Economics 512 – Homework 6

Due January 21, 2019

This assignment extends Problem 7.4 in in Miranda & Fackler. Consider competitive price-taking firm that maximizes discounted sum of expected future profits from harvesting a non-renewable resource. For example, suppose a lumber company is deciding how many trees to harvest in a forest where trees do not grow back. The discount factor of our lumber company is $\delta = 0.95$. The firm earns revenue of $p \cdot x$ per period if it has harvested amount x and market price was p. To harvest x the firm incurs a convex cost $0.2 \cdot x^{1.5}$. The firm is small relative to the market, and has rational expectations that the price of lumber will follow an AR(1) process:

$$p_t = p_0 + \rho \cdot p_{t-1} + u \tag{1}$$

Where $p_0 = 0.5$, $\rho = 0.5$, and u is a mean-zero normal disturbance with standard deviation $\sigma_u = 0.1$. The initial stock of lumber may be anything from 0 to 100.

1. Formulate firm's dynamic optimization problem. Specifically, formulate the Bellman equation, identify state and policy variables, their spaces and transition probabilities. Assume initial stock is between 0 and 100.

   Let the initial stock be s_0. Then the Bellman equation is:

   $$V(s,p) = \max_{s' \in [0,s]} \{p(s-s') - 0.2 * (s-s')^{1.5} + \delta \mathbb{E}[V(s',p')|p]\}$$

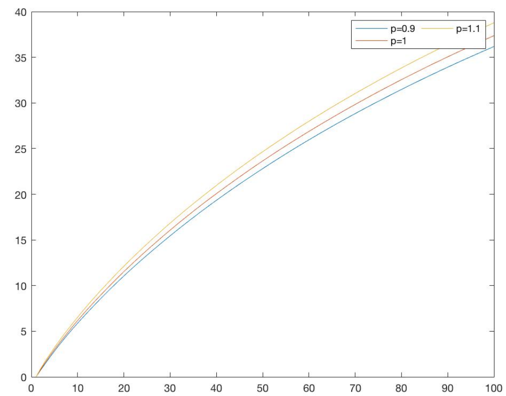
   The state variables are (s,p), the current stock and current market price. Policy variable is s', the stock to hold for next period. The state space is $[0, s_0] \times [0, \infty]$. The policy space is $[0, s]$. The market price follows an AR(1) process, so the transition probability to any point state is zero. However, we can approximate the AR(1) process with a Markov process in question 2.

2. Take a look at `tauchen.m` in the repository (you should know where), use it to generate grid that approximates process for p_t with 21 grid points.

It's done in the programming.

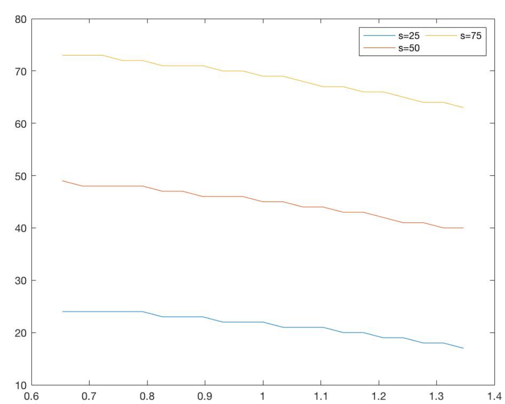
3. Solve the firm's problem using value function iteration. Plot the value of the firm depending on its initial stock (x-axis) and the current price of lumber, for $p \in 0.9, 1, 1.1$.

I solved the dynamic programming problem using the value function iteration. The relationship between firm value and initial stock when holding current price of lumber fixed is plotted as followed:



4. Plot next period optimal stock (or harvest amount if you prefer) as a function of today's price for different amount of lumber left in stock.

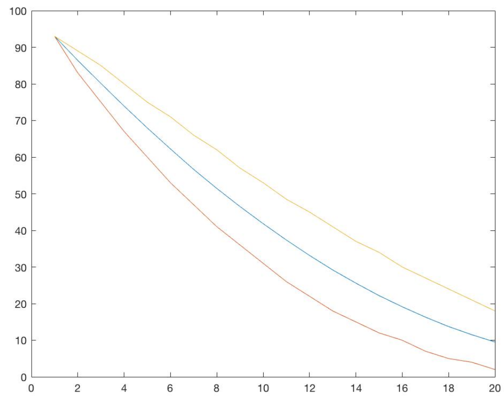
The relationship between next period optimal stock and the price when holding amount of lumber left in stock as 25, 50 or 75 are plotted as:



5. Assume firm starts with stock of 100 and today's price is 1. Plot expected stock over

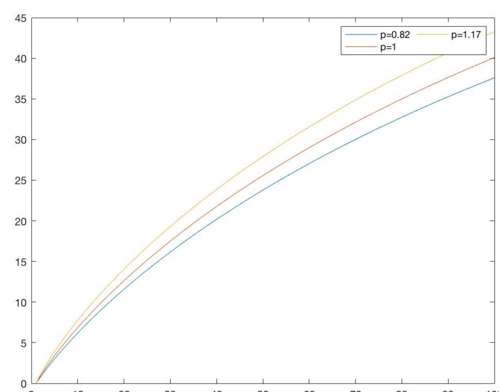
time for 20 periods ahead. Include the 90 percent confidence interval.

The expected stock decreases over time from 93 to about 10. The evolution process is illustrated as:

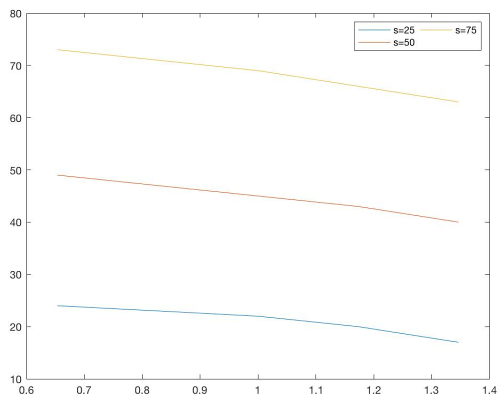


6. Redo the 2-4 for coarse grid of 5 points in Tauchen's representation.

I solved the dynamic programming problem using the value function iteration. The relationship between firm value and initial stock when holding current price of lumber fixed is plotted as followed:



The relationship between next period optimal stock and the price when holding amount of lumber left in stock as 25, 50 or 75 are plotted as:



7. Submit your code together with a pdf of your responses in LaTeX. (Yes, part of this assignment is to get you to embed figures into LaTeX.)