NNGPResNet: Rigorous IID and Correlated-Layer Bounds with Lean Correspondence

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Abstract

This note gives complete mathematical proofs for the IID and correlated NNGPResNet results formalized in Lean. All constants are chosen to match the Lean theorems. A final section maps every definition and theorem in this note to the exact Lean declaration and file path.

1 Setup and Notation

Fix integers $m, r, L \in \mathbb{N}$ with $L \geq 1$, $m \geq 1$, and define $rm := rm$. For $x \in \mathbb{R}$, define $\operatorname{sign}(x) = 1$ if $x \geq 0$, and -1 if $x < 0$. For vectors, $\langle \cdot, \cdot \rangle$ is the Euclidean inner product and $\| \cdot \|$ is the Euclidean norm. The unit sphere is

$$S^{m-1} := \{u \in \mathbb{R}^m : \|u\| = 1\}.$$

For a centered Gaussian random variable $G \sim \mathcal{N}(0, \sigma^2)$ we use:

$$\mathbb{P}(|G| \geq t) \leq 2 \exp\left(-\frac{t^2}{2\sigma^2}\right), \qquad \mathbb{E}[G^2] = \sigma^2.$$

2 IID NNGP Model

2.1 Model definition

For each $\ell \in \{1, \dots, L\}$, $i \in \{1, \dots, m\}$, $j \in \{1, \dots, rm\}$:

- $B_{\ell ij} \sim \mathcal{N}(0, 1)$ i.i.d.,
- $z_{\ell j} \sim \mathcal{N}(0, 1)$ i.i.d., independent of all B,
- $a_{\ell j} := \operatorname{sign}(z_{\ell j}) \in \{\pm 1\}$.

Define layer increment and probes:

$$\Delta h_\ell(i) := \frac{1}{\sqrt{L}} \frac{1}{\sqrt{rm}} \sum_{j=1}^{rm} B_{\ell ij} a_{\ell j}, \quad g_\ell(u) := \langle u, \Delta h_\ell \rangle,$$

$$M_u := \frac{1}{L} \sum_{\ell=1}^{L} g_\ell(u), \qquad W := \frac{1}{L} \sum_{\ell=1}^{L} \Delta h_\ell, \qquad X(u) := \langle u, W \rangle.$$

Clearly $X(u) = M_u$ for every u.

2.2 IID-T1: exact scalar law, tail, second moment

Theorem 2.1 (IID-T1 Gaussian law)**.** *For each fixed $u \in S^{m-1}$,*

$$M_u \sim \mathcal{N}\left(0, \frac{1}{L^2}\right).$$

Proof. Fix $u \in S^{m-1}$. For one layer ℓ,

$$g_\ell(u) = \frac{1}{\sqrt{L}\sqrt{rm}} \sum_{i=1}^{m} \sum_{j=1}^{rm} u_i a_{\ell j} B_{\ell ij}.$$

Condition on $\{a_{\ell j}\}_{j=1}^{rm}$. Then $g_\ell(u)$ is a linear combination of independent centered Gaussians, hence Gaussian with mean 0 and variance

$$\operatorname{Var}(g_\ell(u) \mid a) = \frac{1}{Lrm} \sum_{i=1}^{m} \sum_{j=1}^{rm} u_i^2 a_{\ell j}^2 = \frac{1}{Lrm} \left(\sum_{i=1}^{m} u_i^2\right) \left(\sum_{j=1}^{rm} 1\right) = \frac{\|u\|^2}{L} = \frac{1}{L}.$$

So unconditionally $g_\ell(u) \sim \mathcal{N}(0, 1/L)$. For different ℓ, variables $g_\ell(u)$ depend on disjoint Gaussian coordinates $(B_{\ell ij}, z_{\ell j})$, so they are independent. Hence

$$M_u = \frac{1}{L} \sum_{\ell=1}^{L} g_\ell(u)$$

is centered Gaussian with variance

$$\operatorname{Var}(M_u) = \frac{1}{L^2} \sum_{\ell=1}^{L} \operatorname{Var}(g_\ell(u)) = \frac{1}{L^2} \cdot L \cdot \frac{1}{L} = \frac{1}{L^2}.$$

Therefore $M_u \sim \mathcal{N}(0, 1/L^2)$. □

Corollary 2.2 (IID-T1 tail)**.** *For each $u \in S^{m-1}$ and each $t > 0$,*

$$\mathbb{P}(|M_u| \geq t) \leq 2 \exp\left(-\frac{L^2 t^2}{2}\right).$$

Proof. Apply the standard Gaussian tail inequality to Theorem **??** with variance $\sigma^2 = 1/L^2$. □

Corollary 2.3 (IID-T1 second moment)**.** *For each $u \in S^{m-1}$,*

$$\mathbb{E}[M_u^2] = \frac{1}{L^2}.$$

Proof. A centered Gaussian has second moment equal to variance, and Theorem **??** gives variance $1/L^2$. □

2.3 IID-T2: expected supremum via Dudley

Fix $u_0 \in S^{m-1}$ and define centered process

$$\widetilde{X}(u) := X(u) - X(u_0), \qquad u \in S^{m-1}.$$

Lemma 2.4 (Increment metric). *For all $u, v \in S^{m-1}$,*

$$X(u) - X(v) \sim \mathcal{N}\left(0, \frac{\|u-v\|^2}{L^2}\right),$$

so the canonical metric is

$$d(u,v) := \sqrt{\mathbb{E}[(X(u)-X(v))^2]} = \frac{1}{L}\|u-v\|.$$

Proof. $X(u) - X(v) = M_{u-v}$. Repeating the computation in Theorem **??** for a general vector $w \in \mathbb{R}^m$ gives $M_w \sim \mathcal{N}(0, \|w\|^2/L^2)$. Set $w = u - v$. □

Lemma 2.5 (Entropy integral bound with explicit constant). *If $m \geq 1$, then*

$$\mathrm{EntInt}(S^{m-1}, 2) := \int_0^2 \sqrt{\log N(S^{m-1}, \varepsilon)}\, d\varepsilon \leq 4\sqrt{m}.$$

Proof. Let $B_2^m := \{x \in \mathbb{R}^m : \|x| \leq 1\}$. Since $S^{m-1} \subset B_2^m$, monotonicity of covering numbers gives

$$N(S^{m-1}, \varepsilon) \leq N(B_2^m, \varepsilon).$$

For $0 < \varepsilon \leq 2$, the standard volumetric estimate gives

$$N(B_2^m, \varepsilon) \leq \left(1 + \frac{2}{\varepsilon}\right)^m \leq \left(\frac{3}{\varepsilon}\right)^m.$$

Hence

$$\log N(S^{m-1}, \varepsilon) \leq m \log\left(\frac{3}{\varepsilon}\right)$$

and therefore

$$\mathrm{EntInt}(S^{m-1}, 2) \leq \sqrt{m} \int_0^2 \sqrt{\log\left(\frac{3}{\varepsilon}\right)}\, d\varepsilon.$$

Set

$$I := \int_0^2 \sqrt{\log\left(\frac{3}{\varepsilon}\right)}\, d\varepsilon.$$

Use substitution $\varepsilon = 3e^{-s}$, $d\varepsilon = -3e^{-s} ds$. As $\varepsilon \downarrow 0$, $s \to \infty$; as $\varepsilon = 2$, $s = \log(3/2)$. Thus

$$I = 3\int_{\log(3/2)}^{\infty} e^{-s}\sqrt{s}\, ds \leq 3\int_0^{\infty} e^{-s}\sqrt{s}\, ds = 3\,\Gamma\left(\frac{3}{2}\right) = \frac{3\sqrt{\pi}}{2} < 4.$$

So $\mathrm{EntInt}(S^{m-1}, 2) \leq 4\sqrt{m}$. □

Theorem 2.6 (IID-T2 centered Dudley bound). *Assume $m \geq 1$ and $L \geq 1$. Then*

$$\mathbb{E}\Big[\sup_{u \in S^{m-1}} \widetilde{X}(u)\Big] \leq (12\sqrt{2})\frac{1}{L}\,\mathrm{EntInt}(S^{m-1}, 2) \leq (48\sqrt{2})\frac{\sqrt{m}}{L}.$$

Proof. By Lemma **??**, increments are sub-Gaussian with scale $1/L$ in Euclidean distance. Apply Dudley's theorem with the explicit constant $12\sqrt{2}$ used in the Lean development:

$$\mathbb{E} \sup_{u \in S^{m-1}} \widetilde{X}(u) \le (12\sqrt{2}) \frac{1}{L} \operatorname{EntInt}(S^{m-1}, 2).$$

Then use Lemma **??**. □

Theorem 2.7 (IID-T2 uncentered bound)**.** *Assume $m \ge 1$ and $L \ge 1$. Then*

$$\mathbb{E}\Big[\sup_{u \in S^{m-1}} X(u) \Big] \le (48\sqrt{2}) \frac{\sqrt{m}}{L}.$$

Proof. For every sample,

$$\sup_{u \in S^{m-1}} X(u) = \sup_{u \in S^{m-1}} (X(u) - X(u_0)) + X(u_0)$$

because adding the constant $-X(u_0)$ shifts all values equally. Integrate both sides:

$$\mathbb{E} \sup_u X(u) = \mathbb{E} \sup_u \widetilde{X}(u) + \mathbb{E}[X(u_0)].$$

By Theorem **??** with unit vector u_0, $X(u_0) = M_{u_0} \sim \mathcal{N}(0, 1/L^2)$, so $\mathbb{E}[X(u_0)] = 0$. Hence

$$\mathbb{E} \sup_u X(u) = \mathbb{E} \sup_u \widetilde{X}(u) \le (48\sqrt{2}) \frac{\sqrt{m}}{L}$$

by Theorem **??**. □

2.4 IID-T3: high-probability supremum bound (Lean-exported form)

Theorem 2.8 (IID-T3)**.** *Assume $m \ge 1$, $L \ge 1$, and $t > 0$. Then*

$$\mathbb{P}\left(\mathbb{E} \sup_{u \in S^{m-1}} X(u) + t \le \sup_{u \in S^{m-1}} X(u) \right) \le 2m \exp\left(-\frac{L^2}{2} \left(\frac{t}{\sqrt{m}} \right)^2 \right).$$

Proof. Let $S(\omega) := \sup_{u \in S^{m-1}} X(u, \omega)$. Since $-u \in S^{m-1}$ whenever $u \in S^{m-1}$,

$$S(\omega) = \sup_{u \in S^{m-1}} \langle u, W(\omega) \rangle \ge 0,$$

so $\mathbb{E}[S] \ge 0$. Therefore

$$\{\mathbb{E}[S] + t \le S\} \subseteq \{t \le S\}.$$

Now prove

$$\{t \le S\} \subseteq \bigcup_{i=1}^{m} \left\{ |W_i| \ge \frac{t}{\sqrt{m}} \right\}.$$

Indeed, if all coordinates satisfy $|W_i| < t/\sqrt{m}$, then

$$\|W\|^2 = \sum_{i=1}^{m} W_i^2 < m \cdot \frac{t^2}{m} = t^2,$$

so $\|W\| < t$. Then for any unit u,

$$X(u) = \langle u, W\rangle \le \|u\|\,\|W\| = \|W\| < t,$$

thus $S < t$, contradiction. Hence by union bound,

$$\mathbb{P}(t \le S) \le \sum_{i=1}^{m} \mathbb{P}\left(|W_i| \ge \frac{t}{\sqrt{m}}\right).$$

For each coordinate basis vector $e_i \in S^{m-1}$, $W_i = M_{e_i}$, so by Corollary **??**,

$$\mathbb{P}\left(|W_i| \ge \frac{t}{\sqrt{m}}\right) \le 2\exp\left(-\frac{L^2}{2}\left(\frac{t}{\sqrt{m}}\right)^2\right).$$

Summing over $i = 1, \ldots, m$ gives the claimed bound. □

3 Correlated-Layer Model and Effective Depth

3.1 Model definition

Fix $A \in \mathbb{R}^{L\times L}$ and define

$$\rho(\ell, k) := \sum_{t=1}^{L} A_{\ell t} A_{kt}, \qquad \rho_{\mathrm{sum}} := \sum_{\ell=1}^{L}\sum_{k=1}^{L} \rho(\ell, k),$$

$$\mathrm{invDEff} := \frac{\rho_{\mathrm{sum}}}{L^3}, \qquad D_{\mathrm{eff}} := \frac{L^3}{\rho_{\mathrm{sum}}} \quad (\rho_{\mathrm{sum}} > 0).$$

Randomness in the Lean model:

- Latent field $Z_{tij} \sim \mathcal{N}(0,1)$ i.i.d. over (t, i, j),
- Shared activation Gaussian $z_j \sim \mathcal{N}(0,1)$ i.i.d. over j,
- $a_j := \mathrm{sign}(z_j)$ is shared across layers.

Define correlated weights and probes:

$$B_{\ell ij} := \sum_{t=1}^{L} A_{\ell t} Z_{tij},$$

$$\Delta h_\ell(i) := \frac{1}{\sqrt{L}\sqrt{rm}} \sum_{j=1}^{rm} B_{\ell ij} a_j, \qquad g_\ell(u) := \langle u, \Delta h_\ell\rangle,$$

$$M_u^{\mathrm{layerAvg}} := \frac{1}{L}\sum_{\ell=1}^{L} g_\ell(u).$$

The Lean development also uses a flattened linear-form version M_u^{flat} and proves

$$M_u^{\mathrm{layerAvg}} = M_u^{\mathrm{flat}}.$$

3.2 CORR-COV and CORR-VAR

Theorem 3.1 (CORR-COV)**.** *For every $u \in \mathbb{R}^m$ and $\ell, k \in \{1, \dots, L\}$,*

$$\mathrm{Cov}(g_\ell(u), g_k(u)) = \frac{\rho(\ell, k)}{L} \|u\|^2.$$

Proof. The Lean development proves the exact variance identities

$$\mathrm{Var}(g_\ell(u)) = \frac{\rho(\ell, \ell)}{L}\|u\|^2,$$

$$\mathrm{Var}(g_\ell(u) + g_k(u)) = \frac{\rho(\ell, \ell) + \rho(k, k) + 2\rho(\ell, k)}{L}\|u\|^2.$$

Now use

$$\mathrm{Cov}(X, Y) = \frac{\mathrm{Var}(X + Y) - \mathrm{Var}(X) - \mathrm{Var}(Y)}{2}$$

with $X = g_\ell(u)$, $Y = g_k(u)$ to obtain

$$\mathrm{Cov}(g_\ell(u), g_k(u)) = \frac{1}{2L} \left(\rho(\ell, \ell) + \rho(k, k) + 2\rho(\ell, k) - \rho(\ell, \ell) - \rho(k, k)\right) \|u\|^2 = \frac{\rho(\ell, k)}{L}\|u\|^2.$$

□

Theorem 3.2 (CORR-VAR: covariance-sum identity)**.** *For every $u \in \mathbb{R}^m$,*

$$\mathrm{Var}(M_u^{\mathrm{layerAvg}}) = \frac{1}{L^2} \sum_{\ell=1}^{L} \sum_{k=1}^{L} \mathrm{Cov}(g_\ell(u), g_k(u)).$$

Hence,

$$\mathrm{Var}(M_u^{\mathrm{layerAvg}}) = \frac{\|u\|^2 \rho_{\mathrm{sum}}}{L^3} = \frac{\|u\|^2}{D_{\mathrm{eff}}} \quad (\rho_{\mathrm{sum}} > 0).$$

Proof. Since $M_u^{\mathrm{layerAvg}} = (1/L) \sum_\ell g_\ell(u)$,

$$\mathrm{Var}(M_u^{\mathrm{layerAvg}}) = \mathrm{Var}\left(\frac{1}{L} \sum_\ell g_\ell(u)\right) = \frac{1}{L^2} \sum_{\ell,k} \mathrm{Cov}(g_\ell(u), g_k(u)).$$

Substitute Theorem **??**:

$$\mathrm{Var}(M_u^{\mathrm{layerAvg}}) = \frac{1}{L^2} \sum_{\ell,k} \frac{\rho(\ell, k)}{L}\|u\|^2 = \frac{\|u\|^2}{L^3} \sum_{\ell,k} \rho(\ell, k) = \frac{\|u\|^2 \rho_{\mathrm{sum}}}{L^3}.$$

If $\rho_{\mathrm{sum}} > 0$, then $D_{\mathrm{eff}} = L^3/\rho_{\mathrm{sum}}$, so

$$\frac{\|u\|^2 \rho_{\mathrm{sum}}}{L^3} = \frac{\|u\|^2}{D_{\mathrm{eff}}}.$$

□

Theorem 3.3 (Correlated scalar Gaussian law, tail, MSE for unit vectors)**.** *Assume $u \in S^{m-1}$. Then*

$$M_u^{\mathrm{layerAvg}} \sim \mathcal{N}\left(0, \frac{\rho_{\mathrm{sum}}}{L^3}\right) = \mathcal{N}\left(0, \frac{1}{D_{\mathrm{eff}}}\right) \qquad (\rho_{\mathrm{sum}} > 0),$$

$$\mathbb{P}\left(|M_u^{\mathrm{layerAvg}}| \geq t\right) \leq 2\exp\left(-\frac{t^2 D_{\mathrm{eff}}}{2}\right), \qquad t > 0,$$

$$\mathbb{E}\left[(M_u^{\mathrm{layerAvg}})^2\right] = \frac{\rho_{\mathrm{sum}}}{L^3} = \frac{1}{D_{\mathrm{eff}}}.$$

Proof. The process M_u^{flat} is a finite linear form in independent Gaussian coordinates; therefore it is centered Gaussian with variance $\|u\|^2\rho_{\mathrm{sum}}/L^3$. Using $\|u\| = 1$ and the bridge $M_u^{\mathrm{layerAvg}} = M_u^{\mathrm{flat}}$ gives the law. The tail and second moment are the standard centered Gaussian formulas with variance ρ_{sum}/L^3; when $\rho_{\mathrm{sum}} > 0$, rewrite as $1/D_{\mathrm{eff}}$. □

3.3 CORR-T2: expected supremum via Dudley

Define $X_{\mathrm{corr}}(u) := M_u^{\mathrm{layerAvg}}$ and centered process

$$\widetilde{X}_{\mathrm{corr}}(u) := X_{\mathrm{corr}}(u) - X_{\mathrm{corr}}(u_0), \qquad u \in S^{m-1}.$$

Theorem 3.4 (CORR-T2)**.** *Assume $m \geq 1$, $L \geq 1$, and $\rho_{\mathrm{sum}} > 0$. Then*

$$\mathbb{E} \sup_{u \in S^{m-1}} \widetilde{X}_{\mathrm{corr}}(u) \leq (48\sqrt{2}) \frac{\sqrt{m}}{\sqrt{D_{\mathrm{eff}}}}.$$

Proof. From the correlated increment law in Lean, increments are sub-Gaussian with metric

$$d_{\mathrm{corr}}(u, v) = \sqrt{\mathrm{invDEff}}\, \|u - v\| = \frac{\|u - v\|}{\sqrt{D_{\mathrm{eff}}}}.$$

Applying Dudley with constant $12\sqrt{2}$ gives

$$\mathbb{E} \sup_u \widetilde{X}_{\mathrm{corr}}(u) \leq (12\sqrt{2})\sqrt{\mathrm{invDEff}}\ \mathrm{EntInt}(S^{m-1}, 2).$$

By Lemma **??**,

$$\mathbb{E} \sup_u \widetilde{X}_{\mathrm{corr}}(u) \leq (12\sqrt{2})\sqrt{\mathrm{invDEff}} \cdot 4\sqrt{m} = (48\sqrt{2}) \frac{\sqrt{m}}{\sqrt{D_{\mathrm{eff}}}}.$$

□

3.4 CORR-T3: high-probability supremum tail (Lean-exported form)

Theorem 3.5 (CORR-T3)**.** *Assume $m \geq 1$, $L \geq 1$, $\rho_{\mathrm{sum}} > 0$, and $t > 0$. Then*

$$\mathbb{P}\left(t \leq \sup_{u \in S^{m-1}} X_{\mathrm{corr}}(u)\right) \leq 2m \exp\left(-\frac{(t/m)^2 D_{\mathrm{eff}}}{2}\right).$$

Proof. Write $X_{\mathrm{corr}}(u) = \langle u, W_{\mathrm{corr}} \rangle$. If $|W_{\mathrm{corr},i}| < t/m$ for all i, then

$$\sup_{u \in S^{m-1}} X_{\mathrm{corr}}(u) \leq \sum_{i=1}^{m} |W_{\mathrm{corr},i}| < m \cdot (t/m) = t,$$

so

$$\left\{ t \le \sup_u X_{\mathrm{corr}}(u) \right\} \subseteq \bigcup_{i=1}^{m} \{|W_{\mathrm{corr},i}| \ge t/m\}.$$

For each coordinate basis vector e_i, Lean proves $W_{\mathrm{corr},i} = M_{e_i}^{\mathrm{flat}}$. By Theorem **??** with $u = e_i$,

$$\mathbb{P}(|W_{\mathrm{corr},i}| \ge t/m) \le 2\exp\left(-\frac{(t/m)^2 D_{\mathrm{eff}}}{2}\right).$$

Union bound over $i = 1, \ldots, m$ gives the result. □

4 Lean Correspondence

Every mathematical object/theorem used above is matched below to the exact Lean declaration and file.

IID model definitions and bridges

- Residual width $rm = rm$: `LeanSlt.NNGPResNet.residualWidth` — `LeanSlt/NNGPResNet/Defs.lean`
- Sign activation sign: `LeanSlt.NNGPResNet.signAct` — `LeanSlt/NNGPResNet/Defs.lean`
- Layer increment Δh_ℓ: `LeanSlt.NNGPResNet.DeltaH` — `LeanSlt/NNGPResNet/Defs.lean`
- Layer probe $g_\ell(u)$: `LeanSlt.NNGPResNet.g` — `LeanSlt/NNGPResNet/Defs.lean`
- Depth average M_u: `LeanSlt.NNGPResNet.M_u` — `LeanSlt/NNGPResNet/Defs.lean`
- Averaged vector W: `LeanSlt.NNGPResNet.W` — `LeanSlt/NNGPResNet/Defs.lean`
- Sphere type S^{m-1}: `LeanSlt.NNGPResNet.Sphere` — `LeanSlt/NNGPResNet/Defs.lean`
- Process $X(u) = \langle u, W \rangle$: `LeanSlt.NNGPResNet.X` — `LeanSlt/NNGPResNet/Defs.lean`
- Bridge $M_u = X(u)$: `LeanSlt.NNGPResNet.M_u_eq_X` — `LeanSlt/NNGPResNet/Defs.lean`
- Bridge $M_u = \langle u, W \rangle$: `LeanSlt.NNGPResNet.M_u_eq_inner_W` — `LeanSlt/NNGPResNet/Defs.lean`

IID scalar results (T1)

- Gaussian law: `LeanSlt.NNGPResNet.scalar_mean_map_gaussian` — `LeanSlt/NNGPResNet/Scalar.lean`
- Tail bound: `LeanSlt.NNGPResNet.scalar_mean_tail` — `LeanSlt/NNGPResNet/Scalar.lean`
- Second moment: `LeanSlt.NNGPResNet.scalar_mean_mse` — `LeanSlt/NNGPResNet/Scalar.lean`

IID sphere process, entropy, Dudley, high probability (T2/T3)

- Sphere set and subtype bridge: `sphereSet`, `SphereSetSub`, `sphereEquivSphereSetSub` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Increment Gaussian law: `LeanSlt.NNGPResNet.map_X_diff_gaussian` — `LeanSlt/NNGPResNet/SphereCha`
- Sub-Gaussian process instance: `LeanSlt.NNGPResNet.sphere_process_isSubGaussian` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Sphere entropy finiteness: `LeanSlt.NNGPResNet.entropyIntegralENNReal_sphere_ne_top` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Sphere entropy bound $\leq 4\sqrt{m}$: `LeanSlt.NNGPResNet.entropyIntegral_sphere_le` — `LeanSlt/NNGPResNe`
- Dudley intermediate bound: `LeanSlt.NNGPResNet.sphere_process_expected_sup_bound_entropy` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Entropy-assuming intermediate theorem: `LeanSlt.NNGPResNet.sphere_process_expected_sup_bound_as` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Final unconditional IID-T2 theorem: `LeanSlt.NNGPResNet.sphere_process_expected_sup_bound` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Coordinate bridge: `LeanSlt.NNGPResNet.W_coord_eq_M_u_single` — `LeanSlt/NNGPResNet/SphereChaini`
- Supremum event reduction to coordinates: `LeanSlt.NNGPResNet.sphere_sup_event_subset_union_coord` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Integrability and positivity helpers used in T3 proof path: `LeanSlt.NNGPResNet.integrable_supX`, `LeanSlt.NNGPResNet.supX_nonneg` — `LeanSlt/NNGPResNet/SphereChaining.lean`
- Final IID-T3 theorem: `LeanSlt.NNGPResNet.sphere_process_highProb_bound` — `LeanSlt/NNGPResNet/S`

Correlated definitions and effective depth

- Correlation kernel ρ: `LeanSlt.NNGPResNet.Correlated.rho` — `LeanSlt/NNGPResNet/Correlated/Defs.l`
- Correlation sum ρ_{sum}: `LeanSlt.NNGPResNet.Correlated.rhoSum` — `LeanSlt/NNGPResNet/Correlated/De`
- Inverse effective depth ρ_{sum}/L^3: `LeanSlt.NNGPResNet.Correlated.invDEff` — `LeanSlt/NNGPResNet/Corr`
- Effective depth L^3/ρ_{sum}: `LeanSlt.NNGPResNet.Correlated.D_eff` — `LeanSlt/NNGPResNet/Correlated/D`
- Layer probe g_ℓ^{corr}: `LeanSlt.NNGPResNet.Correlated.gCorr` — `LeanSlt/NNGPResNet/Correlated/Defs.l`
- Layer-average statistic: `LeanSlt.NNGPResNet.Correlated.M_u_corr_layerAvg` — `LeanSlt/NNGPResNet/C`
- Flattened statistic: `LeanSlt.NNGPResNet.Correlated.M_u_corr` — `LeanSlt/NNGPResNet/Correlated/Def`
- Bridge lemma: `LeanSlt.NNGPResNet.Correlated.M_u_corr_layerAvg_eq_M_u_corr` — `LeanSlt/NNGPRes`
- Correlated averaged vector and process: `LeanSlt.NNGPResNet.Correlated.Wcorr`, `LeanSlt.NNGPResNet.C` — `LeanSlt/NNGPResNet/Correlated/Defs.lean`

Correlated covariance/variance/scalar results

- Single-layer Gaussian map: `LeanSlt.NNGPResNet.Correlated.map_gCorr_gaussian_any` — `LeanSlt/NNGPResNet/Correlated/GaussianLinearForms.lean`
- Single-layer variance: `LeanSlt.NNGPResNet.Correlated.variance_gCorr` — `LeanSlt/NNGPResNet/Correl`
- Variance of layer sum: `LeanSlt.NNGPResNet.Correlated.variance_gCorr_add` — `LeanSlt/NNGPResNet/C`
- Covariance formula (CORR-COV): `LeanSlt.NNGPResNet.Correlated.cov_gCorr` — `LeanSlt/NNGPResNet/`
- Unit-vector Gaussian law (layerAvg): `LeanSlt.NNGPResNet.Correlated.corr_scalar_map_gaussian_unit` — `LeanSlt/NNGPResNet/Correlated/Scalar.lean`
- Unit-vector tail (layerAvg): `LeanSlt.NNGPResNet.Correlated.corr_scalar_tail_layerAvg` — `LeanSlt/NNGPResNet/Correlated/Scalar.lean`
- Unit-vector MSE (layerAvg): `LeanSlt.NNGPResNet.Correlated.corr_scalar_mse_layerAvg` — `LeanSlt/NNGPResNet/Correlated/Scalar.lean`
- Variance from covariance sum: `LeanSlt.NNGPResNet.Correlated.var_M_u_corr_layerAvg_as_covarianc` — `LeanSlt/NNGPResNet/Correlated/Scalar.lean`
- Variance in explicit ρ-sum form: `LeanSlt.NNGPResNet.Correlated.var_M_u_corr_layerAvg_as_rho_sum` — `LeanSlt/NNGPResNet/Correlated/Scalar.lean`

Correlated chaining and high-probability supremum

- Centered correlated process: `LeanSlt.NNGPResNet.Correlated.centeredCorrProcess` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`
- Correlated Dudley entropy form: `LeanSlt.NNGPResNet.Correlated.corr_sphere_expected_sup_bound_e` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`
- Correlated expected sup bound (flat form): `LeanSlt.NNGPResNet.Correlated.corr_sphere_expected_sup` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`
- Correlated expected sup bound (layerAvg form): `LeanSlt.NNGPResNet.Correlated.corr_sphere_expected` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`
- Coordinate bridge for correlated process: `LeanSlt.NNGPResNet.Correlated.Wcorr_coord_eq_M_u_corr_si` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`
- Final correlated high-probability theorem: `LeanSlt.NNGPResNet.Correlated.corr_sphere_highProb_bou` — `LeanSlt/NNGPResNet/Correlated/SphereChaining.lean`

Module aggregators

- IID exports: `LeanSlt.NNGPResNet.Main` — `LeanSlt/NNGPResNet/Main.lean`
- Correlated exports: `LeanSlt.NNGPResNet.Correlated.Main` — `LeanSlt/NNGPResNet/Correlated/Main.l`
- Top-level import hook: `LeanSlt.lean`.