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Calculus Exercise Week 15 (9.5, 10.1, 10.2)

9.5.30 Solve the second-order equation $xy'' + 2y' = 12x^2$ by making the substitution $u = y'$.

9.5.35 Let $P(t)$ be the performance level of someone learning a skill as a function of the training time t. The graph of P is called a learning curve. In Exercise 9.1.27 we proposed the differential equation

$$\frac{dP}{dt} = k[M - P(t)]$$

as a reasonable model for learning, where k is a positive constant. Solve it as a linear differential equation and use your solution to graph the learning curve.

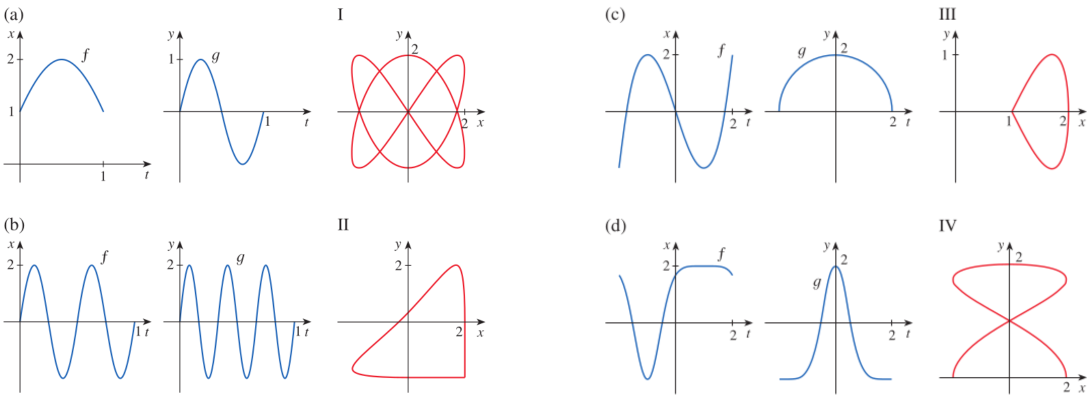
9.5.41 (a) Show that the substitution $z = 1/P$ transforms the logistic differential equation $P' = kP(1 - P/M)$ into the linear differential equation

$$z' + kz = \frac{k}{M}$$

(b) Solve the linear differential equation in part (a) and thus obtain an expression for $P(t)$. Compare with the following equation.

$$P(t) = \frac{M}{1 + Ae^{-kt}}, \text{ where } A = \frac{M - P_0}{P_0}.$$

10.1.30 Match each pair of graphs of equations $x = f(t)$, $y = g(t)$ in (a)-(d) with one of the parametric curves $x = f(t)$, $y = g(t)$ labeled I–IV. Give reasons for your choices.



10.1.49 Let P be a point at a distance d from the center of a circle of radius r. The curve traced out by P as the circle rolls along a straight line is called a **trochoid**. (Think of the motion of a point on a spoke of a bicycle wheel.) The cycloid is the special case of a trochoid with $d = r$. Using the same parameter θ as for the cycloid, and assuming the line is the x-axis and $\theta = 0$ when P is at one of its lowest points, show that parametric equations of the trochoid are

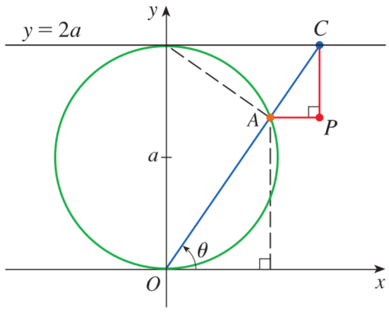
$$x = r\theta - d\sin\theta \qquad y = r - d\cos\theta$$

Sketch the trochoid for the cases $d < r$ and $d > r$.

10.1.53 A curve, called a **witch of Maria Agnesi**, consists of all possible positions of the point P in the figure. Show that parametric equations for this curve can be written as

$$x = 2a\cot\theta,\ \ y = 2a\sin^2\theta$$

Sketch the curve.



10.2.18 Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

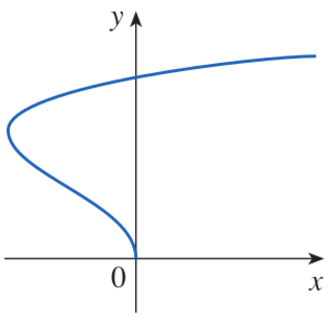
$$x = t^2 + 1,\ \ y = e^t - 1$$

10.2.31 (a) Find the slope of the tangent line to the trochoid $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$ in term of θ. (See Exercise 10.1.49).

(b) Show that if $d < r$, then the trochoid does not have a vertical tangent.

10.2.38 Find the area enclosed by the given parametric curve and the y-axis.

$$x = t^2 - 2t, \; y = \sqrt{t}$$



10.2.50 Find the exact length of the curve.

$$x = 3\cos t - \cos 3t, \qquad y = 3\sin t - \sin 3t, \qquad 0 \leqslant t \leqslant \pi$$

10.2.73 Find the exact area of the surface obtained by rotating the given curve about the x-axis.

$$x = a\cos^3\theta, \qquad y = a\sin^3\theta, \qquad 0 \leqslant \theta \leqslant \pi/2$$