7.1.2 Evaluate the integral using integration by parts with the indicated choices of u and dv.

$$\int \sqrt{x}\ln x dx;\ u = \ln x,\ dv = \sqrt{x} dx$$

7.1.48 First make a substitution and then use integration by parts to evaluate the integral.

$$\int \frac{\sin^{-1}(\ln x)}{x} dx$$

7.1.54 Prove the reduction formula.

(a)

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

(b) Use part (a) to evaluate $\int \cos^2 x dx$.

(c) Use parts (a) and (b) to evaluate $\int \cos^4 x dx$.

7.1.60 Use integration by parts to prove the reduction formula.

$$\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx \ (n \neq 1)$$

7.1.78 (a) Use integration by parts to show that

$$\int f(x)dx = xf(x) - \int xf'(x)dx$$

(b) If f and g are inverse functions and f' is continuous, prove that

$$\int_a^b f(x)dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)dy$$

(c) In the case where f and g are positive functions and $b > a > 0$, draw a diagram to give a geometric interpretation of part (b).

(d) Use part (b) to evaluate $\displaystyle\int_1^e \ln x dx$.

7.2.32 Evaluate the integral.

$$\int \tan^2 x \sec x dx$$

7.2.56 Evaluate the integral.

$$\int \frac{1}{1+\sec\theta} d\theta$$

7.2.62 (a) Prove the reduction formula

$$\int \tan^{2n} x dx = \frac{\tan^{2n-1} x}{2n-1} - \int \tan^{2n-2} x dx$$

(b) Use this formula to find $\int \tan^8 x dx$

7.2.63 Find the average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$.

7.2.71 Find the volume obtained by rotating the region bounded by the curves about the given axis.

$$y = \sin x,\ y = \cos x,\ 0 \leqslant x \leqslant \frac{\pi}{4};\quad \text{about } y = 1$$

7.3.30 Evaluate the integral.

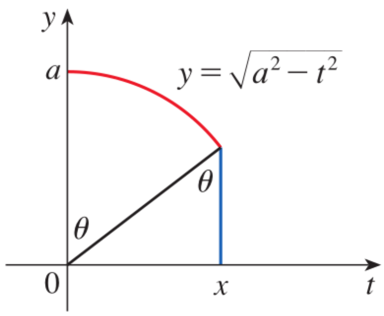
$$\int_0^1 \sqrt{x - x^2} dx$$

7.3.44 Find the volume of the solid obtained by rotating about the line $x = 1$ the region under the curve $y = x\sqrt{1-x^2},\ 0 \leqslant x \leqslant 1$.

7.3.45 (a) Use trigonometric substitution to verify that

$$\int_0^x \sqrt{a^2 - t^2}dt = \frac{1}{2}a^2 \sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}x\sqrt{a^2 - x^2}$$

(b) Use the figure to give trigonometric interpretations of both terms on the right side of the equation in part (a).



7.3.47 A torus is generated by rotating the circle $x^2 + (y - R)^2 = r^2$ about the x-axis. Find the volume enclosed by the torus.