Rebuttal

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1 Proof for Question 2: Why Our Error Rate Holds

We now explain why the $O(1/M)$ rate applies to our time-inhomogeneous MJD SDEs, structured in three steps:

1.1 Step 1: Results on Time-Homogeneous MJD SDEs

For time-homogeneous MJD SDEs, the error term ϵ_t^E of the standard EM method is $O(1/M)$, with an exponential growth term $O(e^t)$. This is supported by the following: (a) Theorem 2.2 in [PT97] establishes the $O(1/M)$ rate; (b) Sec. 4-5 of [PT97] and Theorem 2.1 of [Bic81] shows that ϵ_t^E grows exponentially regarding time with a big-O factor $O(e^t)$. Particularly, the time-dependent term in the error bound $e^{K_p(t)}$ used in the proof of [PT97] is rooted in their Lemma 4.1, which is proven in a more general setting in [Bic81], e.g., Eq. (2.16) in [Bic81] discusses concrete forms of $K_p(t)$ which can be absored into $O(e^t)$.

1.2 Step 2: Results on Time-Inhomogeneous MJD SDEs

Our paper considers time-inhomogeneous MJD SDEs, with parameters fixed within each interval $[\tau - 1, \tau)$ ($\tau \in \mathbb{N}$, $\tau \geq 1$). This happens to align with the Euler-Peano scheme for general time-inhomogeneous SDEs approximation. As a specific case of time-varying Lévy processes, our MJD SDEs retain the same big-O bounds as the time-homogeneous case. This can be justified by extending Section 5 of [PT97] that originally proves the EM's weak convergence for time-homogeneous Lévy processes. Specifically, the core technique lies in the Lemma 4.1 of [PT97], which, based on [Bic81], is applicable to both time-homogeneous and Euler-Peano-style inhomogeneous settings (see Remark 3.3.3 in [Bic81]). Therefore, equivalent weak convergence bounds could be attained by extending Lemma 4.1 of [PT97] with proofs from [Bic81] thanks to the Euler-Peano formulation.

1.3 Step 3: Restarted EM Solver Error Bound

We now discuss the error bound for the restarted EM solver, ϵ_t^R. Thanks to explicit solutions for future states $\{S_1, S_2, \ldots, S_{T_f}\}$, we can analytically compute their mean $\mathbb{E}[S_\tau]$, $\tau \geq 1$, based on Eq. (13), which greatly simplifies the analysis.

Using the restart mechanism in line 10 of Algorithm 2, we ensure that $\mathbb{E}[\bar{S}\tau]$ from our restarted EM solver closely approximates $\mathbb{E}[S\tau]$ at restarting times. ϵ_t^R is significantly reduced when restart happens, then it grows again at the same rate as the standard EM method until the next restart timestep. This explains the $O(e^{t-\rho_t})$ difference in the error bounds of ϵ_t^R and ϵ_t^E, where ρ_t is the last restart time.

References

[Bic81] Klaus Bichteler. "Stochastic integrators with stationary independent increments". In: *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete* 58.4 (1981), pp. 529–548.

[PT97] Philip Protter and Denis Talay. "The Euler scheme for Lévy driven stochastic differential equations". In: *The Annals of Probability* 25.1 (1997), pp. 393–423.