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| COMP 2121 **DISCRETE MATHEMATICS** | Assignment 1  Fall 2024 |

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| Name: Jiarui Xing A01354731 | Set: 2D | |
| Name: Maksim Sadreev A0???? | Set: 2D | |
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| Section | Total | Actual |
| Question 1 | 40 |  |
| Question 2 | 15 |  |
| Question 3 | 10 |  |
| Question 4 | 10 |  |
| Question 5 | 10 |  |
| Total | 85 |  |

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| C:\Users\A00141222\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\ARBZC6KC\MC900432530[1].png | | | Instructions |
|  |  |  | * Assignment must be done using Microsoft Word or an alternative word processor – **type** your work in this document. * Handwritten assignments will not be marked. * The header of every page has math templates and logic symbols that are needed. You can copy them into your text |
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|  |  |  | * The assignment must be done in a **group** of two students – no individual assignments will be accepted. |
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|  |  |  | * Just the answer will not give you credit for a problem. * When you solve a problem, you must provide necessary **explanations** – yes this means explanations in English. Normally one paragraph is sufficient, but it may take more depending on a question. |
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|  |  |  | * Do not evaluate final answer unless the question asks you to do that. Leave it as a **formula**, following the format in lectures and labs. |
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|  |  |  | * **PRINT** the completed assignment – you are handing in a paper copy. |
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|  |  |  | * **Due** at the beginning of the **Lecture** on **October 2, 2024**. * No late assignments will be accepted. * Electronic copies will not be accepted. |
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**Q1)** The System Administrator has set the following rules for the password:

* The password is a string made of 14 characters.
* The available characters are Hexadecimal digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.
* Repetition is allowed unless otherwise stated.

\*\*\* Do not evaluate expressions \*\*\*

1. How many passwords have exactly seven A’s and at least five B’s?

Explanation:

If B = 5:

C(14, 7) \* C(7,5) \*

If B = 6:

C(14, 7) \* C(7, 6) \* 15

If B = 7:

C(14, 7) \* C(7, 7)

= C(14, 7)

Total: C(14, 7) \* C(7,5) \*\* C(14, 7) \* C(7, 6) \* 15 \* C(14, 7)

= C(14, 7) \* [C(7,5) \*+ C(7, 6) \* 15 + 1]

Answer:

**[]**

1. How many passwords have exactly two A’s and exactly three B’s, so that the three B’s are sandwiched between the A’s? 010**A**1 **B**8**BB** 3112**A** is an example of such a string.

Explanation:

Assume first A at i, second A at j (i > j)

∵ 3B inside need at least 4 bits,

∴ i ∈ [1,10], j ∈ [i+4,14]

∴ i has 10 – 1 + 1 = 10 possible bit, j has 14 – i – 4 +1 = 11 – i

∴ **The total possibility of A’s position has**

,

= = **55**

` ∵ The bits between i and j is

j – i – 1

also ∵ j – i ∈[4, 13]

∴ j – i – 1 ∈ [3, 12]

∴ **The total possibility of B’s position has:**

C(j – i - 1, 3)

∵ j – i – 1 ∈ [3, 12]

∴ C(j – i - 1, 3)

=

= 1 + 4 + 10 + 20 + 35 + 56 + 84 + 120 + 165 + 220

= **715**

**∵** The remaining position has

14 – 2 – 3 = 9

∴ **The possibility of remaining position has:**

**∴The total possibility has:**

**55 \* 715 \***

Answer:

**55 \* 715 \***

1. How many passwords have at least one A, at least one B, at least C, and have no other characters?

Explanation:

Total possibility:

Only 2 of them:

3 \*

Only 1 of them:

3 \* 1

Total:

**3**

Answer:

**3**

1. How many passwords with exactly 3 B’s, have the sum of all digits equal to 40 and have no adjacent B’s? Examples of such passwords are 00230 **B**11**B** 000**B**0, **B**1101 0**B**00 00**B**04, etc.

Explanation:

∵ 3 B, total 14 bits

∴ has 14 – 3 = 11 bits to put non-B chars

∵ No adjacent B

∴ has 11 + 1 = 12 bits to put B

**C(12, 3)**

∵ The sum of all digits equal to 40

Also ∵ 3 \* B = 33

∴ Remaining value 40 - 33 = 7

∴ available number ∈ [0, 7]

∴ Possible situations:

7 \* 1 and 4 \* 0: 　　　　　 **C(11, 7) = 330**

1 \* 7 and 10 \* 0: 　　　　 **C(11, 1) = 11**

1 \* 6, 1 \* 1 and 9 \* 0:　　 **C(11, 1) \* C(10, 1) = 11 \* 10 = 110**

1 \* 5, 2 \* 1 and 8 \* 0:　　 **C(11, 1) \* C(10, 2) = 11 \* 45 = 495**

1 \* 5, 1 \* 2 and 9 \* 0:　　 **C(11, 1) \* C(10, 1) = 11 \* 10 = 110**

1 \* 4, 1 \* 3 and 9 \* 0:　　 **C(11, 1) \* C(10, 1) = 11 \* 10 = 110**

1 \* 4, 1 \* 2, 1 \* 1 and 8 \* 0: **C(11, 1) \* C(10, 1) \* C(9, 1) = 11 \* 10 \* 9 = 990**

1 \* 4, 3 \* 1 and 7 \* 0:　　 **C(11, 1) \* C(10, 3) = 11 \* 120 = 1320**

2 \* 3, 1 \* 1 and 8 \* 0:　　 **C(11, 2) \* C(9, 1) = 55 \* 9 = 495**

1 \* 3, 2 \* 2 and 8 \* 0:　 **C(11, 1) \* C(10, 2) = 11 \* 45 = 495**

1 \* 3, 1 \* 2, 2 \* 1 and 7 \* 0: **C(11, 1) \* C(10, 1) \* C(9, 2) = 11 \* 10 \* 36 = 3960**

1 \* 3, 4 \* 1 and 6 \* 0:　 **C(11, 1) \* C(10, 4) = 11 \* 210 = 2310**

3 \* 2, 1 \* 1 and 7 \* 0:　 **C(11, 3) \* C(8, 1) = 165 \* 8 = 1320**

2 \* 2, 3 \* 1 and 6 \* 0:　 **C(11, 2) \* C(9, 3) = 55 \* 84 = 4620**

1 \* 2, 5 \* 1 and 5 \* 0:　 **C(11, 1) \* C(10, 5) = 11 \* 252 = 2772**

∴ Total possibility

**C(12,3)∗(sum of these numbers)**

Answer:

**C(12,3)∗(sum of these numbers)**

**Q2)** Consider the following programming segment. Your answer must rely on a combination structure. Answers that use sigma notation will not be accepted.

counter = 100

for i = 4 to (n+3) do {

counter = counter + 11

for j = i+1 to (3n+15) do {

counter = counter + 22

for k = j+1 to (n+8) do {

counter = counter + 33

}

}

}

// assume n ≥ 10

**a)** Determine the value of the variable counter after the segment is executed. Provide your answer as a function of n (i.e., a formula which depends on n). Make sure to explain how/why the parts of the formula relate to counting.

Explanation:

1. counter += (n + 3 – 4 + 1) \* 11 => += 11n
2. counter += [3n + 15 – (i + 1) + 1] \* 22 => +=

=

=

=

1. counter += [n + 8 – (j + 1) + 1] \* 33 => += (∵j ≤ n + 7)

=

=

**counter =**

Answer:

**counter =**

**b)** Evaluate your answer in part a) for n = 50. Show the work

counter = 100 +

=

=

**c)** Check your answer in part b) by implementing the code in a programming language of your choice. Use the value n = 50 and print the variable counter after the code execution. You must provide two screenshots: implementation and output.

文本

描述已自动生成

图形用户界面, 应用程序

描述已自动生成

1. What do you conclude?

**Nested loops are complicated; computers and programming languages are truly great inventions.**

**Q3)** Use the truth table to show that the following argument is NOT valid.

Clearly, **1) indicate in red/bold in the table** what makes you come to that conclusion, and then **2) explain your answer below the table**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | a∨b | a∨c | (a∨b)→(a→c) |
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | **1** | **0** |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | **1** | **0** |
| 1 | 1 | 1 | 1 | 1 | 1 |

**Explanation:**

In some cases, when (a∨b)→(a→c) is 0, a∨c is also 1.

**Q4)** Use rules of inference and direct proof to prove the argument is valid.

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|  |  |  |
| --- | --- | --- |
| STEPS | | REASON |
| 1 |  | Given conditions |
| 2 |  | Given conditions |
| 3 | q | Rule of Disjunctive Syllogism |
|  |  | Given conditions |
| 3 |  | Modus Ponens |
| 4 |  | Rule of Disjunctive Syllogism |
|  |  | Given conditions |
|  |  | Conjunction |
| 5 | x | Modus Ponens |

**Q5)** Use rules of inference and proof by contradiction to prove the argument is valid.

Note: contradiction pointing to and will not be accepted. In other words, if you start with , then independently prove x (which is Q4), and say lines and are in contradiction – proof will not be accepted. Instead, follow instructions from the lecture/lab.

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| --- | --- | --- |
| STEPS | | REASON |
| 1 |  | Assume |
| 2 | t | Given conditions |
| 3 |  | Given conditions |
| 4 |  | Rule of implication |
| 5 | w | Rule of conjunction |
| 6 |  | Given conditions |
| 7 |  | Rule of disjunctive syllogism |
| 8 |  | Rule of implication |
| 9 |  | Rule of conjunction |
| 10 |  | Given conditions |
| 11 |  | *Rule of disjunctive syllogism* |
| 12 | x | Contradiction |