Empirical Homework 1

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Excercise 1

Time is infinite. An infinitely lived household has capital a_0 at time 0. Household income at each time period $\{y_t\}$ follows AR(1) process $y_{t+1} = (1-\rho)e + \rho y_t + u_t$ where u_t is standard normally distributed. For each time period, the household makes decisions on consumption and saving subject to his budget constraint. Gross return on saving is R. There is no borrowing constraint and the household has utility form $\sum_{t=0}^{\infty} \beta^t U(c_t)$ where $\beta \in (0,1)$ and c_t is consumption at time t. Assume that $U(c) = ln(c)$, $a_0 = 10$, $\beta = 0.9$, $R = 1.06$, $\rho = 0.8$, $e = 6$ and $y_0 = 6$.

(1) Formulate the sequence problem and write down its recursive formulation. What are state variables and control variables?

(2) Solve the household's problem using value function iteration. Plot the household value depending on the initial capital a_0 for $\beta \in \{0.8, 0.9, 0.95\}$ when holding other parameters fixed as assumed.

(3) Plot the value function depending on state variables in a 3D fashion.

(4) Plot the policy function for saving in a 3D fashion.

(5) Plot expected household capital over time for next 20 periods.

Excercise 2

Consider a stochastic growth model with labor supply. Time is infinite. An infinitely lived representative household has capital a_0 at time 0. The production function at time t is $y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$ where A_t is total production factor, k_t is capital and l_t is labor. Maximum labor supply per period is normalized to 1. lnA_t is stochastic and follows AR(1) process $lnA_{t+1} = \rho lnA_t + u_t$ where u_t is standard normally distributed. Depreciation rate over capital is δ. The household's lifetime utility is $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$.
Assume that $U(c, l) = c^{0.5}(1 - l)^{0.5}$, $a_0 = 3$, $\delta = 0.95$, $\beta = 0.9$, $\alpha = 0.3$, $A_0 = 1$, and $\rho = 0.9$.

(1) Formulate the sequence problem and write down its recursive formulation. What are state variables and control variables?

(2) Solve the household's problem using endogenous grid points method. Report the grid set and the convergence criterion.

(3) Plot the policy function for labor and saving in a 3D fashion.

(4) Plot the value function depending on state variables in a 3D fashion.

(5) Plot expected household capital over time for next 20 periods.

(6) Suppose that there are many identical households whose productivity shocks are independent of each other. Does a stable distribution of their capitals exists? How can one find the stable distribution of their capitals if it exists? Plot the distribution if the stable distribution exists.

Excercise 3

Consider a social planner's version of neoclassical growth model. Time is discrete and infinite. A representative household (social planner) has preference over consumption given by $\sum_{t=1}^{\infty} \beta^t \log C_t$ where $\beta \in (0, 1)$ is the discount factor. The household has initial wealth K_0 at time 0. The production technology for each period is $Y_t = A_t^{1-\alpha} K_t^{\alpha} N_t^{1-\alpha}$. Depreciation rate for capital is δ. Maximum labor supply per period is $\bar{N}$. The social planner chooses optimally each period the consumption and saving to maximize its lifetime utility subject to its budget constraint. The labor productivity $A_t = (1 + g)^t A_0$.

(1) Set up the problem and characterize the Euler equation and resource restriction. What is the balanced growth path for consumption and capital?

(2) Assume that $g = 0.2, \alpha = 0.3, \beta = 0.9, \delta = 0.5, K_0 = 1, \bar{N} = 1, A_0 = 1$. Graph a phase diagram for consumption and capital and indicate the likely form of a saddle path for $K_0 = 1$. Use forward shooting algorithm to find the consumption C_0 on the saddle path.