Problem 2

(1)

Denote $z = \ln A$. Then z is stochastic and follows AR(1) process

$$z_{t+1} = \rho z_t + \varepsilon_t$$

where $\varepsilon_t \sim N(0, 0.06)$.

Sequence Problem

$$\max_{\{k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sqrt{c_t}$$

$$\text{subject to } c_t + k_{t+1} \leq e^{z_t} k_t^{\alpha} + (1-\delta)k_t,$$

$$\text{and } k_t \geq 0, \forall t, \text{ with } k_0 \text{ given.}$$

Recursive Formulation

$$V(k,z) = \max_{k' \in \Gamma(k,z)} \sqrt{e^z k^{\alpha} + (1-\delta)k - k'} + \beta \int V(k', z') Q(z, dz'),$$

$$\text{where } \Gamma(k,z) = [0, e^z k^{\alpha} + (1-\delta)k]$$

Hence, state variables are k and z, and control variable is k'.

(2)

The steady state capital is easy to be found as

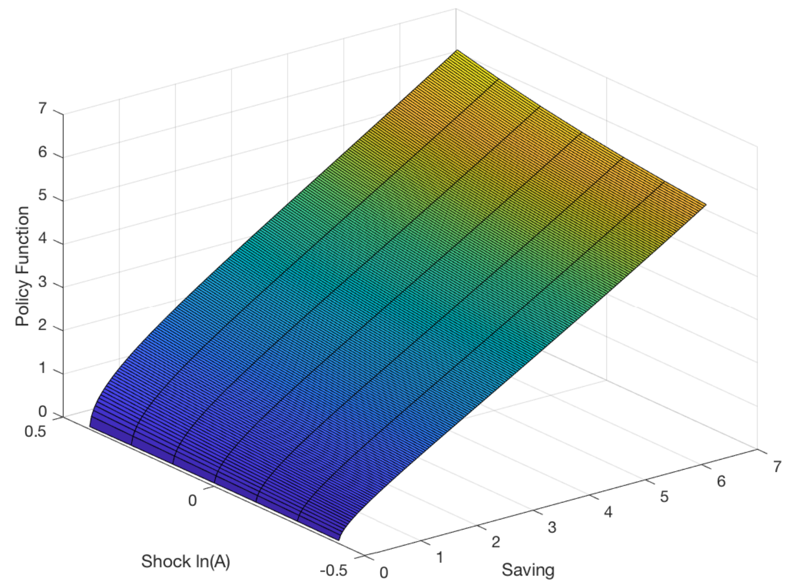
$$k^* \approx 3.2767.$$

The grid set used for k' is thus set as

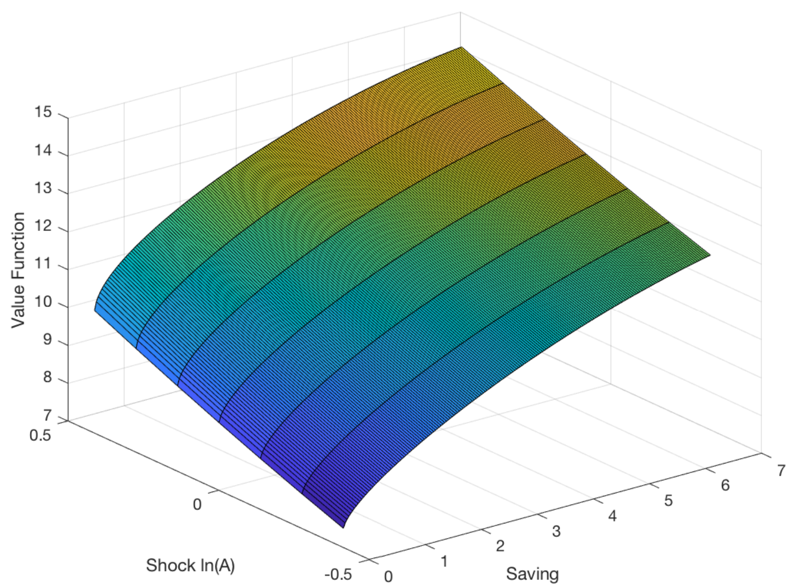
$$[0, 2k^*] = [0, 6.5536],$$

with 200 grid points. The convergence criterion is when the two policy function has a maximum distance smaller than 10^{-6}.

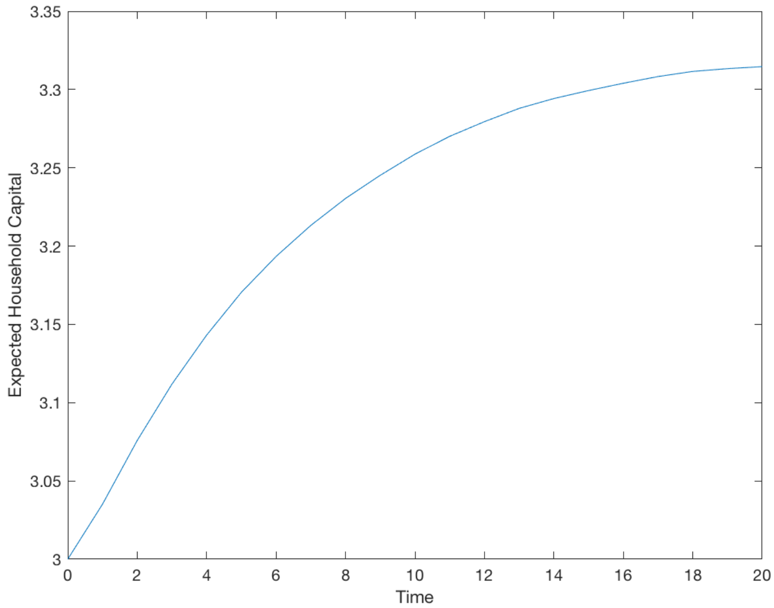
(3)


Policy Function
Shock ln(A)
Saving
0
1
2
3
4
5
6
7
0.5
0
-0.5

(4)


Value Function
Shock ln(A)
Saving
7
8
9
10
11
12
13
14
15
0.5
0
-0.5
0
1
2
3
4
5
6
7

(5)



(6)

Yes. The stable distribution of household capitals can be found by simulating for many times the household capital for a very long period. Here, 10000 households are simulated for 400 periods.