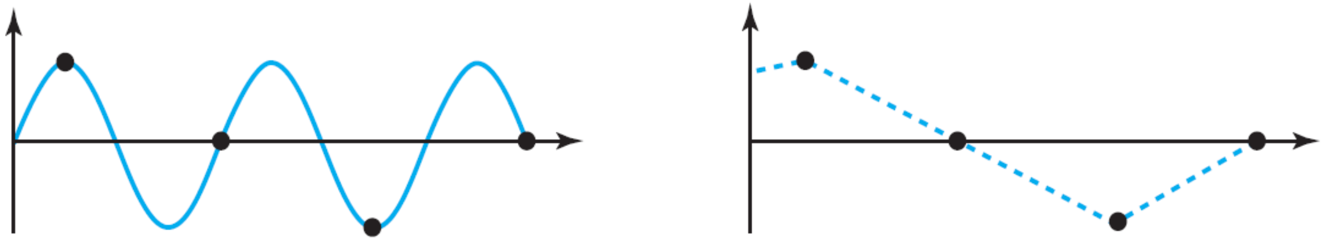


b. Oversampling: $f_s = 4f$

Same approximation but redundant and unnecessary



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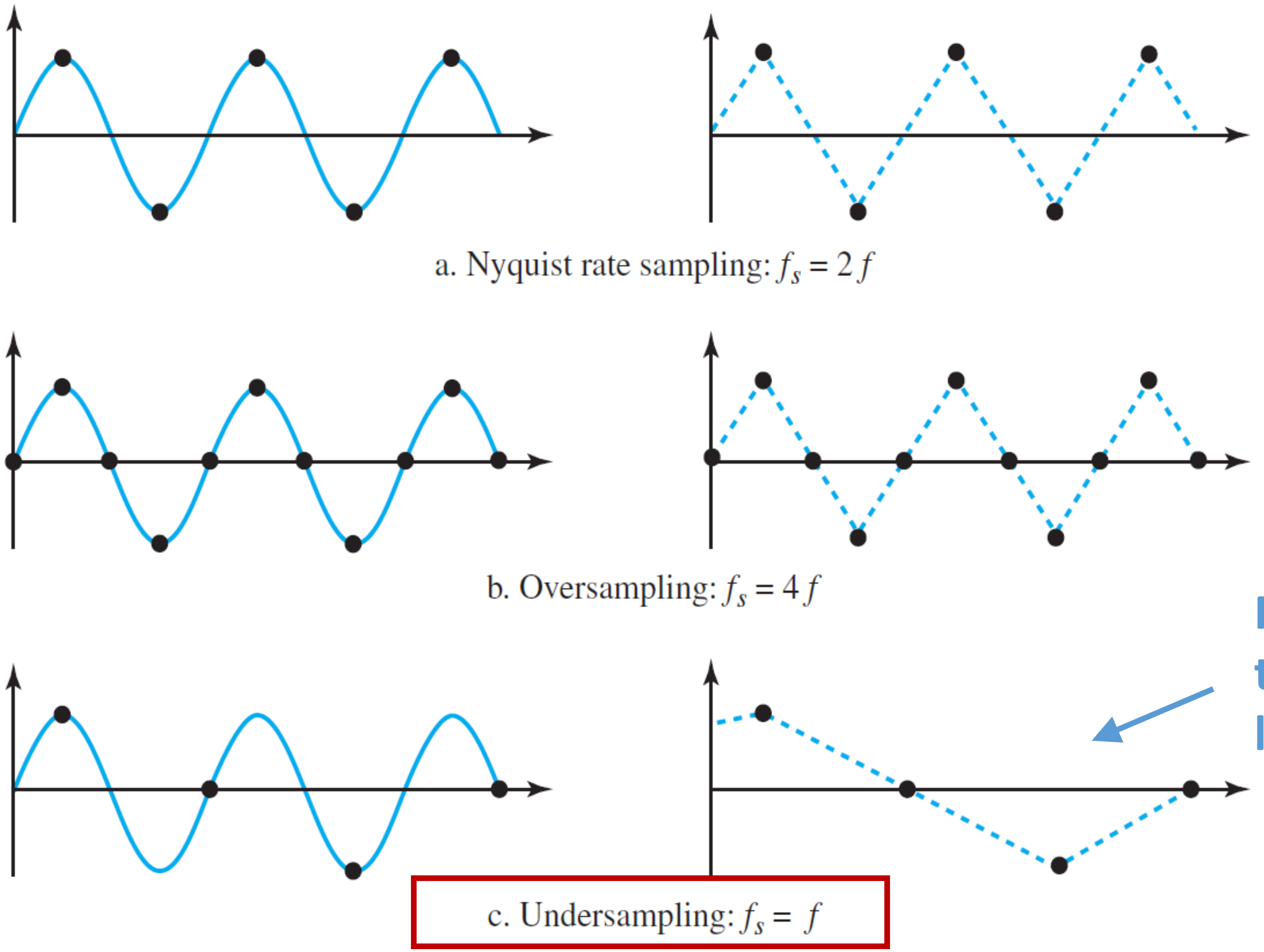


Produces a signal that does not look like the original one

# Nyquist Theorem and Sampling Rate – Example

- What is the minimum sampling rate for a **low-pass signal** that has a bandwidth of 100 kHz?

# Nyquist Theorem and Sampling Rate – Example

- What is the minimum sampling rate for a **low-pass signal** that has a bandwidth of 100 kHz?

- **Answer**:
- For a **low-pass** signal, bandwidth = highest frequency ($f_{max}$)
- Therefore,
- **Minimum sampling rate** = 2 x 100000 = **200000 samples per second**

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# Nyquist Theorem and Sampling Rate – Example

- What is the minimum sampling rate for a **bandpass signal** that has a bandwidth of 100 kHz?

- **Answer**:
- We **cannot find** the **minimum sampling rate** because we do not know the **maximum frequency** of the signal.
- We do not know the maximum frequency in the signal. (so, what to do?)
  - We should either try to find or measure the maximum frequency or obtain additional information about the signal to make this determination.

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  $$\Delta = \frac{V_{\max} - V_{\min}}{L}$$
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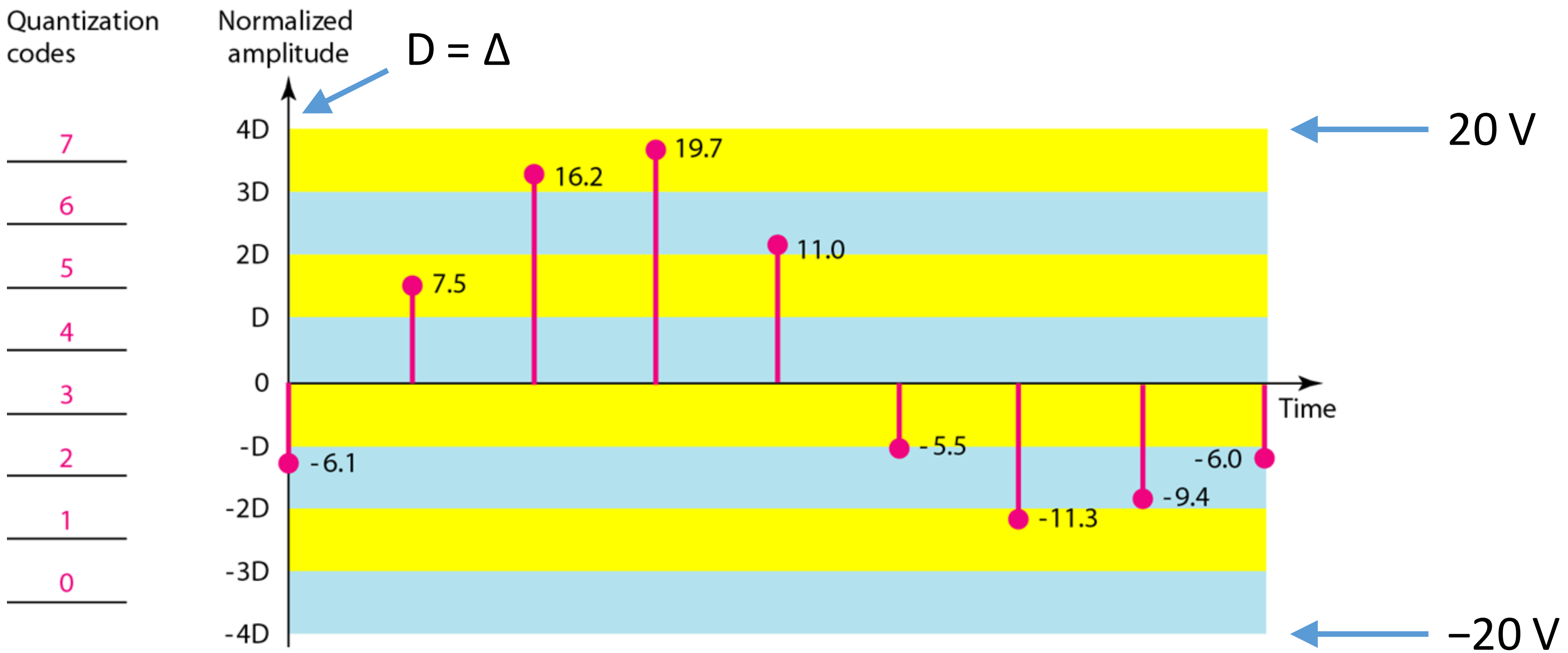
  4. Approximate the value of the sample amplitude to the quantized values.

# Quantization – Example




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Normalized PAM values | -1.22 | 1.50 | 3.24 | 3.94 | 2.20 | -1.10 | -2.26 | -1.88 | -1.20 |
| Normalized quantized values | -1.50 | 1.50 | 3.50 | 3.50 | 2.50 | -1.50 | -2.50 | -1.50 | -1.50 |
| Normalized error | -0.28 | 0 | +0.26 | -0.44 | +0.30 | -0.40 | -0.24 | +0.38 | -0.30 |
| Quantization code | 2 | 5 | 7 | 7 | 6 | 2 | 1 | 2 | 2 |
| Encoded words | 010 | 101 | 111 | 111 | 110 | 010 | 001 | 010 | 010 |

$L = 8$

$\Delta = 5$ V

**Zones**

7

6

5

4

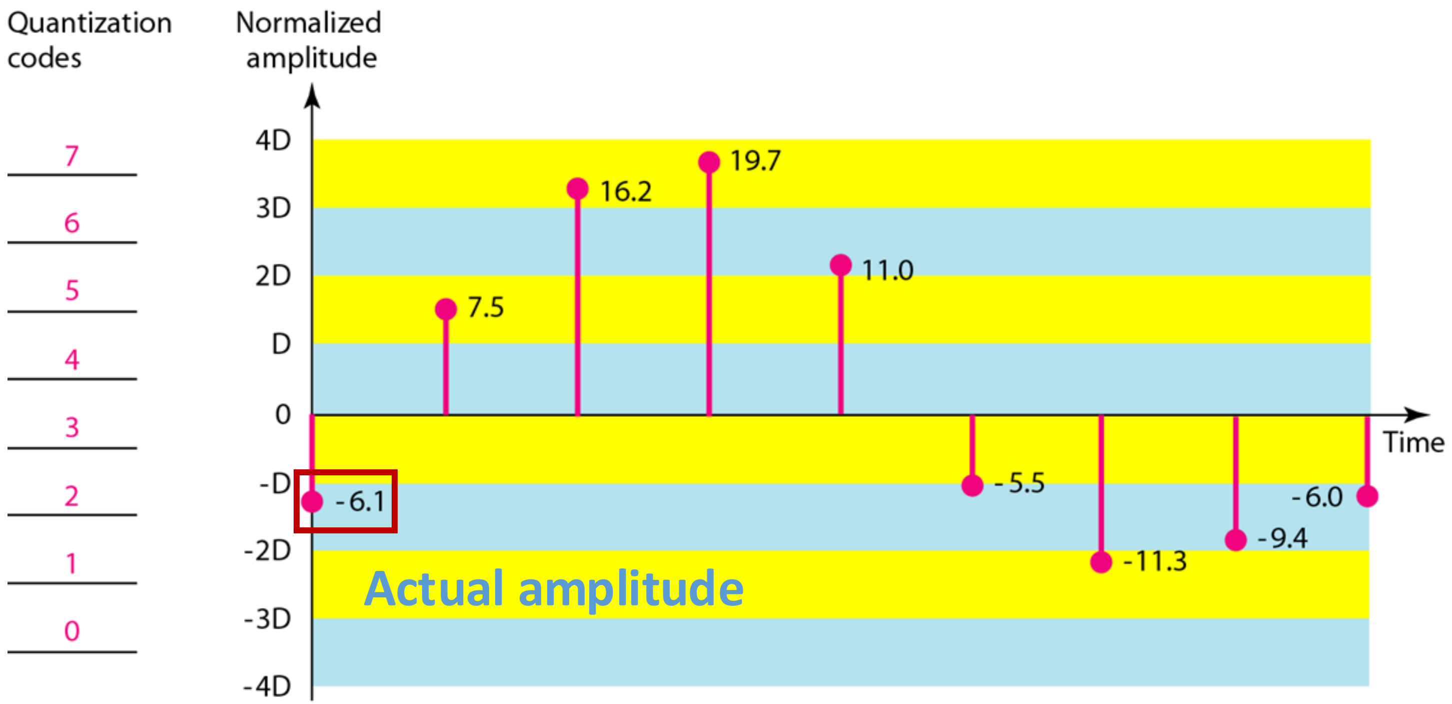
3

2

1

0

# Quantization – Example

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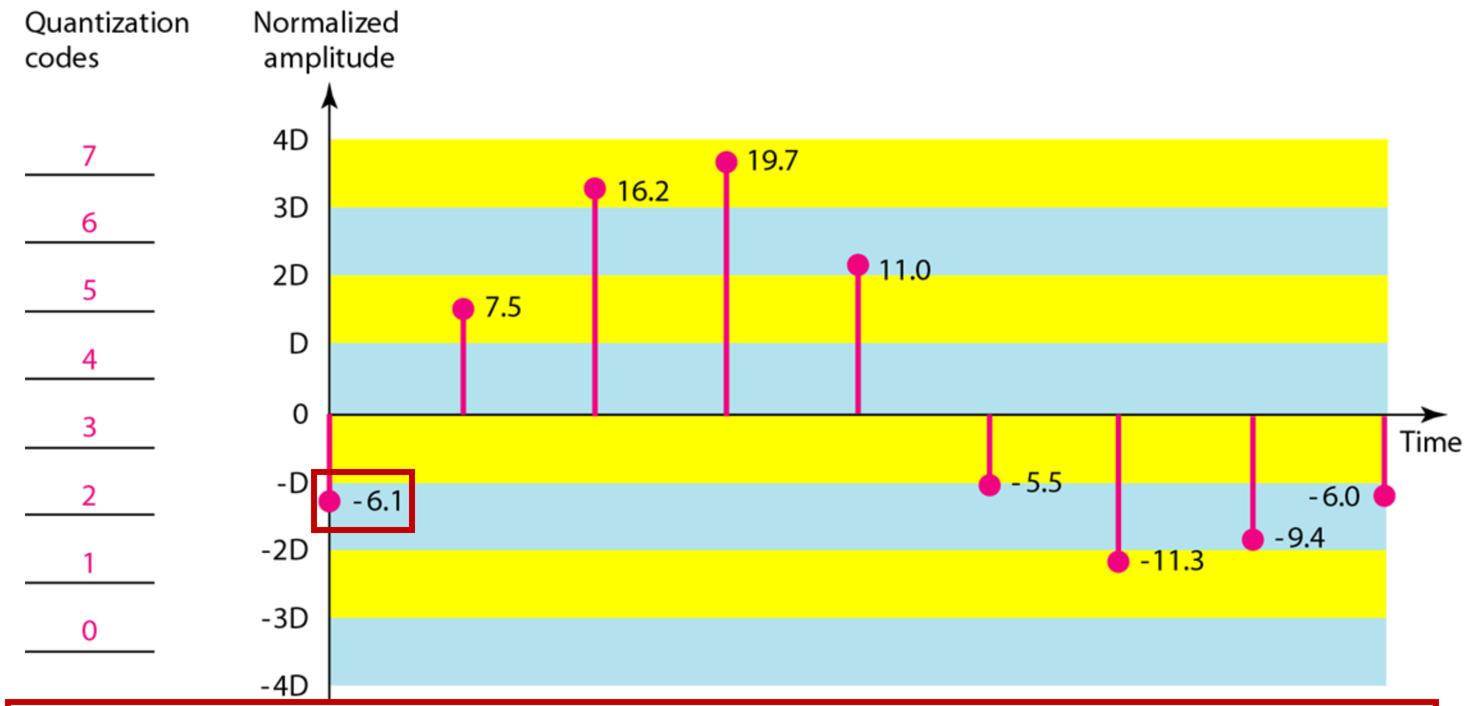


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**Normalized PAM values**
= Actual amplitude/Δ
E.g., −6.1/5 = **−1.22**

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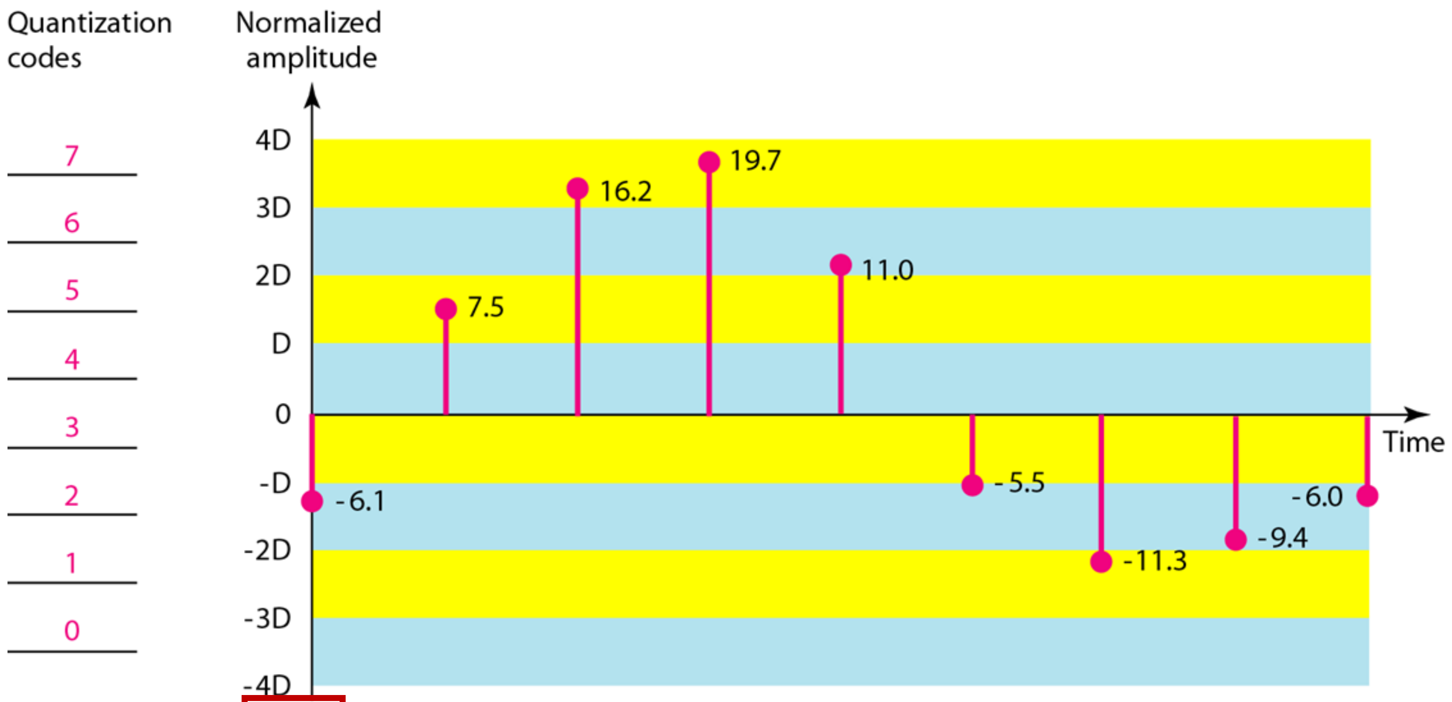


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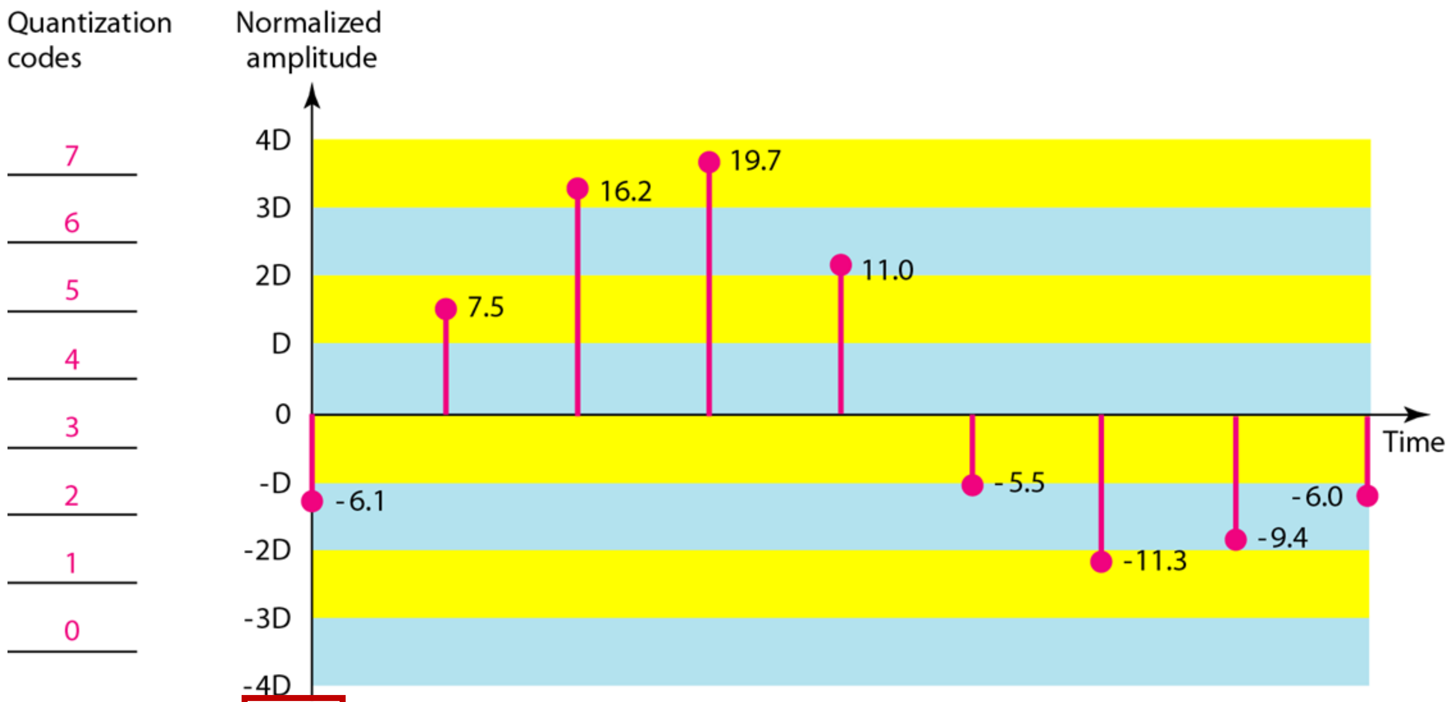
**–1.22** belongs to **zone 2** → **Normalized quantized value** = Midpoint of this zone = **–1.50**

# Quantization – Example

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**Normalized error** is the difference between **normalized quantized value** and **normalized PAM value**.

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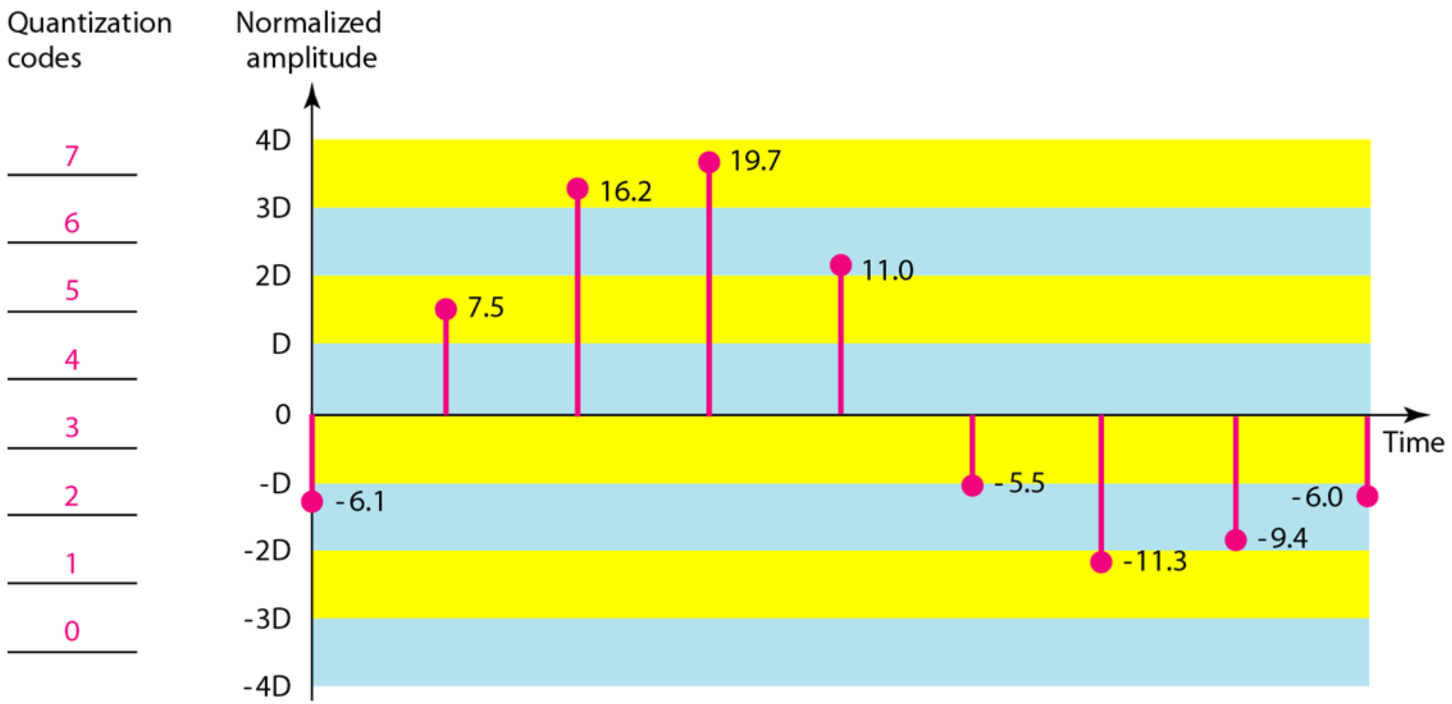


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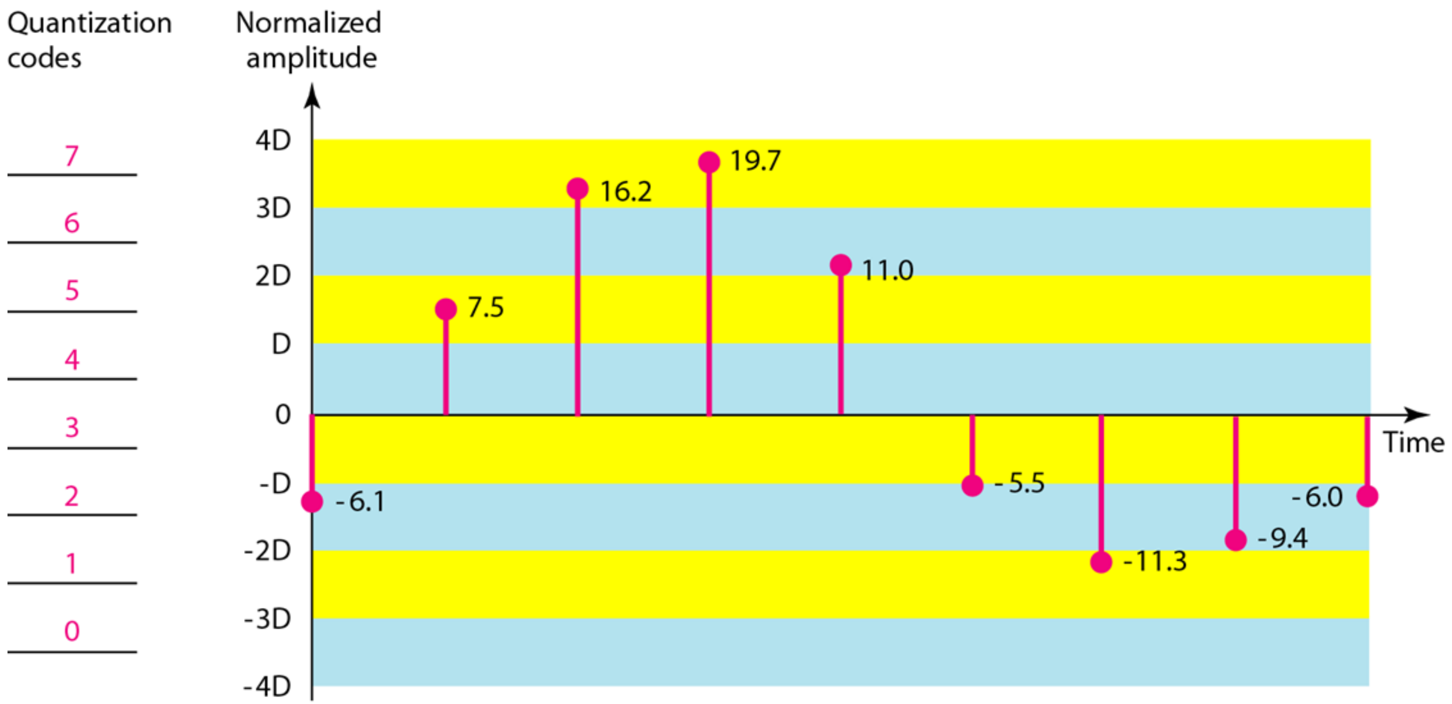
–1.50 → midpoint of zone 2

# Quantization – Example

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| Normalized quantized values | -1.50 | 1.50 | 3.50 | 3.50 | 2.50 | -1.50 | -2.50 | -1.50 | -1.50 |
| Normalized error | -0.28 | 0 | +0.26 | -0.44 | +0.30 | -0.40 | -0.24 | +0.38 | -0.30 |
| Quantization code | 2 | 5 | 7 | 7 | 6 | 2 | 1 | 2 | 2 |
| Encoded words | 010 | 101 | 111 | 111 | 110 | 010 | 001 | 010 | 010 |

Encoding (Last step of PCM)

# Encoding

- Each quantized sample can be changed to an **$n_b$-bit code word**. (L is the number of quantization levels/zones)

$$n_b = \log_2 L$$

$$\boxed{\text{Number of bits per sample} = n_b = \log_2 L}$$

**Bit rate** = Sampling rate × Number of bits per sample

$$= f_s \times n_b$$

# Quantization Levels

- Choosing ***L*** (the number of quantization levels) depends on:
  - The range of the amplitudes of the analog signal.
  - The required accuracy of recovering the signal.
- If the amplitude of a signal fluctuates between two values only, $L = 2$.
- For a **signal with many amplitude values**, e.g., voice, more quantization levels are needed.
- In audio digitizing, $L = 256$.
- Choosing **lower values of *L*** increases the **quantization error** if there is **a lot of fluctuation** in the signal.

# Quantization Error (Noise)

- $-\Delta/2 <=$ quantization error $<= \Delta/2$
- The **quantization error changes the SNR of the signal** → the upper limit capacity (bit rate) is **decreased** (according to Shannon Capacity).
- The contribution of the **quantization error** to the **$\mathrm{SNR_{dB}}$** of the signal depends on the number of quantization levels ***L***, or the bits per sample $\boldsymbol{n_b}$.

$$\boxed{\mathrm{SNR_{dB}} = 6.02 n_b + 1.76\mathrm{dB}}$$

# Uniform Quantization

- Issues with **uniform quantization**
  - Only optimal for uniformly distributed signal.
  - Often the distribution of the instantaneous amplitudes in the analog signal is not uniform.
  - Applications such as speech and music (real audio signals) are more concentrated near zeros (lower amplitudes).

- In **nonuniform quantization**, **height of Δ is not fixed**; it is greater near the lower amplitudes and less near the higher amplitudes.

# PCM Bandwidth

- Given the bandwidth of a **low-pass analog signal**, we want to find the **new minimum bandwidth of the channel that can pass the digitized version of this signal**.

$$B_{\min} = c \times N \times \frac{1}{r} = c \times (n_b \times f_s) \times \frac{1}{r} = c \times n_b \times 2f_{\max} \times \frac{1}{r}$$

$$B_{\min} = c \times n_b \times 2 \times B_{\text{analog}} \times \frac{1}{r}$$

- If $c$ = 1/2 (average case) and $r$ = 1 (NRZ or bipolar line coding):

$$B_{\min} = n_b \times B_{\text{analog}}$$

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The minimum bandwidth of the digital signal is $n_b$ times greater than the bandwidth of the analog signal. This is the price we pay for digitization.

# Summary

- PCM is a technique for analog-to-digital conversion.
- PCM includes sampling, quantizing and encoding.
- PCM requires more bandwidth than the bandwidth of the input analog signal.

# References

[1] Behrouz A.Forouzan, Data Communications & Networking with TCP/IP Protocol Suite, 6th Ed, 2022, McGraw-Hill companies.

# Reading

- Chapter 2 of the textbook, section 2.3.2.
- Chapter 2 of the textbook, section 2.8 (Practice Test)