ELEC2146

Electrical Engineering Modelling and Simulation

Introduction to Modelling and Simulation

Dr Ray Eaton

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Lecture notes written by Dr Julien Epps

Overview

- Simulation
  - Why ?
  - Types of simulation
  - Simulation in Electrical Engineering
- Modelling
  - Why ?
  - Types of models
  - Applications of models

Simulation

- Definition: ‘simulate’
  - “Imitate or reproduce the appearance, character or conditions of”
  - Latin ‘simulare’ – to copy or represent
    - Oxford Dictionary
- A mock system that artificially recreates the behaviour of a real system
  - Or at least some approximation of this behaviour
- Almost always computer-based

Why simulate ?

- In many cases in engineering (perhaps the majority), building the final system, or even a prototype, is time-consuming and expensive or dangerous
  - Not the kind of thing you do just to validate a design
- In some cases, there is no analytical solution to a problem
- Some things are too slow, too fast, too small or too large to observe in real life
- Extrapolation or prediction

Why simulate ?

- Constraints on problem are complex
  - E.g. choose R_{B1}, R_{B2}, R_E, R_C to maximise output voltage swing for given V_{CC}, assuming a possible range of $\beta \in [90,110]$
  - Common in engineering
- Sensitivity analysis or parameter optimization
- Simulations can be almost infinitely adapted to test different possible configurations
  - Quickly and repeatably
  - Want to test millions of parameter values

Types of simulations

- Simulation of existing systems
  - Known behaviour
    - Traffic management algorithm and resulting traffic flow
    - Electronic circuits
  - Partially known behaviour
    - Producing synthetic speech
      - Don’t know the excitation signal at the vocal folds
    - Financial simulations
      - ‘Stress tests’ of European banks
  - Future behaviour
    - Climate change model
    - Power system fault simulation
    - Flight simulation

Types of simulations

- Simulation during design
  - Known behaviour
    - Based on previous design
    - Validation, compliance testing
  - Partially known behaviour
    - New design
    - Performance testing
    - Fault finding
    - Part of design process
  - Unknown behaviour (research)
    - Simulation may produce new insights

Simulation in this course

- Analog circuits and continuous physical systems with known behaviour
  - Impulse response, step response
  - Frequency response
  - State-space representation
  - Transfer function, poles and zeros
- Principles of simulating analog systems using a digital computer
  - Numerical solution of differential equations
- Some simulation of stochastic systems

Modelling

- Definition: ‘model’
  - “**1** . . . **4** a simplified mathematical description of a system or process, used to assist calculations and predictions. . . . **8**”
    - Oxford Dictionary
- A model need not necessarily be mathematical
  - Models are by definition abstract
- In this course, all models are mathematical descriptions of systems or signals and their behaviour
  - In terms of external and internal quantities

Modelling

- A comment from Richard Hamming...
  - The brain processes and organises incoming stimuli in such a way that “we do not actually think about the real external world but rather we ‘think’ in terms of our impressions of the stimuli. Thus, all thought is, in some sense, modeling.”

Why model ?

- Modelling produces an abstraction of a system that allows us to reason about its behaviour
  - Produces insight into the system
- Modelling is fundamental to the scientific process
  - Hypothesis $\rightarrow$ model $\rightarrow$ experimental observations $\rightarrow$ reasoning $\rightarrow$ improved model $\rightarrow$ etc
- Modelling expresses the limitations of our knowledge of a system
  - Deviation from ‘ideal’ or ‘linear’ behaviour
- Learn new rules
  - World changes $\rightarrow$ observed data changes
  - Model structure may remain, but new parameters estimated

Why model ?

- Models can usually be feasibly simulated
  - To predict the likely response to particular types of inputs or conditions
- Control and optimization applications
- Model parameters can be used for classification
- Model parameters provide a compact representation of the system
- Models can make sense of data
  - Data from Google searches may one day provide new models for human cognition

Types of models

- Qualitative and quantitative
  - Qualitative
    - cognitive models (underly human reasoning, learning etc) and
    - normative models (define specified or desired function, e.g. a standard)
    - Derived from categorical data (e.g. contingency table)
  - Quantitative (this course)
    - Described by parameters, derived from numerical data
- Analytical and numerical
  - Analytical: closed-form solution, e.g. circuit equations
  - Numerical: no exact solution, or analytical solution is not feasible

Types of models

- Linear and non-linear
  - Most undergraduate EE courses: linear theory
    - Most of this course too
- Deterministic and stochastic
  - Deterministic: Every input has a known output(s)
  - Stochastic: Outputs may be described using a pdf
    - Try to see some of each in this course
- Static and dynamic
  - Dynamic: describe the change in relationship between modelled quantities as a function of time
    - Output still changing after input is held constant
  - Much of this course deals with dynamic models

Types of models

- Descriptive and functional models
  - Descriptive: explains behaviour of system/process
    - e.g. a circuit produces a decaying sinusoid at the output; descriptive model finds parameters for frequency, decay time constant
    - Could *reconstruct* output signal (input does not change)
  - Functional (generative): action and control oriented
    - e.g. a circuit produces a decaying sinusoid at the output; functional model tries to find all parameters of the circuit
    - Could *generate* outputs (for diff. inputs) *and* internal variables
- General and specific
  - General: describes broad principles concisely and effectively for many practical purposes
    - Only roughly accurate
  - Specific: detailed, often with many parameters
    - May only be valid under limited conditions

Types of models

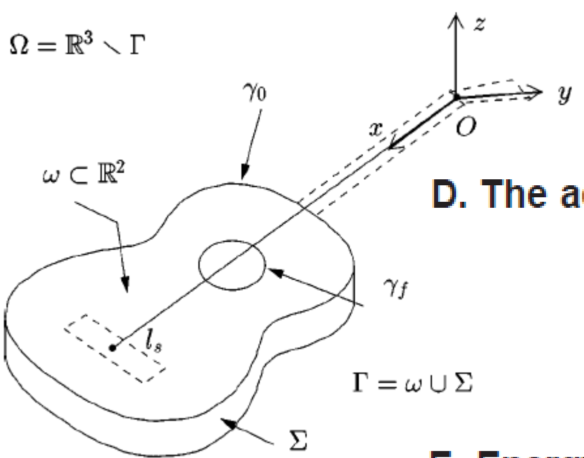
- General and specific
  - Model of a guitar → sound produced

B. The string

$$\rho_s \frac{\partial^2 u_s}{\partial t^2} - T\left(1 + \eta_s \frac{\partial}{\partial t}\right)\frac{\partial^2 u_s}{\partial x^2} + \rho_s R_s \frac{\partial u_s}{\partial t} = f_s(x,t), \quad \text{in }]0, l_s[.$$

C. The soundboard

$$a\rho_p \frac{\partial^2 u_p}{\partial t^2} + \left(1 + \eta_p \frac{\partial}{\partial t}\right) \operatorname{div} \underline{\operatorname{Div}}\, a^3 \mathbf{C}\underline{\underline{\varepsilon}}(\underline{\nabla} u_p) + a\rho_p R_p \frac{\partial u_p}{\partial t} = \mathcal{F} - [p]_\omega, \quad \text{in } \omega, \qquad (5)$$



D. The acoustic field

$$\frac{\partial p}{\partial t} = -c_a^2 \rho_a \operatorname{div}(\underline{\mathbf{v}}_a) \quad \text{in } \Omega, \qquad (9a)$$

$$\rho_a \frac{\partial \underline{\mathbf{v}}_a}{\partial t} = -\underline{\nabla} p \quad \text{in } \Omega, \qquad (9b)$$

E. Energy decay

$$E^s(t) = \frac{1}{2}\int_0^{l_s} \rho_s \left|\frac{du_s}{dt}\right|^2 dx + \frac{1}{2}\int_0^{l_s} T \left|\frac{du_s}{dx}\right|^2 dx,$$

$$E^p(t) = \frac{1}{2}\int\int_\omega \rho_p a \left|\frac{du_p}{dt}\right|^2 dx\, dy + \frac{1}{2}\int\int_\omega a^3 \mathbf{C}\underline{\underline{\varepsilon}}(\underline{\nabla} u_p) : \underline{\underline{\varepsilon}}(\underline{\nabla} u_p)\, dx\, dy, \qquad (12)$$

$$E^a(t) = \frac{1}{2}\int\int\int_\Omega \rho_a |\underline{\mathbf{v}}_a|^2 dx\, dy\, dz + \frac{1}{2}\int\int\int_\Omega \frac{1}{c_a^2 \rho_a} |p|^2 dx\, dy\, dz.$$

Parameters: ρ_s, T, η_s, R_s, ρ_p, η_p, R_p, ρ_a, c_a, plus initial conditions

Derveaux *et al.*: Time-domain simulation of a guitar

J. Acoust. Soc. Am., Vol. 114, No. 6, Pt. 1, Dec. 2003

Types of models

- General and specific
  - Model of a guitar sound
    - Plucked string $\rightarrow$ simple harmonic motion
      - $\therefore$ form of solution is sinusoidal

$$x(t) = A_1 \sin(\omega_0 t + \phi_1) + A_2 \sin(2\omega_0 t + \phi_2) + \ldots$$

      - decays with time (friction losses)

$$x(t) = e^{-\alpha t}(A_1 \sin(\omega_0 t + \phi_1) + A_2 \sin(2\omega_0 t + \phi_2) + \ldots)$$

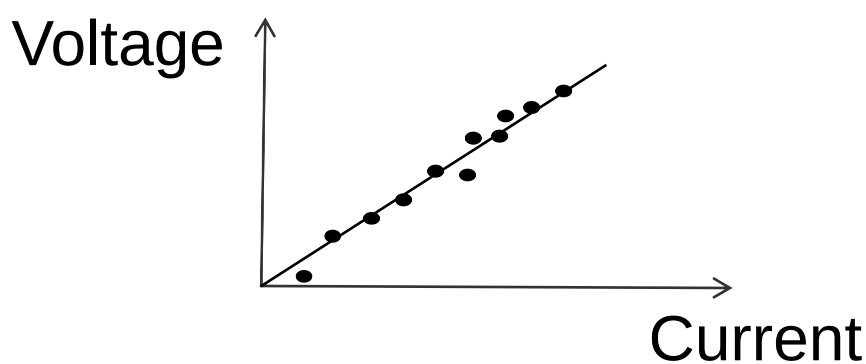
      - suppose all amplitudes equal, phases zero
    - Sound box acts as a resonator cavity
      - Like an LC circuit (resonant filter)

$$x(t) \longrightarrow \boxed{\text{resonant filter}} \longrightarrow y(t)$$

    - Parameters: α, ω_0, parameters of resonator (e.g. L, C)
    - Fewer parameters, applies to other string+resonator systems, but less accurate

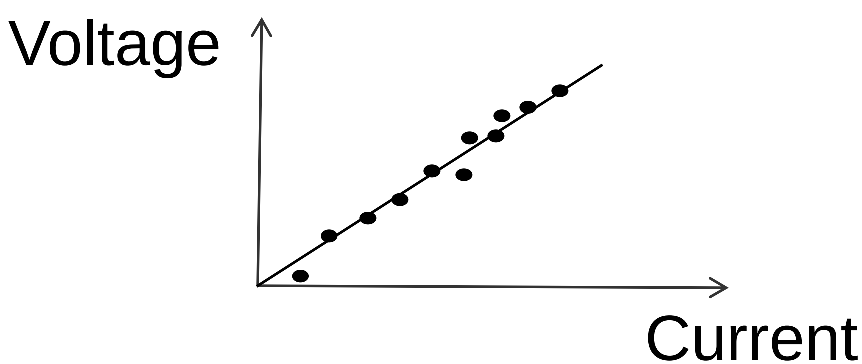
Examples: Linear, static model

- Ohm's Law
  - Actually a model of the relationship between voltage and current in a resistor
  - Initially, Ohm would have created this model from a series of experimental results
    - Observational model
    - Voltage and current are model variables
    - Resistance is model parameter



Examples: Linear, static model

- Ohm's Law (cont'd)
  - Data looks like it follows a straight line relationship: linear regression
    - Slope (resistance) is the model parameter
  - Treat it as an analytical model
    - i.e. "fact" or "law", once everybody agrees with it
  - Examine how closely the model fits the data
    - Unless you're an economist
    - Several techniques available



Examples: Linear, dynamic models

- Capacitive voltage-current relationship

$$i(t) = C\frac{dv(t)}{dt}$$

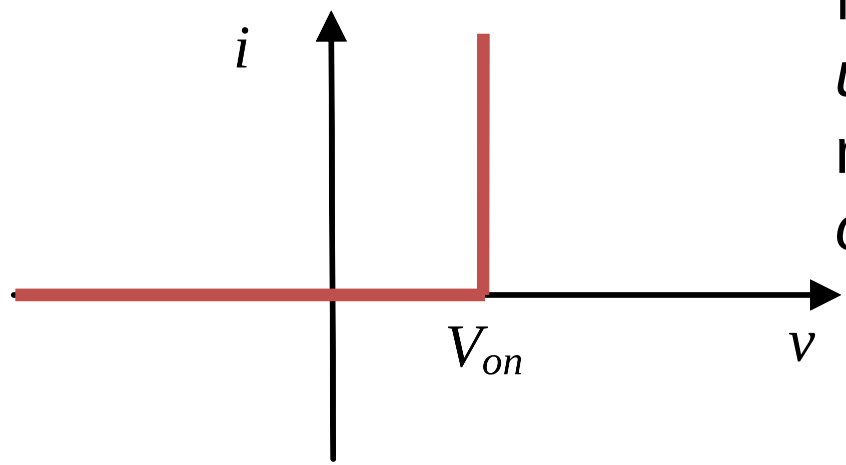
- Inductive voltage-current relationship

$$v(t) = L\frac{di(t)}{dt}$$

  – *L*, *C* are the parameters for each model

Examples: non-linear, static model

- Diode voltage-current relationship
  - Constant voltage drop model
  - V_{on} is the parameter



Note: *System is unique*, but there may be *more than one model*

  - Convenient approximation to the actual analytical relationship (also non-linear, static)

$$I = I_S\left(\exp\left(\frac{qV}{nkT}\right) - 1\right)$$

Examples: non-linear, dynamic model

- Heart rate response to treadmill walking exercise
  – Cheng, Savkin, Celler, Wang and Su, 2007

$$\begin{aligned}\dot{x}_1(t) &= -a_1 x_1(t) + x_2(t) + g(u(t)) \\ \dot{x}_2(t) &= -a_4\big(x_2(t) - \tanh(x_2(t))\big) + a_5 x_1(t) \qquad (1) \\ y(t) &= x_1(t)\end{aligned}$$

where

$$g(u(t)) := \frac{a_2 u^2(t)}{1 + \exp\big(-u(t) + a_3\big)}, \quad u(t) := \begin{cases} v & \text{for } t \le t_s \\ 0 & \text{for } t > t_s \end{cases}$$

  – $y(t)$: change in heart rate from rest
  – $u(t)$: speed of treadmill
  – a_1, a_2, a_3, a_4, a_5, v, t_s : model parameters

Note on dynamic models

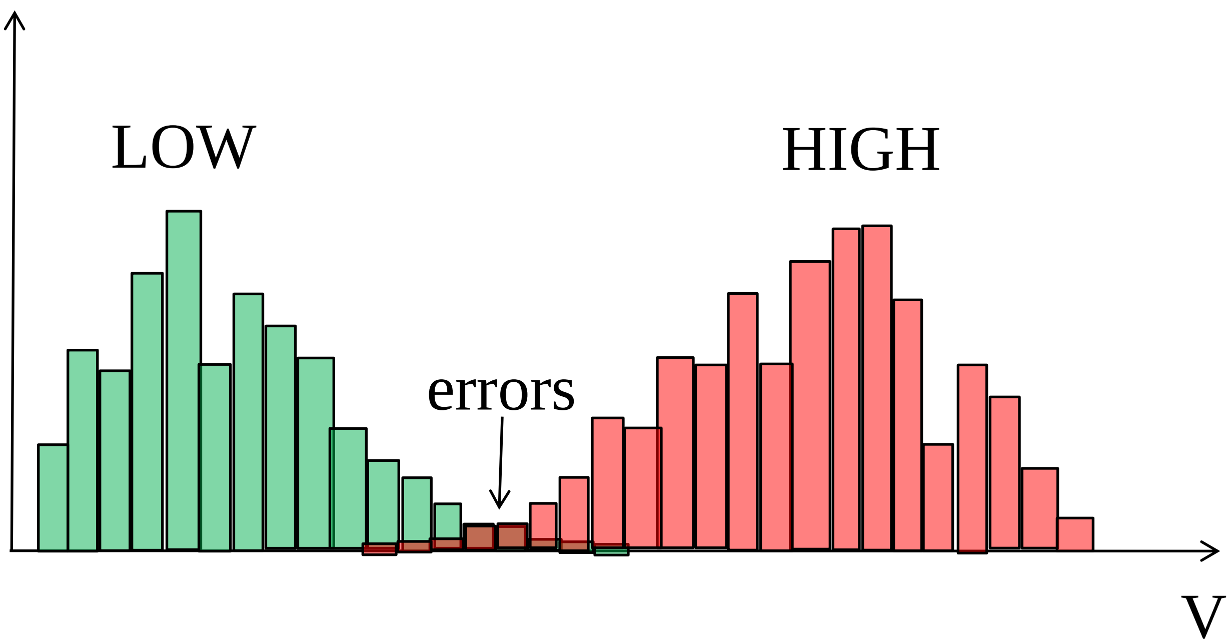
- Modelling change
  - future_value = present_value + change
- An important paradigm, used in
  - Approximating derivatives
    - Instantaneous_change ≈ future_value – present_value
  - Gradient descent approaches
    - Nonlinear least squares
- Example:
  - Interest paid at 1% per month on $1000
    - present_value = $1000
    - change = $10
    - future_value = $1010
    - $a_{n+1} = a_n + 0.01a_n$
    - $a_{n+1} = 1.01a_n$

Examples: stochastic model

- Previous examples have all been deterministic
- Stochastic example: Noisy sensor
  - Normally
    - LOW : < 2.5V, HIGH : > 2.5V
  - Due to noise, a LOW might have a voltage > 2.5V, while a HIGH might have a voltage < 2.5 V
    - Guess that 2.5 V may not be such a good threshold any more
  - Want to know how to classify a given input voltage
    - 1. Collect some experimental data
    - 2. Form stochastic models
    - 3. Base classification on stochastic models

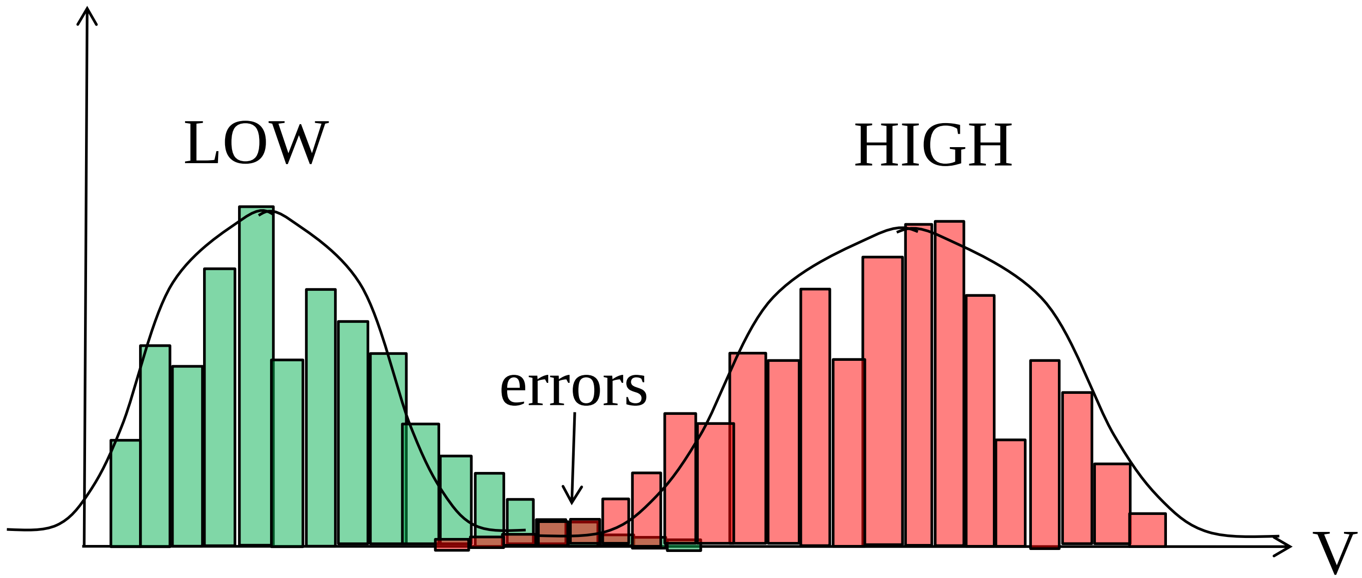
Examples: stochastic model

- 1. Collect experimental data
  - Under controlled conditions (know whether high/low)



Examples: stochastic model

- 2. Form stochastic models
  - Example below: Gaussian models
    - could choose others

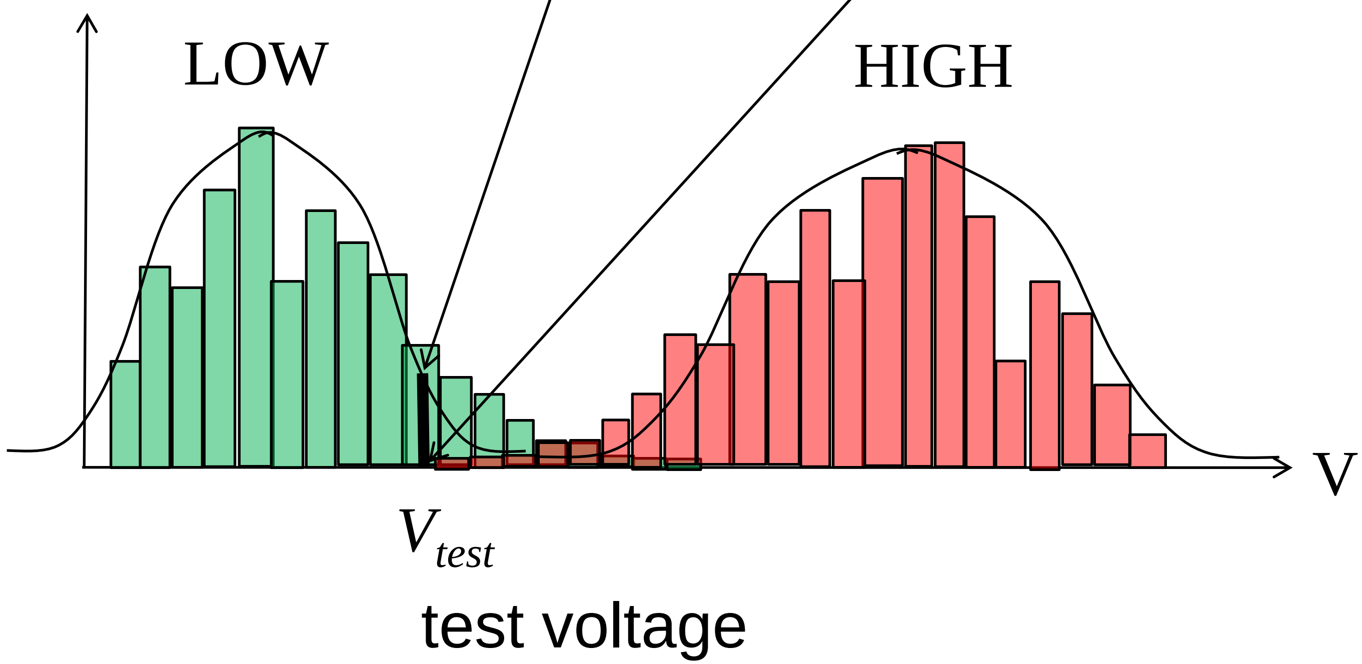


$$p_{LOW}(V) = \frac{1}{\sqrt{2\pi}\sigma_{LOW}} \exp\left(-\frac{1}{2} \frac{(V-\mu_{LOW})^2}{\sigma_{LOW}^2} \right) \qquad p_{HIGH}(V) = \frac{1}{\sqrt{2\pi}\sigma_{HIGH}} \exp\left(-\frac{1}{2} \frac{(V-\mu_{HIGH})^2}{\sigma_{HIGH}^2} \right)$$

model parameters

Examples: stochastic model

- 3. Base classification on stochastic models
  - Choose LOW if $p_{LOW}(V_{test}) > p_{HIGH}(V_{test})$



How good is a model ?

- Model quality is related to use
  - Typically, a model will be good for one application and not so good for others
- Ability to reproduce behaviour
  - A model’s predicted or simulated output should be in agreement with the outputs produced by the system being modelled
- Stability
  - How consistently can the model be reproduced from different measured data sets ? (produced by the same system)

How good is a model ?

- In general
  - A model is inexact
  - How do we measure the accuracy of a model ?
  - Different model complexities capture different levels of detail in a physical system
    - First-order model: Captures key behaviour, ignores higher order effects, has low complexity (few parameters)
    - Higher-order model: Captures details of most or all behaviour, has high complexity (many parameters)
  - In most practical situations, there is a trade-off between model accuracy and complexity
- In this course
  - We look at model evaluation criteria: goodness of fit

Sources of Error

- Formulation error
  - Model is deficient in some respect
    - Often simpler than can fully explain a given set of signals
- Truncation error
  - Numerical approximation
    - e.g. $dx/dt \approx (x_{final} - x_{initial})/T$
- Round off error
  - If 1/9 is 0.11111111, 9*1/9 = 0.99999999
  - Error is 10^{-8}
- Measurement error
  - Imprecise data collection
    - Due to equipment or human error

Model construction

- Identify problem
- Make assumptions
  - Identify and classify variables
  - Determine interrelationships between variables
    - And submodels, if they apply
- Solve the model
- Verify the model
  - Does it address the problem ?
  - Does it make common sense ?
  - Test it with real-world data
- Implement and maintain model

Source: Giordano, Weir and Fox, 2003

Modelling in this course

- Focus on linear, dynamic systems
  - Then look at some stochastic models
- Some key topics:
  - Numerical models of continuous systems
  - Parameter estimation
  - Choice of model structure/order
  - Goodness of fit

Course Logistics

- Lectures
- Tutorials
  - Attempt before class, bring questions
- Labs
  - Assessed live: 20% of assessment
  - Many labs have preparation: come prepared !
- Project
  - 20% of assessment
- Assignment
  - 10% of assessment
- Exams
  - Mid-session: 10% of assessment
  - Final: 40% of assessment