Calculus Exercise

Week 11 (6.1, 6.2, 6.3, 6.5)

ID: Name:

6.1.18 Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

$$4x + y^2 = 12, \ x = y$$

6.1.31 Sketch the region enclosed by the given curves and find tis area.

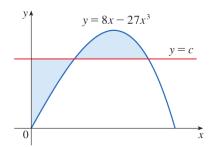
$$y = x^4, \ y = 2 - |x|$$

6.1.42 Use calculus to find the area of the triangle with the given vertices: (2,0), (0,2), (-1,1).

6.1.43 Evaluate the integral and interpret it as the area of a region. Sketch the region.

$$\int_0^{\frac{\pi}{2}} |\sin(x) - \cos(2x)| dx$$

6.1.69 The figure shows a horizontal line y = c intersecting the curve  $y = 8x - 27x^3$ . Find the number c such that the areas of the shaded regions are equal.

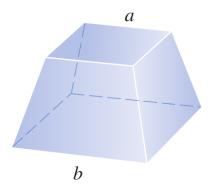


6.2.44 Set up an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Then use a calculator or computer to evaluate the integral correct to five decimal places.

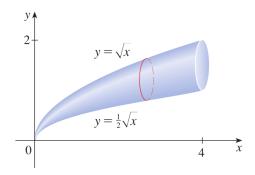
$$y = x^2, \ x^2 + y^2 = 1, \ y \geqslant 0$$

(a) About the x-axis (b) About the y-axis

6.2.62 A frustum of a pyramid with square base of side b, square top of side a, and height b. What happens if a = b? What happens if a = 0?

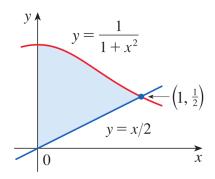


6.2.74 Cross-sections of the solid S in planes perpendicular to the x-axis are circles with diameters extending from the curve  $y=\frac{1}{2}\sqrt{x}$  to the curve  $y=\sqrt{x}$  for  $0\leqslant x\leqslant 4$ .



6.2.86 Suppose that a region R has area A and lies above the x-axis. When R is rotated about the x-axis, it sweeps out a solid with volume  $V_1$ . When R is rotated about the line y=-k (where k is a positive number), it sweeps out a solid with volume  $V_2$ . Express  $V_2$  in terms of  $V_1$ , k, and A.

- 6.3.47 A solid is obtained by rotating the shaded region about the y-axis.
  - (a) Set up an integral using any method to find the volume of the solid.
  - (b) Evaluate the integral to find the volume of the solid.



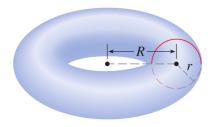
6.3.55 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

$$y^2 - x^2 = 1$$
,  $y = 2$ ; about the *x*-axis

6.3.56 Same as 6.3.55.

$$y^2 - x^2 = 1$$
,  $y = 2$ ; about the y-axis

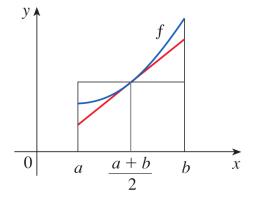
6.3.62 Use cylindrical shells to find the volume of the solid torus.



6.5.13 If f is continuous and  $\int_1^3 f(x)dx = 8$ , show that f takes on the value 4 at least once on the interval [1, 3].

6.5.25 Use the diagram to show that if f is concave upward on [a, b], then

$$\frac{1}{b-a} \int_{a}^{b} f(x) dx > f\left(\frac{a+b}{2}\right)$$



6.5.26 Let  $f_{\text{avg}}$  denote the average value of f on the interval [a, b]. Show that if a < c < b, then

$$f_{\text{avg}}[a, b] = \left(\frac{c - a}{b - a}\right) f_{\text{avg}}[a, c] + \left(\frac{b - c}{b - a}\right) f_{\text{avg}}[c, b]$$