

Today

o Course pack p. 13, 15

111: https://berkeley.zoom.us/rec/share/pqu7ij90xllaSlpPB7fiLXhsVGrpb7A8tTY3omSAOqXZCwll1oTFTQ_TvbBc5H5.aJXOfPKozG3C9Ahv Passcode: 839E%7dh112: https://berkeley.zoom.us/rec/share/kb9vWGrY4052_783-MIQQSuT7TkhtIQ_2hQdgEdKrdO9as5qCNI8N8bbbMxy1hlf.0a3Ztdv_L25PgLsg Passcode: 906X?Kn\$

4.3 Problems Q7

5.3 Problems Q5

7. Income today $m_1 = 100$ tomorrow $m_2 = 200$ interest rate $r = 10\%$ inflation $\pi = 5\%$

$$U = C_1 C_2$$

(a) Intertemporal budget constraint: Good 1 : consumption today
Good 2 : consumption tomorrow

from today's point of view

$$C_2 = \left[\underbrace{m_2}_{\text{tomorrow's income}} + \underbrace{(m_1 - C_1)}_{\text{leftover from today}} \underbrace{(1+r)}_{\text{interest}} \right] \underbrace{\frac{1}{1+\pi}}_{\text{inflation: tomorrow, \$100 can only buy a good of \$95 value from today's perspective}}$$

$$\Rightarrow (1+\pi) C_2 = m_2 + (m_1 - C_1)(1+r) \quad \leftarrow \text{multiply } (1+\pi) \text{ on both sides}$$

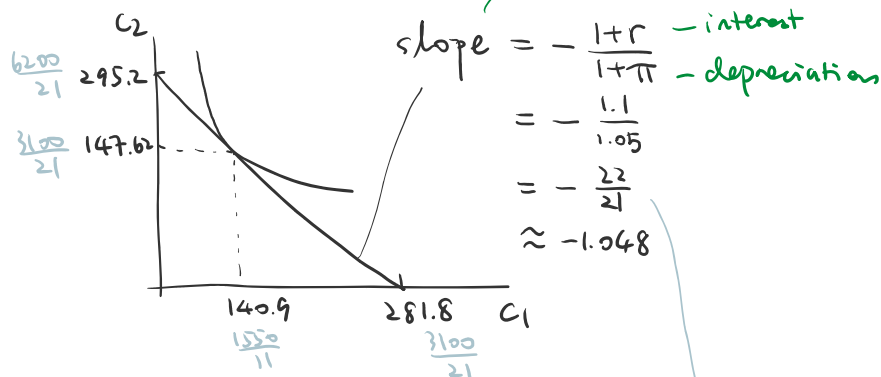
$$\Rightarrow \frac{1+\pi}{1+r} C_2 = \frac{m_2}{1+r} + m_1 - C_1 \quad \leftarrow \text{divide } (1+r) \text{ on both sides}$$

$$\Rightarrow C_1 + \frac{1+\pi}{1+r} C_2 = m_1 + \frac{m_2}{1+r} \quad \leftarrow \text{rearrange terms}$$

$$\Rightarrow C_1 + \frac{1.05}{1.1} C_2 = 100 + 200 \frac{1}{1.1} \quad \leftarrow \text{plug in numbers}$$

$$\Rightarrow C_1 + \frac{21}{22} C_2 = \frac{3100}{11}$$

"Price ratio": spend \$1 today v.s.
save \$1 for tomorrow



$$(b) \quad MRS = \frac{MU_1}{MU_2} = \frac{C_2}{C_1}$$

$$\text{Let } \frac{C_2}{C_1} = \frac{22}{21}$$

$$\Rightarrow C_2^* = \frac{22}{21} C_1^*$$

Intertemporal budget constraint:

$$C_1^* + \frac{21}{22} \cdot \frac{22}{21} C_1^* = \frac{3100}{11}$$

$$C_1^* = \frac{1550}{11} \approx 140.9$$

Saver or borrower?

Recall: $m_1 = 100$
 $m_2 = 200$

$$C_1^* > m_1$$

$$C_1^* = \frac{1550}{11} \approx 140.9$$

$$C_2^* \approx \frac{1550}{11} \cdot \frac{22}{21} = \frac{3100}{21} \approx 147.62$$

$$m_2 = 200$$

$$C_1^* > m_1$$

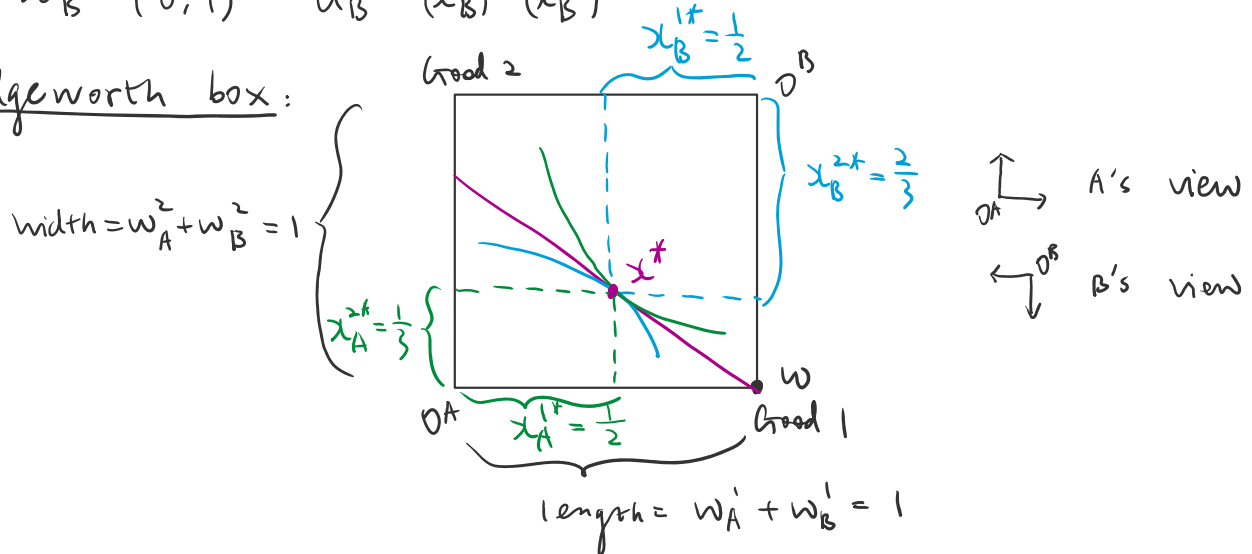
$$\Rightarrow \text{borrower}$$

5. Exchange economy: consumers meet in market \rightarrow price \rightarrow trade
(no production yet)

$$w_A = (1, 0) \quad U_A = (x_A^1)^{\frac{1}{2}} (x_A^2)^{\frac{1}{2}}$$

$$w_B = (0, 1) \quad U_B = (x_B^1)^{\frac{1}{3}} (x_B^2)^{\frac{2}{3}}$$

Edgeworth box:



Competitive equilibrium (Walrasian):

- ① Consumers behave optimally (utility maximizers)
 - ② Market clearance ($\sum \text{consumption} = \sum \text{endowment for all goods}$)
- \rightarrow general equilibrium
- "invisible hand"
- market will come up w/ $\frac{p_1}{p_2}$ to enable trade
- outcome: bundles + prices

① Utility max

$$U_A = (x_A^1)^{\frac{1}{2}} (x_A^2)^{\frac{1}{2}} \rightarrow p_1 w_A^1 + p_2 w_A^2 = p_1$$

$$\Rightarrow (x_A^1)^* = \frac{m_A}{2} \cdot \frac{1}{p_1} = \frac{1}{2}$$

$$(x_A^2)^* = \frac{m_A}{2} \cdot \frac{1}{p_2} = \frac{p_1}{2p_2}$$

$$U_B = (x_B^1)^{\frac{1}{3}} (x_B^2)^{\frac{2}{3}} \rightarrow p_1 w_B^1 + p_2 w_B^2 = p_2$$

$$\Rightarrow (x_B^1)^* = \frac{m_B}{3} \cdot \frac{1}{p_1} = \frac{p_2}{3p_1}$$

$$(x_B^2)^* = \frac{2}{3} m_B \cdot \frac{1}{p_2} = \frac{2}{3}$$

② Market clear:

$$(x_A^1)^* + (x_B^1)^* = w_A^1 + w_B^1 \Rightarrow \frac{1}{2} + \frac{p_2}{3p_1} = 1 \Rightarrow \frac{p_1}{p_2} = \frac{2}{3}$$

$$(x_A^1)^* + (x_B^1)^* = W_A^1 + W_B^1 \Rightarrow \frac{1}{2} + \frac{1}{3P_1} = 1 \Rightarrow \frac{1}{P_2} = \frac{2}{3}$$

$$(x_A^2)^* + (x_B^2)^* = W_A^2 + W_B^2 \Rightarrow \frac{P_1}{2P_2} + \frac{2}{3} = 1 \Rightarrow \frac{P_1}{P_2} = \frac{2}{3}$$

$$\Rightarrow (x_A^2)^* = \frac{1}{2} \frac{P_1}{P_2} = \frac{1}{3}$$

$$(x_B^1)^* = \frac{1}{3} \frac{P_2}{P_1} = \frac{1}{2}$$

By Walras' Law:

there are n markets }
 $n-1$ markets clear } \Rightarrow

the n^{th} market must also clear

Competitive eqn:

$$x_A^* = \left(\frac{1}{2}, \frac{1}{3} \right)$$

$$x_B^* = \left(\frac{1}{2}, \frac{2}{3} \right)$$

$$\frac{P_1}{P_2} = \frac{2}{3}$$

} bundles

\rightarrow prices

Q1: Why the budget line has to go through w point?

A: Because endowment point is always feasible

Q2: Are consumers better off after trade?

A: Yes, because if trade is harmful, consumers can always opt out and stick to the endowment.

Q3: Is the competitive outcome (Pareto) efficient?

Yes! (First fundamental theorem of welfare economics)