

2.2.4 Use the given graph of  $f$  to state the value of each quantity, if it exists. If it does not exist, explain why.

(a)  $\lim_{x \rightarrow 2^-} f(x)$

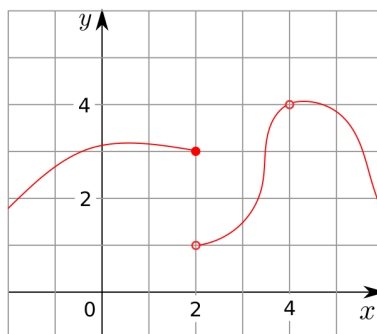
(b)  $\lim_{x \rightarrow 2^+} f(x)$

(c)  $\lim_{x \rightarrow 2} f(x)$

(d)  $f(2)$

(e)  $\lim_{x \rightarrow 4} f(x)$

(f)  $f(4)$

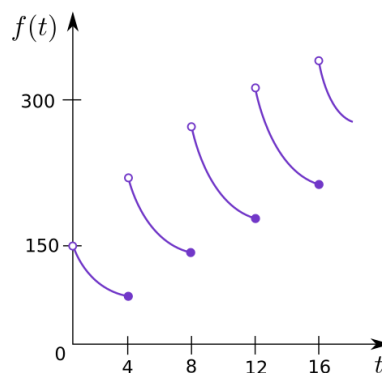


2.2.10 A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount  $f(t)$  of the drug in the bloodstream after  $t$  hours.

Find

$$\lim_{t \rightarrow 12^-} f(t), \quad \lim_{t \rightarrow 12^+} f(t)$$

and explain the significance of these one-sided limits.



2.2.16 Sketch the graph of an example of a function  $f$  that satisfies all of the given conditions.

$$\lim_{x \rightarrow 0} f(x) = 4, \quad \lim_{x \rightarrow 8^-} f(x) = 1, \quad \lim_{x \rightarrow 8^+} f(x) = -3, \quad f(0) = 6, \quad f(8) = -1$$

2.2.38 Determine the infinite limit.

$$\lim_{x \rightarrow 3^-} \frac{x^2 + 4x}{x^2 - 2x - 3}$$

2.2.42 (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

2.3.2 The graphs of  $f$  and  $g$  are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

(a)  $\lim_{x \rightarrow 2} [f(x) + g(x)]$

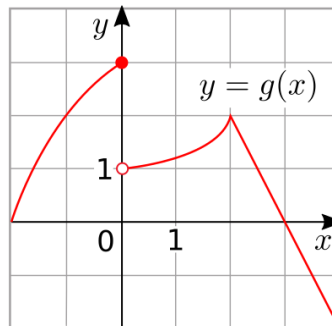
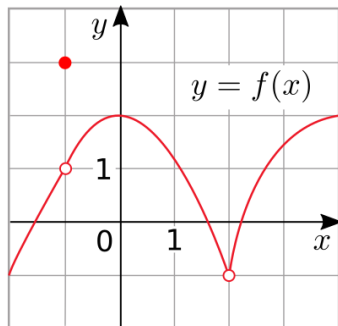
(b)  $\lim_{x \rightarrow 0} [f(x) - g(x)]$

(c)  $\lim_{x \rightarrow -1} [f(x)g(x)]$

(d)  $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$

(e)  $\lim_{x \rightarrow 2} [x^2 f(x)]$

(f)  $f(-1) + \lim_{x \rightarrow -1} g(x)$



2.3.26 Evaluate the limit, if it exists.

$$\lim_{h \rightarrow 0} \frac{(-2 + h)^{-1} + 2^{-1}}{h}$$

2.3.34 Evaluate the limit, if it exists.

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

2.3.54 Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

(a) Evaluate each of the following, if it exists.

(i)  $\lim_{x \rightarrow 1^-} g(x)$

(ii)  $\lim_{x \rightarrow 1} g(x)$

(iii)  $g(1)$

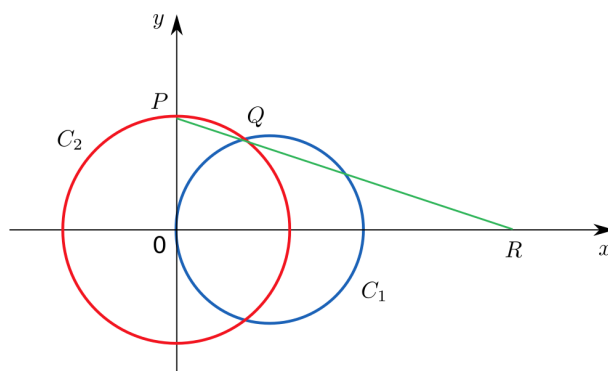
(iv)  $\lim_{x \rightarrow 2^-} g(x)$

(v)  $\lim_{x \rightarrow 2^+} g(x)$

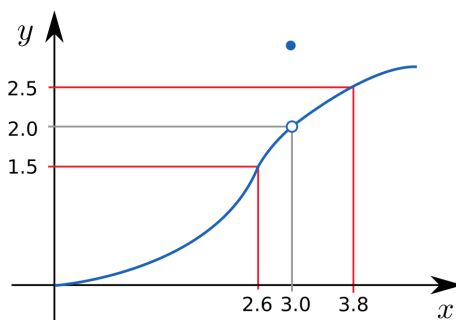
(vi)  $\lim_{x \rightarrow 2} g(x)$

(b) Sketch the graph of  $g$ .

2.3.68 The figure shows a fixed circle  $C_1$  with equation  $(x - 1)^2 + y^2 = 1$  and a shrinking circle  $C_2$  with radius  $r$  and center the origin.  $P$  is the point  $(0, r)$ ,  $Q$  is the upper point of intersection of the two circles, and  $R$  is the point of intersection of the line  $PQ$  and the  $x$ -axis. What happens to  $R$  as  $C_2$  shrinks, that is, as  $r \rightarrow 0^+$ ?



2.4.2 Use the given graph of  $f$  to find a number  $\delta$  such that if  $0 < |x - 3| < \delta$  then  $|f(x) - 2| < 0.5$



2.4.14 Given that  $\lim_{x \rightarrow 2} (5x - 7) = 3$ , illustrate the precise definition of a limit by finding values of  $\delta$  that correspond to  
 (a)  $\epsilon = 0.1$  (b)  $\epsilon = 0.05$  (c)  $\epsilon = 0.01$ .

2.4.28 Prove that  $\lim_{x \rightarrow -6^+} \sqrt[8]{6 + x} = 0$  using the  $\epsilon, \delta$  definition of a limit.

2.4.42 Use the precise definition of an infinite limit to prove that  $\lim_{x \rightarrow -3} \frac{1}{(x+3)^4} = \infty$ .

2.4.44 Suppose that  $\lim_{x \rightarrow a} f(x) = \infty$  and  $\lim_{x \rightarrow a} g(x) = c$ , where  $c \in \mathbb{R}$ . Prove each statement.

- (a)  $\lim_{x \rightarrow a} [f(x) + g(x)] = \infty$
- (b)  $\lim_{x \rightarrow a} [f(x)g(x)] = \infty$  if  $c > 0$
- (c)  $\lim_{x \rightarrow a} [f(x)g(x)] = -\infty$  if  $c < 0$