exercise2 (Score: 0.0 / 13.0)

Lab 3

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

       name = "我的名字"
       student_id= "B06201000"

3. 演算法的實作可以參考lab-3 (https://yuanyuyuan.github.io/itcm/lab-3.html), 有任何問題歡迎找助教詢問。
4. **Deadline: 10/30(Wed.)**

In [1]:

```
name = ""
student_id = ""
```

Exercise 2

It is known that when interpolating a function $f(x)$ with a polynomial p_{m+1} of degree m that using x_j for $j = 0, 1, ..., m$ as interpolation points the error has the form

$$|f(x) - p_{m+1}(x)| = \frac{\left|f^{(m+1)}(\xi_x)\right|}{(m+1)!} \left|\prod_{k=0}^{m}(x - x_k)\right|,$$

where $\xi_x \in [x_0, x_m]$.

Therefore, the polynomial $\omega_m(t) := \prod_{k=0}^{m}(t - x_k)$ influences the size of the interpolation error.

1. Put $m+1$ *distinct equidistant points* in the interval $[-1, 1]$, and plot $\omega_m(t)$ for $m = 5, 10, 15, 20.$

Part 0. Import libraries.

In [2]:

```python
import matplotlib.pyplot as plt
import numpy as np
```

Part 1. Define $\omega_m(t)$ function.

In [3]:

(Top)

```python
def omega_m(t, x):

    # ===== 請實做程式 =====

    # ====================
```

Comments:
No response.

```
  File "<ipython-input-3-f8b44062141d>", line 5
    # ====================
                          ^
SyntaxError: unexpected EOF while parsing
```

In [4]:

omega

(Top)

```python
# Test
print('w_5(0.5) =', omega_m(0.5, np.linspace(-1, 1, 6)))

### BEGIN HIDDEN TESTS
from random import random

rd_number = random()
x = np.linspace(-1, 1, 11)

m = len(x)
product = 1

for i in range(m):
    product *= (rd_number - x[i])

assert omega_m(rd_number, np.linspace(-1, 1, 11)) == product, 'omega_m is wrong!'
### END HIDDEN TESTS
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-4-13cf93e5d9dd> in <module>
      1 # Test
----> 2 print('w_5(0.5) =', omega_m(0.5, np.linspace(-1, 1, 6)))
      3
      4 ### BEGIN HIDDEN TESTS
      5 from random import random

NameError: name 'omega_m' is not defined
```

Part 2. Define the equidistant points function.

For example, if $m = 4$, then $m + 1$ distinct equidistant points in the interval $[-1, 1]$ should be $[-1, -0.5, 0, 0.5, 1]$.

So the results of **equidistant_points(4) will be [-1. -0.5 0.   0.5 1.].**

In [5]:

```
                                                              (Top)

def equidistant_points(m):
    # =====  請實做程式  =====

    # ====================
```

Comments:
No response.

```
  File "<ipython-input-5-d276f9285ec7>", line 4
    # ====================
                          ^
SyntaxError: unexpected EOF while parsing
```

In [6]:

```
          points                                             (Top)

# Test
m = 4
print("Equidistant points:", equidistant_points(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(equidistant_points(m)) - np.linspace(-1, 1, m+1)) < 1e-7, 'equidistant_points is
wrong!'
### END HIDDEN TESTS
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-6-4826dd8929be> in <module>
      1 # Test
      2 m = 4
----> 3 print("Equidistant points:", equidistant_points(m))
      4
      5 ### BEGIN HIDDEN TESTS

NameError: name 'equidistant_points' is not defined
```

Part 3. plot $\omega_m(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* ".

(Top)

```python
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))

# Plot the function w_5(x), w_10(x), w_15(x) and w_20(x)
#
# Hint: ax.plot( x_points, y_points, color='?', label='?')
# =====  請實做程式  =====

# ====================

# Add other text and items
ax.set_title(r'$\omega_{5}(x), \omega_{10}(x), \omega_{15}(x)$ and $\omega_{20}(x)$')
plt.legend(loc='upper right')
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```
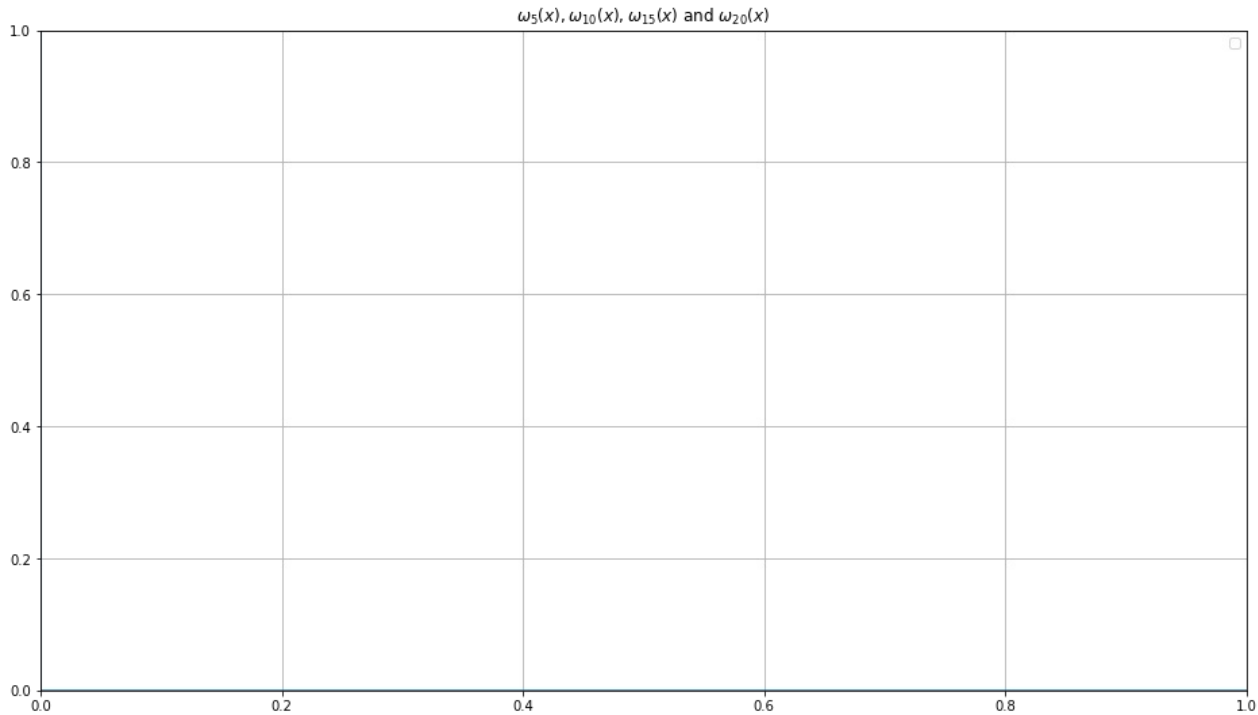
Comments:
No response.

```
No handles with labels found to put in legend.
```


ω₅(x), ω₁₀(x), ω₁₅(x) and ω₂₀(x)

Part 4. What's your observation of the above figure?

(Top)

請點此cell兩下開始作答（如要打文字記得選Markdown, 寫程式則選Code, 一個cell不夠可以再新增在下方）

Comments:
No response.

2. Redo " *Problem 1.* " using *zeros of the Chebyshev polynomial (Chebyshev nodes)* as the interpolation points.

Part 1. Define Chebyshev nodes.

Please refer the part of Chebyshev nodes in " *lagrange.ipynb* **".**

In [8]:

```
                                                                    (Top)

 def chebv_nodes(m):
     # =====  請實做程式  =====

     # ====================
```

Comments:
No response.

```
  File "<ipython-input-8-04e181bf0f14>", line 4
    # ====================
                          ^
SyntaxError: unexpected EOF while parsing
```

In [9]:

```
        chebv_nodes                                                 (Top)

# Test
m = 5
print("Chebyshev nodes:", chebv_nodes(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(chebv_nodes(m)) - np.cos(np.linspace(0, np.pi, m+1))) < 1e-7, 'chebv_nodes is wro
ng!'
### END HIDDEN TESTS
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-9-5a81ada23eb5> in <module>
      1 # Test
      2 m = 5
----> 3 print("Chebyshev nodes:", chebv_nodes(m))
      4
      5 ### BEGIN HIDDEN TESTS

NameError: name 'chebv_nodes' is not defined
```

Part 2. plot $\omega(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* **".**

(Top)

```
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))

# Plot the function w_5(x), w_10(x), w_15(x) and w_20(x)
#
# Hint: ax.plot( x_points, y_points, color='?', label='?')
# =====  請實做程式 =====

# ====================

# Add other text and items
ax.set_title(r'$\omega_{5}(x), \omega_{10}(x), \omega_{15}(x)$ and $\omega_{20}(x)$')
plt.legend(loc='upper right')
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```
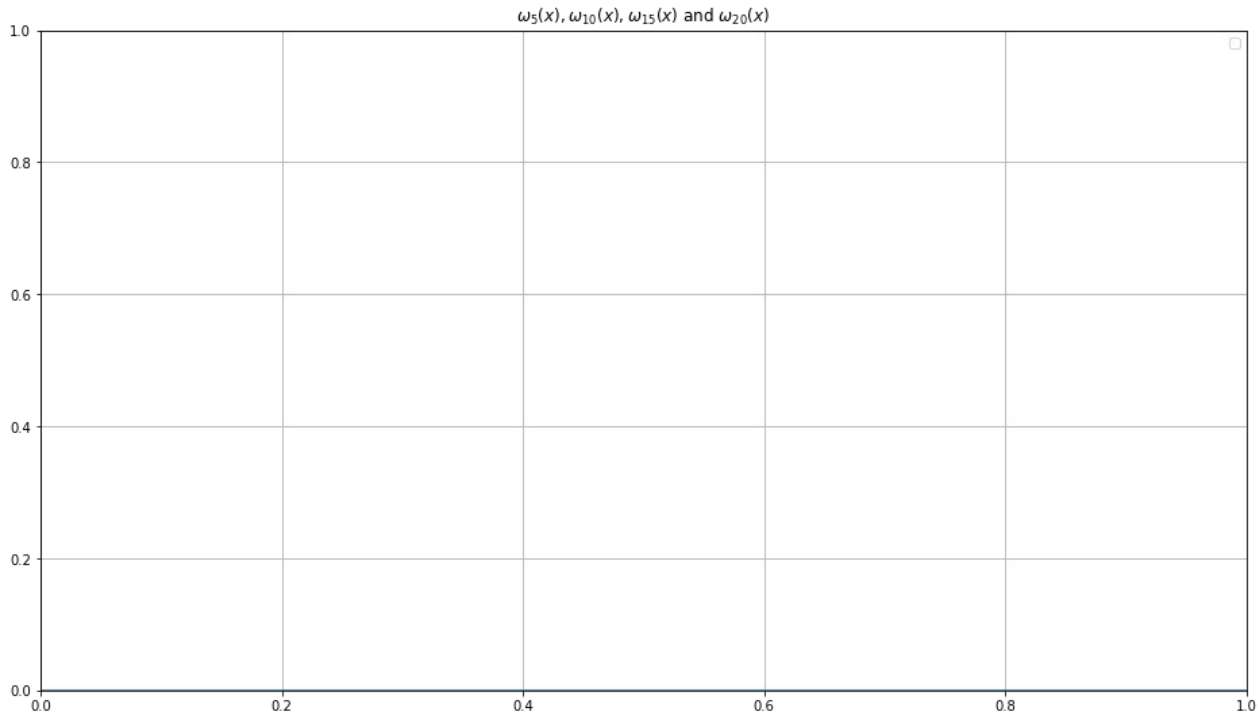
Comments:
No response.

```
No handles with labels found to put in legend.
```


$\omega_5(x), \omega_{10}(x), \omega_{15}(x)$ and $\omega_{20}(x)$

Part 3. What's your observation of the above figure?

(Top)

請點此cell兩下開始作答（如要打文字記得選Markdown, 寫程式則選Code, 一個cell不夠可以再新增在下方）

Comments:
No response.