APPENDIX

A. Algorithm Derivation

In this subsection, the proposed Bayesian controller reduction algorithm as shown in Algorithm 1 will be explain in detail. As introduced before, the actor-critic algorithms based on deterministic policy gradient can be applied over continuous action spaces [9]. And the effective reduction for actor network would be beneficial to real-time inference. For clear explanation, we will take deep deterministic policy gradient algorithm (DDPG), one representative actor-critic algorithm, as the example to elaborate our method. We use $\mathbf{W}$ to represent the parameters of a single layer of actor network θ^{μ}. Firstly, the definition of optimization objective will be illustrated. we will give the proof of proposition 1.

Proof: Given the likelihood with exponential family distribution:

$$p(\mathbf{A} \mid \mathbf{W}, \mathbf{S}, \gamma) \sim \exp\left(-E_M(\mathbf{A}; \mathrm{Net}(\mathbf{S}; \mathbf{W}); \gamma)\right) \tag{A.1}$$

The sparse prior with Gaussian distribution for $\mathbf{W}$ is supposed to be:

$$\begin{aligned} p(\mathbf{W}) &= \prod_{i=1}^{\aleph} \mathcal{N}(\mathbf{W}|0,\Gamma)\varphi(\gamma_i) \\ &= \max_{\gamma \succ \mathbf{0}} \mathcal{N}(\mathbf{W}|0,\Gamma)\varphi(\gamma), \end{aligned} \tag{A.2}$$

where

$$\gamma = [\gamma_1, \ldots, \gamma_{\aleph}] \in \mathbb{R}^{\aleph},\ \Gamma = \mathrm{diag}\,[\gamma]\,.$$

where $\aleph$ means the number of groups that $\mathbf{W}$ could be divided into for reduction; diag denote the operation to get diagonal elements of matrix. The marginal likelihood could be calculated as:

$$\begin{aligned} &\int p(\mathbf{A}|\mathbf{W})\mathcal{N}(\mathbf{W}|\mathbf{0},\Gamma)\prod_{i=1}^{\aleph}\varphi(\gamma_i)d\mathbf{W} \\ =&\int \exp\{-E(\mathbf{W})\}\mathcal{N}(\mathbf{W}|\mathbf{0},\Gamma)\prod_{i=1}^{\aleph}\varphi(\gamma_i)d\mathbf{W} \end{aligned} \tag{A.3}$$

However, it is intractable to achieve analytical solution for Eq A.3. $E_M(\mathbf{W})$ can be expanded around $\mathbf{W}^*$ by performing a Taylor series:

$$E(\mathbf{W}) \approx E(\mathbf{W}^*) + (\mathbf{W}-\mathbf{W}^*)^{\top}\mathbf{g}(\mathbf{W}^*) + \frac{1}{2}(\mathbf{W}-\mathbf{W}^*)^{\top}\mathbf{H}(\mathbf{W}^*)(\mathbf{W}-\mathbf{W}^*) \tag{A.4}$$

where

$$\mathbf{g}(\mathbf{W}^*) \triangleq \nabla E_M(\mathbf{W})|_{\mathbf{W}^*}, \tag{A.5a}$$

$$\mathbf{H}(\mathbf{W}^*) \triangleq \nabla\nabla E_M(\mathbf{W})|_{\mathbf{W}^*}. \tag{A.5b}$$

To derive the cost function in Eq (7), we introduce the posterior mean and covariance:

$$\mathbf{m} = \sigma^2 \cdot [\mathbf{g}(\mathbf{W}^*) + \mathbf{H}(\mathbf{W}^*)\mathbf{W}^*], \tag{A.6a}$$

$$\sigma^2 = \left[\mathbf{H}(\mathbf{W}^*) +^{\top} \Gamma^{-1}\right]^{-1}. \tag{A.6b}$$

Then define the following quantities

$$b(\mathbf{W}^*) \triangleq \exp\left\{-\left(\frac{1}{2}\mathbf{W}^{*\top}\mathbf{H}(\mathbf{W}^*)\mathbf{W}^* - \mathbf{W}^{*\top}\mathbf{g}(\mathbf{W}^*) + E(\mathbf{W}^*)\right)\right\}, \tag{A.7a}$$

$$c(\mathbf{W}^*) \triangleq \exp\left\{\frac{1}{2}\hat{\mathbf{g}}(\mathbf{W}^*)^{\top}\mathbf{H}(\mathbf{W}^*)\hat{\mathbf{g}}(\mathbf{W}^*)\right\}, \tag{A.7b}$$

$$d(\mathbf{W}^*) \triangleq \sqrt{|\mathbf{H}(\mathbf{W}^*)|}, \tag{A.7c}$$

$$\hat{\mathbf{g}}(\mathbf{W}^*) \triangleq \mathbf{g}(\mathbf{W}^*) - \mathbf{H}(\mathbf{W}^*)\mathbf{W}^*. \tag{A.7d}$$

Now the approximated likelihood $p(\mathbf{A}|\mathbf{W})$ is a exponential of quadratic:

$$
\begin{aligned}
& p(\mathbf{A}|\mathbf{W}) \\
=& \exp\{-E(\mathbf{W})\} \\
\approx& \exp\left\{-\left(\frac{1}{2}(\boldsymbol{W}-\boldsymbol{W}^*)^\top \mathbf{H}(\boldsymbol{W}^*)(\boldsymbol{W}-\boldsymbol{W}^*)+(\boldsymbol{W}-\boldsymbol{W}^*)^\top \mathbf{g}(\boldsymbol{W}^*)+E(\mathbf{W}^*)\right)\right\} \\
=& \exp\left\{-\left(\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top\left[\mathbf{g}(\mathbf{W}^*)-\mathbf{H}(\mathbf{W}^*)\mathbf{W}^*\right]\right)\right\} \\
& \cdot \exp\left\{-\left(\frac{1}{2}\mathbf{W}^{*\top}\mathbf{H}(\mathbf{W}^*)\mathbf{W}^*-\mathbf{W}^{*\top}\mathbf{g}(\mathbf{W}^*)+E(\mathbf{W}^*)\right)\right\} \\
=& b(\mathbf{W}^*)\cdot \exp\left\{-\left(\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top \hat{\mathbf{g}}(\mathbf{W}^*)\right)\right\} \\
& \cdot \exp\left\{\frac{1}{2}\hat{\mathbf{g}}(\mathbf{W}^*)^\top \mathbf{H}(\mathbf{W}^*)\hat{\mathbf{g}}(\mathbf{W}^*)-\frac{1}{2}\hat{\mathbf{g}}(\mathbf{W}^*)^\top \mathbf{H}(\mathbf{W}^*)\hat{\mathbf{g}}(\mathbf{W}^*)\right\} \\
=& b(\mathbf{W}^*)\cdot c(\mathbf{W}^*) \\
& \cdot \exp\left\{-\left(\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top \hat{\mathbf{g}}(\mathbf{W}^*)+\frac{1}{2}\hat{\mathbf{g}}(\mathbf{W}^*)^\top \mathbf{H}(\mathbf{W}^*)\hat{\mathbf{g}}(\mathbf{W}^*)\right)\right\} \\
=& (2\pi)^{M/2} b(\mathbf{W}^*)c(\mathbf{W}^*)d(\mathbf{W}^*)\cdot \mathcal{N}(\mathbf{W}|\hat{\mathbf{W}}^*,\mathbf{H}^{-1}(\mathbf{W}^*)) \\
\triangleq& A(\mathbf{W}^*)\cdot \mathcal{N}(\mathbf{W}|\hat{\mathbf{W}}^*,\mathbf{H}^{-1}(\mathbf{W}^*)),
\end{aligned}
\tag{A.8}
$$

where

$$
\begin{aligned}
A(\mathbf{W}^*) &= (2\pi)^{M/2} b(\mathbf{W}^*)c(\mathbf{W}^*)d(\mathbf{W}^*), \\
\hat{\mathbf{W}}^* &= -\mathbf{H}^{-1}(\mathbf{W}^*)\hat{\mathbf{g}}(\mathbf{W}^*) = \mathbf{W}^* - \mathbf{H}^{-1}(\mathbf{W}^*)\mathbf{g}(\mathbf{W}^*).
\end{aligned}
$$

We can write the approximate marginal likelihood as

$$
\begin{aligned}
& A(\mathbf{W}^*)\int \mathcal{N}(\mathbf{W}|\hat{\mathbf{W}}^*,\mathbf{H}^{-1}(\mathbf{W}^*))\cdot \mathcal{N}(\mathbf{W}|\mathbf{0},\Gamma)\prod_{i=1}^{\aleph}\varphi(\gamma_i)d\mathbf{W} \\
=& b(\mathbf{W}^*)\cdot \int \exp\left\{-\left(\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top \hat{\mathbf{g}}(\mathbf{W}^*)\right)\right\}\mathcal{N}(\mathbf{W}|\mathbf{0},\Gamma)\prod_{i=1}^{\aleph}\varphi(\gamma_i)d\mathbf{W} \\
=& \frac{b(\mathbf{W}^*)}{(2\pi)^{\aleph/2}|\Gamma|^{1/2}}\int \exp\{-\hat{E}(\mathbf{W})\}d\mathbf{W}\prod_{i=1}^{\aleph}\varphi(\gamma_i)
\end{aligned}
\tag{A.9}
$$

where

$$
\hat{E}(\mathbf{W}) = \frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top \hat{\mathbf{g}}(\mathbf{W}^*)+\frac{1}{2}\mathbf{W}^\top \Gamma^{-1}\mathbf{W}. \tag{A.10}
$$

Equivalently, we get

$$
\hat{E}(\mathbf{W}) = \frac{1}{2}(\mathbf{W}-\mathbf{m})^\top (\sigma^2)^{-1}(\mathbf{W}-\mathbf{m})+\hat{E}(\mathbf{A}), \tag{A.11}
$$

From (A.6a) and (A.6b), the data-dependent term can be re-expressed as

$$
\begin{aligned}
\hat{E}(\mathbf{A}) =& \frac{1}{2}\mathbf{m}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{m}+\mathbf{m}^\top\ \mathbf{g}(\mathbf{W}^*)+\frac{1}{2}\mathbf{m}^\top \Gamma^{-1}\mathbf{m} \\
=& \min_{\mathbf{W}}\left[\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top \hat{\mathbf{g}}(\mathbf{W}^*)+\frac{1}{2}\mathbf{W}^\top \Gamma^{-1}\mathbf{W}\right] \\
=& \min_{\mathbf{W}}\left[\frac{1}{2}\mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W}+\mathbf{W}^\top\left(\mathbf{g}(\mathbf{W}^*)-\mathbf{H}(\mathbf{W}^*)\mathbf{W}^*\right)+\frac{1}{2}\mathbf{W}^\top \Gamma^{-1}\mathbf{W}\right].
\end{aligned}
\tag{A.12}
$$

Using (A.11), we can evaluate the integral in (A.9) to obtain

$$
\int \exp\left\{-\hat{E}(\mathbf{W})\right\}d\mathbf{W} = \exp\left\{-\hat{E}(\mathbf{A})\right\}(2\pi)^{\aleph}|\sigma^2|^{1/2}. \tag{A.13}
$$

Applying a $-2\log(\cdot)$ transformation to (A.9), we have

$$
\begin{aligned}
&-2\log\left[\frac{b(\mathbf{W}^*)}{(2\pi)^{\aleph/2}|\Gamma|^{1/2}}\int \exp\{-\hat{E}(\mathbf{W})\}d\mathbf{W}\prod_{i=1}^{\aleph}\varphi(\gamma_i)\right]\\
\propto& -2\log\cdot b(\mathbf{W}^*)+\hat{E}(\mathbf{A})+\log|\Gamma|+\log|\mathbf{H}(\mathbf{W}^*)+^{\top}\Gamma^{-1}|-2\sum_{i=1}^{\aleph}\log\varphi(\gamma_i)\\
\propto& \mathbf{W}^{\top}\mathbf{H}(\mathbf{W}^*)\mathbf{W}+2\mathbf{W}^{\top}\hat{\mathbf{g}}(\mathbf{W}^*)+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}\\
&+\log|\Gamma|+\log|\mathbf{H}(\mathbf{W}^*)+^{\top}\Gamma^{-1}|-2\log\cdot b(\mathbf{W}^*)-2\sum_{i=1}^{\aleph}\log\varphi(\gamma_i).
\end{aligned}
\tag{A.14}
$$

Therefore we get the following cost function to be minimized over $\mathbf{W},\gamma$

$$
\begin{aligned}
\mathscr{L}(\mathbf{W},\gamma)=&\mathbf{W}^{\top}\mathbf{H}(\mathbf{W}^*)\mathbf{W}+2\mathbf{W}^{\top}[\mathbf{g}(\mathbf{W}^*)-\mathbf{H}(\mathbf{W}^*)\mathbf{W}^*]+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}\\
&+\log|\Gamma|+\log|\mathbf{H}(\mathbf{W}^*)+^{\top}\Gamma^{-1}|-2\log b(\mathbf{W}^*)-2\sum_{i=1}^{\aleph}\log\varphi(\gamma_i).
\end{aligned}
$$

It can be easily found that the first line of $\mathscr{L}$ is quadratic programming with ℓ_2 regularize. The second line is all about the hyperparameter γ. Once the estimation on $\mathbf{W}$ and γ are obtained, the cost function is alternatively optimized. The new estimated $\mathbf{W}$ can substitute $\mathbf{W}^*$ and repeat the estimation iteratively.

■

We note that in (A.5), $\mathbf{W}^*$ may not be the mode (i.e., the lowest energy state), which means the gradient term $\mathbf{g}$ may not be zero. Therefore, the selection of $\mathbf{W}_1^*$ remains to be problematic. We give the following Corollary to address this issue, which is more general.

Corollary 1: Suppose

$$
\mathbf{W}^*=\arg\min_{\mathbf{W}} E(\mathbf{W})+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W},
\tag{A.15}
$$

we define a new cost function

$$
\hat{\mathscr{L}}(\mathbf{W},\Gamma)\triangleq E(\mathbf{W})+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}+\log|\Gamma|+\log|\mathbf{H}(\mathbf{W}^*)+^{\top}\Gamma^{-1}|-2\log b(\mathbf{W}^*)-2\sum_{i=1}^{\aleph}\log\varphi(\gamma_i).
\tag{A.16}
$$

Instead of minimizing $\mathscr{L}(\mathbf{W},\gamma)$, we can solve the following optimization problem to get $\mathbf{W},\gamma$,

$$
\min_{\mathbf{W},\gamma,}\hat{\mathscr{L}}(\mathbf{W},\gamma).
$$

Proof: Since the likelihood is

$$
p(\mathbf{A}|\mathbf{W})=\exp\{-E_M(\mathbf{W})\},
$$

then $\min_{\mathbf{W}} E(\mathbf{W})+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}$ is exactly the regularized *maximum likelihood estimation* with ℓ_2 type regularize.

We look at the first part of $\mathscr{L}(\mathbf{W},\gamma)$ in Eq (7), and define them as

$$
\mathscr{L}_0(\mathbf{W},\gamma)\triangleq\mathbf{W}^{\top}\mathbf{H}(\mathbf{W}^*)\mathbf{W}+2\mathbf{W}^{\top}[\mathbf{g}(\mathbf{W}^*)-\mathbf{H}(\mathbf{W}^*)\mathbf{W}^*]+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W},
$$

then

$$
\begin{aligned}
&\min_{\mathbf{W}}\mathscr{L}_0(\mathbf{W},\gamma)\\
=&\min_{\mathbf{W}}\frac{1}{2}(\boldsymbol{W}-\boldsymbol{W}^*)^{\top}\mathbf{H}(\boldsymbol{W}^*)(\boldsymbol{W}-\boldsymbol{W}^*)+(\boldsymbol{W}-\boldsymbol{W}^*)^{\top}\mathbf{g}(\boldsymbol{W}^*)+E_M(\mathbf{W}^*)+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}\\
\approx&\min_{\mathbf{W}}E_M(\mathbf{W})+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}
\end{aligned}
\tag{A.17}
$$

This provides the solution to find the optimal objective at iteration t

$$
\begin{aligned}
&\min_{\mathbf{W}}E_M(\mathbf{W}^t)+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}\\
=&\min_{\mathbf{W}}\frac{1}{2}(\mathbf{W}-\mathbf{W}^t)^{\top}\mathbf{H}(\mathbf{W}^t)(\mathbf{W}-\mathbf{W}^t)+(\mathbf{W}-\mathbf{W}^t)^{\top}\mathbf{g}(\mathbf{W}^t)+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W}
\end{aligned}
\tag{A.18}
$$

Suppose

$$
\mathbf{W}^*=\arg\min_{\mathbf{W}} E(\mathbf{W})+\mathbf{W}^{\top}\Gamma^{-1}\mathbf{W},
$$

then inject $\mathbf{W}^*$ into $\min_{\mathbf{W},\gamma,}\mathscr{L}(\mathbf{W},\gamma)$, the original optimization problem Eq (7),i.e could be substituted with (A.16)

■

B. *Updating parameter* $\mathbf{W}$ *and hyper-parameters* γ

In this Section, we propose iterative optimization algorithms to estimate $\mathbf{W}$ and γ alternatively. The $\mathbf{H}(\mathbf{W}^*)$ is a known positive semidefinite symmetric matrix. The proof for proposition 2 will be elaborated firstly.

Proof: Fact on convexity: the function

$$\begin{aligned}u(\mathbf{W},\Gamma) =& \mathbf{W}^\top \mathbf{H}^*(\mathbf{W})\mathbf{W} + 2\mathbf{W}^\top [\mathbf{g}(\mathbf{W}^*) - \mathbf{H}(\mathbf{W}^*)\mathbf{W}^*] + \mathbf{W}^\top \Gamma^{-1}\mathbf{W} \\ \propto& (\mathbf{W}-\mathbf{W}^*)^\top \mathbf{H}(\mathbf{W}^*)(\mathbf{W}-\mathbf{W}^*) + 2\mathbf{W}^\top \mathbf{g}(\mathbf{W}^*) + \mathbf{W}^\top \Gamma^{-1}\mathbf{W}\end{aligned} \tag{B.1}$$

is convex jointly in $\mathbf{W}$, Γ due to the fact that $f(\mathbf{S},Y) = \mathbf{S}\mathbf{A}^{-1}\mathbf{S}$ is jointly convex in $\mathbf{x}$, $\mathbf{A}$ (see, [?, p.76]).

Fact on concavity: the function

$$v(\Gamma) = \log|\Gamma| + \log|\Gamma^{-1} + \mathbf{H}(\mathbf{W}^*)| \tag{B.2}$$

is jointly concave in Γ, $\mathbf{\Pi}$. We exploit the properties of the determinant of a matrix

$$|A_{22}||A_{11} - A_{12}A_{22}^{-1}A_{21}| = \left|\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{pmatrix}\right| = |A_{11}||A_{22} - A_{21}A_{11}^{-1}A_{12}|.$$

Then we have

$$\begin{aligned}v(\Gamma) &= \log|\Gamma| + \log|\Gamma^{-1} + \mathbf{H}(\mathbf{W}^*)| \\ &= \log\left(|\Gamma||\Gamma^{-1} + \mathbf{H}(\mathbf{W}^*)|\right) \\ &= \log\left|\begin{pmatrix}\mathbf{H}(\mathbf{W}^*) & \\ & -\Gamma\end{pmatrix}\right| \\ &= \log\left|\Gamma + \mathbf{H}^{-1}(\mathbf{W}^*)^\top\right| + \log|\mathbf{H}(\mathbf{W}^*)|\end{aligned} \tag{B.3}$$

which is a log-determinant of an affine function of semidefinite matrices $\mathbf{\Pi}$, Γ and hence concave.

Therefore, we can derive the iterative algorithm solving the CCCP. We have the following iterative convex optimization program by calculating the gradient of concave part.

$$\mathbf{W}^t = \arg\min_{\mathbf{W}} u(\mathbf{W},\Gamma^{t-1},\mathbf{H}(\mathbf{W}^*)), \tag{B.4}$$

$$\gamma^t = \arg\min_{\gamma\succeq\mathbf{0}} u(\mathbf{W},\Gamma^{t-1},\mathbf{H}(\mathbf{W}^*)) + \nabla_\gamma v(\gamma^{t-1},\mathbf{H}(\mathbf{W}^*))^\top \gamma^{t-1}. \tag{B.5}$$

■

Using basic principles in convex analysis, we then obtain the following analytic form for the negative gradient of $v(\gamma)$ at γ is (using chain rule):

$$\begin{aligned}\boldsymbol{\alpha}^t \triangleq& \nabla_\gamma v(\gamma,\mathbf{H}(\mathbf{W}^*))^\top|_{\gamma=\gamma^t} \\ =& \nabla_\gamma\left(\log|\Gamma^{-1} + \mathbf{H}(\mathbf{W}^*)| + \log|\Gamma|\right)^\top|_{\gamma=\gamma^t} \\ =& -\mathrm{diag}\left\{(\Gamma^t)^{-1}\right\}\circ \mathrm{diag}\left\{\left((\Gamma^t)^{-1} + \mathbf{H}(\mathbf{W}^*)\right)^{-1}\right\}\circ \mathrm{diag}\left\{(\Gamma^t)^{-1}\right\} \\ &+ \mathrm{diag}\left\{(\Gamma^k)^{-1}\right\} \\ =& \begin{bmatrix}\alpha_1^t & \cdots & \alpha_\aleph^t\end{bmatrix}\end{aligned} \tag{B.6}$$

Then we have:

$$\alpha_i^t = -\frac{((\Gamma^t)^{-1} + \mathbf{H}(\mathbf{W}^*))^{-1}}{(\gamma_i^t)^2} + \frac{1}{\gamma_i^t}. \tag{B.7}$$

Therefore, the iterative procedures (B.4) and (B.5) for $\mathbf{W}^t$ and γ^t can be formulated as

$$\begin{aligned}&\left[\mathbf{W}^{t+1},\gamma^{t+1}\right] \\ =& \arg\min_{\Gamma\succeq\mathbf{0},\mathbf{W}} (\mathbf{W}-\mathbf{W}^*)^\top \mathbf{H}(\mathbf{W}^*)(\mathbf{W}-\mathbf{W}^*) + 2\mathbf{W}^\top \mathbf{g}(\mathbf{W}^*) + \sum_{i=1}^{\aleph}\left(\frac{\mathbf{W}^\top\mathbf{W}}{\gamma_i} + \alpha_i^t\gamma_i\right) \\ =& \arg\min_{\mathbf{W}} \mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W} + 2\mathbf{W}^\top\left(\mathbf{g}(\mathbf{W}^*) - \mathbf{H}(\mathbf{W}^*)\mathbf{W}^*\right) + \sum_{i=1}^{\aleph}\left(\frac{\mathbf{W}^\top\mathbf{W}}{\gamma_i} + \alpha_i^t\gamma_i\right).\end{aligned} \tag{B.8}$$

Or in the compact form

$$\left[\mathbf{W}^{t+1},\gamma^{t+1}\right] = \arg\min_{\mathbf{W}} \mathbf{W}^\top \mathbf{H}(\mathbf{W}^*)\mathbf{W} + 2\mathbf{W}^\top\left(\mathbf{g}(\mathbf{W}^*) - \mathbf{H}(\mathbf{W}^*)\mathbf{W}^*\right) + \mathbf{W}^\top\Gamma^{-1} + \sum_{i=1}^{\aleph}\alpha_i^t\gamma_i. \tag{B.9}$$

Since

$$\frac{\mathbf{W}^\top \mathbf{W}}{\gamma_i} + \alpha_i^t \gamma_i \geq 2 \left| \sqrt{\alpha_i^t} \cdot \mathbf{W} \right|,$$

the optimal γ can be obtained as:

$$\gamma_i = \frac{|\mathbf{W}|}{\sqrt{\alpha_i^t}}, \forall i. \tag{B.10}$$

If we define:

$$\omega_i^t \triangleq \sqrt{\alpha_i^t} = \sqrt{-\frac{((\Gamma^t)^{-1} + \mathbf{H}(\mathbf{W}^*)^t)^{-1}}{(\gamma_i^k)^2} + \frac{1}{\gamma_i^t}}. \tag{B.11}$$

$\mathbf{W}^t$ can be obtained as follows

$$\mathbf{W}^{t+1} = \arg\min_{\mathbf{W}} \frac{1}{2} \mathbf{W}^\top \mathbf{H}(\mathbf{W}^*) \mathbf{W} + \mathbf{W}^\top \left(\mathbf{g}(\mathbf{W}^*) - \mathbf{H}(\mathbf{W}^*) \mathbf{W}^* \right) + \sum_{i=1}^{\aleph} \| \omega_i^t \cdot \mathbf{W} \|_{\ell_1}. \tag{B.12}$$

It should be noted here that $\|\omega_i^t \cdot \mathbf{W}\|_{\ell_1}$ represents the regularization term in Eq A.16. We can then inject this into (B.10), which yields

$$\gamma_i^{t+1} = \frac{|\mathbf{W}^{t+1}|}{\omega_i^t}, \forall i. \tag{B.13}$$

We notice that the update for $\mathbf{W}^t$ is to use the *lasso* or ℓ_1-*regularised* regression type optimization. Referring to DDPG[9], the pseudo code combined with DDPG [9] is summarized in Algorithm 2.

Algorithm 2 Bayesian Controller Reduction Algorithm in DDPG [9]

Randomly initialize critic network $Q(s,a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$, θ^μ is denoted as $\mathbf{W}$
Initialize ω and λ, $\forall l = 1, \dots, L$, $(\omega^\ell)^0 = I$; $\lambda^\ell \in \mathbb{R}^+$;
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s,a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i} + \lambda^\ell \nabla_{\theta^\mu} R(\omega^\ell \circ \mathbf{W}^\ell)$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau \theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau \theta^\mu + (1-\tau)\theta^{\mu'} \lambda^\ell$$

    Update the Hessian recursively for each layer of the actor network.
    Update hyper-parameters:
    $(\gamma^\ell)^t \leftarrow \text{Update}((\omega^\ell)^t, (\boldsymbol{W}^\ell)^t), (\Gamma^\ell)^t = [(\gamma^\ell)^t]$ {update rules are in Table I}
    $(C^\ell)^t \leftarrow (((\Gamma^\ell)^t)^{-1} + \mathrm{H}^\ell)^t)^{-1}$
    $(\alpha^\ell)^t$ is given by Eq B.7
    $(\omega^\ell)^{t+1} \leftarrow \text{Update}((\alpha^\ell)^t)$
  end for
end for

C. Hessian computation

The mathematical operation in a Fully Connected (FC) layer could be formulated as:

$$h^\ell = a^{\ell-1}\mathbf{W}^\ell, \quad a^\ell = \sigma(h^\ell) \tag{C.1}$$

where h_ℓ is the pre-activation value for layer ℓ and a_ℓ is the activation value. $\sigma()$ is the element-wise activation function. In [25], a recursive method is proposed to compute the Hessian $\mathbf{H}$ in a FC layer:

$$\mathbf{H}^\ell = (a^{\ell-1})^\top \cdot a^{\ell-1} \otimes H^\ell \tag{C.2}$$

where $\otimes$ stands for Kronecker product; H^ℓ denotes the pre-activation Hessian and could be computed with known $H^{\ell+1}$:

$$H^\ell = B^\ell \mathbf{W}^{\ell+1} H^{\ell+1} (\mathbf{W}^{\ell+1})^T B^\ell + D^\ell \tag{C.3}$$

$$B^\ell = \mathrm{diag}(\sigma'(h^\ell)), \quad D^\ell = \mathrm{diag}(\sigma''(h^\ell)\frac{\partial L}{\partial a^\ell}) \tag{C.4}$$

diag() means the operation to expand the input vector into a square matrix by assigning the principal diagonal values with input variable. L stands for the loss function. It can be found that the pre-activation Hessian H^ℓ needs to be computed recursively for each layer before computing the Hessian H.

In order to reduce computation complexity, the original matrix multiplication in Eq C.2-C.3 could be reduced to the vector multiplication without accuracy deterioration:

$$\mathbf{H}^\ell = (a^{\ell-1})^2 \otimes H^\ell \tag{C.5}$$

$$H^\ell = (B^\ell)^2 \circ (H^{\ell+1}(\mathbf{W}^{\ell+1})^2) + D^\ell \tag{C.6}$$

$$B^\ell = \sigma'(h^\ell), \quad D^\ell = \sigma''(h^\ell) \circ \frac{\partial L}{\partial a^\ell} \tag{C.7}$$