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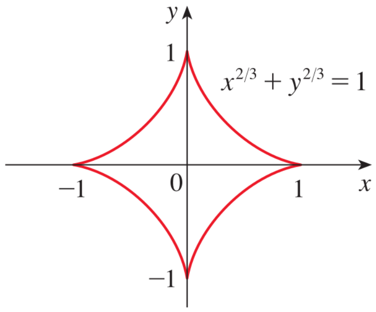
8.1.39 Find the length of the astroid.



8.1.40 (a) Sketch the curve $y^3 = x^2$.

(b) Use the two types of arc length formula

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx,\quad \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

to set up two integrals for the arc length from $(0,0)$ to $(1,1)$. Observe that one of these is an improper integral and evaluate both of them.

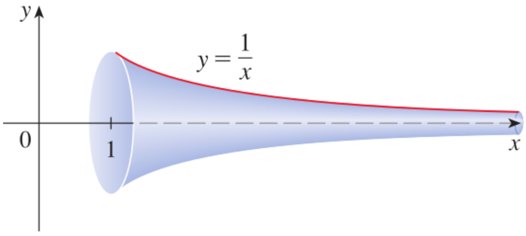
(c) Find the length of the arc of this curve from $(-1,1)$ to $(8,4)$.

8.1.43 Find the arc length function for the curve $y = \sin^{-1} x + \sqrt{1 - x^2}$ with starting point $(0, 1)$.

8.1.46 A steady wind blows a kite due west. The kite's height above ground from horizontal position $x = 0$ to $x = 80$ ft is given by $y = 150 - \frac{1}{40}(x - 50)^2$. Find the distance traveled by the kite.

8.2.28 Find the exact area of the surface obtained by rotating the curve $y = \sqrt{x^2 + 1}$, $0 \leqslant x \leqslant 3$, about the x-axis.

8.2.33 **Gabriel's Horn** The surface formed by rotating the curve $y = \frac{1}{x}, x \geqslant 1$, about the x-axis is known as *Gabriel's horn*. Show that the surface area is infinite (although the enclosed volume is finite.)



8.2.37 (a) The ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \;\; a > b$$

is rotated about the x-axis to form a surface called an *ellipsoid*, or *prolate spheroid*. Find the surface area of this ellipsoid.

(b) If the ellipse in part (a) is rotated about its minor axis (the y-axis), the resulting ellipsoid is called an *oblate spheroid*. Find the surface area of this ellipsoid.

8.2.42 **Zone of a Sphere** A *zone of a sphere* is the portion of the sphere that lies between two parallel planes.

Show that the surface area of a zone of a sphere is $S = 2\pi Rh$, where R is the radius of the sphere and h is the distance between the planes. (Notice that S depends only on the distance between the planes and not on their location, provided that both planes intersect the sphere.)

9.1.19 (a) What can you say about a solution of the equation $y' = -y^2$ just by looking at the differential equation?

(b) Verify that all members of the family $y = \frac{1}{x+C}$ are solutions of the equation in part (a).

(c) Can you think of a solution of the differential equation $y' = -y^2$ that is not a member of the family in part (b)?

(d) Find a solution of the initial-value problem $y' = -y^2,\ y(0) = 0.5$.

9.1.21 A population is modeled by the differential equation

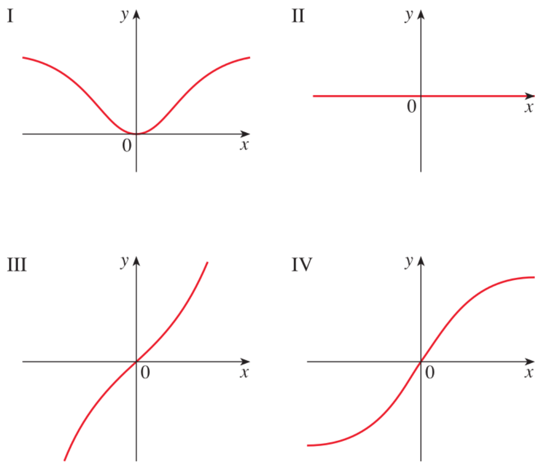
$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

(a) For what values of P is the population increasing?

(b) For what values of P is the population decreasing?

(c) What are the equilibrium solutions?

9.1.25 Match the differential equations with the solution graphs labeled I-IV. Give reasons for your choices.
(a) $y' = 1 + x^2 + y^2$ (b) $y' = xe^{-x^2-y^2}$ (c) $y' = \frac{1}{1+e^{x^2+y^2}}$ (d) $y' = \sin(xy)\cos(xy)$



9.1.29 Differential equations have been used extensively in the study of drug dissolution for patients given oral medications. One such equation is the Weibull equation for the concentration $c(t)$ of the drug:

$$\frac{dc}{dt} = \frac{k}{t^b}(c_s - c)$$

where k and c_s are positive constants and $0 < b < 1$.

Verify that

$$c(t) = c_s(1 - \exp(-\alpha t^{1-b}))$$

is a solution of the Weibull equation for $t > 0$, where $\alpha = \frac{k}{1-b}$. What dose the differential equation say about how drug dissolution occurs?

9.3.42 In elementary chemical reaction, single molecules of two reactants A and B from a molecule of the product C: A + B $\longrightarrow$ C. The law of mass action states that the rate of reaction is proportional to the product of the concentrations of A and B:

$$\frac{d[\mathrm{C}]}{dt} = k[\mathrm{A}][\mathrm{B}]$$

Thus if the initial concentrations are $[\mathrm{A}] = a$ moles/L and $[\mathrm{B}] = b$ moles/L and we write $x = [\mathrm{C}]$, then we have

$$\frac{dx}{dt} = k(a-x)(b-x)$$

(a) Assuming that $a \neq b$, find x as a function of t. Use the fact that the initial concentration of C is 0.

(b) Find $x(t)$ assuming that $a = b$. How does this expression for $x(t)$ simplify if it is known that $[\mathrm{C}] = \frac{a}{2}$ after 20 seconds?

9.3.44 A sphere with radius 1 m has temperature 15°C. It lies inside a concentric sphere with radius 2 m and temperature 25°C. The temperature $T(r)$ at a distance r from the common center of the spheres satisfies the differential equation

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

If we let $S = \frac{dT}{dr}$, then S satisfies a first-order differential equation. Solve it to find an expression for the temperature $T(r)$ between the spheres.

9.3.48 The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run?

9.3.54 A model for tumor growth is given by the Gompertz equation

$$\frac{dV}{dt} = a(\ln b - \ln V)V$$

where a and b are positive constants and V is the volume of the tumor measured in mm^3.

(a) Find a family of solutions for tumor volumes as a function of time.

(b) Find the solution that has an initial tumor volume of $V(0) = 1\text{mm}^3$.