

2.2.4 Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \rightarrow 2^-} f(x)$

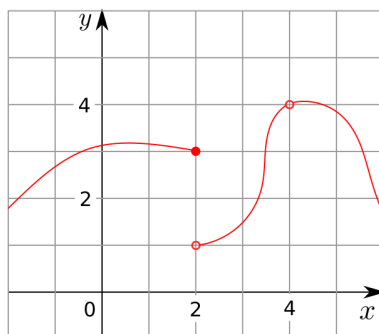
(b) $\lim_{x \rightarrow 2^+} f(x)$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $f(2)$

(e) $\lim_{x \rightarrow 4} f(x)$

(f) $f(4)$

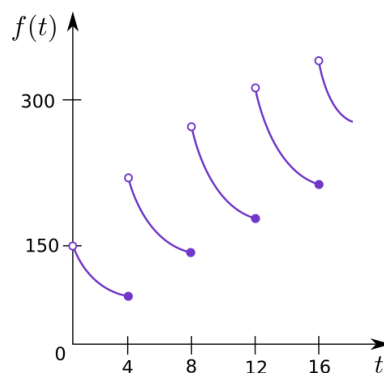


2.2.10 A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours.

Find

$$\lim_{t \rightarrow 12^-} f(t), \quad \lim_{t \rightarrow 12^+} f(t)$$

and explain the significance of these one-sided limits.



2.2.16 Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x \rightarrow 0} f(x) = 4, \quad \lim_{x \rightarrow 8^-} f(x) = 1, \quad \lim_{x \rightarrow 8^+} f(x) = -3, \quad f(0) = 6, \quad f(8) = -1$$

2.2.38 Determine the infinite limit.

$$\lim_{x \rightarrow 3^-} \frac{x^2 + 4x}{x^2 - 2x - 3}$$

2.2.42 (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

2.3.2 The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow 2} [f(x) + g(x)]$

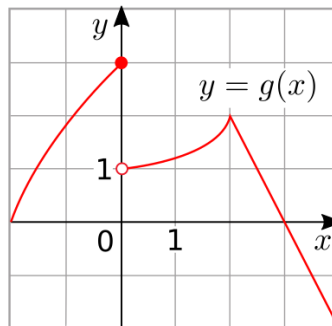
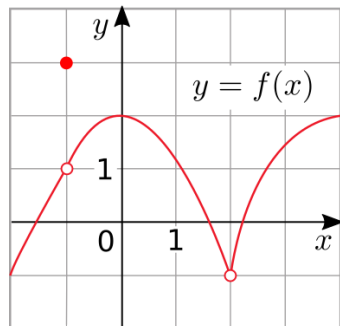
(b) $\lim_{x \rightarrow 0} [f(x) - g(x)]$

(c) $\lim_{x \rightarrow -1} [f(x)g(x)]$

(d) $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$

(e) $\lim_{x \rightarrow 2} [x^2 f(x)]$

(f) $f(-1) + \lim_{x \rightarrow -1} g(x)$



2.3.26 Evaluate the limit, if it exists.

$$\lim_{h \rightarrow 0} \frac{(-2 + h)^{-1} + 2^{-1}}{h}$$

2.3.34 Evaluate the limit, if it exists.

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

2.3.54 Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

(a) Evaluate each of the following, if it exists.

(i) $\lim_{x \rightarrow 1^-} g(x)$

(ii) $\lim_{x \rightarrow 1} g(x)$

(iii) $g(1)$

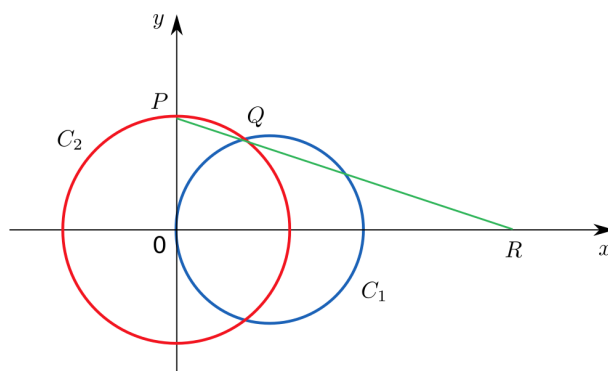
(iv) $\lim_{x \rightarrow 2^-} g(x)$

(v) $\lim_{x \rightarrow 2^+} g(x)$

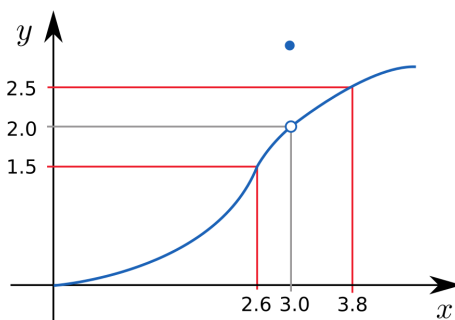
(vi) $\lim_{x \rightarrow 2} g(x)$

(b) Sketch the graph of g .

2.3.68 The figure shows a fixed circle C_1 with equation $(x - 1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x -axis. What happens to R as C_2 shrinks, that is, as $r \rightarrow 0^+$?



2.4.2 Use the given graph of f to find a number δ such that if $0 < |x - 3| < d$ then $|f(x) - 2| < 0.5$



2.4.14 Given that $\lim_{x \rightarrow 2} (5x - 7) = 3$, illustrate the precise definition of a limit by finding values of δ that correspond to
 (a) $\epsilon = 0.1$ (b) $\epsilon = 0.05$ (c) $\epsilon = 0.01$.

2.4.28 Prove that $\lim_{x \rightarrow -6^+} \sqrt[8]{6 + x} = 0$ using the ϵ, δ definition of a limit.

2.4.42 Use the precise definition of an infinite limit to prove that $\lim_{x \rightarrow -3} \frac{1}{(x+3)^4} = \infty$.

2.4.44 Suppose that $\lim_{x \rightarrow a} f(x) = \infty$ and $\lim_{x \rightarrow a} g(x) = c$, where $c \in \mathbb{R}$. Prove each statement.

- (a) $\lim_{x \rightarrow a} [f(x) + g(x)] = \infty$
- (b) $\lim_{x \rightarrow a} [f(x)g(x)] = \infty$ if $c > 0$
- (c) $\lim_{x \rightarrow a} [f(x)g(x)] = -\infty$ if $c < 0$