ID: ______________

Calculus Exercise Week 13 (7.4, 7.5, 7.8) Name: ______________

7.4.27 Evaluate the integral.

$$\int \frac{4x}{x^3 + x^2 + x + 1} dx$$

7.4.50 Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{\sqrt{1 + \sqrt{x}}}{x} dx$$

7.4.58 Use integration by parts, together with the techniques of this section, to evaluate the integral.

$$\int x \tan^{-1} x dx$$

7.4.60 Evaluate $\displaystyle\int \frac{dx}{x^2+k}$ by considering several cases for the constant k.

7.4.63 **Weierstrass Substitution** The German mathematical Karl Weierstrass (1815 - 1897) noticed that the substitution $t = \tan(\frac{x}{2})$ will convert any rational function of $\sin x$ and $\cos x$ into an ordinary rational function of t.

(a) If $t = \tan(\frac{x}{2})$, $-\pi < x < \pi$, sketch a right triangle or use trigonometric identities to show that

$$\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}} \text{ and } \sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}.$$

(b) Show that

$$\cos x = \frac{1-t^2}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}.$$

(c) Show that

$$dx = \frac{2}{1+t^2}dt.$$

7.5.8 Three integrals are given that, although they look similar, may require different techniques of integration. Evaluate the integrals.

(a) $\int e^x \sqrt{e^x - 1}dx$ (b) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ (c) $\int \frac{1}{\sqrt{e^x - 1}}dx$

7.5.27 Evaluate the integral $\int e^{x+e^x} dx$.

7.5.44 Evaluate the integral $\int \frac{1+\sin x}{1+\cos x}dx$.

7.5.76 Evaluate the integral $\int \frac{x^2}{x^6 + 3x^3 + 2} dx$.

7.5.93 Evaluate the integral $\int_0^{\frac{\pi}{6}} \sqrt{1 + \sin 2\theta} d\theta$.

7.8.68 **Improper Integrals that Are Both Type 1 and Type 2**

The integral $\int_a^{\infty} f(x)dx$ is improper because the interval $[a, \infty)$ is infinite. If f has an infinite discontinuity at a, then the integral is improper for a second reason. In this case we evaluate the integral by expressing it as a sum of improper integrals of Type 2 and Type 1 as follows:

$$\int_0^{\infty} f(x)dx = \int_a^c f(x)dx + \int_c^{\infty} f(x)dx, \; c > a$$

Evaluate the given integral if it is convergent.

$$\int_2^{\infty} \frac{1}{x\sqrt{x^2 - 4}} dx$$

7.8.74 The *average speed* of molecules in an ideal gas is

$$\bar{v} = \frac{4}{\sqrt{\pi}} \left(\frac{M}{2RT}\right)^{\frac{3}{2}} \int_0^\infty v^3 e^{-\frac{Mv^2}{2RT}}\, dv,$$

where M is the molecular weight of the gas, R is the gas constant, T is the gas temperature, and v is the molecular speed.

Show that

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}}.$$

7.8.75 We know that the region $R = \{(x, y) | x \geqslant 1, 0 \leqslant y \leqslant \frac{1}{x}\}$ has infinite area. Show that by rotating R about the x-axis we obtain a solid (called *Gabriel's horn*) with finite volume.

7.8.80 A radioacitve substance decays exponentially: The mass at time t is $m(t) = m(0)e^{kt}$, where $m(0)$ is the initial mass and k is a negative constant. The *mean life* M of an atom in the substance is

$$M = -k \int_0^\infty t e^{kt} dt.$$

For the radioactive carbon isotope, ${}^{14}C$, used in radiocarbon dating, the value of k is -0.000121. Find the mean life of a ${}^{14}C$ atom.