Peano Theorem of Existence

Theorem

If D is an open subset of $\mathbb{R} \times \mathbb{R}$, $f : D \to \mathbb{R}$ is a continuous function, and $y'(x) = f(x, y)$ is an explicit first order ODE,

then, $\forall (x_0, y_0) \in D$, the initial value problem $y(x_0) = y_0$ must have a local solution.

Notes:

- D is a region on the xy plane. It is a set of initial conditions for which we guarantee a local solution.
- $\mathbb{R} \times \mathbb{R}$ is the Cartesian product of the real number field with itself. Basically, it is the set of all 2-tuples (x, y) such that $x \in R$ and $y \in R$.
- By saying that f is continuous, we refer to the continuity of a multivariable function.
- A local solution is a solution defined on a neighborhood. By saying that the IVP $y(x_0) = y_0$ has a local solution, we guarantee the existence of a positive ε such that at least one solution of the IVP is defined on $(x_0 - \varepsilon, x_0 + \varepsilon)$.