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## 1 Basic

### 1.1 Default code

```
#include<bits/stdc++.h>
#include<chrono> // for timing
#pragma GCC optimize("O3,unroll-loops")
#pragma target optimize("avx2,bmi,bmi2,lzcnt,popcnt")
#define IO ios_base::sync_with_stdio(0);cin.tie(0);cout
        .tie(0);
#define pii pair<int,int>
#define ft first
#define sd second
#define int long long
#define double long double
#define PI acos(-1)
#define SZ(x) (int)x.size()
#define all(v) (v).begin(), (v).end()
#define _for(i,a,b) for(int i=(a);i<(b);++i)
using namespace std;
template<typename T>
ostream& operator<<(ostream& os,const vector<T>& vn){
    for(int i=0;i<vn.size();++i)os<<vn[i]<<" ";
    return os;
}
template<typename T>
ostream& operator<<(ostream& os,const set<T>& vn){
    for(typename set<T>::iterator it=vn.begin();it!=vn.
        end();++it)os<<*it<<" ";
    return os;
}
mt19937 mt(hash<string>{}("Mashu_AC_Please")); //mt();
// mt19937 mt(chrono::steady_clock::now().
//     time_since_epoch().count());
// g++ a.cpp -Wall -Wshadow -fsanitize=undefined -o a.
//     exe
// ./a.exe
const int MXN=2e5+5;
const int INF=INT_MAX;
void sol() {}
signed main() {
    // auto start=chrono::high_resolution_clock::now();
    // #ifdef LOCAL
    //     freopen("input.txt","r",stdin);
    //     freopen("output.txt","w",stdout);
    // #endif
    IO
    int t=1;
    cin>>t;
    while(t--) {sol();}
    // auto stop = chrono::high_resolution_clock::now()
    //     ;
    // auto duration = chrono::duration_cast<chrono::
    //     milliseconds>(stop - start);
    // cerr<<"Time:"<<duration.count()<<" ms\n";
}
```

### 1.2 Misc

```
iota(vec.begin(),vec.end(),1);// 產生1~size的整數列
stoi(s.begin(),s.end(),k);// 法1,字串轉成k進位int
string s;cin>>s;
int x=stoi(s,0,2); // 法2,2可以改其他進位
__builtin_popcountll // 二進位有幾個1
__builtin_clzll // 左起第一個1前0的個數
__builtin_parityll // 1的個數的奇偶性
__builtin_mul_overflow(a,b,&res) // a*b是否溢位

// double 轉整數 請加 int b=round(a)
// 或是 int b =floor(a+0.5) (floor向下取整)
```

### 1.3 Fast read & write

```
inline int read() {
    char c = getchar(); int x = 0, f = 1;
    while(c < '0' || c > '9') {if(c == '-') f = -1; c =
        getchar();}
```

```

while(c >= '0' && c <= '9') x = x * 10 + c - '0', c
    = getchar();
return x * f;
}
inline void write(int x){
    if(x<0) putchar('-'),x=-x;
    if(x>9) write(x/10);
    putchar(x%10+'0');
}

```

## 1.4 Sort cmp

```

struct cmp{inline bool operator()(const int a,const int
    b){return a<b;}}; //common use
auto cmp= [](vector<int> a, vector<int> b) {return a[1]<
    b[1];}; //for set use
set<vector<int>, decltype(cmp)> prepare, done;

```

## 1.5 Discretization

```

vector<int> vec;
sort(vec.begin(),vec.end());
vec.resize(unique(vec.begin(),vec.end())-vec.begin());
for(int i=0;i<n;++i){ //+1是讓 index是1到N 可以不要
    arr[i]=lower_bound(vec.begin(),vec.end(),ll[i])-vec
        .begin()+1;
}

```

## 1.6 Custom unordered\_map

```

struct Type{
    int x;
    string y;
    bool operator==(const Type &other) const {
        return (x == other.x && y == other.y);
    }
};
struct hashes{
    size_t operator()(const Type &o) const {
        return ((hash<int>()(o.x)^(hash<string>()(o.y)
            <<1))>>1);
    }
};
//unordered_map<Type,int,hashes> map;

```

## 1.7 \_\_int128 read

```

// __int128_t p;
// lll n=qr(p);
#define lll __int128
template<class type_name> inline type_name qr(type_name
    sample)
{
    type_name ret=0,sgn=1;
    char cur=getchar();
    while(!isdigit(cur))
        sgn=(cur=='-'?-1:1),cur=getchar();
    while(isdigit(cur))
        ret=(ret<<1)+(ret<<3)+cur-'0',cur=getchar();
    return sgn==-1?-ret:ret;
}
inline void print(__int128 x){
    if(x < 0){
        putchar('-');
        x = -x;
    }
    if(x > 9)
        print(x / 10);
    putchar(x % 10 + '0');
}

```

## 1.8 字典序 a 嚴格小於 b

```

template<class T> //字典序a嚴格小於b
bool lexicographicallySmaller(const vector<T> &a,const
    vector<T> &b){
    int n=a.size();
    int m=b.size();
    int i;
    for(int i=0;i<n && i<m;++i){
        if(a[i]<b[i])return true;
        else if(b[i]<a[i])return false;
    }
    return (i==n && i<m);
}

```

## 1.9 Radom

```

mt19937 gen(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

```

## 2 對拍

### 2.1 run.bat

```

@echo off
g++ ac.cpp -o ac.exe
g++ wa.cpp -o wa.exe
g++ gen1.cpp -o gen.exe

:loop
    echo %%x
    gen.exe > input
    ac.exe < input > ac
    wa.exe < input > wa
    fc ac wa
if not errorlevel 1 goto loop

```

### 2.2 run.sh

```

for ((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac < input > ac.out
    ./wa < input > wa.out
    diff ac.out wa.out || break
done

```

## 3 Flow & Matching

### 3.1 Dicnic

```

// flow.init(n,s,t):有n個點(0~n-1)，起點s終點t
// flow.add_edge(u,v,f):建一條邊，從u點到v點流量為f
// flow.solve():回傳網路最大流答案
//時間複雜度: O(V^2*E)
struct Dinic{
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].push_back({v,f,(int)(E[v]).size()});
        E[v].push_back({u,0,(int)(E[u]).size()-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){

```

```

        int u = que.front(); que.pop();
        for (auto it : E[u]){
            if (it.f > 0 && level[it.v] == -1){
                level[it.v] = level[u]+1;
                que.push(it.v);
            } } }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            } }
        if (!res) level[u] = -1;
        return res;
    }
    int solve(int res=0){
        while ( BFS() )
            res += DFS(s,2147483647);
        return res;
    } }flow;
}

```

### 3.2 ZKW FLOW

```

//最大流量上的最小花費
//最大流量優先，相同才是找最小花費，複雜度 $O(V^2 \cdot E^2)$ 
// flow.init(n,s,t):有n個點(0~n-1)，起點s終點t
// flow.add_edge(u,v,f,c):建一條邊，從u點到v點流量為f，
// 每一單位流量的花費為c
// flow.solve():回傳一個pair(maxFlow,minCost)
// 限制：圖不能有負環
// 網路最大流的add_edge(u,v,f)可以無痛轉成最大流量上的
// 最小花費add_edge(u,v,1,f)即建立一條從u到v的邊流量為
// 1，單位流量花費為f
#define ll long long
struct zkwflow{
    static const int maxN=20000;
    struct Edge{ int v,f,re; ll w;};
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        for(int i=0;i<n;i++) E[i].clear();
    }
    void add_edge(int u,int v,int f,ll w){
        E[u].push_back({v,f,(int)E[v].size(),w});
        E[v].push_back({u,0,(int)E[u].size()-1,-w});
    }
    bool SPFA() {
        fill_n(dis, n, LLONG_MAX);
        fill_n(vis, n, false);
        queue<int> q;
        q.push(s); dis[s]=0;
        while(!q.empty()) {
            int u = q.front(); q.pop();
            vis[u] = false;
            for(auto &it: E[u]){
                if(it.f>0 && dis[it.v]>dis[u]+it.w){
                    dis[it.v] = dis[u]+it.w;
                    if(!vis[it.v]) {vis[it.v] = true; q
                        .push(it.v);}
                }
            }
        }
        if(dis[t]==LLONG_MAX) return false;
        // 不管流量是多少，花費不能是正數時加上這行（最
        // 小花費可行流）
        // if(dis[t] >= 0) return false;
        return true;
    }
    int DFS(int u, int nf) {
        if(u==t) return nf;
        int res = 0; vis[u] = true;
        for(int &i=ptr[u] ; i<(int)E[u].size() ; i++) {
            auto &it = E[u][i];

```

```

            if(it.f>0 && dis[it.v]==dis[u]+it.w && !vis
                [it.v]) {
                int tf = DFS(it.v, min(nf, it.f));
                res += tf;
                nf-=tf;
                it.f-=tf;
                E[it.v][it.re].f += tf;
                if(nf==0) { vis[u]=false; break; }
            }
        }
        return res;
    }
    pair<int,ll> solve(){
        int flow = 0; ll cost = 0;
        while (SPFA()){
            fill_n(ptr, n, 0);
            int f = DFS(s, INT_MAX);
            flow += f;
            cost += dis[t]*f;
        }
        return {flow, cost};
    } // reset: do nothing
} flow;

```

### 3.3 Hungarian

```

//匈牙利演算法-二分圖最大匹配
//記得每次使用需清空vis數組
//O(nm)
//其中Map為鄰接表(Map[u][v]為u和v是否有連接) S為紀錄這
//個點與誰匹配(S[i]為答案i和誰匹配)
const int M=505, N=505;
bool Map[M][N] = {0};
int S[N];
bool vis[N];
bool dfs(int u){
    for(int i=0;i<N;i++){
        if(Map[u][i]&&!vis[i]){ //有連通且未拜訪
            vis[i]=1; //紀錄是否走過
            if(S[i]==-1||dfs(S[i])){ //紀錄匹配
                S[i]=u;
                return true; //反轉匹配邊以及未匹配邊
                的狀態
            }
        }
    }
    return false;
}
//此二分圖為左邊M個點右邊N個點，跑匈牙利只要跑1~M就可以
//了，(S[右邊的點] -> 左邊的點)
memset(S,-1,sizeof(S));
int ans = 0;
for(int i=0;i<M;i++){
    memset(vis,0,sizeof(vis));
    if(dfs(i)) ans++;
} //跑匈牙利
cout<<ans<<"\n";
for(int i=0 ; i<N ; i++) {
    if(S[i]!=-1) cout<<"pair: "<<S[i]<<" "<<i<<"\n";
}

```

### 3.4 KM

```

//二分圖最大權完美匹配
//二分圖左邊的點都要匹配到右邊的點，且每條邊都有權重，
//求權重最大值，複雜度 $O(V^3)$ 
// graph.init(n):二分圖左右各n個點
// graph.add_edge(u,v,w):建一條邊，從u點到v點權重為w
// graph.solve():回傳最大權重
struct KM{ // max weight, for min negate the weights
    int n, mx[MXN], my[MXN], pa[MXN];
    ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
    bool vx[MXN], vy[MXN];
    void init(int _n) { // 1-based, N個節點
        n = _n;

```

```

    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
}
void add_edge(int x, int y, ll w) {g[x][y] = w;} //
// 左邊的集合節點x連邊右邊集合節點y權重為w
void augment(int y) {
    for(int x, z; y; y = z)
        x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
}
void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
        while(q.size()) {
            int x=q.front(); q.pop(); vx[x]=1;
            for(int y=1; y<=n; ++y) if(!vy[y]){
                ll t = lx[x]+ly[y]-g[x][y];
                if(t==0){
                    pa[y]=x;
                    if(!my[y]){augment(y);return;}
                    vy[y]=1, q.push(my[y]);
                }else if(sy[y]>t) pa[y]=x, sy[y]=t;
            }
        }
        ll cut = INF;
        for(int y=1; y<=n; ++y)
            if(!vy[y]&&cut>sy[y]) cut=sy[y];
        for(int j=1; j<=n; ++j){
            if(vx[j]) lx[j] -= cut;
            if(vy[j]) ly[j] += cut;
            else sy[j] -= cut;
        }
        for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
        }
    }
}
11 solve(){ // 回傳值為完美匹配下的最大總權重
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
        // 1-base
        lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
}
} graph;

```

## 4 Graph

### 4.1 BCC

//無向圖上，不會產生割點的連通分量稱為點雙連通分量，  
0base

```

#define PB push_back
#define REP(i, n) for(int i = 0; i < n; i++)
struct BccVertex {
    int n, nScc, step, dfn[MXN], low[MXN];
    vector<int> E[MXN], sccv[MXN];
    int top, stk[MXN];
    void init(int _n) {
        n = _n;
        nScc = step = 0;
        for (int i = 0; i < n; i++)
            E[i].clear();
    }
    void addEdge(int u, int v) {
        E[u].PB(v); E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v : E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v, u);

```

```

                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            }
            else low[u] = min(low[u], dfn[v]);
        }
    }
    vector<vector<int>> solve() { //回傳每個點雙聯通分量
        vector<vector<int>> res;
        for (int i = 0; i < n; i++)
            dfn[i] = low[i] = -1;
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i, i);
            }
        REP(i, nScc) res.PB(sccv[i]);
        return res;
    }
} graph;

```

### 4.2 SCC

//在有向圖裡的任兩點 $u$ 、 $v$ ，皆存在至少一條 $u$ 到 $v$ 的路徑  
以及 $v$ 到 $u$ 的路徑  
//fill zero 注意多筆測資要改fill  
//注意要0base

```

#define PB push_back
#define FZ(x) memset(x, 0, sizeof(x))
const int MXN = 1e5;
struct Scc {
    int n, nScc, vst[MXN], bln[MXN]; //nScc 有幾個強連通分量
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v) {
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u) {
        vst[u] = 1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u) {
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve() {
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i = 0; i < n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]) {rDFS(v); nScc++;}
    }
} scc;

```

### 4.3 2SAT

有 $N$ 個 boolean 變數 $a_1 \dots a_N$

ex: 滿足  $(\neg a_1 \text{ or } a_2) \text{ and } (a_2 \text{ or } a_3) \text{ and } (\neg a_3 \text{ or } \neg a_4)$  的解

```
//給定一張有向圖，邊上有權重，要找到一個環其平均權重最小
/* minimum mean cycle  $O(VE)$  */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int n )
```



```

{ n = _n; m = 0; }
// WARNING: TYPE matters
//建一條單向邊 (u, v) 權重為 w
void addEdge( int vi , int ui , double ci )
{ e[ m++ ] = { vi , ui , ci }; }
void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}
double solve(){//回傳值為最小平均權重 (小數)
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.
        clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        if(rho.empty()) return inf;
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} }mmc;

```

## 4.7 Dominator Tree

```

// 給一張有向圖，圖上有一個起點 s 可以走到所有點。
// 定義 "支配" 為從起點 s 出發，所有能走到節點 x 的路徑
// 的最後一個必經點
// 最後 idom[i] 為點 i 的支配點
struct DominatorTree{ // O(n+m)
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ]; //idom[u] s到u的最後一個必經點
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if(cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ))
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    //節點數量，起點編號 1-base
    void init( int _n , int _s ){
        ts = 0; n = _n; s = _s;
        REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );

```

```

        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){// 建立支配樹
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue ;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i , 2 , n ){
            int u = nfd[ i ];
            if( u == 0 ) continue ;
            if( idom[ u ] != sdom[ u ] )
                idom[ u ] = idom[ idom[ u ] ];
        } }domT;

```

## 5 DP

### 5.1 數位 DP

```

// dp[位數][狀態]
// dp[pos][state]: 定義為目前位數在前導狀態為state的時
// 候的計數
// ex: 求數字沒有出現66的數量 l~r
// -> dp[pos][1] 可表示計算pos個位數在前導出現一個6的計
// 數 -> dp[3][1] 則計算 6XXX
// 模板的pos是反過來的，但不影響(只是用來dp記憶用)

// pos: 目前位數
// state: 前導狀態
// lead: 是否有前導0 (大部分題目不用但有些數字EX:00146
// 如果有影響時要考慮)
// limit: 是否窮舉有被num限制
vector<int> num;
int dp[20][state];
int dfs(int pos, int state, bool lead, bool limit) {
    if(pos==num.size()) {
        //有時要根據不同state回傳情況
        return 1;
    }
    if(limit==false && lead==false && dp[pos][state]
        !=-1) return dp[pos][state];
    int up = limit?num[pos]:9;
    int ans = 0;
    for(int i=0 ; i<=up ; i++) {
        //有時要考慮那些狀況要continue
        ans += dfs(pos+1, state||(check[i]==2), lead&&i
            ==0, limit&&i==num[pos]);
    }
    if(limit==false && lead==false) dp[pos][state] =
        ans;
    return ans;
}

```

## 6 Math

### 6.1 Formulas

```
//五次方幂次和
a(n) = n^2*(n+1)^2*(2*n^2+2*n-1)/12
//四次方幂次和
a(n) = n*(n+1)*(2n+1)*(3n^3+3n-1)/30
```

### 6.2 Primes

```
1097774749, 1076767633, 100102021, 999997771
1001010013, 1000512343, 987654361, 999991231
999888733, 98789101, 987777733, 999991921, 1010101333
```

### 6.3 Quick Pow

```
// a^b
const int MOD = 1e9+7;
int qpow(int n, int k, int p) {
    int ret = 1;
    for(; k >= 1, n = n * n % p) if(k & 1) ret = ret
        * n % p;
    return ret;
}
// a^(b^c) = a^(q*(p-1)+r) = a^r so let b^c mod p-1
bc = qpow(b, c, p-1);
ans = qpow(a, bc, p);
```

### 6.4 Mat quick Pow

```
struct mat{
    long long a[200][200], r, c; // resize
    mat(int _r, int _c){r=_r; c=_c; memset(a, 0, sizeof(a))
    };
    void build(){for(int i=0; i<r; ++i) a[i][i]=1;}
};
mat operator * (mat &x, mat &y){
    mat z(x.r, y.c);
    for(int i=0; i<x.r; ++i) for(int j=0; j<y.c; ++j) for(int
        k=0; k<y.c; ++k)
        z.a[i][j] = (z.a[i][j] + x.a[i][k] * y.a[k][j] % MOD) %
            MOD;
    return z;
}
mat qpow(mat a, int k){
    mat r(a.r, a.r); r.build(); while(k){if(k&1) r=r*a; a=a*
        a; k>>=1;} return r;
}
```

### 6.5 Primes Table

```
int np[MXN];
vector<int> vec;
void sol(){
    np[0]=np[1]=1;
    for(int i=2; i<MXN; ++i){
        if(!np[i]){
            for(int j=i; j<MXN; j+=i){
                np[j]=1;
            }
            vec.push_back(i);
        }
    }
}
```

### 6.6 Factor Table

```
int arr[MXN];
void init(){
    for(int i=1; i<MXN; ++i) for(int j=i; j<MXN; j+=i) arr[
        j]++;
}
```

### 6.7 Catalan Number

```
// O(N), 要記得開Long Long 跟設定 MOD
cat[0]=1; cat[1]=1;
for(ll i=1; i<N; i++) {
    cat[i+1] = cat[i]*(i+2)%MOD*qpow(i+2, MOD-2)%MOD;
}
```

### 6.8 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmses <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={}
bool witness(LL a, LL n, LL u, int t){
    if(!a) return 0;
    LL x = mypow(a, u, n);
    for(int i=0; i<t; i++){
        LL nx = mul(x, x, n);
        if(nx==1 && x!=1 && x!=n-1) return 1;
        x = nx;
    }
    return x!=1;
}
bool miller_rabin(LL n) {
    int s = (magic number size)
    // iterate s times of witness on n
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u = n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a = magic[s] % n;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}
```

### 6.9 PollardRho

```
// does not work when n is prime O(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x, x, mod), 1, mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}
```

### 6.10 PrimeFactor0(logn)

```
#define i64 __int64
vector<i64> ret;
void fact(i64 x) {
    if (miller_rabin(x)) {
        ret.push_back(x);
        return;
    }
    i64 f = pollard_rho(x);
    fact(f); fact(x/f);
}
```

## 6.11 $O(1)$ mul

```
LL mul(LL x,LL y,LL mod){
    LL ret=x*y-(LL)((long double)x/mod*y)*mod;
    // LL ret=x*y-(LL)((long double)x*y/mod+0.5)*mod;
    return ret<0?ret+mod:ret;
}
```

## 6.12 Josephus Problem

```
//base1 n people count k find lastone  $O(n)$ 
int jo(int n, int k){return n>1?(jo(n-1,k)+k-1)%n+1:1;}
//base0 when  $k<n$   $O(k\log n)$ 
int jo(int n, int k) {
    if (n == 1) return 0;
    if (k == 1) return n - 1;
    if (k > n) return (jo(n - 1, k) + k) % n;
    int f = jo(n - n / k, k) - n % k;
    return f + (f < 0 ? n : (f / (k - 1)));
}
//base1 when  $k=2$  fast find mth
int jo2(int n, int m, int f=0){
    if(n == 1) return 1;
    int kill = (n + f) / 2;
    if(m <= kill) return 2 * m - f;
    return 2 * jo2(n - kill, m - kill, (n ^ f) & 1) -
        (1 ^ f);
}
```

## 6.13 Harmonic Sum

```
struct Harmonic{
    const double gamma = 0.5772156649;
    //求第N個調和級數
    double nthHarmonic(int n){
        double result = log(n)+gamma;
        return result;
    }
    //求項數n的 $S_n > k$ 
    int findNearstN(int k){
        int n = exp(k-gamma)+0.5;
        return n;
    }
    // 16n
    //  $n/1 + n/2 + n/3 + \dots + n/n$ 
    //就是這東西
    [20,10,6,5,4,3,2,2,2,1,1,1,1,1,1,1]
    //這是N以下的全因數和
    int nthHarmonicSum9(int n){
        int inv2=qpow(2,MOD-2,MOD),ans=0;
        for(int i=1;i<=n;){
            int v = n/i; int j = n/v;
            int area=((j-i+1)%MOD)*((j+i)%MOD)%MOD*
                inv2%MOD; //梯形
            ans=(ans+v*area%MOD)%MOD;
            i=j+1;
        }
        return ans;
    }
};
```

# 7 Data Structure

## 7.1 BIT

```
//注意值域
#define lowbit(x) (x & -x)
const int N = 1e5+5;
int bit[N];
struct BIT {
    int n;
    void init(int n){this->n = n;}
    void update(int x, int val) {
        for (; x <= n; x += lowbit(x))
            bit[x] += val;
    }
}
```

```
int query(int x) {
    int res = 0;
    for (; x; x -= lowbit(x))
        res += bit[x];
    return res;
}
int query(int L, int R) { return query(R) - query(L
    - 1); }
```

## 7.2 Sparse Table

```
//st[i][j]表示[i,i+2^j-1]的最值,區間最大長度為 $\log_2(n)$ 
//i為base
const int N = 5e4+5;
int stMax[N][20],stMin[N][20],a[N];
struct ST{
    int k;
    void build(int n,int a[]){
        k=log2(n);
        for(int i = 1; i <= n; i++) stMin[i][0] =
            stMax[i][0] = a[i];
        for(int j = 1; j <= k; j++){
            for(int i = 1; i + (1 << j) - 1 <= n; i++){
                stMax[i][j] = max(stMax[i][j - 1],
                    stMax[i + (1 << (j - 1))][j - 1]);
                stMin[i][j] = min(stMin[i][j - 1],
                    stMin[i + (1 << (j - 1))][j - 1]);
            }
        }
    }
    int queryMax(int l,int r){
        int j = log2(r-l+1);
        return max(stMax[l][j],stMax[r-(1<<j)+1][j]);
    }
    int queryMin(int l,int r){
        int j = log2(r-l+1);
        return min(stMin[l][j],stMin[r-(1<<j)+1][j]);
    }
}st;
```

## 7.3 Segment Tree

```
struct seg {
    #define left (index<<1)
    #define right (index<<1|1)
    static const int MXN = 200005;
    int val[MXN*4], tag[MXN*4];
    int a[MXN];
    void push(int index, int l, int r) {
        if(tag[index]!=0) {
            val[index]+=tag[index]*(r-l+1);
            if(l!=r) {
                tag[left] += tag[index];
                tag[right] += tag[index];
            }
            tag[index]=0;
        }
    }
    void pull(int index, int l, int r) {
        int mid = l+r>>1;
        push(left, l, mid);
        push(right, mid+1, r);
        val[index] = val[left]+val[right];
    }
    void build(int index, int l, int r) {
        if(l==r) {
            val[index] = a[l];
            return;
        }
        int mid = (l+r)>>1;
        build(left, l, mid);
        build(right, mid+1, r);
        pull(index, l, r);
    }
    void add(int index, int s, int e, int l, int r, int
        v) {
        if(e<l || r<s) return;
        if(l<=s && e<=r) {
```



```

        tag[index] += v;
        push(index, s, e);
        return;
    }
    int mid = (s+e)>>1;
    push(index, s, e);
    add(left, s, mid, l, r, v);
    add(right, mid+1, e, l, r, v);
    pull(index, s, e);
}
int query(int index, int s, int e, int l, int r) {
    if(e<l || r<s) return 0;
    if(l<=s && e<=r) {
        push(index, s, e);
        return val[index];
    }
    push(index, s, e);
    int mid = (s+e)>>1;
    return query(right, mid+1, e, l, r)
        +query(left, s, mid, l, r);
}
} tree;

```

```

timing.push_back(build(1, n));
for(int i=0; a < i<n ; i++) {
    cin>>a; num[i] = a; sor.push_back(a);
}
// add: 1 1 1 2 1
// num: 3 3 3 4 3
// sor: 3 4
sort(sor.begin(), sor.end());
sor.erase(unique(sor.begin(), sor.end()), sor.end());
for(int i=0 ; i<n ; i++) {
    int pos = lower_bound(sor.begin(), sor.end(),
        num[i]) - sor.begin() + 1;
    //printf("mp[%d] = %d\n", pos, num[i]);
    mp[pos] = num[i];
    num[i] = pos;
    add(num[i], 1);
}
while(q--){
    int a, b, c; cin>>a>>b>>c;
    cout<<mp[query(a, b, c)]<<endl;
}
}

```

## 7.4 Time Segment Tree

```

#include <bits/stdc++.h>
#define int long long int
using namespace std;
int n, q;
struct node{
    int val;
    node *l, *r;
    node(int v) {val=v; l=r=nullptr;}
    node() {val=0; l=r=nullptr;}
};
vector<node*> timing;
node* build(int s, int e) {
    node *ret = new node();
    if(s==e) return ret;
    int mid = (s+e)>>1;
    ret->l = build(s, mid);
    ret->r = build(mid+1, e);
    ret->val = ret->l->val + ret->r->val;
    return ret;
}
node* update(node* pre, int s, int e, int pos, int v) {
    node *ret = new node();
    if(s==e) {ret->val=pre->val+v; return ret;}
    int mid = (s+e)>>1;
    if(pos<=mid) {
        ret->l = update(pre->l, s, mid, pos, v);
        ret->r = pre->r;
    } else {
        ret->r = update(pre->r, mid+1, e, pos, v);
        ret->l = pre->l;
    }
    ret->val = ret->l->val + ret->r->val;
    return ret;
}
void add(int pos, int v) {
    timing.push_back(update(timing.back(), 1, n, pos, v));
}
int que(node* pre, node* now, int l, int r, int k) {
    if(l==r) return r;
    int mid = (l+r)>>1;
    int diff = now->l->val - pre->l->val;
    //printf("now %d~%d diff %d\n", l, r, diff);
    if(diff>=k) return que(pre->l, now->l, l, mid, k);
    else return que(pre->r, now->r, mid+1, r, k-diff);
    return -1;
}
int query(int l, int r, int k) {
    l--;
    return que(timing[l], timing[r], 1, n, k);
}
int num[100005];
vector<int> sor;
map<int, int> mp;
signed main() {
    cin>>n>>q;

```

## 7.5 Treap

```

struct Treap {
    int sz, val, pri, tag;
    Treap *l, *r;
    Treap(int _val){
        val=_val; sz=1;
        pri=rand(); l=r=NULL; tag=0;
    }
};
int Size(Treap *a) {return a->sz>0;}
void pull(Treap *a) {
    a->sz = Size(a->l) + Size(a->r) + 1;
}
//val of a is always bigger than val of b
Treap* merge(Treap *a, Treap *b) {
    if(!a || !b) return a ? a : b;
    if(a->pri>b->pri) {
        a->r = merge(a->r, b);
        pull(a);
        return a;
    } else {
        b->l = merge(a, b->l);
        pull(b);
        return b;
    }
}
// a<k, b>=k
void split(Treap *t, int k, Treap*&a, Treap*&b){
    if(!t) {a=b=NULL; return;}
    if(k <= t->val) {
        b = t;
        split(t->l, k, a, b->l);
        pull(b);
    }
    else {
        a = t;
        split(t->r, k, a->r, b);
        pull(a);
    }
}
Treap* add(Treap *t, int v) {
    Treap *val = new Treap(v);
    Treap *l = NULL, *r = NULL;
    split(t, v, l, r);
    return merge(merge(l, val), r);
}
Treap* del(Treap *t, int v) {
    Treap *l, *mid, *r, *temp;
    split(t, v, l, temp);
    split(temp, v+1, mid, r);
    return merge(l, r);
}
// base 1
int position(Treap *t, int p) {
    if(Size(t->l)+1==p) return t->val;

```

```

    if(Size(t->l)<p) return position(t->r, p-Size(t->l)-1);
    else return position(t->l, p);
}

//num of >= k
int query(Treap *t, int k) {
    if(!t) return 0;
    if(t->val==k) return Size(t->l)+1;
    if(t->val>k) return query(t->l, k);
    return Size(t->l)+1+query(t->r, k);
}

```

## 7.6 PBDS

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#define ordered_set tree<int, null_type, less<int>,
    rb_tree_tag, tree_order_statistics_node_update>
using namespace __gnu_pbds;
// ordered_set s;
// s.insert(1); s.erase(s.find(1));
// order_of_key(k) : Number of items strictly smaller
// than k .
// find_by_order(k) : K-th element in a set (counting
// from zero). (return iterator)

```

## 8 String

### 8.1 SA

```

#pragma GCC optimize("O3,unroll-loops")
#pragma target optimize("avx2,bmi,bmi2,lzcnt,popcnt")
#include<bits/stdc++.h>
#include<chrono>
#define mid (l + r) / 2
using namespace std;
const int N = 100010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=a; i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmxx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);

```

```

        MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]]] = p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmxx+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxx + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
    }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}
bool check(string &s,string &t,int p){
    for(int i=0;i<t.size() && i+p<s.size();++i){
        if(t[i]<s[i+p])return 1;
        else if(t[i]>s[i+p]) return 0;
    }
    if(t.size()>s.size()-p) return 0;
    return 1;
}
//example for finding patterns in a string
string s,t;
int ip[N],len;
int main(){
    int n;
    cin>>s>>n;
    len = s.length();
    for(int i=0;i<len;++i) ip[i]=(int)s[i];
    ip[len] = 0;
    suffix_array(ip,len);
    int l,r;
    for(int i=0;i<n;++i){
        cin>>t;
        l = 0, r = s.size()-1;
        while(l==r){
            if(check(s,t,SA[mid])) r=mid;
            else l = mid+1;
        }
        bool f=1;
        if(t.size()>s.size()-SA[l]){
            cout<<"NO\n",f=0;
            continue;
        }
        for(int j=0;j<t.size();++j){
            if(t[j]!=s[j+SA[l]]){
                cout<<"NO\n",f=0;
                break;
            }
        }
        if(f) cout<<"YES\n";
    }
}

```

### 8.2 KMP

```

// 回傳所有匹配成功的起始位置，s為文本，t為匹配字串
// nxt表示為匹配失敗時要退回的位置，也是t字串的相等前綴
// 後綴的最大長度
// *注意前綴後綴為長度最多為n-1的子字串
// nxt[j] = -1 if j=0
// 0 if 沒有相等的前綴後綴
// K k 為相等前綴後綴的最大長度
// 以下為例子
// j: 0 1 2 3 4 5 6
// t: a b a a b e
// nxt[j]: -1 0 0 1 1 2 0

```

```
// O(n+m), n為s長, m為t長
const int MXN = 1e6+5;
int nxt[MXN];
vector<int> KMP(string s, string t){
    int slen = s.length(), tlen = t.length(), i=0, j=0, k=-1;
    nxt[0]=-1;
    while(j<tlen){//build nxt
        if(k==-1 || t[j]==t[k]) nxt[++j] = ++k;
        else k=nxt[k];
    }
    i=0, j=0;
    vector<int> ret;
    while(i<slen){//matching
        if(j==-1 || s[i]==t[j]) i++, j++;
        else j=nxt[j];
        if(j==tlen){
            ret.push_back(i-tlen+1); //1-base
            j=nxt[j];
        }
    }
    return ret;
}
```

### 8.3 Single Hash

```
//字串雜湊前的idx是0-base, 雜湊後為1-base
//H[R] - H[L-1] * p^(R-L+1)
//cmp的+modl是為了防止負數
//記得build完之後要buildPow
//小心遇到hash出負數要記得+modl
#define int long long
const int p = 75577, modl = 1e9 + 7, MXN = 1e6+5;
int Hash[MXN], qpow[MXN];
void build(const string& s) {
    Hash[0]=0;
    for(int i=1; i<=s.size(); i++){
        Hash[i] = (Hash[i-1] * p + s[i-1]) % modl;
    }
}
void buildPow(){
    qpow[0]=1;
    for(int i=1; i<MXN; ++i) qpow[i]=qpow[i-1]*p%modl;
}
bool cmp(int i, int j, int len) {
    return (Hash[i+len-1] - Hash[i-1] * qpow[len] % modl + modl) % modl ==
        (Hash[j+len-1] - Hash[j-1] * qpow[len] % modl + modl) % modl;
}
int get(int i, int j) {
    return (Hash[j]-Hash[i-1]*qpow[j-i+1]%modl+modl)%modl;
}
```

### 8.4 Double Hash

```
//字串雜湊前的idx是0-base, 雜湊後為1-base
//即區間為 [0, n-1] -> [1, n]
//若要取得區間[L, R]的值則
//H[R] - H[L-1] * p^(R-L+1)
//cmp為比較從i開始長度為Len的字串和從j開始長度為Len的字串是否相同
//(h[i+len-1] - h[i-1] * qpow(p, Len) % modl + modl)
#define int long long
#define x first
#define y second
const int P1 = 75577, P2 = 17, MOD = 1e9 + 7, MXN = 1e6+5;
pair<int, int> Hash[MXN];
int qpow[2][MXN];
void build(const string& s){
    pair<int, int> val = make_pair(0, 0);
    Hash[0]=val;
    for(int i=1; i<=s.size(); i++){
        val.x = (val.x * P1 + s[i-1]) % MOD;
        val.y = (val.y * P2 + s[i-1]) % MOD;
        Hash[i] = val;
    }
}
```

```
}
void buildPow(){
    qpow[0][0]=qpow[1][0]=1;
    for(int i=1; i<MXN; ++i){
        qpow[0][i]=qpow[0][i-1]*P1%MOD;
        qpow[1][i]=qpow[1][i-1]*P2%MOD;
    }
}
bool cmp(int i, int j, int len) {
    return ((Hash[i+len-1].x-Hash[i-1].x*qpow[0][len]%MOD+MOD)%MOD == (Hash[j+len-1].x-Hash[j-1].x*qpow[0][len]%MOD+MOD)%MOD)
    && ((Hash[i+len-1].y-Hash[i-1].y*qpow[1][len]%MOD+MOD)%MOD == (Hash[j+len-1].y-Hash[j-1].y*qpow[1][len]%MOD+MOD)%MOD);
}
pair<int, int> get(int i, int j) {
    return {(Hash[j].x-Hash[i-1].x*qpow[0][j-i+1]%MOD+MOD)%MOD, (Hash[j].y-Hash[i-1].y*qpow[1][j-i+1]%MOD+MOD)%MOD};
}
```

### 8.5 Trie

```
//cnt為記錄有多少個一樣的单詞且end的時候才有數字
const int MXN=1e6+5; //MXN取文本長
int trie[MXN][26], cnt[MXN], tot=0; //0 base
void update(string s){
    int p=0; //0 base
    for(int i=0; i<s.size(); ++i){
        int ch = s[i] - 'a';
        if(!trie[p][ch]) trie[p][ch]=++tot;
        p = trie[p][ch];
    }
    cnt[p]++;
}
int query(string s){
    int p=0;
    for(int i=0; i<s.size(); ++i){
        int ch=s[i] - 'a';
        p = trie[p][ch];
        if(!p) return 0;
    }
    return cnt[p];
}
void visualizeTrie(int node = 0, int depth = 0) { //for debug
    for (int i = 0; i < 26; ++i) {
        if (trie[node][i]) {
            for (int j = 0; j < depth; ++j) cout << " ";
            cout << (char)('a' + i) << " (" << cnt[trie[node][i]] << ")\n";
            visualizeTrie(trie[node][i], depth + 1);
        }
    }
}
```

### 8.6 Z value

```
// O(n)
//z[i] = Lcp(s[1...], s[i...])
//1base
int z[MXN];
void Z_value(const string& s) {
    int i, j, left, right, len = s.size();
    left=right=0; z[0]=len;
    for(i=1; i<len; i++){
        j=max(min(z[i-left], right-i), 0);
        for(; i+j<len&&s[i+j]==s[j]; j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}
```

## 8.7 MinRotation

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
//For example,rotations of acab are acab, caba, abac,
//and baca.
//find Lexicographically minimal rotation of a string
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    for(int b=0;b<N;b++) for(int k=0;k<N;k++) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}
```

## 8.8 Manacher

```
// O(N)求以每個字元為中心的最長回文半徑
// 頭尾以及每個字元間都加入一個
// 沒出現過的字元，這邊以 '@' 為例
// s為傳入的字串，len為字串長度
// z為儲存以每個字元為中心的回文半徑+1(有包含 '@' 要小心)
// ex: s = "abaac" -> "@a@b@a@a@c@"
// z = [12141232121]
const int MXN = 1e6+5;
int z[2*MXN];
char s[2*MXN];
void z_value_pal(char *s,int len,int *z){
    len=(len<<1)+1;
    for(int i=len-1;i>=0;i--){
        s[i]=i&1?s[i>>1]:'@';
        z[0]=1;
        for(int i=1,l=0,r=0;i<len;i++){
            z[i]=i<r?min(z[l+l-i],r-i):1;
            while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
                ++z[i];
            if(i+z[i]>r) l=i,r=i+z[i];
        }
    }
    // cin>>s;
    // z_value_pal(s,strlen(s),z);
    // int mx=-1,mxi=0;
    // for(int i=0;i<=strlen(s);++i)
    //     if(mx<z[i]) mx = z[i], mxi = i;
    // mx--;
    // for(int i=mxi-mx;i<=mxi+mx;++i)
    //     if(s[i]!='@') cout<<s[i];
}
```

## 8.9 PalTree

```
// Len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26],fail[MXN],len[MXN];
    int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
    int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
    char s[MXN]={-1};
    int newNode(int l,int f){
        len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
        memset(nxt[tot],0,sizeof(nxt[tot]));
        diff[tot]=(l>0?1-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v],dp[fail[v]]);
        return dp[v]+1;
    }
}
```

```
int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(!(lst=nxt[np][c])){
        lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
        nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst;len[v]>0;v=sfail[v])
        fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
}
void init(const char *_s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
}
} palt;
// state 數組
// state[i] 代表第 i 個字元為結尾的最長回文編號(編號
// 是甚麼不重要)
// S = "abacaaba"
// 以第 2(0-base) 個字元為結尾的最長回文是 aba
// 以第 7(0-base) 個字元為結尾的最長回文是 aba
// 兩個最長回文都相同，因此 state[2] 會等於 state[7]
// Len 數組
// 求出某個 state 的長度
// S = "aababa"
// (0-base)
// Len[state[1]] = 2 ("aa")
// Len[state[3]] = 3 ("aba")
// Len[state[5]] = 5 ("ababa")
// num 數組
// 某個state的回文有幾個回文後綴
// 假設某個 state 代表的回文為 "ababa" 為例
// state 代表的回文的 num = 3
// -> ababa -> aba -> a
// cnt 數組
// 某個 state 的回文在整個字串中出現次數
// S = "aababaa"
// state[3] 代表的回文為 "aba" 在整個字串中出現 2
// 次
// 因此 cnt[state[3]] = 2
```

```
// fail數組
// 每個 state 的次長回文後綴的 state 編號
// S = "ababa"
// Len[fail[4]] = 3 (fail[4] = "aba")
// Len[fail[2]] = 1 (fail[2] = "a")
// Len[fail[0]] = 0 (fail[0] = "" 空字串)
// 0 所代表的 state 是空字串
```

## 9 Tree

### 9.1 TreeHash

```
// 1. dfs 先做子樹
// 2. 葉節點的hash值為1
// 3. 對於節點x，其hash值為紀錄x的所有子樹的hash值(紀錄
// 到temp)，然後由小排到大(排除子樹的隨機問題)
// 4. n表示節點x有幾個子樹，p和MOD通常為一個很大的質
// 數，由此算出x的hash值
// 5. 樹根的hash值即為整顆樹的hash值，若兩顆樹的hash值
// 相同，則兩棵樹就是同構
const int MXN = 200005;
```

```

int subtree_sz[MXN];
int hash_[MXN];
int base = 44560482149;
int MOD = 274876858367;
int dfs(int x, int fa, vector<int>* edge){
    vector<int> temp;
    subtree_sz[x] = 1;
    for(int child : edge[x]){
        if(child==fa) continue;
        temp.push_back(dfs(child, x, edge));
        subtree_sz[x] += subtree_sz[child];
    }
    sort(temp.begin(), temp.end());
    int ret = subtree_sz[x];
    for(int v : temp){
        ret = (((ret * base + v + ret) % MOD + ret) %
            MOD + v) % MOD ;
    }
    hash_[x] = ret;
    return ret;
}

```

## 9.2 輕重鏈剖分

```

const int MXN = 2e5+7;
int top[MXN], son[MXN], dfn[MXN], rnk[MXN], dep[MXN],
    father[MXN];
vector<int> edge[MXN];
int dfs1(int v, int fa, int d) {
    int maxsz = -1, maxu, total = 1;
    dep[v] = d;
    father[v] = fa;
    for(int u: edge[v]) {
        if(fa == u) continue;
        int temp = dfs1(u, v, d+1);
        total += temp;
        if(temp>maxsz) {
            maxsz = temp;
            maxu = u;
        }
    }
    if(maxsz==-1) son[v] = -1;
    else son[v] = maxu;
    return total;
}

int times = 1;
void dfs2(int v, int fa) {
    rnk[times] = v;
    dfn[v] = times++;
    top[v] = (fa==-1 || son[fa] != v ? v : top[fa]);
    if(son[v]!=-1) dfs2(son[v], v);
    for(int u: edge[v]) {
        if(fa == u || u == son[v]) continue;
        dfs2(u, v);
    }
}

//rnk: 剖分後的編號 (rnk[時間] = 原點)
//dfn: 剖分後的編號 (dfn[原點] = 時間)
//top: 剖分的頭頭
//son: 剖分的重兒子

```

## 10 Geometry

### 10.1 Definition2D

```

struct Point{
    double x,y;
    Point(double x=0,double y=0):x(x),y(y){}
    bool operator==(const Point &other) const {
        return x == other.x && y == other.y;
    }
};
typedef Point Vector;
Vector operator + (Vector a,Vector b){return Vector(a.x
    +b.x,a.y+b.y);}
Vector operator - (Vector a,Vector b){return Vector(a.x
    -b.x,a.y-b.y);}

```

```

Vector operator * (Vector a,double p){return Vector(a.x
    *p,a.y*p);}
Vector operator / (Vector a,double p){return Vector(a.x
    /p,a.y/p);}
const double eps=1e-10;
int dcmp(double x){if(fabs(x)<eps) return 0;else return
    x<0?-1:1;}
//bool operator == (const Point& a,const Point& b){
    return dcmp(a.x-b.x)==0&&dcmp(a.y-b.y)==0;}//高精度
bool operator < (const Point& a,const Point& b){return
    a.x<b.x||(a.x==b.x&&a.y<b.y);}
bool operator > (const Point& a,const Point& b){return
    a.x>b.x||(a.x==b.x&&a.y>b.y);}
double Dot(Vector a,Vector b){return a.x*b.x+a.y*b.y;}
double Cross(Vector a,Vector b){return a.x*b.y-a.y*b.x
    ;}
double Length(Vector a){return sqrt(Dot(a,a));}

double Angle(Vector a,Vector b){return acos(Dot(a,b)/
    Length(a)/Length(b));};//弧度
double Degree(Vector a,Vector b){return Angle(a,b)*180/
    acos(-1);};//角度
double Area2(Point a,Point b,Point c){return Cross(b-a,
    c-a);};//(a,b)X(a,c)的面積
Vector Rotate(Vector a,double rad){return Vector(a.x*
    cos(rad)-a.y*sin(rad),a.x*sin(rad)+a.y*cos(rad));}
//逆時針旋轉,rad為弧度
Vector Normal(Vector a){double L=Length(a);return
    Vector(-a.y/L,a.x/L);};//單位法向量,確保a不是零向量

```

### 10.2 Basic

```

getLineIntersection
//確保兩直線P+tv和Q+tw有唯一交點且Cross(v,w)非零
Point getLineIntersection(Point P,Vector v,Point Q,
    Vector w){
    Vector u=P-Q;
    double t=Cross(w,u)/Cross(v,w);
    return P+v*t;
}

distanceToLine
//點到直線距離
double distanceToLine(Point p,Point a,Point b){
    Vector v1=b-a,v2=p-a;
    return fabs(Cross(v1,v2)/Length(v1));
}

distanceToSegment
//點到線段距離
double distanceToSegment(Point p,Point a,Point b){
    if(a==b) return Length(p-a);
    Vector v1=b-a,v2=p-a,v3=p-b;
    if(dcmp(Dot(v1,v2))<0) return Length(v2);
    else if(dcmp(Dot(v1,v3))>0) return Length(v3);
    else return fabs(Cross(v1,v2)/Length(v1));
}

GetLineProjection
//點到直線投影
Point GetLineProjection(Point p,Point a,Point b){
    Vector v=b-a;
    return a+v*(Dot(v,p-a)/Dot(v,v));
}

getSymmetryPoint
//點p於直線ab的對稱點
Point getSymmetryPoint(Point p,Point a,Point b){
    Point q=getLineProjection(p,a,b);
    return q*2-p;
}

isSegmentProperIntersection
//判斷線段相交(剛好交一點),若兩線段共線->c1=c2=0
bool isSegmentProperIntersection(Point a1,Point a2,
    Point b1,Point b2){
    double c1=Cross(a2-a1,b1-a1),c2=Cross(a2-a1,b2-a1),
        c3=Cross(b2-b1,a1-b1),c4=Cross(b2-b1,a2-b1);
    return dcmp(c1)*dcmp(c2)<0&&dcmp(c3)*dcmp(c4)<0;
}

```



```

}

isSegmentNotProperIntersection
//判斷線段相交(只要有交點即可)
bool isSegmentNotProperIntersection(Point a1,Point a2,
    Point b1,Point b2){
    return max(a1.x,a2.x)>=min(b1.x,b2.x)&&max(b1.x,b2.
        x)>=min(a1.x,a2.x)&&max(a1.y,a2.y)>=min(b1.y,b2
            .y)&&max(b1.y,b2.y)>=min(a1.y,a2.y)
        &&dcmp(Cross(a1-b1,a2-b1))*dcmp(Cross(a1-b2,a2-b2))
            <=0&&dcmp(Cross(b1-a1,b2-a1))*dcmp(Cross(b1-a2,
                b2-a2))<=0;
}

isOnSegment
//點是否在線段上
bool isOnSegment(Point p,Point a1,Point a2){
    return dcmp(Cross(a1-p,a2-p))==0&&dcmp(Dot(a1-p,a2-
        p))<=0;
}

```

### 10.3 PolygonArea

```

//須注意Long Long 及 加上絕對值
double polygonArea(Point* p,int n){
    double area=0;
    for(int i=1;i<n-1;++i){
        area+=Cross(p[i]-p[0],p[i+1]-p[0]);
    }
    return area/2;
}

```

### 10.4 IsPointInPolygon

```

//判斷點是否在多邊形內部
int isPointInPolygon(Point p,Point* poly,int n){
    int wn=0;
    for(int i=0;i<n;++i){
        if(isOnSegment(p,poly[i],poly[(i+1)%n])) return
            -1;//在邊界上
        int k=dcmp(Cross(poly[(i+1)%n]-poly[i],p-poly[i
            ]));
        int d1=dcmp(poly[i].y-p.y);
        int d2=dcmp(poly[(i+1)%n].y-p.y);
        if(k>0&&d1<=0&&d2>0) wn++;
        if(k<0&&d2<=0&&d1>0) wn--;
    }
    if(wn!=0) return 1;//內部
    return 0;//外部
}

```

### 10.5 ConvexHull

```

//回傳凸包頂點數
//輸入不能有重複點，注意h的點未排序！
//如果有在邊上的輸入點，要把<=改成<
//若要求高精度用dcmp比較
Point ch[N];
int convexHull(Point* p,int n){
    sort(p,p+n);
    int m=0;
    for(int i=0;i<n;++i){//downHull
        while(m>1&&Cross(ch[m-1]-ch[m-2],p[i]-ch[m-2])
            <=0) m--;
        ch[m++]=p[i];
    }
    int k=m;
    for(int i=n-2;i>0;--i){//upHull
        while(m>k&&Cross(ch[m-1]-ch[m-2],p[i]-ch[m-2])
            <=0) m--;
        ch[m++]=p[i];
    }
    if(n>1) m--;
    return m;
}

```

### 10.6 Polar Sort

```

//極角排序，從270度開始逆時針排序
bool cmp(const Point& lhs,const Point&rhs){
    if(Cross((lhs < Point()),(rhs < Point())){
        return (lhs < Point()) < (rhs < Point());
    }
    return Cross(lhs,rhs) > 0;
}
/* 若要以p[i]為原點排序->計算v=p[j]-p[i]
for(int j=0;j<n;++j){
    if(i!=j){
        Vector v = p[j]-p[i];
        node[nodeSz++] = {v,j};
    }
}
sort(node,node+nodeSz,cmp);
*/

```

### 10.7 PickTheorm

```

int area,in,on;//area:多邊形面積 in:內部格點數 on:邊界
格點數
void PickTheorm(Point* p,int n){
    area=polygonArea(p,n);
    for(int i=0;i<n;++i){
        on+=__gcd(abs((int)p[i].x-(int)p[(i+1)%n].x),
            abs((int)p[i].y-(int)p[(i+1)%n].y));
    }
    in=abs(area)+1-on/2;
}

```

### 10.8 最近點對

```

//最近點對距離注意若整數要define double Long Long
double closestEuclideanDistance(Point* p,int n){
    sort(p,p+n);
    set<Point> s={{p[0].y,p[0].x}};
    int j = 0;
    Point t;
    double dd=LLONG_MAX,d;
    for(int i=1;i<n;++i){
        d = sqrt(dd);
        while(j<i && p[j].x < p[i].x-d){
            s.erase({p[j].y,p[j++].x});
        }
        auto l = s.lower_bound({p[i].y-d,p[i].x-d});
        auto u = s.upper_bound({p[i].y+d,p[i].x+d});
        for(auto it=l;it!=u;it++){
            t = {it->y,it->x};
            dd =min(dd, Dot(p[i]-t,p[i]-t));
        }
        s.emplace(p[i].y,p[i].x);
    }
    return dd;
}

```

### 10.9 幾何中位數

```

//回傳為到每個頂點距離和最小的點
Point weiszfeld(const Point *p,int n){
    double nn=n;
    Point cur = p[0];
    for(int i=1;i<n;++i){
        cur.x+=p[i].x, cur.y+=p[i].y;
    }
    cur.x/=nn, cur.y/=nn;
    Point next;
    double w,numerX,numerY,denomin;
    while(1){
        numerX=numerY=denomin=0;
        bool update=0;
        double d;
        for(int i=0;i<n;++i){
            d=Length(cur-p[i]);
            if(d>eps){
                w = 1.0/d;

```



```

        numerX+=w*p[i].x;
        numerY+=w*p[i].y;
        denomin+=w;
        update=1;
    }else{
        next = p[i];
        break;
    }
}
if(update){
    next.x = numerX/denomin;
    next.y = numerY/denomin;
}
if(Length(cur-next)<eps) break;

cur = next;
}
return next;
}
}

```

## 10.10 矩陣掃描線

```

#include <bits/stdc++.h>
#define int long long int
using namespace std;
int n, st[1000005<<2], lazy[1000005<<2], old
    [1000005<<2];
vector<tuple<int, int, int, int>> v;
vector<int> sor;
void pull(int index, int l, int r) {
    if(lazy[index]) st[index] = old[index];
    else if(l==r) st[index] = 0;
    else st[index] = st[index<<1|1]+st[index<<1];
    // printf("pull %lld~%lld, %lld\n", l, r, st[index]);
    return;
}
void insert(int index, int s, int e, int l, int r, int k) {
    //printf("insert: range %lld~%lld, query %lld~%lld\n", s, e, l, r);
    if(l<=s && e<=r) {
        lazy[index] +=k;
        pull(index, s, e);
        return;
    }
    int mid = (s+e)/2;
    if(l<=mid) insert(index<<1, s, mid, l, r, k);
    if(mid<r) insert(index<<1|1, mid+1, e, l, r, k);
    pull(index, s, e);
}
void input(int index, int l, int r) {
    if(l==r) {
        old[index] = sor[l]-sor[l-1];
        return;
    }
    int mid = (l+r)/2;
    input(index<<1, l, mid);
    input(index<<1|1, mid+1, r);
    old[index] = old[index<<1] + old[index<<1|1];
    //cout<<l<<" to "<<r<<" is "<<old[index]<<endl;
    return;
}
// int diff=1000005;
signed main(){
    cin >> n;
    int l, r, d, u;
    for (int i = 0; i < n; i++){
        cin >> l >> d >> r >> u;
        // l+=diff;
        // d+=diff;
        // r+=diff;
        // u+=diff;
        sor.push_back(d);
        sor.push_back(u);
        v.push_back({l, d, u, 1});
        v.push_back({r, d, u, -1});
    }
    set<int> temp(sor.begin(), sor.end());
    sor = vector<int>(temp.begin(), temp.end());
    sort(sor.begin(), sor.end());
}

```

```

for(int i=0 ; i<v.size() ; i++) {
    auto [a, b, c, k] = v[i];
    v[i] = make_tuple(a, (int)(lower_bound(sor.
        begin(), sor.end(), b)-sor.begin()), (int)(
        lower_bound(sor.begin(), sor.end(), c)-sor.
        begin()), k);
}
input(1, 1, sor.size()-1);
// cout<<"get: ";
// for(int i: sor) cout<<i<<" "; cout<<endl;
sort(v.begin(), v.end());
int pre=0;
int ans=0;
for(auto [pos, a, b, k]: v) {
    if(pre!=pos) {
        ans+=(pos-pre)*st[1];
        pre = pos;
    }
    insert(1, 1, sor.size()-1, a+1, b, k);
    // printf("now act: pos %lld, %lld~%lld, act:
    // %lld\n", pos, a+1, b, k);
    // printf("now ans: %lld\n", st[1]);
}
cout<<ans<<endl;
}

```

## 11 特殊題目

### 11.1 包含子字串計數

```

// * 給一個字串s
// * 求長度為len且有包含s的字串有幾種
// * 呼叫solve(s, len)
const int len = 1005;
int aut[len][26];
int dp[len][len];
const int mod = 1e9+7;
void prefix(string &s, vector<int> &pi) {
    for(int i=1, j=0 ; i<s.size() ; i++) {
        while(j>0 && s[i]!=s[j]) j = pi[j-1];
        if(s[i]==s[j]) j++;
        pi[i] = j;
    }
}
void automata(string &s, vector<int> &pi) {
    for(int i=0 ; i<s.size() ; i++) {
        for(int c=0 ; c<26 ; c++) {
            if(i>0 && c+'A' != s[i]) aut[i][c] = aut[pi[i-1]][c];
            else aut[i][c] = i + (c + 'A'==s[i]);
        }
    }
}
int quai(int x, int n) {
    if(n==0) return 1;
    int mid = quai(x, n/2);
    mid = mid*mid%mod;
    if(n&1) return mid*x%mod;
    return mid;
}
int solve(string s, int len) {
    vector<int> pi(s.size(), 0);
    prefix(s, pi);
    automata(s, pi);
    int n = s.size(), ans = quai(26, len);
    dp[0][0] = 1;
    for(int i=0 ; i<len ; i++) {
        for(int j=0 ; j<n ; j++) {
            for(int c=0 ; c<26 ; c++) {
                dp[i+1][aut[j][c]] += dp[i][j];
                dp[i+1][aut[j][c]] %= mod;
            }
        }
    }
    for(int i=0 ; i<n ; i++) ans = (ans - dp[len][i] +
        mod)%mod;
    return ans;
}

```

## 12 Python

### 12.1 Decimal

```
from decimal import Decimal, getcontext, ROUND_FLOOR
getcontext().prec = 250 # set precision (MAX_PREC)
getcontext().Emax = 250 # set exponent limit (MAX_EMAX)
getcontext().rounding = ROUND_FLOOR # set round floor
itwo,two,N = Decimal(0.5),Decimal(2),200
pi = angle(Decimal(-1))
```

### 12.2 Fraction

```
from fractions import Fraction
import math
"""專門用來表示和操作有理數，可以進行算"""
frac1 = Fraction(1) # 1/1
frac2 = Fraction(1, 3) # 1/3
frac3 = Fraction(0.5) # 1/2
frac4 = Fraction('22/7') # 22/7
frac5 = Fraction(8, 16) # 自動約分為 1/2
frac9 = Fraction(22, 7)
frac9.numerator # 22
frac9.denominator # 7
x = Fraction(math.pi)
y2 = x.limit_denominator(100) # 分母限制為 100
print(y2) # 311/99
float(x) #轉換為浮點數
```

### 12.3 Misc

```
# 轉為高精度整數比，(分子，分母)
x=0.2
x.as_integer_ratio() # (8106479329266893,
9007199254740992)
x.is_integer() # 判斷是否為整數
x.__round__() # 四捨五入
int(eval(num.replace("/", "//"))) # parser string num
```

A full-page sheet of white graph paper. The grid consists of thin black horizontal and vertical lines forming small squares. There are 20 columns and 20 rows of these small squares, creating a large rectangular area for drawing or writing. The margins are uniform on all sides.

[illegible]