Lecture Summary: Jan. 17, 2020

Two-sided (B.5)
$H_0: \beta_1 = \beta_{10} (\delta = 0)$
$H_a: \beta_1 \neq \beta_{10} (\delta \neq 0)$

$t = \frac{\hat\beta_1 - \beta_{10}}{s.e.(\hat\beta_1)} \sim t_{n-2}(\delta)$

$\delta = \frac{|\beta_1 - \beta_{10}|}{s.d.(\hat\beta_1)}$

$\delta = 0: t \sim t_{n-2}$

$\delta \neq 0: t \sim t_{n-2}(\delta)$

- Power of the test

It is the probability of rejecting H_0 when the alternative holds at a given value. $P(\text{reject } H_0 \mid H_a \text{ is true } \& \ \beta_1 = 1) = P(|t| > \underbrace{t_{1-\alpha/2, n-2}}_{c.t} \mid \delta = 2)$ non-central t-dis

Power calculation:

Using Table B.5. Example: If $\beta_{10} = 0, \beta_1 = 1$, s.d.$(\hat\beta_1) = 0.5$, we have $\delta = 1/0.5 = 2.0$. Let $\alpha = 0.05$. From Table B.5 we find the power is about 0.48. $n = 25$

Derivation (not required for exams):

Example (Toluca). $H_0 : \beta_1 = 0$; $H_1 : \beta_1 > 0$; $\alpha = 0.05$. Suppose that wishes to compute the power at the alternative $\beta_1 = 1$. Under the given alternative, the test statistic can be written as

$$\begin{aligned} t &= \frac{\hat\beta_1}{\text{s.e.}(\hat\beta_1)} \\ &= \frac{\hat\beta_1 - 1}{\text{s.e.}(\hat\beta_1)} + \frac{1}{\text{s.e.}(\hat\beta_1)} \\ &= \frac{\hat\beta_1 - \beta_1}{\text{s.e.}(\hat\beta_1)} + \frac{1}{\text{s.e.}(\hat\beta_1)} \\ &= T + \frac{1}{\text{s.e.}(\hat\beta_1)}, \end{aligned} \tag{1}$$

where

$$T = \frac{\hat\beta_1 - \beta_1}{\text{s.e.}(\hat\beta_1)} \sim t_{n-2}, \tag{2}$$

as previously derived.

The power is the probability of rejecting the null hypothesis when $\beta_1 = 1$, which is

$$\mathrm{P}(t > t_{n-2,\alpha}) \quad = \quad \mathrm{P}[T + 1/\text{s.e.}(\hat\beta_1) > t_{23,0.05}]$$

$$
\begin{aligned}
&= \mathrm{P}[T > 1.71 - 1/\mathrm{s.e.}(\hat{\beta}_1)] \\
&= \mathrm{P}(t_{23} > 1.71 - 1/0.35) \\
&= \mathrm{P}(t_{23} > -1.15) \\
&\approx 0.87.
\end{aligned}
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