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Calculus Exercise | Week 8 (4.5, 4.7, 4.9) | Name: __________

4.5.11 Sketch the curve

$$y = \frac{x - x^2}{2 - 3x + x^2}$$

4.5.37 Sketch the curve

$$y = \sin(x) + \sqrt{3}\cos(x), \ -2\pi \leqslant x \leqslant 2\pi$$

4.5.54 Sketch the curve

$$y = \tan^{-1}\left(\frac{x-1}{x+1}\right)$$

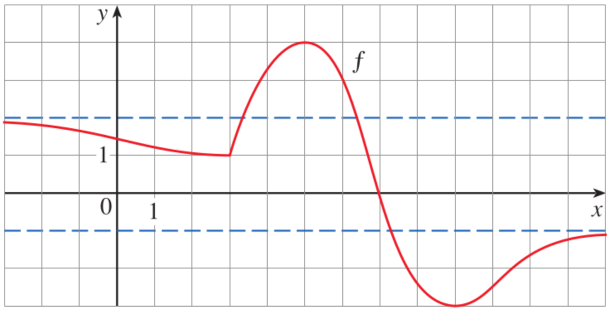
4.5.56 The graph of a function f is shown. (The dashed lines indicate horizontal asymptotes.) Find each of the following for the given function $g(x) = \sqrt[3]{f(x)}$.

(a) The domains of g and g'

(b) The critical numbers of g

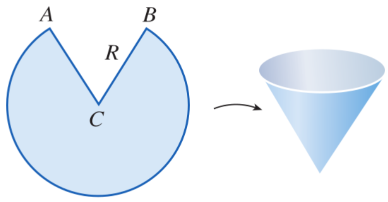
(c) The approximate value of $g'(6)$

(d) All vertical and horizontal asymptotes of g



4.5.75 Show that the curve $y = x - \tan^{-1}(x)$ has two slant asymptotes: $y = x + \frac{\pi}{2}$ and $y = x - \frac{\pi}{2}$. Use this fact to help sketch the curve.

4.7.47 A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

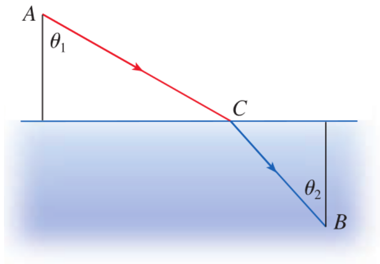


4.7.57 An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying pipe is \$400,000/km over land to a point P on the north bank and \$800,000/km under the river to the tanks. To minimize the cost of the pipeline, where should P be located?

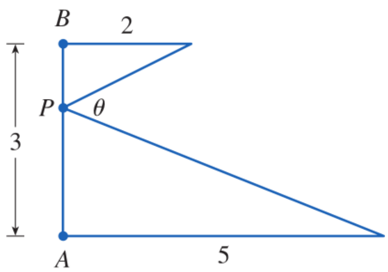
4.7.77 Let v_1 be the velocity of light in air and v_2 the velocity of light in water. According to Fermat's Principle, a ray of light will travel from a point A in the air to a point B in the water by a path ACB that minimizes the time taken. Show that

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2}$$

where θ_1 (the angle of incidence) and θ_2 (the angle of refraction) are as shown. This equation is known as Snell's Law.



4.7.83 Where should the point P be chosen on the line segment AB so as to maximize the angle θ?



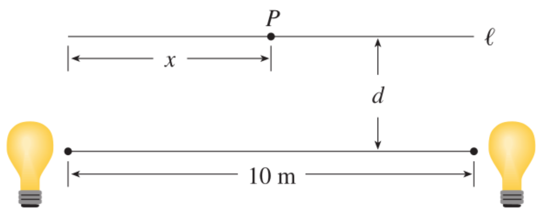
4.7.88 Two light sources of identical strength are placed 10 m apart. An object is to be placed at a point P on a line l, parallel to the line joining the light sources and at a distance d meters from it (see the figure). We want to locate P on l so that the intensity of illumination is minimized. We need to use the fact that the intensity of illumination for a single source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source.

(a) Find an expression for the intensity $I(x)$ at the point P.

(b) If $d = 5$m, use graphs of $I(x)$ and $I'(x)$ to show that the intensity is minimized when $x = 5$m, that is, when P is at the midpoint of l.

(c) If $d = 10$m, show that the intensity (perhaps surprisingly) is *not* minimized at the midpoint.

(d) Somewhere between $d = 5$ m and $d = 10$ m there is a transitional value of d at which the point of minimal illumination abruptly changes. Estimate this value of d by graphical methods. Then find the exact value of d.



4.9.4 Find an antiderivative of the function

$$f(x) = \frac{1}{x}$$

4.9.12 Find the most general antiderivative of the function

$$f(x) = 3x^{0.8} + x^{-2.5}$$

4.9.24 Find the most general antiderivative of the function

$$f(x) = 2\cos(x) - \frac{3}{\sqrt{1-x^2}}$$

4.9.42 Find f

$$f'(x) = \frac{x+1}{\sqrt{x}},\ f(1) = 5$$

4.9.52 Find f

$$f''(x) = \sqrt[3]{x} - \cos(x),\ f(0) = f(1) = 2$$