ELEC3104: Digital Signal Processing

S1, 2017

Tutorial-Laboratory Problem Sheet 1

Question 1
[Analytical Component]

A. Evaluate the DTFT (discrete time Fourier transform) of

$$x[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & otherwise \end{cases}$$

B. Show that,

$$y[n] = \begin{cases} 2N+1+n, & -2N \le n \le 0 \\ 2N+1-n, & 0 \le n \le 2N \\ 0, & otherwise \end{cases}$$

Is given by $y[n] = x[n] * x[n]$. Hence use the DTFT of $x[n]$ to determine the DTFT of $y[n]$.

[Laboratory Component]

C. Assume $N = 20$ and plot $x[n]$ and its magnitude and phase spectra, $|\hat{x}(\theta)|$ and $\angle\hat{x}(\theta)$. You need to implement the code required to compute the DTFT:

$$\hat{x}(\theta) = \sum_{n} x[n]e^{-j\theta n}$$

Do not use any built-in MATLAB functions in your implementation, you should be able to do it using for-loops. Additionally, you cannot compute the DTFT at all values of θ (since there are infinitely many values it can take) but you should do so for a large enough number of points to give you a good idea of the magnitude and phase spectra. For e.g., compute it at 10,000 equally spaced points between $-\pi$ and π.

D. Using $\hat{x}(\theta)$, the DTFT of $x[n]$, that you just computed, estimate $\hat{y}(\theta)$, the DTFT of $y[n] = x[n] * x[n]$. Based on $\hat{y}(\theta)$, compute $\tilde{y}[n]$ as its inverse DTFT (again, implement this in MATLAB without making use of any built-in functions). Compare your results to a direct realisation of $y[n]$ based on the expression given in question B above

Question 2
[Analytical Component]

A. Figure 1(a) shows the front-end of a simple data acquisition system and the input filter is an analogue low-pass filter used to band limit the signal before it is sampled at the rate of F_s.

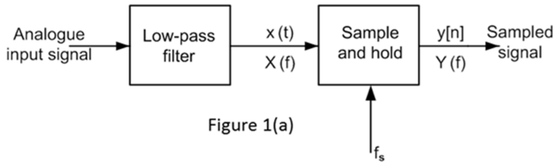


Figure 1(a)

The magnitude response of the low-pass filter is given by

$$|H(\omega)| = \frac{1}{\left[1 + \left(\frac{\omega}{2\pi f_c}\right)^2\right]^{\frac{1}{2}}}, \qquad where\ f_c = 2kHz$$

Determine the minimum sampling rate, F_s, so as to give an aliasing error of less than 2% of the signal level within the pass-band of the analogue input filter. Note that the cut-off frequency of this low pass filter that determines its pass-band is defined as the frequency at which the gain of the filter is 3dB less than the gain at DC.

B. If the input signal to this data acquisition system is given by $\cos(2\pi f_1 t) + \sin(2\pi f_2 t) + \cos(2\pi f_3 t)$, where $f_1 = 700Hz$, $f_2 = 1kHz$ and $f_3 = 3kHz$, write down an expression for the sampled signal $y[n]$, obtained using a sampling rate of F_s and derive an expression for its DTFT, $\hat{y}(\theta)$.

[Laboratory Component]

C. Write a MATLAB script/function that computes 100,000 samples of $y[n]$ and plot its magnitude spectrum for any given sampling rate, F_s. You can modify the code you wrote to compute DTFT in the previous question to show frequency components present at $\theta > \pi$ (Why would components corresponding to $\theta > \pi$ be of interest in this case?). Use this code to observe the magnitude spectra obtained when using the sampling rate you estimated in part A and ensure that aliasing is indeed limited less than 2%.

D. Use your code to compare the magnitude spectrum of $y[n]$ as you vary the sampling rate. Specifically, determine the sampling rates at which the magnitude spectrum has only two spectral components within the Nyquist range, $0 \leq \theta \leq \pi$.

E. Listen to the $y[n]$ that you generated for different sampling rates. Is aliasing audible? Try listening to a sinusoidal signal of a single frequency sampled at differing rates, can you detect the change in frequency caused by aliasing?

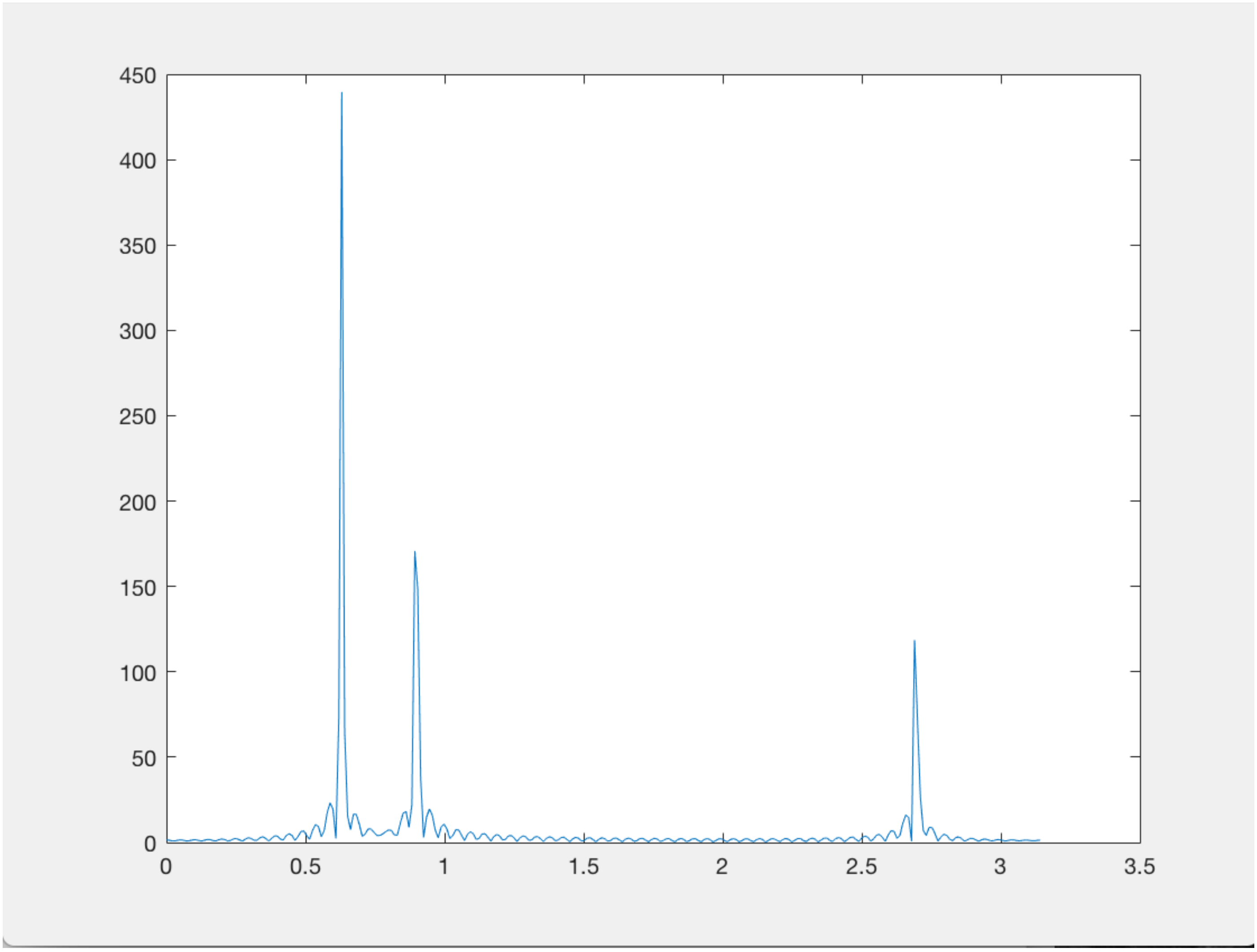
Question 3

[Analytical Component]

A. Show that the spectrum, $\hat{x}(\theta)$, of discrete-time sinusoid, $x[n]$, of frequency f_1, sampled at F_s, only has two spectral components in the interval $-\pi \leq \theta \leq \pi$ and these components are located at $\theta = \pm 2\pi \frac{f_1}{F_s}$

Q3 C

with 10,000 samples



100 samples

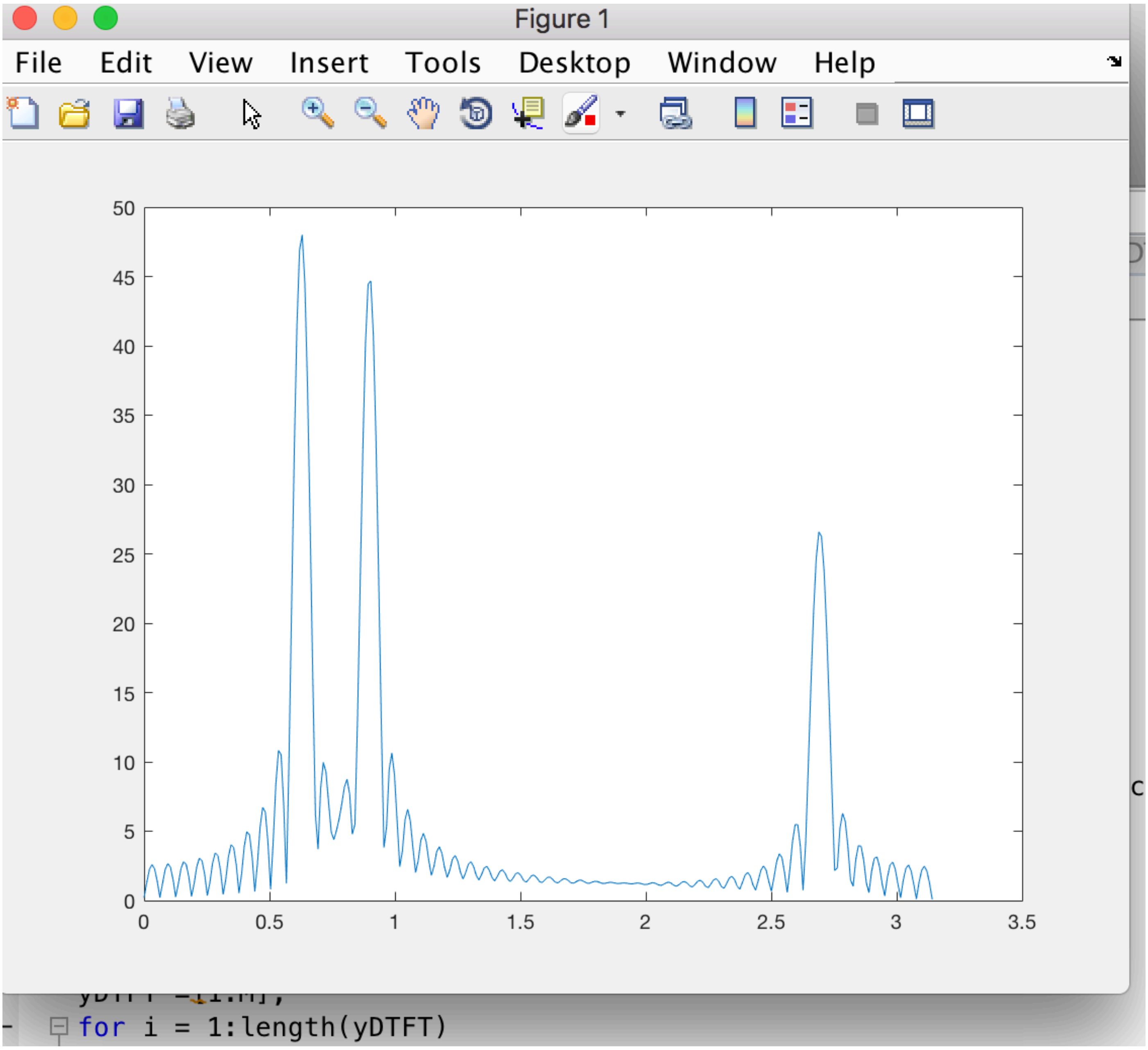
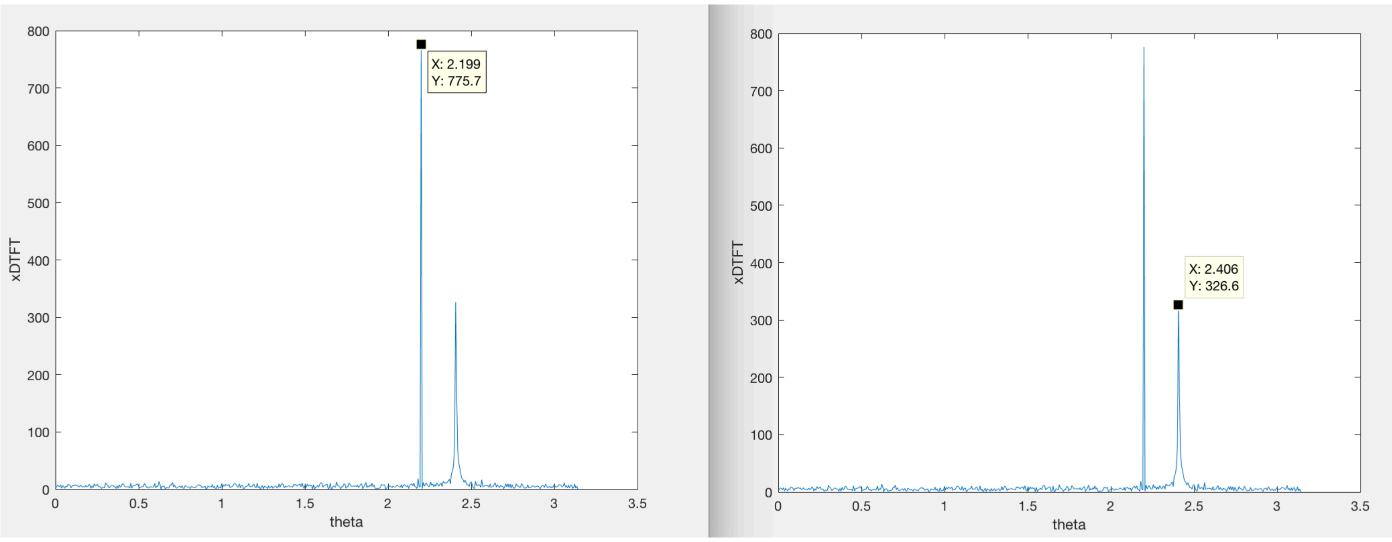


Figure 1
File Edit View Insert Tools Desktop Window Help
50
45
40
35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
for i = 1:length(yDTFT)

Q3 D



$2\pi \times f1/Fs = 2.199 \longrightarrow f1 = 2100$ HZ
$2\pi \times f2/Fs = 2.406 \longrightarrow f2 = 2298$ HZ

B. If you only had N consecutive samples of $x[n]$ to analyse, determine and sketch the magnitude spectrum you would infer from these samples. Given that in most applications you will only work on segments of a signal, what are the implications of the size of the segment on spectral analyses?

will still have two spectrum but with some noise, just like in the plot of q2

[Laboratory Component]

C. In MATLAB compare the magnitude spectra of the a sinusoidal signal of length N obtained for varying values of N. Do your observations match your expectations from your response to part B above?

D. The file '*s1_q3d.wav*' (on Moodle) contains a signal of the form:

$$x[n] = \cos\left(2\pi n\frac{f_1}{F_s}\right)(u[n] - u[n - N_1]) + \cos\left(2\pi n\frac{f_2}{F_s}\right)(u[n - N_1] - u[n - N_2])$$
$$+ \eta[n](u[n] - u[n - N_1]$$

Where, $F_s = 6000Hz$, $N_2 = 6000$ and $\eta[n]$ denotes a small amount of white noise (it is referred to as 'white' noise since there the power is distributed evenly across all frequencies and references is made to white light being composed of all visible frequencies). Determine f_1, f_2 and N_1 and justify your answers based on your spectral analyses.

f1= 2100, f2 =2298

Question 4

[Laboratory Component]

A discrete -time signal $x[n] = \cos 2\pi f_1\, n/fs\ +\ \cos 2\pi f_2\, n/fs$, where $f_1 = 50Hz$, and $f_2 = 150Hz$ modulates the amplitude of the carrier, $x_c[n] = \cos 2\pi f_c\, n/fs$, where $f_c = 3000Hz$, $fs = 10000\ samples/sec$. The resulting amplitude modulated signal is $x_{am}[n] = \mathrm{x}[\mathrm{n}]x_c[n]$.

A. Write MATLAB code to generate 1000 samples of $x[n], x_c[n]\ and\ x_{am}[n]$, and then plot the first 50 milliseconds of these signals (time axis in millisecond).

B. Write a MATLAB code to plot the magnitude spectra of the signals $x[n]$, $x_c[n]$ and $x_{am}[n]$.

C. Repeat plots using 256 and 128 samples of these signals. Compare the results with those obtained in part B and explain.

[Analytical Component]

D. Explain the results obtained in Parts B by deriving the spectrum of the amplitude-modulated signal and comparing it with the experimental results.

Question 5

[Analytical Component]

A. Consider a discrete-time system described by the input-output relation,

$$y[n] = nx[n]$$

Show that this system is linear.

B. Each linear system (H1, H2 and H3) has the input-output pairs shown in Figure 2 below. Determine the following and explain your answers for each of the system H1, H2 and H3 shown in Fig 2.

i. Is the system Causal?
ii. Is the system time invariant?
iii. Is the system memoryless?

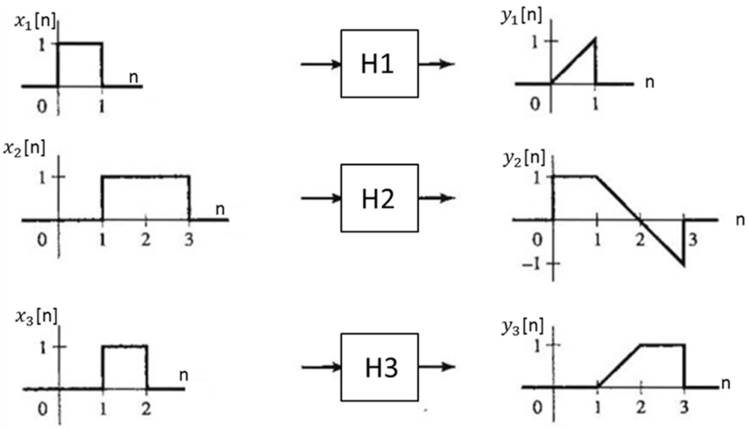


Figure 2

[Laboratory Component]

C. A simple digital differentiator is given by $y[n] = x[n] - x[n-1]$ which computes a backward first-order difference of the input sequence. Implement this differentiator on the following sequences using MATLAB and plot the results. Comment on the appropriateness of this simple differentiator.

i. $x[n] = 5\{u[n] - u[n-20]\}; a\ rectangular\ pulse$
ii. $x[n] = sin\left(\frac{\pi n}{25}\right)\{u[n] - u[n-100]\}; a\ sinusoidal\ pulse$

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ELEC3104 LAB1

ELEC 3104 lab 1

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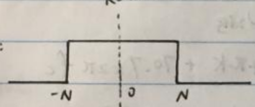
A.

$$\hat{X}(\theta) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\theta n}$$
$$= \sum_{n=-N}^{N} (e^{-j\theta})^{N}$$
$$= e^{jN\theta} \cdot \frac{1-(e^{-j\theta})^{2N+1}}{1-e^{-j\theta}}$$
$$= \frac{e^{jN\theta} - e^{-j(N+1)\theta}}{1-e^{-j\theta}} \times \frac{e^{j\frac{1}{2}\theta}}{e^{j\frac{1}{2}\theta}}$$
$$= \frac{e^{j(N+\frac{1}{2})\theta} - e^{-j(N+\frac{1}{2})\theta}}{e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}}$$
$$= \frac{[\cos(N+\frac{1}{2})\theta + j\sin(N+\frac{1}{2})\theta] - [\cos(N+\frac{1}{2})\theta - j\sin(N+\frac{1}{2})\theta]}{(\cos\frac{\theta}{2} + j\sin\frac{\theta}{2}) - (\cos\frac{\theta}{2} - j\sin\frac{\theta}{2})}$$
$$= \frac{2j\sin[(N+\frac{1}{2})\theta]}{2j\sin\frac{\theta}{2}} = \frac{\sin(N+\frac{1}{2})\theta}{\sin(\theta/2)}$$

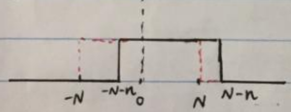
B.

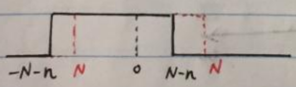
$$y[n] = x[n] * x[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k] \cdot x[n-k]$$

$x[-k]$:



$x[n-k] = x[-(k-n)]$





1° $N-n < -N$ no overlap

↳ $n > 2N$, $y[n] = 0$

2° $-N-n > N$ no overlap

↳ $n < -2N$ $y[n] = 0$

3° $\begin{cases} -N-n \le N \\ -N-n \ge -N \end{cases} \Rightarrow -2N \le n \le 0$

$$y[n] = N - (-N-n) + 1$$
$$= 2N+1+n$$

4. $\begin{cases} N-n \ge -N \\ N-n \le N \end{cases} \Rightarrow 0 \le n \le 2N$

$$y[n] = (N-n) - (-N) + 1$$
$$= 2N+1-n$$

Question 2

A: Find W_{3dB} : $|H(\omega)| = \dfrac{1}{\left[1+\left(\frac{W_{3dB}}{2\pi f_c}\right)^2\right]^{\frac{1}{2}}} = \dfrac{1}{\sqrt{2}}$

$$1+\left(\frac{W_{3dB}}{2\pi f_c}\right)^2 = 2$$

$$W_{3dB} = 2\pi f_c = 4\pi\ k \text{ rad/s}$$

2% error : $\dfrac{1}{\left[1+\left(\frac{W_{3dB}}{2\pi f_c}\right)^2\right]^{\frac{1}{2}}} = \dfrac{1}{\sqrt{2}} \times 2\%$ square both side

$$1\Big/\left[1+\left(\frac{W_{3dB}}{2\pi f_c}\right)^2\right] = 0.0002$$

$$5000 = 1+\left(\frac{W_{3dB}}{2\pi f_c}\right)^2$$

$$\left|\frac{W_{3dB}}{2\pi f_c}\right| = -\sqrt{4999} \quad \text{(take the negative side)}$$

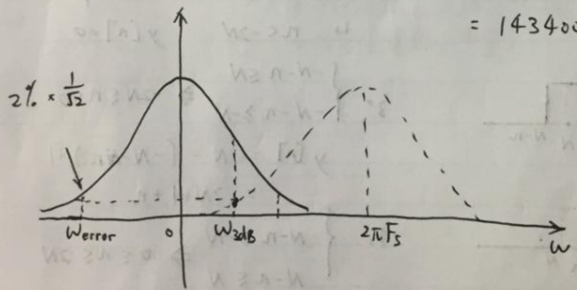
$$W_{error} = -70.7 \times 2\pi f_c$$

$$W_{error} + 2\pi F_s = W_{3dB}$$

$$2\pi F_s = 4\pi k + 70.7 \times 2\pi f_c$$

$$F_s = 2k + 70.7 f_c$$

$$= 143400 \text{ HZ}$$



B: $t = nT = \dfrac{n}{F_s}$, as $F_s = 143400$, $f_1 = 700$. $f_2 = 1k$, $f_3 = 3k$ HZ

$$y(t) = \cos(2\pi f_1 t) + \sin(2\pi f_2 t) + \cos(2\pi f_3 t)$$

$$y[n] = \cos\left[2\pi \frac{f_1}{F_s} n\right] + \sin\left[2\pi \frac{f_2}{F_s} n\right] + \cos\left[2\pi \cdot \frac{f_3}{F_s} n\right]$$

$$= \cos[0.031n] + \sin[0.044n] + \cos[0.13n]$$

$$\hat{y}(\theta) = \sum_{n=-\infty}^{\infty} \cos[0.031n] e^{-jn\theta} + \sum_{n=-\infty}^{\infty} \sin[0.044n] e^{-jn\theta} + \sum_{n=-\infty}^{\infty} \cos(0.13n) e^{-jn\theta}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{j0.031n} + e^{-j0.031n}}{2}\right) e^{-jn\theta} + \sum_{n=0}^{\infty} \left(\frac{e^{j0.044n} - e^{-j0.044n}}{2j}\right) e^{-jn\theta}$$

$$+ \sum \left(\frac{e^{j0.13n} + e^{-j0.13n}}{}\right) e^{-jn\theta}$$

∴ Question 3

~~assume~~ $x[n] = \cos[an]$

$$x[n] = \frac{e^{jan} + e^{-jan}}{2}$$

$$\hat{X}(\theta) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(e^{jan} + e^{-jan}\right)e^{-j\theta n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(e^{j(an-\theta n)} + e^{-j(an+\theta n)}\right) \text{ frequency shift}$$

$$= \pi \sum_{k=-\infty}^{\infty} \delta\, \theta -$$

Assume ~~$x[n] = \cos$~~ [illegible]

So ~~$x[n] = \cos$~~

Assume $x[n] = \cos\left[2\pi \frac{f_1}{F_s} n\right]$

$$x[n] = \frac{e^{j2\pi\frac{f_1}{F_s}n} + e^{-j2\pi\frac{f_1}{F_s}n}}{2}$$

$$\hat{X}(\theta) = \sum_{n=-\infty}^{\infty} \frac{e^{j2\pi\frac{f_1}{F_s}n} + e^{-j2\pi\frac{f_1}{F_s}n}}{2} \cdot e^{-j\theta n}$$

$$= \pi \sum_{k=-\infty}^{\infty}\left\{\delta\left(\theta - 2\pi\frac{f_1}{F_s} - 2k\pi\right) + \delta\left(\theta + 2\pi\frac{f_1}{F_s} - 2k\pi\right)\right\}$$

For $k = 0$, we have $\hat{X}(\theta)$ which $-\pi < \theta < \pi$

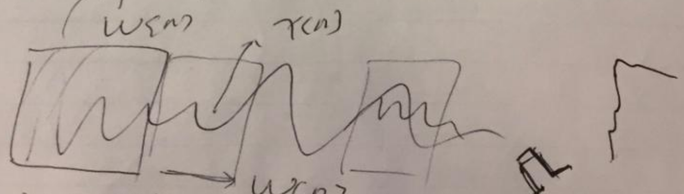
$$\hat{X}(\theta) = \pi\left(\delta\left(\theta - 2\pi\frac{f_1}{F_s}\right) + \delta\left(\theta + 2\pi\frac{f_1}{F_s}\right)\right)$$

$F_s > 2f_1$, so $2\pi\frac{f_1}{F_s} < \pi$

Hence we only have two spectrum in $-\pi < \theta < \pi$

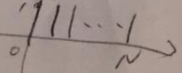
if $k \neq 0$, $2\pi\frac{f_1}{F_s} < \pi$ is not valid

STFT



$w[n] = u(n) - u(n-N)$

$\hat{X}[n] = x(n) \cdot w[n]$



Q3 C

$$x[n] = \cos[2\pi f_1 n/f_s] + \cos[2\pi f_2 n/f_s]$$

$$x_c[n] = \cos[2\pi f_c n/f_s]$$

$$x_{am}[n] = \cos[2\pi f_1 n/F_s] \cdot \cos[2\pi f_c n/F_s] + \cos[2\pi f_2 n/F_s] \cos(2\pi f_c n/F_s]$$

$$= \frac{1}{2}\Big[\cos(2\pi(f_1+f_2)n/F_s) + \cos[2\pi(f_1-f_2)n/F_s) + \cos[2\pi(f_2+f_c)n/F_s) + \cos[2\pi(f_2-f_c)n/F_s]\Big]$$

$$\hat{X}_{am}(\theta) = \delta(\theta - 2\pi(f_1+f_2)n/F_s) + \delta(\theta - 2\pi(f_1-f_2)n/F_s) + \delta(\theta - 2\pi(f_2+f_c)n/F_s) + \delta(\theta - 2\pi(f_2-f_c)n/F_s)$$

Question 5

A. $H\{a_1 x_1[n] + a_2 x_2[n]\}$

$= n\,[a_1 x_1[n] + a_2 x_2[n]]$

$= a_1(n x_1[n]) + a_2(n x_2[n]))$

$= a_1 y_1[n] + a_2 y_2[n]$

B. $y_1[n] = n X[n]$

$y_2[n] = 1 - nX[n+1]$

$y_3[n] = n X[n] + X[n-1]$

For $y_1[n]$ i. is Causal

ii. Not time invariant

iii. is memoryless

For $y_2[n]$ i. is Causal

ii. is not time invariant

iii. is not memoryless

For $y_3[n]$ i. is Causal

ii. Not time invariant

iii Not memoryless