I.O.S.

ESTIMATION OF WAVE SPECTRA FROM WAVE HEIGHT AND PERIOD

by
D.J.T. CARTER

I.O.S. Report No. 135
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NATURAL ENVIRONMENT RESEARCH COUNCIL
INSTITUTE OF OCEANOGRAPHIC SCIENCES

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WORMLEY

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C O N T E N T S

SUMMARY

Formulae are derived for estimating single-peaked sea surface wave spectra as a function of frequency from values of significant wave height and wave period. The formulae are based upon a general spectral formula suggested by Bretschneider - recommended by the International Ship Structures Congress' committee on environmental conditions - and include the Pierson-Moskowitz and JONSWAP spectra.

D.J.T. Carter is a member of the IOS Marine Physics Group which undertakes research into various aspects of ocean waves.

NOTATION

$\left.\begin{matrix}A\\B\end{matrix}\right\}$	Parameters of Bretschneider spectrum
D	limiting water depth
$E(f)$	one dimensional frequency spectrum of sea surface variance
f	frequency (Hz)
f_m	frequency of spectral peak (corresponding to maximum $E(f)$)
F	f/f_m
g	acceleration due to gravity
H_s	significant wave height
m_n	nth moment of spectrum $E(f)$
q	function of f incorporated in the JONSWAP peak enhancement function
T_1	mean wave period
T_m	peak wave period
T_s	significant wave period
T_v	visual estimate of wave period
T_z	zero-up-crossing wave period
t	arbitrary dummy variable
u	wind speed at 10 m above surface
U	numerical value of wind speed at 10 m above surface in m s^{-1}
$u_{19\cdot 5}$	numerical value of wind speed at 19.5 m above surface in m s^{-1}
x	wind fetch
z	arbitrary dummy variable
α	spectral parameter
γ	JONSWAP spectral peak enhancement parameter
γ^q	JONSWAP spectral peak enhancement function
σ	JONSWAP spectral parameter
Γ	Gamma function
λ_m	wavelength of peak frequency

INTRODUCTION

Estimates of sea conditions are required by engineers designing offshore structures. Commonly these conditions are described by the significant wave height and a wave period, such as the zero-up-crossing period. The values of these parameters can be obtained from wave measurements or from visual estimates of wave height and period, or from a knowledge of the wind field, using for example the wave prediction curves of Darbyshire and Draper (1963).

Sometimes a more detailed description of the sea surface is required; the engineer may need to evaluate the stress upon his structure from all the wave energy components that would be present across the frequency spectrum, from low-frequency swell to high frequency sea waves with periods of a few seconds.

Spectra showing sea surface variance, proportional to wave energy, as a function of frequency can be derived from wave measurements. If these are not available, then spectra can be estimated from significant wave height and wave period. This report gives various formulae that can be used for deep water waves, based upon the following form of the spectrum, E, as a function of frequency, f(Hz), proposed by Bretschneider (1959)

$$E(f) = A\, f^{-5} \exp\left[- B\, f^{-4} \right] \tag{1}$$

Three cases are considered:

- the Bretschneider spectrum, which is similar to that proposed by the ITTC (1972) and that recommended by the ISSC (1976) for open ocean conditions,
- the Pierson-Moskowitz spectrum for a fully-developed sea in which the parameter A in equation (1) is fixed,
- the JONSWAP spectrum derived by Hasselmann et al. (1973), obtained by multiplying the Bretschneider formula by a peak enhancement function, and suggested by ISSC (1976) for use in conditions of limited fetch.

All these spectra are single peaked. In practice, wave spectra sometimes have two peaks or more, although Houmb and Due (1978) found that only about 4% of spectra from Waverider buoy measurements off northern Norway (near 71°N 18°E) were multi-peaked. A method of modelling double-peaked spectra is given by Ochi and Hubble (1977).

THEORY

Significant wave height

Given a spectrum of sea surface elevation E(f) where f is frequency (Hz), and defining the nth moment by

$$m_n = \int_0^\infty f^n E(f)\, df \tag{2}$$

then significant wave height is defined by

$$H_s = 4\, m_o^{\frac{1}{2}} \tag{3}$$

Originally, significant wave height was defined (by Sverdrup and Munk, 1947) as the mean height of the highest one-third waves because this value seemed to be close to visual estimates of wave height. The two definitions are almost identical for narrow-band waves (see Longuet-Higgins, 1952) and in general appear to be in reasonable agreement.

Wave period

Mean wave period T_1 is given by

$$T_1 = m_o/m_1 \tag{4}$$

zero-up-crossing wave period T_z by

$$T_z = (m_o/m_2)^{\frac{1}{2}} \tag{5}$$

and the frequency with maximum spectral energy (peak frequency), f_m gives a peak period defined by

$$T_m = {}^1/f_m \tag{6}$$

Visual estimates of wave period T_v have been compared with T_z by Cartwright (1964), but showed poor correlation and he suggested that T_v might be a better estimate of the mean wave period T_1, rather than the zero-up-crossing period T_z. ISSC (1967) supported this suggestion and recommended equating T_v and T_1 but subsequently ISSC (1979) have suggested a better relationship to be

$$T_v = T_m/1.05$$

Derivation

For the spectrum E(f) given by equation (1)

$$m_n = \int_0^\infty A\, f^{n-5} \exp\left[-\, B\, f^{-4}\right] df$$

Substituting $t = B\, f^{-4}$

$$m_n = \frac{A}{4B^{(1-n/4)}} \int_0^\infty t^{-n/4}\, e^{-t} dt$$

and using the gamma function $\Gamma(1-n/4) = \int_0^\infty t^{-n/4}\, e^{-t} dt$

$$m_n = \frac{A}{4B^{(1-n/4)}} \Gamma(1-n/4) \qquad \text{for } n < 4$$

Therefore

$$\left.\begin{aligned} m_o &= A/4B \\ m_1 &\simeq 1.2254A/4B^{\frac{3}{4}} \\ m_2 &\simeq 1.77245A/4B^{\frac{1}{2}} \end{aligned}\right\} \tag{7}$$

and

$$\left.\begin{aligned} T_1 &\simeq 0.8161B^{-\frac{1}{4}} \\ T_z &\simeq 0.7511B^{-\frac{1}{4}} \end{aligned}\right\} \tag{8}$$

The peak frequency is given from equation (1) by

$$f_m^{\ 4} = 4B/5 = 0.8B \tag{9}$$

and

$$T_m \simeq 1.0574B^{-\frac{1}{4}} \tag{10}$$

The ratios of the different periods from equations (8) and (10) are given in Table (1).

Table 1

Ratio of various wave periods for the Bretschneider spectrum (equation 1)

T_1/T_z	1.0864
T_1/T_m	0.7718
T_z/T_m	0.7104

BRETSCHNEIDER SPECTRUM

The Bretschneider spectrum given by equation (1) has two variables A and B, which can be specified by wave height and period. Variable B is given by the period, from equation (8) or (10), and A is then determined from the significant wave height using equations (3) and (7)

(in units of $m^2\ Hz^{-1}$ if H_s is in metres)

$$\left.\begin{aligned} E(f) &= 0.11 H_s^2\ T_1 (T_1 f)^{-5} \exp\left[-0.443/(T_1 f)^4\right] \\ &= 0.080 H_s^2\ T_z (T_z f)^{-5} \exp\left[-0.318/(T_z f)^4\right] \\ &= 0.31 H_s^2\ T_m (T_m f)^{-5} \exp\left[-1.25/(T_m f)^4\right] \end{aligned}\right\} \tag{11}$$

Bretschneider (1977) defines significant wave period T_s by

$$T_s = (0.8)^{\frac{1}{4}}/f_m$$

$$\text{i.e. } T_s = B^{-\frac{1}{4}}$$

$$\text{and so } T_s \simeq 0.946 T_m \simeq 1.23 T_1 \simeq 1.33 T_z$$

Substituting for variable B in terms of T_s leads to the spectrum (in units of $m^2\ Hz^{-1}$)

$$E(f) = 0.25 H_s^2\ T_s (T_s f)^{-5} \exp\left[-1/(T_s f)^4\right] \tag{12}$$

Bretschneider (1977, fig. 1) gives a diagram as well as equations for estimating H_s and T_s, but he states that judging from North Atlantic wave spectra, the estimate of T_s from this diagram is about 10% too high for high wind speeds. He then recommends the following formula for T_s

$$T_s = 0.73 U \tanh\{0.5\ \ell n\left[(1+z)/(1-z)\right]\}^{0\cdot 6}$$

where

$z = 35 H_s/U^2$

H_s = significant wave height (m)

U = 10 minute mean wind speed at 10 m ($m\ s^{-1}$)

PIERSON-MOSKOWITZ SPECTRUM

This spectrum was derived by Pierson and Moskowitz (1964) using Shipborne Wave Recorder traces obtained at the North East Atlantic Ocean Weather Stations when the sea was considered to be fully arisen.

It is given by:

$$E(f) = \alpha\, g^2 (2\pi)^{-4} f^{-5} \exp\left[-\ 1.25 (f/f_m)^{-4}\right] \tag{13}$$

where $\alpha = 0.0081$

and the peak frequency f_m is given by $f_m = 0.8772(g/2\pi u_{19.5})$

with $u_{19.5}$ = wind speed at 19.5 m above sea surface.

Thus this spectrum is of the form of the spectrum given in equation (1)

with $A = \alpha\, g^2 (2\pi)^{-4}$

and $B = 1{\cdot}25 f_m^4 = 0{\cdot}7401(g/2\pi u_{19.5})^4$

Values of significant wave height, H_s (m), derived from equations (3) and (7) with $g = 9.81$ m s^{-2}, and of wave periods (s) from equations (8) and (10) are given in Table 2.

Table 2

Values of significant wave height H_s (m) and various wave periods (s) for the Pierson-Moskowitz spectrum in terms of wind speed at 19.5 m above sea level ($u_{19.5}$) m s^{-1}; also wave periods in terms of H_s (m)

$$H_s = 0.0213 u_{19.5}^2$$

$$T_1 = 0.5635 u_{19.5} = 3.86 H_s^{\frac{1}{2}}$$

$$T_z = 0.5187 u_{19.5} = 3.55 H_s^{\frac{1}{2}}$$

$$T_m = 0.7302 u_{19.5} = 5.00 H_s^{\frac{1}{2}}$$

Since the Pierson-Moskowitz spectrum has only one variable (B), it can be specified either by wave height or by wave period (in units of $m^2\ Hz^{-1}$)

(a) given significant wave height H_S (m)

$$E(f) = 5.00\ 10^{-4} f^{-5} \exp\left[-\ 2.00\ 10^{-3}/H_S^{\,2} f^4\right] \tag{14}$$

(b) given wave period (s)

$$\left.\begin{aligned} E(f) &= 5.00\ 10^{-4} f^{-5} \exp\left[-\ 0.443/(T_1 f)^4\right] \\ &= 5.00\ 10^{-4} f^{-5} \exp\left[-\ 0.318/(T_Z f)^4\right] \\ &= 5.00\ 10^{-4} f^{-5} \exp\left[-\ 1.25/(T_m f)^4\right] \end{aligned}\right\} \tag{15}$$

JONSWAP SPECTRUM

This spectrum was determined by Hasselmann et al. (1973) from observations in the North Sea of fetch-limited waves, that is for a growing sea state in the absence of swell. It is given by

$$E(f) = \alpha\, g^2 (2\pi)^{-4} f^{-5} \exp\left[-\ 1.25 (f/f_m)^{-4}\right] \gamma^q \tag{16}$$

where $\alpha = 0.076 (gx/u^2)^{-0 \cdot 22}$

with u = wind speed at 10 m above sea surface

x = fetch

$$q = \exp\left[-\ (f - f_m)^2 / 2\sigma^2 f_m^{\ 2}\right]$$

$$\text{with } \sigma = \begin{cases} 0.07 & f \leqslant f_m \\ 0.09 & f > f_m \end{cases}$$

and $\gamma = 3.3$.

The value for the peak enhancement parameter (γ) of 3.3 is an average figure derived by Hasselmann et al. (1973). They found individual values within the range of 1-6. Detailed analysis of these γ values by Ochi (1979) showed that they have a normal distribution with a mean of 3.3 and a standard deviation of 0.79, i.e. 95% between 1.75 and 4.85.

So the JONSWAP spectrum is of the general form of the Bretschneider spectrum given by equation (1), multiplied by a peak enhancement function γ^q

$$E(f) = Af^{-5} \exp\left[-\ Bf^{-4}\right] \gamma^q \tag{17}$$

and since γ^q has its only maximum at $f = f_m$, the maximum of $E(f)$ is given from equation (9) i.e.

$$f_m^{\ 4} = 4B/5$$

The moments of this spectrum cannot be determined analytically but may be estimated by numerical integration of the following expression

$$I_n = \int_0^\infty F^n F^{-5} \exp\left[-1.25(F)^{-4}\right]\gamma^q dF \tag{18}$$

where $q = \exp\left[-(F-1)^2/2\sigma^2\right]$

$$\text{with } \sigma = \begin{cases} 0.07 & F \leqslant 1 \\ 0.09 & F > 1 \end{cases}$$

Table 3 provides results of numerical integration of I_n for a range of values of γ (page 13).

Therefore, for the JONSWAP spectrum, letting $f = F f_m$,

$$m_n = \alpha g^2 (2\pi^{-4}) f_m^{n-4} I_n(\gamma) \tag{19}$$

and from equation (3)

$$m_o = \frac{H_s^2}{16} = \alpha g^2 (2\pi)^{-4} f_m^{-4} I_o(\gamma) \tag{20}$$

<u>The JONSWAP spectrum</u> can, therefore, be specified as follows

By substituting equation (20) in equation (16) with $f_m = {}^1/T_m$

$$E(f) = \frac{1}{16 I_o(\gamma)} H_s^2 T_m(T_m f)^{-5} \exp\left[-1.25/(T_m f)^4\right]\gamma^q \tag{21}$$

where $q = \exp\left[-(T_m f-1)^2/2\sigma^2\right]$

$$\text{with } \sigma = \begin{cases} 0.07 & T_m f \leqslant 1 \\ 0.09 & T_m f > 1 \end{cases}$$

(a) <u>In terms of T_m</u>

e.g. if $\gamma = 3.3$

$$E(f) = 0.205 H_s^2 T_m(T_m f)^{-5} \exp\left[-1.25/(T_m f)^4\right] 3.3^q \tag{22}$$

(b) <u>In terms of T_1</u>

From equations (4) and (19)

$$T_1 = T_m \, I_o / I_1 \qquad (23)$$

Substituting for T_m in equation (21) gives the required result

e.g. if $\gamma = 3.3$

$$T_1 = 0.8345 T_m$$

$$\text{and } E(f) = 0.0994 H_s^{\,2} T_1 (T_1 f)^{-5} \exp\left[-\ 0.6062/(T_1 f)^4\right] 3.3^q \qquad (24)$$

$$\text{where } q = \exp\left[-\ (1.20 T_1 f - 1)^2 / 2\sigma^2\right]$$

$$\text{with } \sigma = \begin{cases} 0.07 & 1.20\ T_1 f \leqslant 1 \\ 0.09 & 1.20\ T\ f > 1 \end{cases}$$

(c) <u>In terms of T_z</u>

From equations (5) and (19)

$$T_z = T_m (I_o / I_2)^{\frac{1}{2}} \qquad (25)$$

Substituting for T_m in equation (21) gives the required result

e.g. if $\gamma = 3.3$

$$T_z = 0.7775 T_m$$

$$\text{and } E(f) = 0.0749 H_s^{\,2} T_z (T_z f)^{-5} \exp\left[-\ 0.4567/(T_z f)^4\right] 3.3^q \qquad (26)$$

$$\text{where } q = \exp\left[-\ (1.286 T_z f - 1)^2 / 2\sigma^2\right]$$

$$\text{with } \sigma = \begin{cases} 0.07 & 1.286 T_z f \leqslant 1 \\ 0.09 & 1.286 T_z f > 1 \end{cases}$$

JONSWAP spectra for other values of γ may be similarly derived using the values in Table 3.

Limiting water depth

The JONSWAP measurements from which the spectrum was derived were with off-shore winds. Hasselmann et al. (1973) found that the waves were unaffected by the water depth and did not 'feel' the bottom. The criterion used by them was that water depth was greater than a quarter of the wave length λ_m associated with the peak frequency, i.e. the limiting water depth D was given by

$$D = \lambda_m/4 = g/8\pi \ f_m^{\ 2}$$

therefore if D is in metres and period is in seconds

$$D = 0.390T_m^{\ 2}$$

and for $\gamma = 3.3$

$$D = 0.561T_1^{\ 2}$$

$$= 0.646T_z^{\ 2}$$

If the water depth is less than D then the wave would be affected by the sea floor and the JONSWAP spectrum would be inappropriate.

TABLE 3

Values of I_n (γ) defined by equation 18

γ	I_{-1}	I_0	I_1	I_2	I_0/I_1	$(I_0/I_2)^{\frac{1}{2}}$
1.0	0.171	0.200	0.259	0.396	0.772	0.711
1.2	0.182	0.211	0.270	0.407	0.780	0.719
1.4	0.192	0.221	0.280	0.417	0.788	0.727
1.6	0.202	0.230	0.290	0.427	0.795	0.734
1.8	0.211	0.240	0.300	0.437	0.801	0.741
2.0	0.220	0.249	0.309	0.447	0.807	0.747
2.2	0.229	0.258	0.318	0.456	0.812	0.753
2.4	0.238	0.267	0.327	0.465	0.817	0.758
2.6	0.247	0.276	0.336	0.474	0.821	0.763
2.8	0.255	0.284	0.344	0.483	0.826	0.768
3.0	0.264	0.293	0.353	0.491	0.830	0.772
3.2	0.272	0.301	0.361	0.500	0.834	0.776
3.3	0.2755	0.3050	0.3655	0.5046	0.8345	0.7775
3.4	0.280	0.309	0.370	0.508	0.837	0.780
3.6	0.288	0.318	0.378	0.517	0.840	0.784
3.8	0.296	0.326	0.386	0.525	0.844	0.788
4.0	0.304	0.334	0.394	0.533	0.847	0.791
4.2	0.312	0.341	0.402	0.541	0.850	0.794
4.4	0.319	0.349	0.410	0.549	0.852	0.797
4.6	0.327	0.357	0.418	0.557	0.855	0.801
4.8	0.335	0.365	0.425	0.565	0.858	0.803
5.0	0.342	0.372	0.433	0.573	0.860	0.806
5.2	0.350	0.380	0.441	0.580	0.862	0.809
5.4	0.357	0.387	0.448	0.588	0.865	0.812
5.6	0.365	0.395	0.456	0.596	0.867	0.814
5.8	0.372	0.402	0.463	0.603	0.869	0.817
6.0	0.380	0.410	0.471	0.611	0.871	0.819
6.2	0.387	0.417	0.478	0.618	0.873	0.821
6.4	0.394	0.424	0.485	0.626	0.874	0.824
6.6	0.401	0.432	0.493	0.633	0.876	0.826
6.8	0.408	0.439	0.500	0.641	0.878	0.828
7.0	0.416	0.446	0.507	0.648	0.880	0.830

EXAMPLES

Figures 1 to 4 illustrate examples of wave spectra described in this report.

Figure 1 shows

- the JONSWAP spectrum (with $\gamma = 3.3$) and the Bretschneider spectrum for significant wave height of 3.0 m and peak frequency of 1/7.0 s (i.e. $T_m = 7.0$ s), taken from equations (22) and (11) respectively.

- the Pierson-Moskowitz spectrum with the same peak frequency and significant wave height of 1.96 m (from Table 2).

- the Pierson-Moskowitz spectrum for a significant wave height of 3.0 m.

For the Bretschneider spectrum, the value of T_z is 5.0 s, for the Pierson-Moskowitz spectra it is 5.0 s and 6.1 s respectively (from Table 1) whilst for the JONSWAP spectrum it is 5.4 s (from equation (25)).

Figure 2 shows

- the JONSWAP spectrum ($\gamma = 3.3$) with significant wave height of 3 m and T_z of 4.5 s, 5 s and 6 s.

Using the value for α in terms of wind speed, u, and fetch, x (from equation (16)), and the following equation for f_m (from Hasselmann et al., 1973)

$$f_m = 3.5\ (g^2/xu)^{1/3}$$

it may be shown that the spectrum with $T_z = 4.5$ s would arise from a strong wind over a short fetch (about 40 m s^{-1} over 20 km) and the $T_z = 6$ s spectrum from a lower wind over a long fetch (about 10 m s^{-1} over 200 km).

Figure 3 illustrates

- how the shape of the JONSWAP spectrum with significant wave height of 7 m varies with γ if the peak frequency is held constant.

Figure 4 illustrates

- how the shape of the JONSWAP spectrum with significant wave height of 7 m varies with γ if the zero-up-crossing period is held constant ($\gamma = 1$ corresponds to the Bretschneider spectrum).

Figure 1

H_s = 3.0m T_m = 7.0s

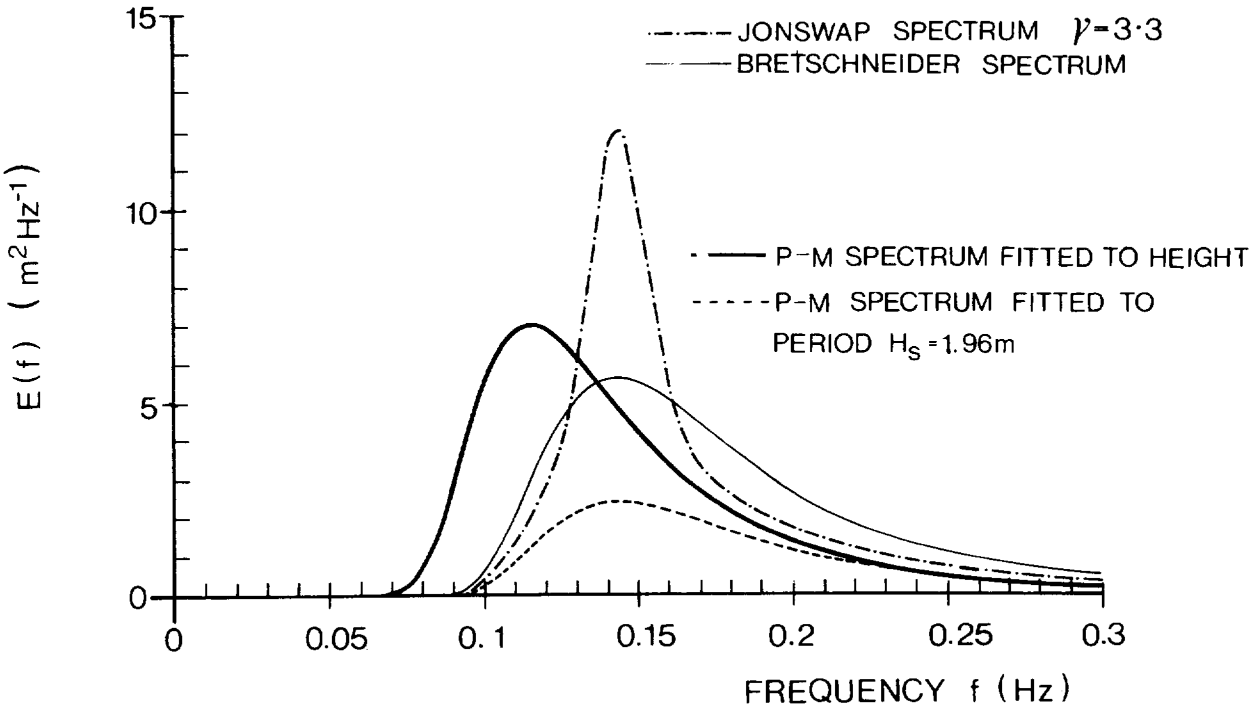


Figure 2

JONSWAP SPECTRUM H_s = 3.0m γ = 3.3

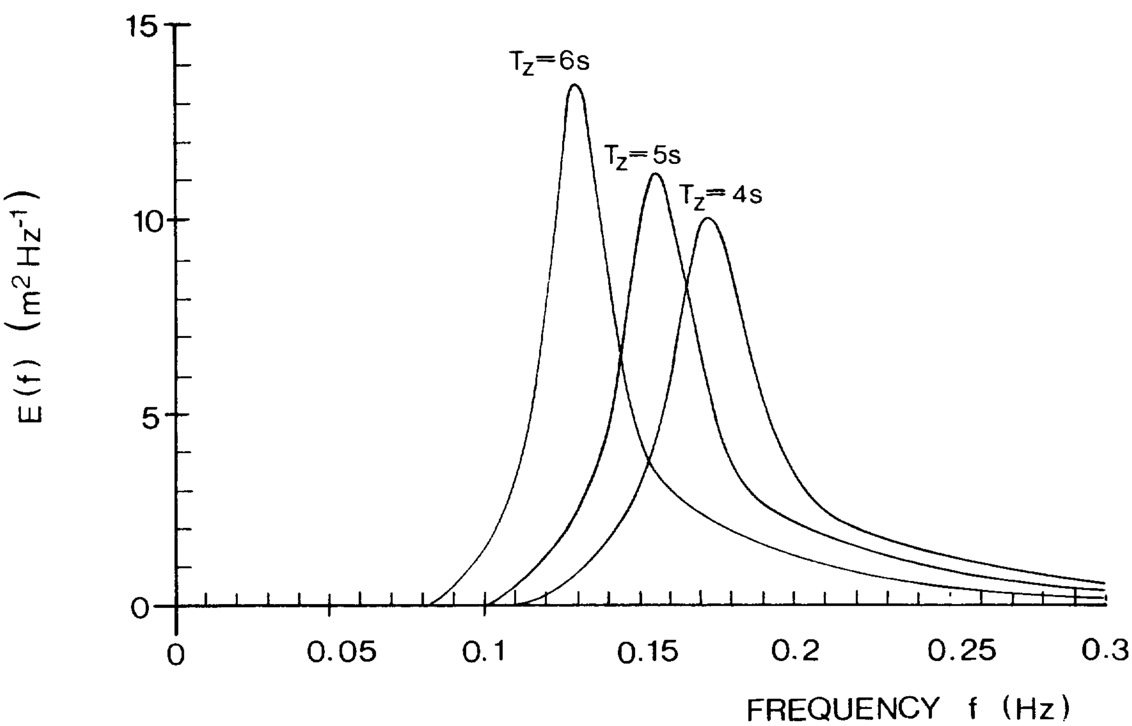


Figure 3

JONSWAP SPECTRUM $H_s = 7.0m$, $T_m = 10.0s$

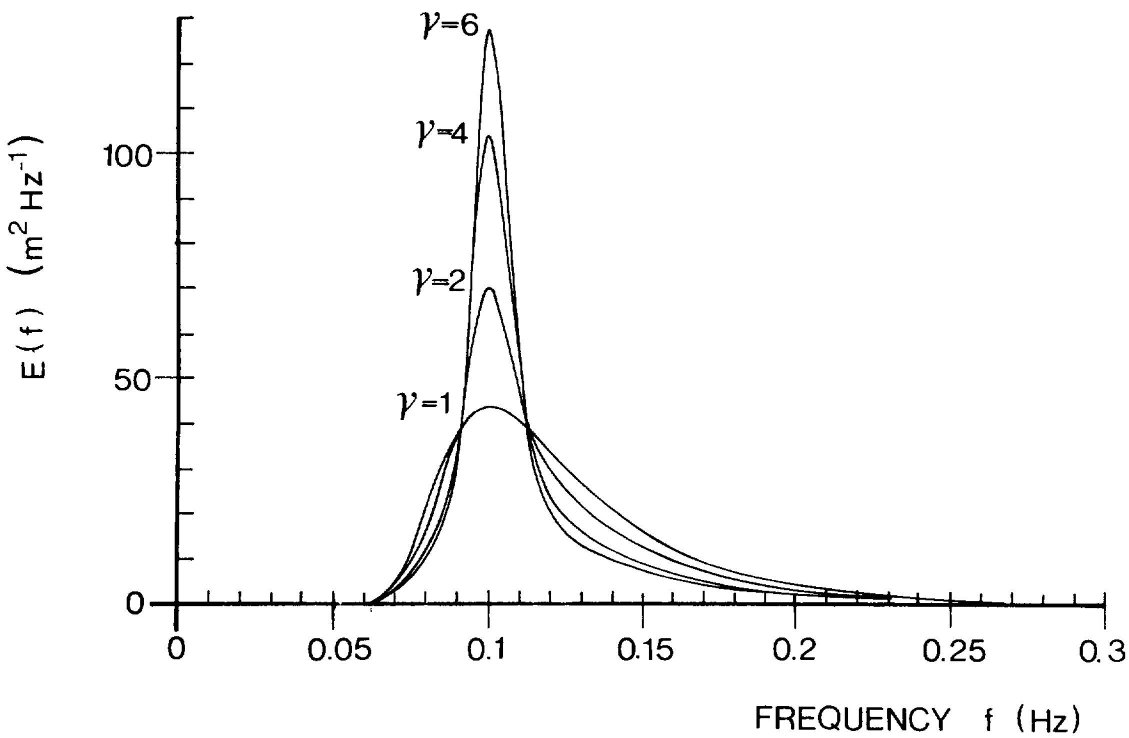
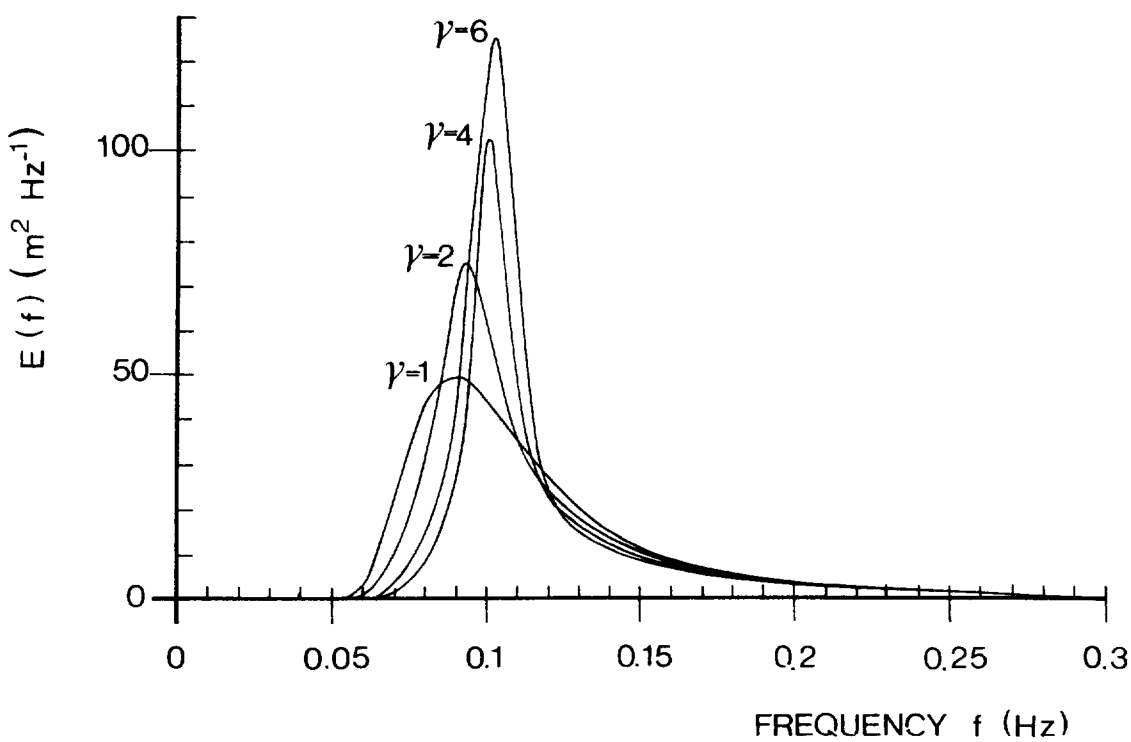


Figure 4

JONSWAP SPECTRUM $H_s = 7.0\,m$, $T_z = 8.0\,s$



CONCLUDING REMARKS

Although the recommendations of the ITTC (1972) and the ISSC (1976, 1979) have been given, no attempt has been made to establish which spectrum should be used. However, the Pierson-Moskowitz spectrum represents a fully developed sea, so is generally not appropriate for very high sea states such as that associated with the 50-year storm; the Bretschneider spectrum is more appropriate for these conditions in the open ocean.

The JONSWAP spectrum was derived from measurements of fetch-limited and growing sea states in the absence of swell; the choice of value for the parameter γ is a problem: Hasselmann et al. (1973) show that the wide range of values they found for γ cannot be explained in terms of fetch and mean wind speed, and suggest that it might be associated with small scale inhomogeneities in the wind field. So it would seem reasonable to use the mean value of 3.3 for γ or, if significant to the engineering criteria, a range of values based upon the normal distribution fitted by Ochi (1979).

Ewing (1980) examines whether the JONSWAP spectrum can be used to describe the wave spectra obtained from measurements in the open ocean about 20 km west of South Uist in the Outer Hebrides. He finds that with easterly winds the spectrum is double-peaked and is well-fitted by the JONSWAP formula (with a range of γ from 1 to 7) plus a swell component; with westerly winds it is difficult to separate the local sea and the swell components but the sea component seems to have values of wave height and period consistent with the Pierson-Moskowitz spectrum.

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EQUATIONS

General relationships of wave height and periods

$$m_n = \int_0^{\infty} f^n E(f)\, df \qquad (2)$$

$$H_s = 4\, m_o^{1/2} \qquad (3)$$

$$T_1 = m_o/m_1 \qquad (4)$$

$$T_z = (m_o/m_2)^{1/2} \qquad (5)$$

$$T_m = {}^{1}/\, f_m \qquad (6)$$

$$m_o = A/4B;\; m_1 \simeq 1.2254A/4B^{3/4};\; m_2 \simeq 1.77245A/4B^{1/2} \qquad (7)$$

$$T_1 \simeq 0.8161B^{-1/4};\; T_z \simeq 0.7511B^{-1/4} \qquad (8)$$

$$f_m^{4} \quad 4B/5 = 0.8B \qquad (9)$$

$$T_m \simeq 1.0574B^{-1/4} \qquad (10)$$

General spectral form (Bretschneider)

$$E(f) = A\, f^{-5} \exp\left[-\, B\, f^{-4}\right] \qquad (1)$$

Bretschneider spectrum in terms of H_s and T_1, T_z, T_m or T_s

$$\left.\begin{aligned} E(f) &= 0.11H_s^{2}\, T_1 (T_1 f)^{-5} \exp\left[-\, 0.443/(T_1 f)^{4}\right] \\ &= 0.080H_s^{2}\, T_z (T_z f)^{-5} \exp\left[-\, 0.318/(T_z f)^{4}\right] \\ &= 0.31H_s^{2}\, T_m (T_m f)^{-5} \exp\left[-\, 1.25/(T_m f)^{4}\right] \end{aligned}\right\} \qquad (11)$$

$$E(f) = 0.25H_s^{2}\, T_s (T_s f)^{-5} \exp\left[-\, 1/(T_s f)^{4}\right] \qquad (12)$$

Pierson-Moskowitz spectrum general form

$$E(f) = \alpha\, g^{2} (2\pi)^{-4} f^{-5} \exp\left[-\, 1.25 (f/f_m)^{-4}\right] \qquad (13)$$

Pierson-Moskowitz spectrum in terms of H_s or T_1, T_z, T_m

$$E(f) = 5.00\; 10^{-4} f^{-5} \exp\left[-\, 2.00\; 10^{-3}/H_s^{2} f^{4}\right] \qquad (14)$$

$$\left.\begin{aligned} E(f) &= 5.00\; 10^{-4} f^{-5} \exp\left[-\, 0.443/(T_1 f)^{4}\right] \\ &= 5.00\; 10^{-4} f^{-5} \exp\left[-\, 0.318/(T_z f)^{4}\right] \\ &= 5.00\; 10^{-4} f^{-5} \exp\left[-\, 1.25/(T_m f)^{4}\right] \end{aligned}\right\} \qquad (15)$$

JONSWAP spectrum general form

$$E(f) = \alpha\, g^{2} (2\pi)^{-4} f^{-5} \exp\left[-\, 1.25 (f/f_m)^{-4}\right] \gamma^{q} \qquad (16)$$

JONSWAP spectrum in terms of H_s and T_m, T_1, T_z

$$E(f) = 0.205H_s^{2}\, T_m (T_m f)^{-5} \exp\left[-\, 1.25/(T_m f)^{4}\right] 3.3^{q} \qquad (22)$$

$$E(f) = 0.0994H_s^{2}\, T_1 (T_1 f)^{-5} \exp\left[-\, 0.6062/(T_1 f)^{4}\right] 3.3^{q} \qquad (24)$$

$$E(f) = 0.0749H_s^{2}\, T_z (T_z f)^{-5} \exp\left[-\, 0.4567/(T_z f)^{4}\right] 3.3^{q} \qquad (26)$$