ID: _____

Calculus Exercise | Week 1 (1.4, 1.5, 2.1) | Name: _____

1.4.1 Use the Laws of Exponents to simplify each expression.

(a) $\dfrac{-2^6}{4^3}$

(b) $\dfrac{(-3)^6}{9^6}$

(c) $\dfrac{1}{\sqrt[4]{x^5}}$

(d) $\dfrac{x^3 \cdot x^n}{x^{n+1}}$

(e) $b^3(3b^{-1})^{-2}$

(f) $\dfrac{2x^2y}{(3x^{-2}y)^2}$

1.4.2 Use the Laws of Exponents to simplify each expression.

(a) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{108}}$

(b) $27^{\frac{2}{3}}$

(c) $2x^2(3x^5)^2$

(d) $(2x^{-2})^{-3}x^{-3}$

(e) $\dfrac{3a^{\frac{3}{2}} \cdot a^{\frac{1}{2}}}{a^{-1}}$

(f) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

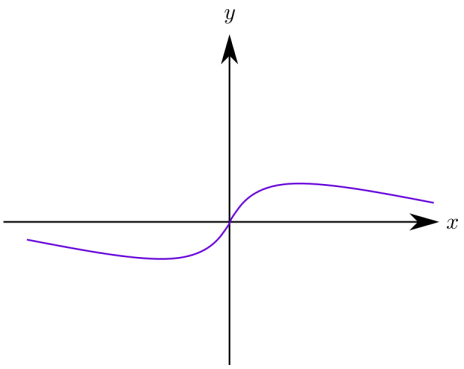
1.4.10 Make a rough sketch of $f(x) = 2\left(\frac{1}{2}\right)^x - 3$ by hand.

1.4.14 Make a rough sketch of $f(x) = e^{|x|}$ by hand.

1.4.21 If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h}\right)$$

1.5.8 Determine the function whether it's one-to-one.



1.5.18 If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

1.5.26 Find the inverse of $f(x) = \dfrac{6 - 3x}{5x + 7}$.

1.5.44 Use the laws of logarithms to expand each expression.

(a) $\ln \sqrt{\dfrac{3x}{x-3}}$

(b) $\log_2 \left((x^3+1) \sqrt[3]{(x-3)^2} \right)$

1.5.58 Solve both an exact value and an approximation to three decimal places for x.

(a) $\log_2(x^2 - x - 1) = 2$

(b) $1 + e^{4x+1} = 20$

2.1.2 A student bought a smartwatch that tracks the number of steps she walks throughout the day. The table shows the number of steps recorded t minutes after 3:00 PM on the first day she wore the watch.

t (min)	0	10	20	30	40
Steps	3438	4559	5622	6536	7398

(a) Find the slopes of the secant lines corresponding to the given intervals (i), (ii), (iii) of t. What do these slopes represent? (i): $[0, 40]$, (ii): $[10, 20]$, (iii): $[20, 30]$.

(b) Estimate the student's walking pace, in steps per minute, at 3:20 PM by averaging the slopes of two secant lines.

2.1.3 The point $P : (2, -1)$ lies on the curve $C : y = \dfrac{1}{1-x}$.

(a) Find the slope of the secant line PQ (correct to six decimal places), where $Q : \left(x, \dfrac{1}{1-x}\right)$ is another point on the curve C for the following values of x:
(i) 1.5, (ii) 1.9, (iii) 1.99, (iv) 1.999, (v) 2.5, (vi) 2.1, (vii) 2.01, (viii) 2.001.

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve C at the point P.

(c) Using the slope from part (b), find an equation of the tangent line to the curve C at the point P.

2.1.6 If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.

(a) Find the average velocity over the given time intervals:
(i) $[1, 2]$, (ii) $[1, 1.5]$, (iii) $[1, 1.1]$, (iv) $[1, 1.01]$ (v) $[1, 1.001]$

(b) Estimate the instantaneous velocity when $t = 1$.

2.1.7 The table shows the position of a motorcyclist after accelerating from rest.

t (seconds)	0	1	2	3	4	5	6
s (meters)	0	1.5	6.3	14.2	24.1	38.0	53.9

(a) Find the average velocity of each time period:
(i) $[2, 4]$, (ii) $[3, 4]$, (iii) $[4, 5]$, (iv) $[4, 6]$

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.