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Calculus Exercise — Week 5 (3.5, 3.6) — Name:

3.5.44 If $x^2 + xy + y^3 = 1$, find the value of y''' at the point where $x = 1$.

3.5.58 Find the value of the number a such that the families of curves $y = (x + c)^{-1}$ and $y = a(x + k)^{\frac{1}{3}}$ are orthogonal trajectories.

3.5.65 Use implicit differentiation to find $\frac{dy}{dx}$ for the equation

$$\frac{x}{y} = y^2 + 1,\ y \neq 0$$

and for the equivalent equation

$$x = y^3 + y,\ y \neq 0$$

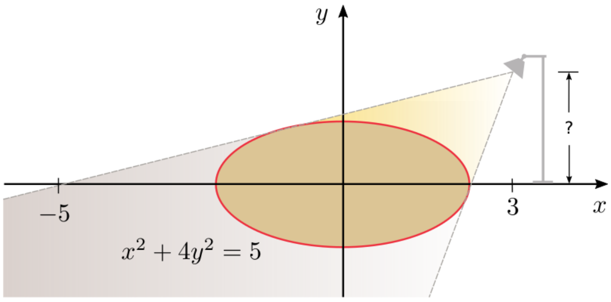
Show that although the expressions you get for $\frac{dy}{dx}$ look different, they agree for all points that satisfy the given equation.

3.5.66 The *Bessel function* of order 0, $y = J(x)$, satisfies the differential equation $xy'' + y' + xy = 0$ for all values of x and its value at 0 is $J(0) = 1$.

(a) Find $J'(0)$.

(b) Use implicit differentiation to find $J''(0)$.

3.5.67 The figure shows a lamp located three units to the right of the y-axis and a shadow created by the elliptical region $x^2 + 4y^2 \leq 5$. If the point $(-5, 0)$ is on the edge of the shadow, how far above the x-axis is the lamp located?



3.6.49 Use logarithmic differentiation to find the derivative of the function $y = x^x$.

3.6.62 Show that $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$ for any $x > 0$.

3.6.82 (a) One way of defining $\sec^{-1}(x)$ is to say that $y = \sec^{-1}(x) \iff \sec(y) = x$ and $0 \leq y < \frac{\pi}{2}$ or $\pi \leq y < \frac{3\pi}{2}$. Show that, with this definition,

$$\frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2 - 1}}.$$

(b) Another way of defining $\sec^{-1}(x)$ that is sometimes used it to say that $y = \sec^{-1}(x) \iff \sec(y) = x$ and $0 \leq y \leq \pi$, $y \neq \frac{\pi}{2}$. Show that, with this definition,

$$\frac{d}{dx} \sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2 - 1}}.$$

3.6.83 **Derivatives of Inverse Functions** Suppose that f is a one-to-one differentiable function and its inverse function f^{-1} is also differentiable. Use implicit differentiation to show that

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

provided that the denominator is not 0.

3.6.85 Use the formula in **3.6.83**. If $f(x) = x + e^x$, find $(f^{-1})'(1)$.