ELEC3104: Digital Signal Processing

S1, 2017

Tutorial-Laboratory Problem Sheet 3

Question 1

A digital IIR bandpass filter is required. The specifications are as follows: sampling rate of 7 kHz, passband edge frequencies at 1.4 kHz and 2.1 kHz, stopband edge frequencies at 1.05 kHz and 2.45 kHz, passband ripple of 0.4 dB and a minimum stopband attenuation of 50dB.

[Analytical Component]

A. Design a Butterworth filter that satisfies these requirements, using bilinear transformation to convert the analogue system to a discrete time system, and obtain its transfer function.

[Laboratory Component]

B. Design this filter in MATLAB and plot its frequency response to verify that the requirements are met. Compare the transfer function obtained in MATLAB to the one you derived in part A.

Question 2

The simplest approach to FIR filter design is to simply truncate (apply a finite-length window) an infinite impulse response to a finite number of terms. The function *fir1*()can be employed to design windowed FIR digital filters in MATLAB and yields a linear-phase design. You are required to design a linear-phase low-pass FIR filter with the following specifications: passband edge = 2 kHz, stopband edge =2.5 kHz, passband ripple of 0.005, stopband ripple of 0.005, and a sampling rate of 16 kHz.

A. Design a linear-phase FIR low-pass filter using a rectangular window to meet the specifications above. Plot its magnitude and phase responses and show the filter coefficients in a tabular form. Does your design meet the specifications? If it does not, adjust the filter order until the design meets the specifications. What is the order of the filter meeting the specifications?

B. Redesign the filter using a Hamming window and compare the magnitude response and impulse response to those obtained in part A. Do you observe what you expect?

C. Record a sample of your voice (about 1 to 3 secs) and the plot the spectrogram.

D. Filter the recorded voice signal using the filters you have designed and plot the spectrogram of the filtered output and compare to the spectrogram of the input.

Question 3

A multiplexer is a device that takes more than one signal as its input and combines them together to create a single multiplexed signal such that at some later stage (usually after transmission) the original signals can be recovered from the multiplexed signal. Your task is to implement a crude multiplexer that takes in three signals sampled at 2000Hz each and combine them to form a single signal that is sampled at 16000Hz. You can accomplish this by resampling each of the three signals to the final sampling rate of 16kHz and then shifting the spectra of the second and third signal such that all three spectra do not overlap and finally add them. i.e., if $x_1[n]$, $x_2[n]$ and $x_3[n]$ are the three inputs to your multiplexer sampled at 2000Hz and $y_1[n]$, $y_2[n]$ and $y_3[n]$ are obtained by resampling $x_1[n]$, $x_2[n]$ and $x_3[n]$ respectively to the new sampling rate of 16kHz, your system must produce an output $x[n]$ sampled at 16kHz such that

$$\hat{x}(\theta) = \begin{cases} \hat{y}_1(\theta), & -\theta_1 \leq \theta \leq \theta_1 \\ \hat{y}_2(\theta - \theta_3), & \theta_2 \leq \theta \leq \theta_4 \\ \hat{y}_2(\theta + \theta_3), & -\theta_4 \leq \theta \leq -\theta_2 \\ \hat{y}_3(\theta - \theta_6), & \theta_5 \leq \theta \leq \theta_7 \\ \hat{y}_3(\theta + \theta_6), & -\theta_7 \leq \theta \leq -\theta_5 \end{cases}$$

Where, $\hat{x}(\theta)$ is the DTFT of $x[n]$, $\hat{y}_k(\theta)$ is the DTFT of $y_k[n]$ and $\theta_n = 2\pi n \cdot \frac{1000}{16000}$.

Let,

$$x_1[n] = \cos\left(2\pi \frac{200}{2000} n\right) + \cos\left(2\pi \frac{450}{2000} n\right)$$
$$x_2[n] = \cos\left(2\pi \frac{700}{2000} n\right) + \cos\left(2\pi \frac{550}{2000} n\right)$$
$$x_3[n] = \cos\left(2\pi \frac{100}{2000} n\right) + \cos\left(2\pi \frac{850}{2000} n\right)$$

(**Hint:** To aid in the shift of the spectra consider using modulation, i.e., analyse the Fourier transform of the product of a signal with a sinusoid: $x[n] \cdot \cos an$)

[Analytical Component]

A. Draw the block diagram of the system that you come up with and sketch the magnitude spectra at all the stages in your system.

[Laboratory Component]

B. Generate 10000 samples of the three input signals and implement the system you came up with. Plot the magnitude spectra of all the signals involved.

Question 4

Telephone touch pads generate dual tone multi-frequency (DTMF) signal to dial a telephone. When any key is pressed, the tones of the corresponding column and row (see table below) are generated, hence a dual tone. As an example, pressing the **8** button generates the tones 852 Hz and 1336 Hz summed together.

Table 1: DTFM frequencies

Frequencies	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	**8**	9
941 Hz	*	0	#

The frequencies were chosen to avoid harmonics and no frequency is a multiple of another, the difference between any two frequencies does not equal any of the frequencies, and the sum of any two frequencies does not equal any of the frequencies. This makes it easier to detect exactly which tones are present in the dial signal in the presence of line distortions.

DTMF Decoding: There are several steps to decoding a DTMF signal:

- Divide the signal into shorter time segments representing individual key presses
- Determine which two frequency components are present in each segment
- Determine which button was pressed (0 to 9, #)

It is possible to decode DTMF signals using simple FIR filter bank (many filters of varying centre frequencies and bandwidths). The filter bank required in this case consists of filters that each passes only one of the DTMF frequencies and when the input is a DTMF signal. When the input to the filter bank is a DTMF signal, the outputs of two filters should be larger than the rest. The two corresponding frequencies must be detected in order to determine the DTMF code. A good measure of the output levels is the average power at the filter outputs. This can be calculated by squaring the filter outputs and averaging over a short time interval.

A. Design the filter bank (7 bandpass filters, centre frequencies are given in Table 1, you may choose the bandwidth for each filter) using any of the filter design techniques that you have learnt. Sampling rate: 8000Hz. (ii) Plot the magnitude response (use freqz command) of all the 7 filters all in one panel to ensure you have designed them correctly.

B. Generate a signal consisting 770 Hz and 1336 Hz (summed together) and pass it through the filter bank and calculate the power at the output of each of the band pass filter. Plot a graph of output signal power (y axis) vs centre frequencies (x axis) of the filters.

C. Check if you are able to detect the two input frequencies. Repeat with various input frequency combinations as per Table 1 to ensure you can detect all keys.