# Moderately large deflection



**Figure 1 Force analysis of a point on buckling cylinder**

As showed in Figure 1, bending moment can be given by

1.1

Former document (understanding\_2017\_05\_26) has shown that



1.2

In the rectangular coordinate system the exact definition of the curvature of a line is given by

1.3

While there is a moderate large deflection in cylinder, curvature still can be approximated as

1.4

Then differential equation 1.1 can be rewritten as

1.5

The solution of differential equation 1.5 can be given by

1.6

Where . Clamped-pin boundary condition is applied to deflection function as follow

1.7

1.8

1.9

1.10

Then, we have

1.11

1.12

1.13

Where is the smallest positive non-zero solution of the transcendental equation.



**Figure 2: Change of length of the beam axis produced by rotation**

As can be shown in Figure 2, the initial and current length element in the undeformed and deformed configuration respectively is denoted by *dx* and *ds.* Then, length of deformed beam axis is given by

1.14

Change of the Cauchy strain measure is adopted to describe length change of beam axis produced by rotation, which is given by

1.15

Length change of buckling beam axis can be divided into two parts: one is compression deformation due to displacement of *a* and the other one is rotation of the compressed beam, which is given by

1.16

Assuming axis force will not change along beam, axis force can be derived from clamped part. The compress strain is given by.

1.17

Integrate differential equation 1.15, we have

1.18

Substituting 1.12 and 1.13 and into 1.17, buckling shape can be derived.

Programming in Matlab, we can have buckling shape as follow



**Figure 3: Buckling shape if end-point displacement equals 5 mm**



**Figure 4: Buckling shape with moderately large deflection**