Concepts in Hypothesis Testing

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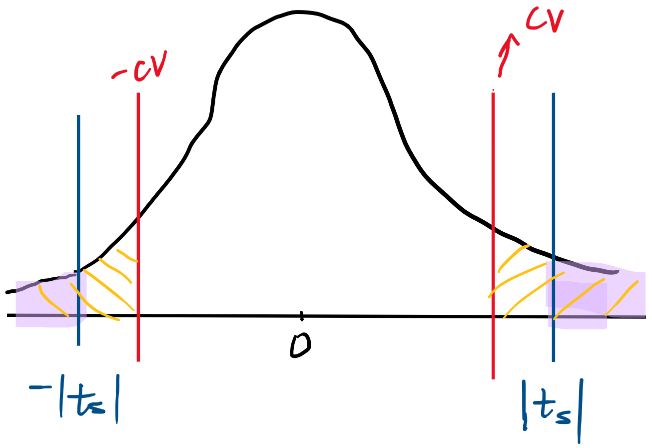
When would you reject H_0 in a test?

$$t_s = \frac{\hat{\beta}_1 - \beta_{10}}{se(\hat{\beta}_1)}$$

① p-value $< \alpha$

② $|t_s| > |cv|$

1. For two-sided test ($H_0: \beta_1 = \beta_{10}$, $H_a: \beta_1 \neq \beta_{10}$)

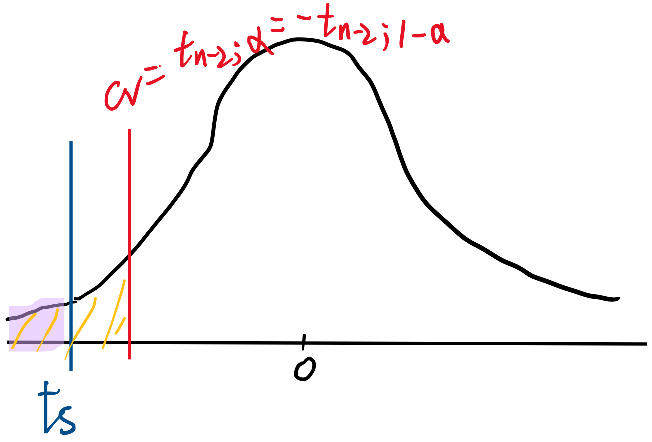


P-value $= 2 \times P(t_{n-2} > |t_s|)$

$cv = t_{n-2;1-\alpha/2}$ "$1-\alpha/2$ quantile of t_{n-2}"

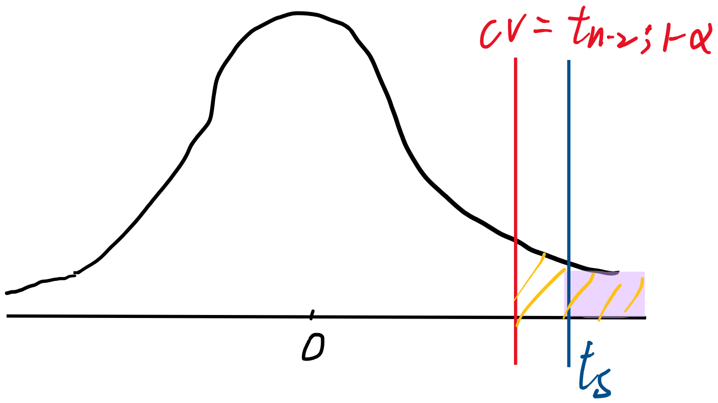
α : Type I error

2. One-sided test ($H_a: \beta_1 < \beta_{10}$)



P-value $= P(t_{n-2} < t_s)$

3. One-sided test ($H_a: \beta_1 > \beta_{10}$)



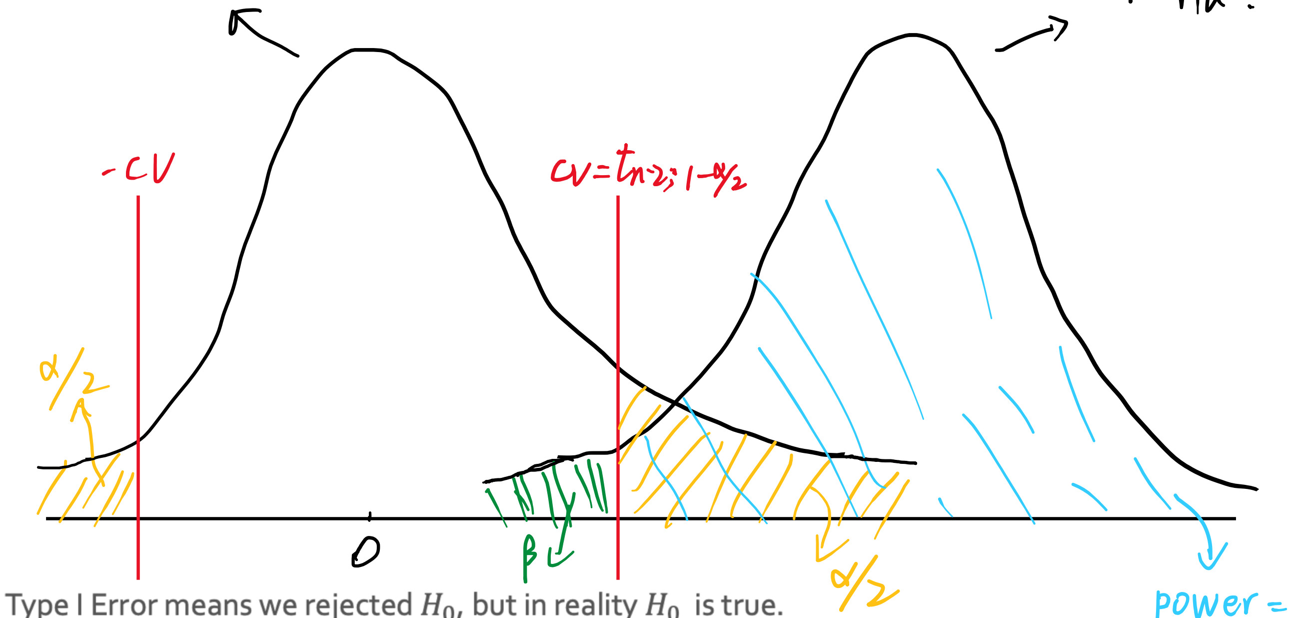
P-value $= P(t_{n-2} > t_s)$

Type 1 error, Type 2 error, power of a Test

$H_0: \beta_1 = \beta_{10}, \quad H_a: \beta_1 \neq \beta_{10}$

under H_0: $T \sim t_{n-2}$

under H_a: $T \sim t_{n-2}(\delta)$, $\delta = \dfrac{\beta_1 - \beta_{10}}{sd(\hat{\beta}_1)}$



power $= 1-\beta = P(|t| > t_{n-2;1-\alpha/2} \mid \delta)$

- Type I Error means we rejected H_0, but in reality H_0 is true.

$\alpha = P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ TRUE})$

- Type II Error means we supported H_0, but in reality H_0 is false.

$\beta = P(\text{Type II error}) = P(\text{FTR } H_0 \mid H_A \text{ TRUE})$

*power : $1-\beta$ = chance of having sufficient evidence for H_A. $= P(\text{Reject } H_0 \mid H_A \text{ TRUE})$