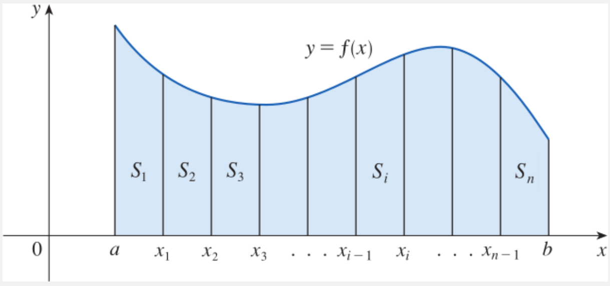
5.1.18 Use the following **Definition** to find an expression for the area under the graph $f(x) = x + \ln x$, $3 \leqslant x \leqslant 8$ as a limit. You don't need to evaluate the limit.

Definition. The **area** A of the region $S = \bigcup_{k=1}^{n} S_n$ that lies under the graph of the continuous function f on the interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x, \quad \Delta x = x_1 - a = b - x_{n-1} = x_i - x_{i-1} \text{ for } 1 \leqslant i \leqslant n.$$



5.1.22 Determine a region whose area is equal to the given limit. You don't need to evaluate the limit.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + \frac{3i}{n}}$$

5.1.34 (a) Let A_n be the area of a polygon with n equal sides inscribed in a circle with radius r. By dividing, the polygon into n congruent triangles with central angle $2\pi/n$, show that

$$A_n = \frac{1}{2}nr^2 \sin\left(\frac{2\pi}{n}\right)$$

(b) Show that

$$\lim_{n\to\infty} A_n = \pi r^2.$$

5.2.23 Show that the definite integral is equal to $\lim_{n\to\infty} R_n$, and then evaluate the limit.

$$\int_0^4 (x - x^2)dx,\ R_n = \frac{4}{n}\sum_{i=1}^{n}\left(\frac{4i}{n} - \frac{16i^2}{n^2}\right)$$

5.2.57 Write as a single integral in the form $\displaystyle\int_a^b f(x)dx$:

$$\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$$

5.2.60 Find $\int_0^5 f(x)dx$ if

$$f(x) = \begin{cases} 3 & , \ x < 3 \\ x & , \ x \geqslant 3 \end{cases}$$

5.2.68 Use the properties of integrals to verify the inequality without evaluating the integrals.

$$\frac{\pi}{12} \leqslant \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x dx \leqslant \frac{\sqrt{3}\pi}{12}$$

5.3.58 Sketch the region enclosed by the given curves and calculate its area.

$$y = 2x - x^2, \ y = 0$$

5.3.68 Find the derivative of the function.

$$g(x) = \int_{1-2x}^{1+2x} t \sin t dt$$

5.3.76 If $f(x) = \displaystyle\int_0^{\sin x} \sqrt{1+t^2} dt$ and $g(y) = \displaystyle\int_3^y f(x)dx$, find $g''(\frac{\pi}{6})$.

5.3.78 Use l'Hospital's Rule to evaluate the limit.

$$\lim_{x\to\infty} \frac{1}{x^2} \int_0^x \ln(1+e^t)dt$$

5.3.93 Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2} dt = 2\sqrt{x}, \ \forall \ x > 0.$$