

7.1.2 Evaluate the integral using integration by parts with the indicated choices of  $u$  and  $dv$ .

$$\int \sqrt{x} \ln x dx; \quad u = \ln x, \quad dv = \sqrt{x} dx$$

7.1.48 First make a substitution and then use integration by parts to evaluate the integral.

$$\int \frac{\sin^{-1}(\ln x)}{x} dx$$

7.1.54 Prove the reduction formula.

(a)

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

(b) Use part (a) to evaluate  $\int \cos^2 x dx$ .

(c) Use parts (a) and (b) to evaluate  $\int \cos^4 x dx$ .

7.1.60 Use integration by parts to prove the reduction formula.

$$\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx \quad (n \neq 1)$$

7.1.78 (a) Use integration by parts to show that

$$\int f(x) dx = xf(x) - \int xf'(x) dx$$

(b) If  $f$  and  $g$  are inverse functions and  $f'$  is continuous, prove that

$$\int_a^b f(x) dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y) dy$$

(c) In the case where  $f$  and  $g$  are positive functions and  $b > a > 0$ , draw a diagram to give a geometric interpretation of part (b).

(d) Use part (b) to evaluate  $\int_1^e \ln x dx$ .

7.2.32 Evaluate the integral.

$$\int \tan^2 x \sec x dx$$

7.2.56 Evaluate the integral.

$$\int \frac{1}{1 + \sec \theta} d\theta$$

7.2.62 (a) Prove the reduction formula

$$\int \tan^{2n} x dx = \frac{\tan^{2n-1} x}{2n-1} - \int \tan^{2n-2} x dx$$

(b) Use this formula to find  $\int \tan^8 x dx$

7.2.63 Find the average value of the function  $f(x) = \sin^2 x \cos^3 x$  on the interval  $[-\pi, \pi]$ .

7.2.71 Find the volume obtained by rotating the region bounded by the curves about the given axis.

$$y = \sin x, \ y = \cos x, \ 0 \leq x \leq \frac{\pi}{4}; \text{ about } y = 1$$

7.3.30 Evaluate the integral.

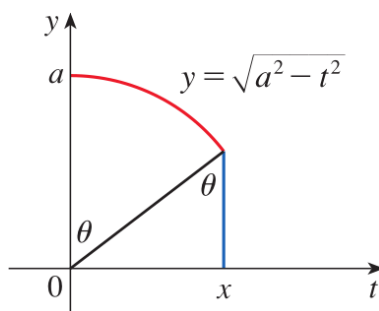
$$\int_0^1 \sqrt{x - x^2} dx$$

7.3.44 Find the volume of the solid obtained by rotating about the line  $x = 1$  the region under the curve  $y = x\sqrt{1-x^2}$ ,  $0 \leq x \leq 1$ .

7.3.45 (a) Use trigonometric substitution to verify that

$$\int_0^x \sqrt{a^2 - t^2} dt = \frac{1}{2}a^2 \sin^{-1} \left( \frac{x}{a} \right) + \frac{1}{2}x\sqrt{a^2 - x^2}$$

(b) Use the figure to give trigonometric interpretations of both terms on the right side of the equation in part (a).



7.3.47 A torus is generated by rotating the circle  $x^2 + (y - R)^2 = r^2$  about the  $x$ -axis. Find the volume enclosed by the torus.