

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/222520056>

Ocean Waves Synthesis and Animation Using Real World Information

Article in *Computers & Graphics* · February 2002

DOI: 10.1016/S0097-8493(01)00161-3 · Source: DBLP

CITATIONS

21

READS

68

2 authors, including:



[Sébastien Thon](#)

Aix-Marseille Université

14 PUBLICATIONS 117 CITATIONS

SEE PROFILE

Ocean waves synthesis and animation using real world information

Sébastien THON and Djamchid GHAZANFARPOUR

MSI Laboratory
ENSIL – Limoges University
Technopole
87068 Limoges Cedex, France
thon@ensil.unilim.fr - ghazanfa@ensil.unilim.fr

Abstract

In Computer Graphics, existing ocean waves models still suffer from lack of realism and/or complexity of use essentially due to empirical definition of parameters by the user. In this paper, we propose an easy to use spectral ocean waves model whose parameters are obtained automatically from real world information. These parameters can be computed in two different ways. The first one makes use of measured oceanographic data and the second one is image based. Both are very fast and simple to use. By using real world information, we can achieve more realistic results than previous models. Ocean waves animation is very easy to perform with our spectral model. In addition, this model is procedural and allows a continuous representation of ocean surface away from shore with any size and any level of detail, while requiring very low memory storage.

Keywords: realistic ocean waves; spectral model; automatic synthesis; animation; image based; buoy data.

1. Introduction

The representation of natural phenomena is an important research field in Computer Graphics. Among these natural phenomena, water is one of the most important because of its omnipresence in our everyday life and the fact that it covers two-thirds of the surface of the earth. Water can be found under many forms, ranging from single drops to the hugeness of an ocean. In this paper, we propose to represent ocean waves with an easy to use spectral model whose parameters are obtained automatically from real world information. The waves generated by our model correspond to ocean waves away from shore, which are the most common waves at the surface of an ocean.

Two kinds of approaches have been used in Computer Graphics for representing ocean waves: empirical models and more or less simplified fluid mechanics models.

The goal of empirical models is to reproduce the appearance of the ocean waves behavior in a precise situation, by giving an explicit description of the shape of the water surface in this situation. Most of the empirical models represent ocean waves by using parametric functions. Max [1] used a model of superposed sinusoidal functions to represent

low amplitude ocean waves. Peachey [2] improved this method by changing waves shapes according to water depth. More complex parametric equations have been used by Fourier and Reeves [3] as well as Gonzato and Le Saëc [4], based on the empirical water wave model of Gerstner established in 1809 [5]. These models are generally fast and require very low memory storage because they describe the water surface with parametric functions. However, one of the most important drawbacks of these models is the empirical definition of their parameters by the user, with no connection to the reality. This is particularly the case for the definition of ocean waves characteristics such as amplitudes, wavelengths and directions of propagation which are crucial parameters for the realism of the ocean surface appearance. As a consequence, the results provided by these empirical models are often unrealistic and very hard to setup by the user.

In order to represent more realistic ocean waves, we could rely on fluid mechanics equations such as those used in scientific simulations. These physical equations are mainly derived from the Navier-Stokes equations introduced in 1827, which are the governing laws of fluid mechanics. The fluid behavior is computed by solving these equations over a finite-element grid representing the studied fluid area. These equations have been used in Computer Graphics either under their complete formulation [6,7] or under more or less simplified formulations [8-10], but only for representing small water surfaces and never for ocean waves. In fact, the representation of large scenes of ocean waves is an open problem for methods based on fluid mechanics, because the computation of water motion over wide areas entails important memory storage and computation time problems. Indeed, a huge amount of memory is required for sampling a large area in a finite-element grid, specially if the user wants to take into account the small details of the waves. This entails also considerable computation time because the larger the size of the finite-element grid, the longer the computation time. Moreover, these models are hard to set up because the user needs to define very precisely initial conditions over the finite-element grid as well as the different forces that act on the water. Thus, fluid mechanics based models are very difficult to use for non-specialists who are not familiar with the underlying equations.

Hence, empirical models of ocean waves suffer from the lack of real parameters and models based on fluid mechanics can hardly be used for representing large ocean waves scenes and are difficult to use for non-specialists. Very few models not based on fluid mechanics, but closer to the reality than purely empirical ones have been proposed. Mastin *et al* [11] used the Pierson-Moskowitz filter [12], a theoretical oceanographic model that describes statistically the frequency characteristics of ocean waves. This filter is used for filtering a 2D white noise in the frequency domain. By the inverse fast Fourier transformation of this filtered noise, a finite dimensions image in the 2D spatial domain is obtained. This image is then directly used as a height field of water waves. This representation of the water as a finite set of heights is the main drawback of this method because it is impossible to obtain any level of details. The filtering of a higher resolution white noise image is needed for a higher level of details.

In [13], we proposed a procedural model that overcomes some of the problems encountered in [11]. Since the definition of this model is procedural (see section 2.1 for a short survey of this model), we can represent continuously any size of water surface while requiring very low memory storage contrarily to [11]. All the parameters of [13] are defined by also using the Pierson-Moskowitz spectrum. However, this model has some drawbacks. In order to use the theoretical Pierson-Moskowitz spectrum, the user needs to provide some parameters of this spectrum that require some knowledge in the field of oceanography. Moreover, the selection of frequency components in the spectrum used in our previous work

does not sufficiently respect the spectrum energy. Consequently, the synthesized ocean surface does not sufficiently correspond to the original spectrum that we sampled. In addition, as the water surface in our model is mainly based on a superposition of periodic functions, it could introduce more or less noticeable unrealistic repetitive patterns.

In this paper, we propose several efficient solutions to the drawbacks of our previous work for achieving a more easy to use model and more realistic results. The main features of this paper are the following. First, we propose two new easy to use techniques to automatically compute the parameters of our model. These techniques are based on real world information and are more intuitive than the theoretical Pierson-Moskowitz spectrum that we previously used:

- The first technique makes use of real measurements obtained from buoys by oceanographers. From these measurements, oceanographers compute a representation of the frequency characteristics of the ocean waves called an *ocean waves spectrum*.
- The second technique is a simple and efficient image-based approach that allows the synthesis of an ocean surface by the spectral analysis of a digitized photograph.

These two techniques are fast and easy to use. By using these real world information, we can automatically achieve more realistic results than previous models. The second innovation of this paper is a new spectrum adaptive sampling method that respects the spectrum energy better than the method in [13], giving more realistic results. The third main feature of this paper is a new method for reducing the unrealistic repetition patterns of ocean waves that can be observed due to the superposition of periodic functions, by using a phase perturbation.

Ocean waves animation can be performed very easily because the formulation of the water surface in our model is time dependent. We use raytracing to render the ocean waves surface produced by our model, so we can easily take into account refraction and reflection of light in the water. The water surface is rendered as an implicit surface, by using the procedural and continuous definition of our model, to obtain any level of details while requiring very low memory storage.

This paper is organized as follows. In section 2, we present our ocean waves model and its two new generation techniques. Efficient solutions to the spectrum sampling and repetition patterns drawbacks are given as well. We show how this model can easily be animated in section 3 and how it can be rendered in section 4. We conclude in section 5 and give some ideas for future works in section 6.

2. The ocean waves model

2.1. The model

According to Fourier transformation theory, the surface of an ocean can be described as the superposition of elementary sine waves with different frequencies, amplitudes and directions of propagation. Thus, the elevation of the water surface at a point of coordinates (x,y) of the reference plane at time t can be computed by equation 1.

$$h(x, y, t) = \sum_{i=1}^n A_i \cdot \cos(k_i(x \cdot \cos \theta_i + y \cdot \sin \theta_i) - \omega_i t + \varphi_i) \quad (1)$$

Where n is the number of elementary sine waves, A_i is the amplitude, $\omega_i = 2\pi f_i$ is the angular frequency, f_i is the frequency, $k_i = 2\pi/\lambda_i$ is the wave number, λ_i is the wavelength, θ_i is the direction of propagation of the wave in the xy horizontal plane and φ_i is the phase.

This formulation is also used in oceanography in the linear theory of ocean waves for describing an ocean surface as the superposition of many wave trains. We proposed in [13] to use this theory as the base of our ocean waves model. Gerstner [5] showed that the motion of each water particle is a circle of radius r around a fixed point (x_0, z_0) , giving a wave profile that can be described by a mathematical function called *trochoid* (equation 2).

$$\begin{cases} x = x_0 + r \cdot \sin(k \cdot x_0 - \omega t) \\ z = z_0 - r \cdot \cos(k \cdot x_0 - \omega t) \end{cases} \quad (2)$$

The use of trochoids allows to obtain more or less sharpened crested waves according to the product $k \cdot r$ (Fig. 1).

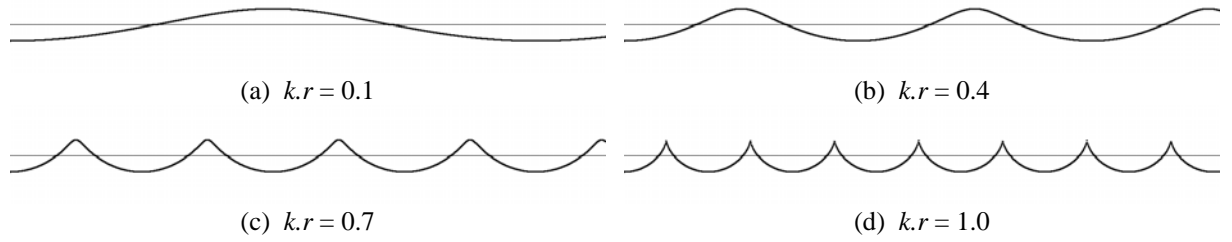


Fig. 1. Shape of a trochoid according to the product $k \cdot r$

This kind of shape is more realistic than simple sine shape in the case of an agitated ocean surface. In order to replace the sinusoidal shape of water waves by a trochoid shape, we replace the cosine function of equation (1) by the *trochoid* function, giving equation 3. We can use trochoids instead of cosines, because these two kinds of functions have more or less similar spectra. The trochoid main frequency corresponds to the cosine frequency and their amplitudes are the same.

$$h(x, y, t) = \sum_{i=1}^n A_i \cdot \text{trochoid}(k_i(x \cdot \cos \theta_i + y \cdot \sin \theta_i) - \omega_i t + \varphi_i) \quad (3)$$

This superposition of 2D trochoids represents the main structure of the ocean waves surface. In [13] we added a 3D turbulence function in order to provide a small scale details level. Thus, the final formulation of our ocean waves model is given by equation 4.

$$h(x, y, t) = \left[\sum_{i=1}^n A_i \cdot \text{trochoid}(k_i(x \cdot \cos \theta_i + y \cdot \sin \theta_i) - \omega_i t + \varphi_i) \right] + A_t \cdot \text{Turbulence3D}(x_t(t), y_t(t), z_t(t), m) \quad (4)$$

Now, the crucial problem is to determine what amplitudes, frequencies and directions values must be used for the trochoids of our model in order to get realistic results. Random values will give chaotic results too far from the reality. In order to determine these values, we propose to use what is called an ocean waves spectrum by oceanographers.

2.2. Spectral generation

In oceanography, the directional analysis of ocean waves consists in determining the spreading of the energy of ocean waves according to their frequencies and directions of propagation. This spectral and angular distribution is represented by the directional energy spectrum $S(f, \theta)$ of the ocean waves, which is a function of both ocean waves frequencies f and directions of propagation θ (Fig. 3). In the spatial domain, this directional spectrum represents a wavy surface corresponding to the superposition of elementary sine waves whose parameters are obtained from the spectrum. Thus, we propose to determine the parameters of our ocean waves model from such a directional spectrum.

In practice, we propose to generate a directional spectrum in two different ways, either by using real oceanographic measurements or by using an image-based technique.

2.2.1. Measured buoy data approach

With this first technique, we propose to generate a directional spectrum from real measurements obtained by buoys. We use measured data from NDBC¹. NDBC provides hourly observations from a network of about 60 buoys around the world. These buoys measure various information such as waves heights and periods, sea surface temperature, wind speed and direction.

The principle of the measure of an ocean waves spectrum with a buoy is the following: the buoy follows the ripples of the water surface (Fig. 2a) and records its vertical displacement with an accelerometer as a function of time (Fig. 2b). A Fast Fourier Transform (FFT) is applied to the recorded data by a processor on board the buoy to transform the data from temporal domain into frequency domain (Fig. 2c). This measured spectrum is then transmitted shore-side by satellite. Thus, we obtain a non-directional ocean waves spectrum which describes the spreading of the ocean waves energy (this energy is proportional to the variance of ocean surface height) as a function of its frequencies.

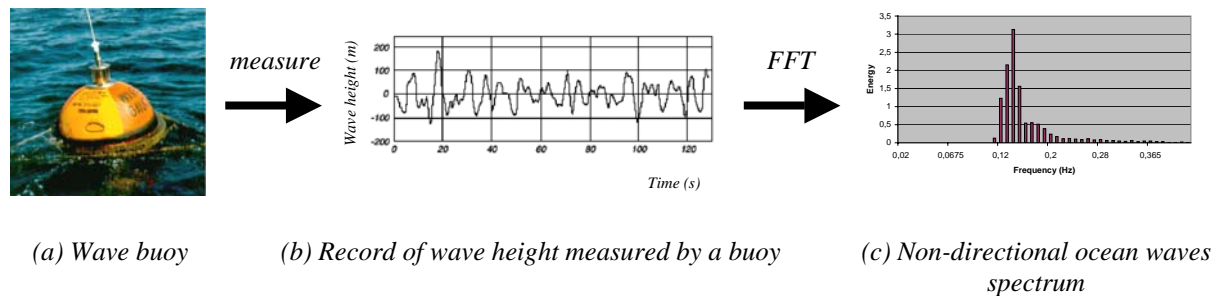


Fig. 2. Measure of an ocean waves spectrum with a buoy.

Some NDBC buoys are also able to measure directional ocean waves spectra² by determining ocean waves directions of propagation with methods described in [14,15]. Such a directional

¹ National Data Buoy Center, a part of the American forecasting organization NWS (National Weather Service).

² A list of these buoys can be found on the Internet at: <http://www.ndbc.noaa.gov/dwa.shtml>

ocean waves spectrum describes the spreading of ocean waves energy as a function of both frequency and direction (Fig. 3). Processors onboard these buoys compute ocean waves spectra over a limited number of frequency bins (typically about 40 fixed frequency bins ranging from 0.03 Hz to 0.40 Hz). For each frequency bin, buoys processors compute mean direction α_1 and principal direction α_2 of the ocean wave corresponding to this frequency bin, as well as its energy value E and R_1 and R_2 which are non-dimensional values describing the distribution of the ocean waves directions of propagation around the principal direction α_2 [16].

To determine the parameters of the trochoids of our model, we propose to use these measured data that can be found in real time on the NDBC site³.

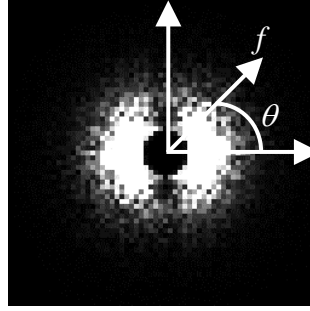


Fig. 3. Directional ocean waves spectrum $S(f, \theta)$ computed from buoy record.

We can directly use these values in the ocean surface equation of our model (equation 4). The number of superposed trochoids is given by the number of frequency bins. The amplitudes and directions of the trochoids are respectively given by the energy value E and mean direction α_1 of the frequency bins. The trochoids frequencies f_i are given by the central frequency values of the frequency bins. The resulting ocean surface equation is given by equation 5.

$$h(x, y, t) = \sum_{i=1}^n [E_i \text{trochoid}(k_i(x \cos \alpha_{1i} + y \sin \alpha_{1i}) - \omega_i t + \phi_i)] + A_t \text{Turbulence3D}(x_t(t), y_t(t), z_t(t), m) \quad (5)$$

Where n is the number of frequency bins.

However, as we can see on Fig. 2c, usually less than ten out of forty frequency bins are really significant, i.e. with sufficient energy values. All other frequency bins are negligible because their energy values are much lower. The limited number of fixed frequency bins used by the buoys is not sufficient, we need a higher frequency resolution for the spectrum. Therefore, in order to get more realistic results, we need an interpolation way to determine more trochoids from the buoy recorded data.

To overcome this problem, we propose to use a sort of data interpolation used in oceanography (equation 6), which consists in reconstructing a directional ocean waves spectrum from buoy recorded data by using a truncated Fourier series decomposition [16].

$$S(f, \theta) = E(f) \cdot D(f, \theta) \quad (6)$$

with: $D(f, \theta) = \frac{1}{\pi} (0.5 + R_1 \cdot \cos(\theta - \alpha_1) + R_2 \cdot \cos(2(\theta - \alpha_2)))$

³ http://www.ndbc.noaa.gov/to_station.shtml

Where $S(f, \theta)$ is the directional spectrum, $E(f)$ are the frequency bins energy values and $D(f, \theta)$ is the Directional Spreading Function (DSF). The DSF describes the directional spreading of ocean waves energy for each frequency f . R_1 , R_2 , α_1 and α_2 are values previously described obtained from the buoy. In this directional spectrum $S(f, \theta)$ we can select more frequency components in order to obtain more trochoids and thus more complex ocean waves surfaces. Our new method for an efficient sampling of the spectrum is described in section 2.3.

Ocean surfaces synthesized by using this technique are very realistic (Fig. 4), because the frequency characteristics of the synthesized ocean waves are measured from real ocean waves. This technique is very fast and easy to use, because the user only needs to use data files recorded by NDBC buoys. This simplicity of use avoids parameterization by the user and the ocean waves appearance only depends on the buoys data.



Fig. 4. Ocean waves generated by using measured data from buoys.

2.2.2. Image based approach

This second directional ocean waves spectrum generation technique is image based. This technique is based on the Fourier transformation that states that any function can be represented as an infinite sum of sines. This is very similar to the oceanographic linear theory of ocean waves that we use (see section 2.1), if we consider that the function is the surface of an ocean. This second spectrum generation technique is based on this similarity. In our case this function is a 2D image of real ocean waves. In order to use this image based technique, the user only needs to provide a digitized photograph of ocean waves used as the natural model (Fig. 5a). This photograph must be taken as vertically as possible in order to avoid perspective distortion and the illumination must be uniform (overcast sky) to avoid shadows and reflections. The user selects in this image a part (Fig. 5b) as a representative natural model of ocean waves. This image part is transformed into the frequency domain by applying a 2D Fast Fourier Transform (FFT). The result is a 2D spectrum (Fig. 5c) very similar to an ocean waves spectrum. This approach is similar to the analytical method that one of the authors has already used to synthesize procedural textures [17]. Like the previous generation technique of section 2.2.1, we select some frequency components in this spectrum by an efficient sampling (see section 2.3) and use these values for the parameters of the trochoids of

our model. We can achieve very realistic results (Fig. 6a and 6b), with similar frequency characteristics than the ocean waves represented on the original photograph (Fig. 5b). It is a very intuitive and easy to use ocean waves generation technique, because the user only needs to provide a simple digitized photograph. From this 2D image, we can synthesize an infinite 3D ocean waves surface. This parameters generation technique is very fast, because the FFT step requires very low computation time and the frequency components selection in the spectrum is also very fast.

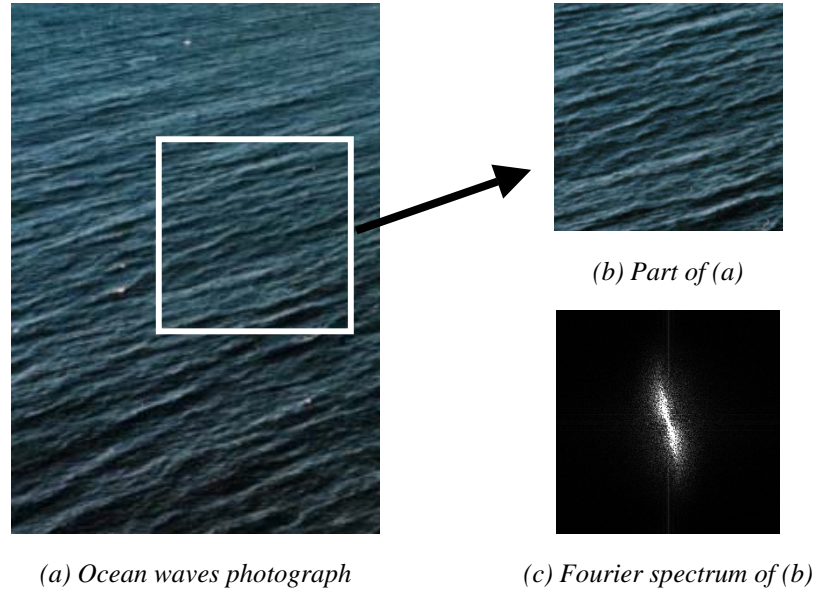


Fig. 5. Spectral analysis of a part of an ocean waves digitized photograph

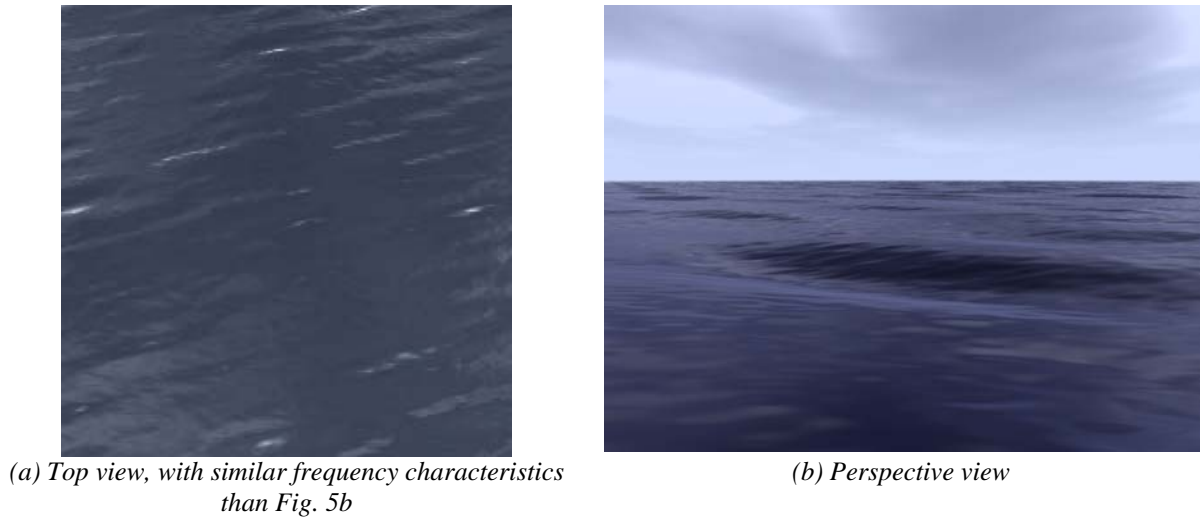


Fig. 6. Ocean waves generated by the spectral analysis of the photograph of Fig. 5b.

2.3. Spectrum sampling

Once we have obtained a directional ocean waves spectrum by either previously described techniques, we select in this spectrum some frequency components whose characteristics of frequency, amplitude and direction are then used in the ocean waves surface equation of our model. We could select a great number of frequency components in the spectrum, but the computation time needed to sum all the corresponding trochoids in our

model would be too expensive. So, we have to select a small number of frequency components that are the most representative of the spectrum. In [13], the selection method cannot represent sufficiently the whole structure of the spectrum, because only individual frequency components having the highest amplitudes are selected. Instead of picking individual components, we need a more precise method that samples the spectrum in some areas covering the whole spectrum.

We propose to sample adaptively the spectrum in areas of approximately equal energy quantity by using a quadtree (Fig. 7). This energy quantity is given by the user as a percentage P_E of the total spectrum energy. We recursively subdivide an area of the spectrum as long as the energy of this area is greater than the percentage P_E given by the user and as long as the size of the area is greater than a single frequency component. Thus, the most representative areas of the spectrum, which contain the highest amplitudes frequency components, will be sampled more finely than less representative areas (Fig. 7). The spectrum is simplified while preserving its main structure. The final number of areas gives the number of superposed trochoids in our model. From each area we compute parameters values for one trochoid. Trochoid amplitude is proportional to the energy of the area. Frequency and direction are given by the polar coordinates of area central point and are proportional respectively to polar coordinates radius and angle.

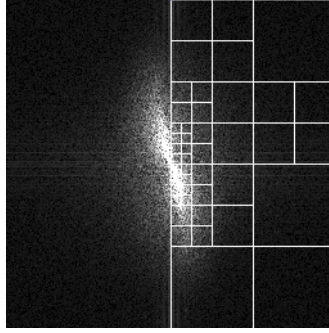


Fig. 7. Adaptive spectrum sampling.

2.4. Repetition pattern reduction

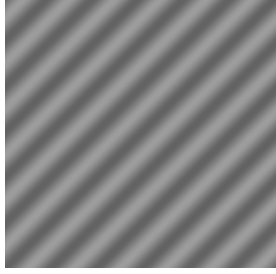
Since our model is based on the superposition of trochoids which are periodic functions, unrealistic repetitive patterns of ocean waves can be noticeable (Fig. 9a). To overcome this problem which was one of the drawbacks of [13] we propose to alter the periodicity of each superposed trochoid (Fig. 8a) by modifying its phase in the spatial domain (Fig. 8b). Good results are obtained by applying a Perlin turbulence function [18] to the phase, because it is fast and allows a realistic fractal perturbation with a great control for the user. This method allows a very efficient reduction of the periodicity artifact (Fig. 9b). The new formulation of our model which takes into account this perturbation is given by equation 7.

$$h(x, y, t) = \left[\sum_{i=1}^n A_i \cdot \text{trochoid}(k_i((x + dx) \cdot \cos \theta_i + (y + dy) \cdot \sin \theta_i) - \omega_i t + \varphi_i) \right] + A_i \cdot \text{Turbulence3D}(x_i(t), y_i(t), z_i(t), m) \quad (7)$$

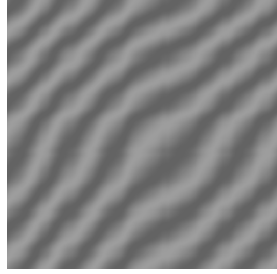
with:

$$\begin{aligned} dx &= A_d \cdot \lambda_i \cdot \text{Turbulence1D}(x \cdot F_d / \lambda_i) \\ dy &= A_d \cdot \lambda_i \cdot \text{Turbulence1D}(y \cdot F_d / \lambda_i) \end{aligned}$$

where A_d and F_d are respectively amplitude and frequency of the turbulence. By modifying these two parameters the user can obtain more or less deformed wave fronts. Turbulence amplitude is proportional to wavelength and turbulence frequency is inversely proportional to wavelength. Thus, the longer is the wavelength, the more important is the amplitude of the deformation and the lower is the frequency of this deformation.

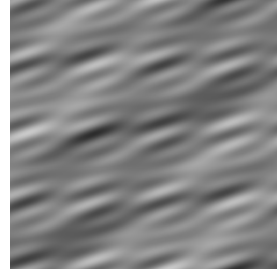


(a)

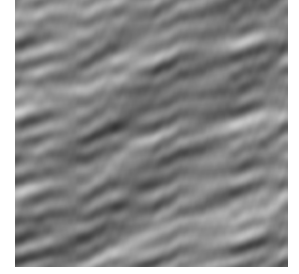


(b)

Fig. 8. A single trochoid before (a) and after (b) periodicity reduction.



(a)



(b)

Fig. 9. Sixteen superposed trochoids before (a) and after (b) periodicity reduction.

3. Animation

Animating ocean waves (Fig. 10) is very straightforward with our model. We consider separately the animation of the water waves main structure (trochoids) and the animation of small water waves (3D turbulence function). The main structure animation is simply done by shifting trochoids phases with respect to the time, entailing the propagation of the water waves. The animation of small water waves is achieved by a simple displacement into the 3D turbulence function. In the xy horizontal plane, the displacement direction is given by the main direction of the ocean waves spectrum, with a speed V_H . We additionally apply a linear displacement along the vertical z axis with speed V_v in order to change the turbulence appearance, giving the impression that all the small waves are oscillating.

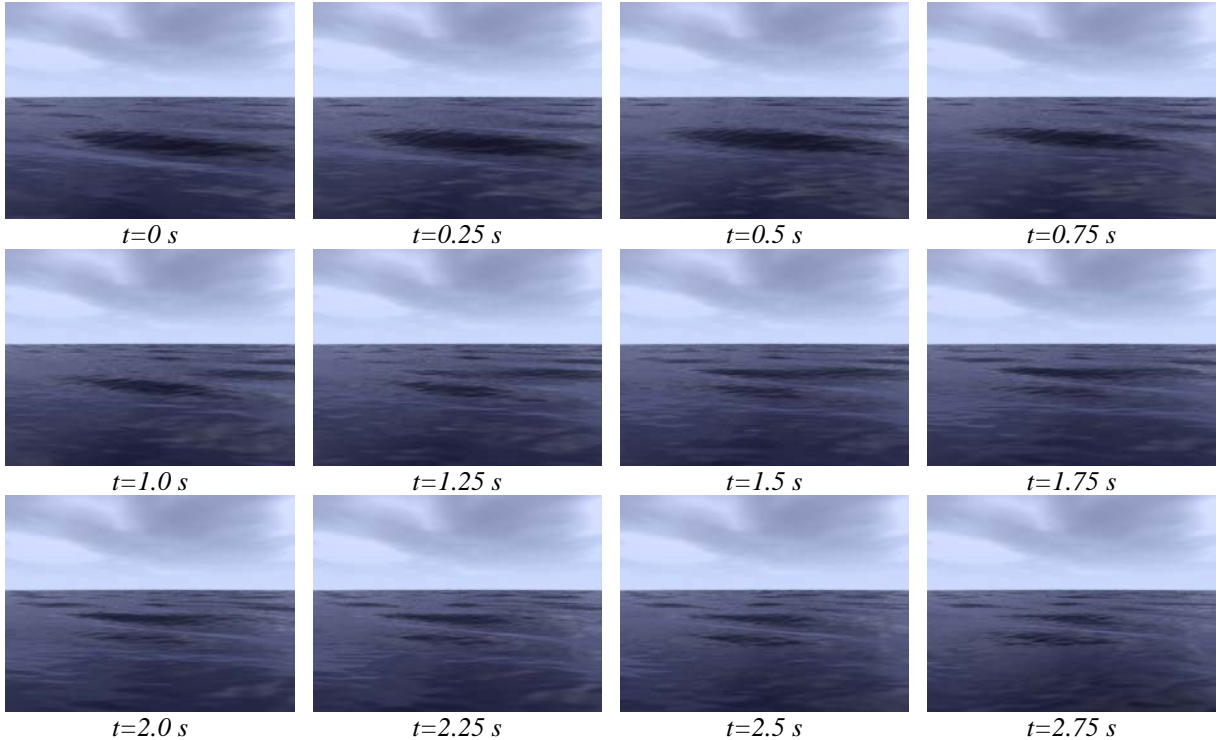


Fig. 10. Some frames from three seconds of animation (look from left to right). Waves parameters are obtained with the image-based method applied to picture 5b.

4. Rendering

The computation step of our model parameters is very fast, for both measured based and image based techniques (less than 1 second on a Pentium II 450 MHz). The next step is the rendering of the ocean surface given by our model. The computation of water elevation at any point by equation (4) is also very fast (in 1 second, we can compute water elevation at 70000 points with 50 superposed trochoids on a Pentium II 450 MHz).

To display the ocean surface, it is possible to use different rendering techniques with our model. For example, z-buffer can be used for a fast rendering, while sacrificing realism. For high quality rendering, raytracing is preferable. We use raytracing for the pictures presented in this paper. We process our model as an implicit surface, using its procedural and continuous definition. Intersections between rays and the implicit surface are determined by a ray-marching process [19,20]. Hence, any level of details is available, and there is no need for huge memory storage. Moreover, raytracing is well adapted to the rendering of water because it allows realistic reflection and refraction of light. However, raytracing is a slow process, especially the ray-marching determination of the ocean surface. Aliasing is reduced by using a kind of low-pass filter described in [13] on the trochoids, similar to the “clamping” method [21] for 2D textures.

5. Conclusions

We have proposed efficient solutions to the drawbacks of our previous work [13], achieving a more easy to use model and more realistic representation of ocean surfaces away from shore. Major innovations of this paper are the automatic determination of our model parameters by two new easy to use spectral techniques, a precise spectrum sampling and a repetition patterns reduction method.

These parameters generation techniques are based on real world information and are more intuitive than the theoretical Pierson-Moskowitz spectrum used in our previous work. The first technique makes use of real measurements realized with buoys by oceanographers. The second one is an image based method that allows to synthesize the surface of an ocean by the spectral analysis of a simple digitized photograph. These two techniques are very fast and very easy to use. It is up to the user to choose either of these techniques, depending on the possession of either NDBC data files or ocean waves appropriate photographs. By using these real world information, we can achieve more realistic results than previous ocean waves models proposed in Computer Graphics. Moreover, we have proposed a new spectrum adaptive sampling method that respects the spectrum energy better than in [13], giving better results. In addition, we have proposed a perturbation method that efficiently reduces the unrealistic repetition patterns of ocean waves due to the superposition of periodic trochoids. However, a single ocean waves spectrum statistically describes the state of the whole surface of an ocean and cannot give different state information for different localized parts of this surface. Consequently, our ocean waves model cannot represent an ocean near the shore, because in shallow water we need to take into account the local influence of the seabed and the shore on the ocean waves that entails water waves refraction, reflection or diffraction. Moreover, an ocean waves spectrum cannot give information about the presence of breaking waves. So, the limitation of our model is that it can only represent open ocean scenes far from the shore, without breaking waves. However, our model can be used to generate the frequency characteristics of waves away from shore and to represent these waves, in conjunction with another model specialized in the propagation and representation of these waves near the shore.

6. Future works

Ocean waves appearance depends on two factors: waves shapes and optical properties of water. There remain unresolved problems in both of these domains, such as realistic breaking waves and waves interactions with objects. These are now the main modeling problems in the field of ocean waves representation in Computer Graphics. Moreover, the rendering of water should be enhanced for more realism. It is necessary to take a particular care in the determination of water color according to its composition as well as interactions between light and water. Foam representation have also to be studied in order to achieve a higher degree of realism.

7. Acknowledgements

We would like to thank the Regional Council of Limousin for their financial support of this work, as well as Danièle Hauser from CETP and Ken Steele from NDBC for their precious help.

References

- [1] [Max N. Vectorized procedural models for natural terrain: waves and islands in the sunset. Computer Graphics \(Siggraph '81\) 1981;15\(3\):317-324.](#)
- [2] [Peachey DR. Modeling waves and surf. Computer Graphics \(Siggraph '86\) 1986;20\(4\):65-74.](#)
- [3] [Fournier A, Reeves WT. A simple model of ocean waves. Computer Graphics \(Siggraph '86\), 1986;20\(4\):75-84.](#)
- [4] [Gonzato JC, Le Saëc B. A phenomenological model of coastal scenes based on physical considerations. Eurographics Workshop on Computer Animation and Simulation '97, 1997;137-148.](#)
- [5] [Gerstner FJ. Theorie der Wellen. Ann. der Physik, 1809;32:412-440.](#)
- [6] [Foster N, Metaxas D. Realistic animation of liquids. Proceedings of Graphics Interface '96, 1996;204-212.](#)
- [7] [Foster N, Metaxas D. Controlling fluid animation. Proceedings of Graphics Interface '97, 1997;178-188.](#)
- [8] [Kass M, Miller G. Rapid, stable fluid dynamics for computer graphics. Computer Graphics \(Siggraph '90\) 1990;24\(4\):49-57.](#)
- [9] [Chen J, Lobo N. Toward interactive-rate simulation of fluids with moving obstacles using Navier-Stokes equations. Graphical Models and Image Processing 1995;57\(2\):107-116.](#)

- [10] O'Brien JF, Hodgins JK. Dynamic simulation of splashing fluids. Proceedings of Computer Animation '95, 1995;198-205.
- [11] Mastin GA, Watterberg PA, Mareda JF. Fourier synthesis of ocean scenes. IEEE Computer Graphics and Applications 1987;7(3):16-23.
- [12] Pierson WJ, Moskowitz L. A proposed spectral form for fully developed wind seas based on the similarity theory of S.A. Kilaigorodskii. Journal of Geophysical Research 1964;5181-5190.
- [13] Thon S, Dischler JM, Ghazanfarpour D. Ocean waves synthesis using a spectrum-based turbulence function. Proceedings of Computer Graphics International 2000;65-72.
- [14] Steele KE, Earle M. Directional ocean wave spectra using buoy azimuth, pitch, and roll derived from magnetic field components. IEEE Journal of Ocean Engineering 1991;16(4):427-433.
- [15] Steele KE, Teng CC, Wang DWC. Wave direction measurements using pitch and roll buoys. Ocean Engineering 1992;19(4):349-375.
- [16] Benoit M, Frigaard P, Schäffer HA. Analysing multidirectional wave spectra: a tentative classification of available methods. Proceedings of IAHR '97, 1997;131-158.
- [17] Ghazanfarpour D, Dischler JM. Spectral analysis for automatic 3D texture generation. Computers and Graphics 1995;19(3):413-422.
- [18] Perlin K. An image synthesizer. Computer Graphics (Siggraph '85) 1985;19(3):287-296.
- [19] Tuy H, Tuy L. Direct 2D display of 3D objects. Computer Graphics and Applications 1984;4(10):29-33.
- [20] Kajiya J, Von Herzen B. Ray tracing volume densities. Computer Graphics (Siggraph '84) 1984;18(3):165-173.
- [21] Norton A, Rockwood A, Skolmoski P. Clamping: a method of antialiasing textured surfaces by bandwidth limiting in object space. Computer Graphics (Siggraph '82) 1982;16(3):1-8.