Working from home, household production, and durable good consumption dynamics*

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Abstract

I decompose the durable good consumption increase during 2020-2021 using a household production model with working-from-home. Durable good consumption is susceptible to business cycles; in past recessions, durable good consumption either decreased or slowed down. However, durable good consumption was very robust during and after the COVID-19 pandemic. I build a household production model with working from home and estimate the model using a Bayesian approach. Using Kalman smoother, we can then decompose the increase in durable good consumption into different channels. Working from home can account for up to one-third of the durable good consumption increase, and substitution between nondurable and durable can account for another one-third of the increase.

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1 Introduction

The durable good consumption is susceptible to business cycles. In every post-WWII recession, we observe a decrease or slow down in durable goods consumption. However, the durable goods consumption increased during the recession caused by the COVID-19 pandemic. In this paper, we propose a model that cooperates working from home and household production to explain the durable good consumption dynamic during the pandemic.

Durable good consumption is a significant part of consumer expenditure. Figure 1 shows the durable good consumption expenditure in real term. The durable good consumption has been increasing since the 1960s, however, during each recession, we observed a decrease or at least a slow down. Berger and Vavra (2015) and Ahmed and Cassou (2016) pointed out that the response of durable good consumptions to economic shocks is state dependent. Durable good consumption response more sluggishly to economic shocks during recessions.

Time spent working from home has been slowly increasing since the early 2000s and jumped at early 2020, as shown in Figure 2. The reason is clear; many companies let their employees work remotely to mitigate the risk of covid. If we only consider the pre-pandemic era, both the fraction of full-time workers who only work from home and the average hours hybrid workers spend working from home have increased. In the United States, full-time workers spend about 4 hours per week working from home pre-pandemic. Among those who solely work from home, the average working hours per day increased from 4 hours in 2003 to 6.4 hours in 2019. The fraction of these workers also increased from 3.7% in 2003 to 7% in 2019, almost doubled, and the fraction of hybrid workers increased from 10% to 12.5% during the same period. Fisher (2007) showed that making household capital complementary to business capital and labor in market production is the key to reconciles RBC theory with the investment dynamics(e.g. household investment leads business cycle). Fisher (2007) argus that labor need to be regenerated by, like the maintenance needed to keeping business capital working. Workers need to rest, relax, and take personal care to supply labor effectively, and all of those activities requires household capital/durable goods. Working from home could have strengthened this complementarity and, therefore, could at least partially explain the sharpe increase in durable goods consumption during the pandemic.

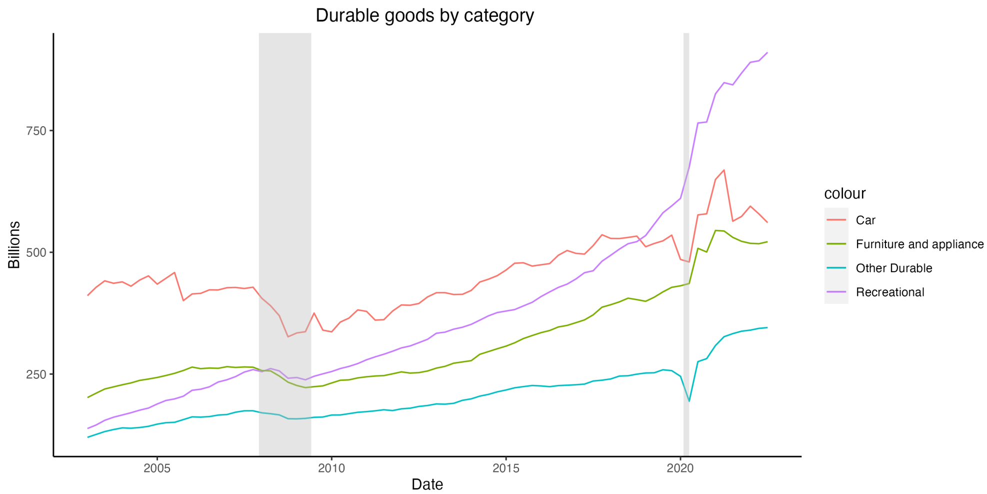

Figure 1: Real durable goods consumption expenditure

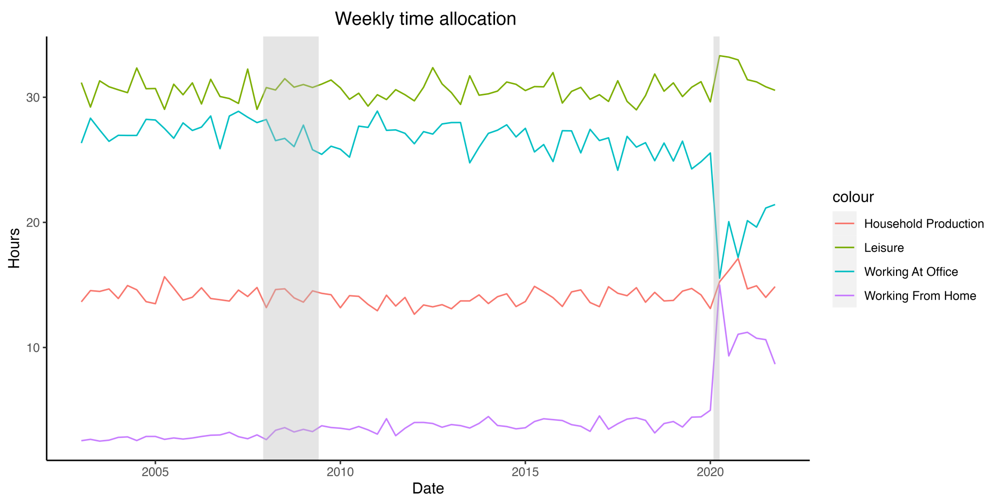

Figure 2: Time allocation

There are two ways of modeling durable goods in the literature. The first one is to assume durable goods stock provides utility flows in each period. Agents do not need to put in any effort/time. Typically, this modeling choice is always combined with an inelastic labor supply; agents are endowed with a unit of labor and do not value leisure. The other way is that agents use durable goods to produce homemade consumption goods. This approach is often seen when labor supply is endogenous and agents value leisure. In this paper, we follow the second modeling choice since one of our assumptions is that labor supply is endogenous, and workers can benefit from household capital while working from home. Household capital is complementary to labor productivity while working from home in two perspectives. First, household capital such as home office equipment directly affects labor productivity. Second, recreational durable goods can help agents produce "effective labor," which is used as an input for market production. This is that labor need to be regenerated.

1.1 Durable goods dynamic and stimulus

In the past two decades, the US experienced three recessions. The government made a direct cash payment to households to mitigate the effect of economic downturns. The 2001 rebate was a payment of the benefit of a new tax bracket; the previous 15 percent tax bracket was reduced to 10 percent. The 2008 rebate is a one-time stimulus payment; even though it was administered through the tax system, similar to the 2020 stimulus, it was not related to any change in tax policy. Many papers have shown that direct cash transfers can stimulate non-durable consumption and do not have much of an effect on durables. The reason is that durable consumption is lumpy, and there is an adjustment cost. This makes them can be easily postponed, especially during recessions.

(Johnson et al., 2006) utilized the random timing of the arrival of the 2001 rebate to estimate how people spent their 2001 rebate. The timing of the sending of each rebate was determined by the second-to-last digit of the SSN. They found that households spent 20-40 percent of their rebates on non-durable goods during the three months they received their rebate. They also found no significant evidence of response in durables such as automobiles or large household equipment, which again might reflect the relatively small size of the average refund per household and the greater volatility of expenditure on such durables. (Shapiro and Slemrod, 2003)report that only 21.8 percent of households increased their spending in response to the 2001 rebate. The amount spent on durable goods would be even smaller. (Agarwal et al., 2007) find that households first improved their balance sheet by saving more or paying off credit card debt, then increased spending in the following nine months. The increased amount is $200 or about 40 percent of the average household rebate.

Recent researches about the effect of the 2020 CARES act stimulus also find similar patterns: most of the stimulus payments are saved or used to pay off debt. (Coibion et al., 2020) document that only 15 percent of recipients say that they spent (or planned to spend) most of their payment, with the large majority of respondents saying instead that they either mostly saved it (33 percent) or used it to pay down debt (52 percent). When asked to provide a quantitative breakdown of how they used their checks, U.S. households report having spent approximately 40 percent of their checks on average, with about 60 percent of the average check being saved or used to pay down debt. Little of the spending went to large durable goods. (Baker et al., 2020) used transaction-level data to analyze how recipients spend their 2020 stimulus payments. Relative to the effects of previous economic stimulus programs in 2001 and 2008, they find faster effects, smaller increases in durables spending, more significant increases in spending on food, and substantial increases in payments like rents, mortgages, and credit cards. They also find substantially smaller impacts on durables spending and confirm this in the survey of users.

Previous research found mixed results on the response of durable spending of stimulus payments. Most papers report either negligible effect see (Shapiro and Slemrod, 2003), (Johnson et al., 2006), (Graziani et al., 2016), (Shapiro and Slemrod, 2009),or auto and related services contribute to most of the durable spending responses, see (Parker et al., 2013), (Kanishka Misra et al., 2014),(Kueng, 2018).

Research has shown that durable spending is less responsive to economic stimulus, and households tend to either save or spend the stimulus payment on nondurables. The sharp increase since early 2020 in durable goods spending can be partially explained by shifting consumer demand; substitute market consumption for household production. Precautions such as social distancing and lockdowns limited consumers' choice of consumption and services. Therefore, they spend more time than usual in and around their home to take care of their children, work or study from home, engage in home production, or enjoy leisure (Tauber and Van Zandweghe, 2021). There is a large literature focusing on household production started by (Becker, 1965). Most papers including household production to explain allocation of capital and labor between market and household activities and their business cycle implications (Heckman, 1974),(McGrattan et al., 1997),(Gomme et al., 2001),(Greenwood, 1991),(Benhabib et al., 1991),(Boerma and Karabarbounis, 2021),(Fang and Zhu, 2017),(Chang, 2000).

The widespread adoption of working from home during the pandemic permanently changed households' consumption choices, time allocation, and housing choices. In the economics literature, residential housing is often modeled as durable goods. There are plenty of papers exploring the impact of working from home at household's housing choice, and residential and commercial real estate. Delventhal et al. (2022), Brueckner et al. (2021), Stanton and Tiwari (2021)showed that jobs moved to the core of the city, while households moved to the periphery; average real estate prices fall, with declines in core locations and increases in the periphery. However, there are no studies exploring the effect of working from home on the consumption of durable goods such as furniture, appliance, recreational, vehicle, etc. This paper shows that the complementarity between household and business capital is an important factor in explaining the durable good consumption dynamics, especially when there is a sudden transition to working from home.

The remainder of this paper is organized as follows. Section 2 presents a model that incorporates household production and complementarity between household capital and market capital. Section 4 describes our estimation strategy, results and discusses their implication. Section 5 concludes.

2 Model

The extended household production model presented in this section consists of households and firms. We first describe the full model, and then we discuss the implication of our assumptions and the economic mechanism of the model. The key difference from the traditional household production model is that households now can choose how to allocate their time between working from home, working at the office, and household production. Following Fisher (2007), we allow labor productivity while working from home to be positively correlated with housheold capital stock.

2.1 Household

The representative household has preference over a consumption good that is purchased from the market; hereafter, we refer to it as market consumption, $c_{m,t}$, a consumption good that is produced at home, $c_{h,t}$, hours spend at working from home, $h_{m_1,t}$, hours spent working at the office, $h_{m_2,t}$, and hours spend on household production, $h_{h,t}$.

Households maximize the following expected utility take interest rate and wage as given.

$$Max\,\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t}\{U(C_j) - V(H_j)\} \tag{1}$$

Where C_j denotes the aggregate consumption at period, j and H_j denote the total hours spent on working and household production. $U(\cdot)$ denotes the instantaneous utility flow from consumption, and $V(\cdot)$ denotes the disutility from working and household production. Household maximize their expected utility, equation 2, conditional on time t information.

$$c_{h,t} = g(k_{h,t}, h_{h,j}) = z_{p,t} k_{h,t}^{\alpha_h} h_{h,t}^{1-\alpha_h} \tag{2}$$

$$C_t = (\omega_t c_{m,t}^{\epsilon_c} + (1-\omega_t) c_{h,t}^{\epsilon_c})^{\frac{1}{\epsilon_c}} \tag{3}$$

$$H_t = h_{m_1,t} + h_{m_2,t} + h_{h,t} \tag{4}$$

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{5}$$

$$V(H_t) = \frac{H_t^{1+\phi}}{1+\phi} \tag{6}$$

Equation 2 and 3 represent the household production technology and the final consumption good aggregator. $k_{h,t}$ is the household capital stock and $h_{h,t}$ is the time spend on household production at period t. The aggregate consumption, C_t, is given by a CES aggregator function, in which ω_t is the weight on market consumption, and ϵ_c governs the elasticity of substitution between market consumption and home produced good. ω_t follows a log AR(1) process such that $log(\omega_t) = (1-\rho_\omega)\mu_\omega + \rho_\omega log(\omega_{t-1}) + \epsilon_\omega$ where $\epsilon_\omega \sim \mathcal{N}(0,\, \sigma_\omega^2)$. This process captures the variation of household production technology required to produce the final consumption good. For example, many consumption goods become either inaccessible or undesirable during the pandemic because of lockdown or the risk of COVID. γ and ϕ are the relative risk aversion and Frisch labor supply elasticity.

Assumption 1 : Each period, household supply one unit of effective labor.

$$1 = (k_{h,t}^{\alpha_l} h_{m_1,t}^{\epsilon_l} + z_t * h_{m_2,t}^{\epsilon_l})^{\frac{1}{\epsilon_l}} \tag{7}$$

ϵ_l governs the elasticity of substitution between working from home and at the office. z_t denotes the relative productivity of working at the office. It follows a log AR(1) process such that $log(z_t) = (1-\rho_z)\mu_z + \rho_z log(z_{t-1}) + \epsilon_z$ where $\epsilon_z \sim \mathcal{N}(0,\, \sigma_z^2)$. If $z_t > 1$ then agents are more productive at their office; however, if $z_t < 1$, then they are more productive working from home. This assumption implies that workers can endogenously decide where to work, home or office; however, they must complete a certain amount of tasks.

Assumption 2 : Household face a quadratic transaction cost when adjusting their household capital stock.

$$A(k_{h,t+1}, k_{h,t}) = \frac{\Phi}{2}(\frac{k_{h,t+1}}{k_{h,t}} - 1)^2 k_{h,t} \tag{8}$$

This assumption implies that if the household decides to maintain its current level of household capital, then there is no transaction cost. However, if the household decides to adjust its household capital stock, either increase or decrease, there will be a transaction cost. For example, there is no transaction cost to maintain a car, but there is a transaction cost to replace a car with a new one. We need transaction costs here because this will prevent households from using durable goods as a saving vehicle. In reality, durable good stock adjustment is lumpy and always comes with a cost.

$$c_{m,t} + k_{m,t+1} + k_{h,t+_1} + A(k_{h,t+1}, k_{h,t}) = (1 + r_t)k_{m,t} + (1 - \delta_h)k_{h,t} + wage_t * EL_t \tag{9}$$

The above equation shows the budget constraint. $k_{m,t}$ is the saving vehicle, and $k_{h,t}$ is the stock of household capital. EL_t represents the effective labor supply; later, we will fix it to one.

2.2 Firms

The market is perfectly competitive, and firms produce market consumption goods, durable goods, and investment goods with capital and effective labor to maximize profit.

$$Y_t = z_{p,t} k_{m,t}^{\alpha_p} * EL_t^{1-\alpha_p} \tag{10}$$
$$r_t = \alpha_p z_{p,t} k_{m,t}^{\alpha_p - 1} EL_t^{1-\alpha_p} - \delta_m \tag{11}$$
$$wage_t = (1 - \alpha_p) z_{p,t} k_{m,t}^{\alpha_p} EL_t^{-\alpha_p} \tag{12}$$

Y_t is the total output of period t, r_t and $wage_t$ are given by the first order conditions. EL_t equals to one for all periods. z_p is the total factor productivity, it follows a log AR(1) process, $log(z_{p,t}) = (1 - \rho_{z_{p,t}})\mu_{z_p} + \rho_{z_p} log(z_{p,t-1}) + \epsilon_{z_p}$ where $\epsilon_{z_p} \sim \mathcal{N}(0,\, \sigma^2_{z_p})$.

$$k_{m,t+1} = (1 - \delta_m)k_{m,t} + I_{m,t} \tag{13}$$
$$k_{h,t+1} = (1 - \delta_h)k_{h,t} + I_{h,t} \tag{14}$$

Equation 13 and 14 shows the law of motion of business capital and household capital. $I_{m,t}$ is the business capital investment and $I_{h,t}$ is the durable good consumption.

2.3 Equilibrium

We first rewrite the household's objective function as follows.

$$\begin{aligned}\mathbf{V}(X,S) = &\max_{c_m,c_h,k'_m,k'_h,h_{m1},h_{m2},h_h} U(C) - V(h_h,h_{m1},h_{m2}) \\ &+ \beta\,\mathbb{E}_{|S}\{\mathbf{V}(X',S')\}\end{aligned}$$

where $X = \{k_m, k_h\}$ and $S = \{z, \omega, z_p, r, wage\}$. The optimal allocation of household's problem is given by the following conditions and budget constraints.

$$\beta\,\mathbb{E}_{|S}[(1+r')U_{C'} * C'_{c'_m}] = U_C * C_{c_m} \tag{15}$$

$$\beta\,\mathbb{E}_{|S}[U_{C'} * C'_{c'_h} * g_{k'_h} + (1-\delta_h - A'_2) * U_{C'} * C'_{c'_m} + \frac{V'_{h_{m_1}} EL'_{k'_h}}{EL'_{h'_{m_1}}}] = U_C * C_{c_m}(1+A_1) \tag{16}$$

$$EL_{h_{m_1}} = EL_{h_{m_2}} \tag{17}$$

$$U_C * C_{c_h} * g_{h_h} = H^{\phi} \tag{18}$$

Equation 15 is the Euler equation that governs the intertemporal substitution if households use market capital as the saving vehicle. Equation 16 is the Euler equation that shows, at the optimal solution, how to allocate resources between market consumption of the current period and household capital of the next period. Notice that there are three parts in the expectation operator of equation 16 because household capital can not only be used for household production but also effective labor. So, investing in the next period's household capital/durable goods can improve the marginal household production and labor productivity while working from home the next period. Equation 17 implies that, at optimal allocation, the household is indifferent between working from home and working at the office; the marginal production of effective labor is the same at either place. Equation 18 implies that the household is indifferent between working, either at home or at the office, and household production.

A recursive equilibrium is given by household policy functions that solve the optimization problem : $f_{c_m}(X,S)$, $f_{c_h}(X,S)$, $f_{h_{m1}}(X,S)$, $f_{h_{m2}}(X,S)$, $f_{h_h}(X,S)$, $f_{k'_m}(X,S)$, $f_{k'_m}(X,S)$. r and $wage$ that maximize firms profit and clear the market.

3 Analysis of the model

Our model allows households to allocate their resources between market consumption, durable good consumption, and saving; and allocate their time between working from home, working at the office, and household production. At optimal solution, households allocate their resources such that the marginal utility from market consumption equals the discounted expected marginal utility of consumption of the next period, shown by Equation 15. There are three ways that households can utilize their durable good stock/household capital. First, they can use it for household production; however, homemade goods are not tradable and cannot be stored across periods. Second, households can liquidize their durable goods stock for market consumption. Lastly and most importantly, household capital can improve labor productivity while working from home. Some durable goods can directly improve labor productivity, such as desks, chairs, computer monitors, etc. Some durable can indirectly improve labor productivity; for example, workers need to rest, relax, and take personal care to supply labor effectively; therefore, recreational durable goods can indirectly increase labor productivity. The indirect channel improves the overall labor productivity; however, in our model, we only include the direct channel since we cannot identify these two channels separately. Fisher (2007) focus on the indirect channel and do not distinguish between working from home and working at the office. In our model, the effective labor supply, EL_t, is exogenously determined. This is consistent with the literature where households are endowed with one unit of time and do not value leisure. However, we allow household to endogenously determined how to allocate their time while the effective labor they have to supply is exogenously determined.

In this section, we discuss the mechanism that the model generates durable consumption increase while the pandemic hit the economy and how agents allocate their time between working from home, working at the office, and household production. The COVID-19 pandemic affected the economy from many perspectives. Many firms have let their employees work from home since March 2020 to mitigate the risk of getting COVID. This transition from working at the office to working from home is endogenous; however, we should treat this transition as a response to a negative relative labor productivity shock. In other words, where to work is an endogenous outcome that firms and their employees jointly determine. In the early stage of the pandemic, many market consumptions became unavailable or undesirable because of the shelter-in-place order, the risk of getting COVID, and many shops being closed or understaffed. Therefore, the pandemic will affect the consumption weight in the final consumption good aggregate function, ω.

We model the process of ω as AR(1); however, it not only captures the fluctuation of consumption but also the relative price change. To see this point, consider the following one-period static consumption choice model where agents allocate their resources between two consumption goods.

$$U(c_1, c_2) = \frac{[\omega c_1^{\epsilon} + (1-\omega)c_2^{\epsilon}]^{\frac{1-\gamma}{\epsilon}}}{1-\gamma} \tag{19}$$

$$p_1 c_1 + p_2 c_2 = w \tag{20}$$

The optimal allocation is that the agent allocate ω proportion of their resources to consumption good 1 and $1-\omega$ proportion of their resources to consumption good 2.

$$c_1 = \frac{\omega w}{p_1} \tag{21}$$

$$c_2 = \frac{(1-\omega)w}{p_2} \tag{22}$$

From the simplified example we can see that we couldn't separately identify the consumption weight ω and the relative price between two consumption goods from the consumption data only.

3.1 Static Analysis of time allocation

The time allocation choice of households is a function of household capital stock. We can obtain the following policy functions using Assumption 1 and equation 17. In our model, households can allocate their time between working from home, working at the office, and household production. h_{m_1} and h_{m_2} represent the time allocated to working from home and working at the office, respectively.

$$h_{m_1} = \frac{EL}{(k_h^{\frac{\epsilon\alpha}{\epsilon-1}} * z^{\frac{1}{1-\epsilon}} + k_h^{\alpha})^{\frac{1}{\epsilon}}} \tag{23}$$

$$h_{m_2} = \frac{EL}{(z^{\frac{\epsilon}{\epsilon-1}} * k_h^{\frac{\alpha}{1-\epsilon}} + z)^{\frac{1}{\epsilon}}} \tag{24}$$

$$\tag{25}$$

From the above two equations, we can see that h_{m_1} is a decreasing function in z, the relative productivity of working at the office. The intuition is that if the relative productivity of working at the office is higher, households will allocate less time to working from home. However, this does not necessarily mean that they will allocate more time to working at the office since h_{m_2} is not monotonically increasing in z.

Proposition 1 *There exist a z^* as function of k_h such that if $z \geq z^*$ then h_{m_2} is decreasing in z; if $z \leq z^*$ then h_{m_2} is increasing in z*

See the proof in the appendix.

The intuition of Proposition 1 is that if workers are very productive at their office, then increasing the relative productivity further will decrease the time they spend on both working from home and working at the office because the effective labor they have to supply is fixed.

We model the COVID-19 pandemic as shocks on the relative labor productivity, TFP, consumption share in the final consumption good aggregator, and household productivity. Later, we will let the effective labor supply vary exogenously, which will capture the unemployment fluctuations

3.2 Benchmark Calibrations

In this section, we calibrate our model to match the pre-pandemic working hours and consumption allocation we have observed in PCE (Personal Consumption Expenditure) and ATUS (American Time Use Survey).

Table 1 shows the quarterly calibrated parameters. Following the literature, we can pre-set many parameters such as β, the discount factor, is set to be 0.98, α_m, the capital share in the market production function, is set to be 0.33, etc. However, we can rarely find any information about parameters such as z, the relative productivity of working in the office, α_l, the capital share in the effective labor aggregate function; or there is a wide range for parameters like ϵ_c which governs the EIS between market consumption and home made goods. In the literature, the range of ϵ_c is between -0.65 and 0.5 Been et al. (2020) Rupert et al. (1995). In our model, the final consumption good is aggregated by market goods and household productions using a CES function. We calibrate ϵ_c to be -0.5 since we do not believe that household production could replace the market consumption very well. Market consumption and household production are more like complements than substitutes. The intuition is that in most cases, we transform a fixed ratio of c_m and c_h to final consumption good; however, if there is a shortage of c_m we certainly cannot produce even near the same amount of final consumption good by simply increase c_h.

Table 1: Parameter Values

Parameter	**Description**	**Value**
$\mu_{hp_{tfp}}$	Household productivity	0.000
μ_z	Relative productivity	0.362
μ_ω	Market Consumption weight	-0.433
μ_{z_p}	TFP	0.000
α_m	Capital weight in Market production	0.330
α_h	Capital weight in Household production	0.140
α_l	Capital weight in EL	0.097
β	Discount factor	0.980
δ_m	Depreciation rate of Market Capital	0.025
δ_h	Depreciation rate of Household Capital	0.025
γ	Relative risk aversion	2.000
EL	Effective labor	1.000
ϕ	Frisch Labor supply elasticity	1.500
ϵ_c	EIS between c_h and c_m	-0.500
ϵ_l	EIS between WFH and WIO	0.950
f_x	Household Capital Adjustment cost	1.000

Table 2: Steady state

Variable	**Value**
kh_i	0.157
cm	2.015
ch	0.512
k_m	19.304
k_h	6.284
h_{m_1}	0.104
h_{m_2}	0.562
h_h	0.333
$output$	2.66

Table 3: CORRELATION OF SIMULATED VARIABLES

$VARIABLE$	I_{k_h}	I_{k_m}	c_m	h_{m_1}	h_{m_2}	h_h
I_{k_h}	1.0000	-0.1712	-0.0732	0.6163	-0.6281	0.3130
I_{k_m}	-0.1712	1.0000	-0.1794	-0.4138	0.4265	0.0770
c_m	-0.0732	-0.1794	1.0000	-0.0325	0.0007	-0.7500
h_{m_1}	0.6163	-0.4138	-0.0325	1.0000	-0.9975	-0.0012
h_{m_2}	-0.6281	0.4265	0.0007	-0.9975	1.0000	-0.0020
h_h	0.3130	0.0770	-0.7500	-0.0012	-0.0020	1.0000

Table 4: Real Data Correlation

$VARIABLE$	I_{k_h}	I_{k_m}	c_m	h_{m_1}	h_{m_2}	h_h
I_{k_h}	1.00	0.54	0.37	0.25	-0.25	-0.01
I_{k_m}	0.54	1.00	0.52	-0.36	0.40	-0.06
c_m	0.37	0.52	1.00	-0.68	0.60	0.09
h_{m_1}	0.25	-0.36	-0.68	1.00	-0.96	-0.07
h_{m_2}	-0.25	0.40	0.60	-0.96	1.00	0.10
h_h	-0.01	-0.06	0.09	-0.07	0.10	1.00

Table 5 shows the variance decomposition .

Table 5: VARIANCE DECOMPOSITION SIMULATING ONE SHOCK AT A TIME (in percent)

	z_l	ω	z_{tfp}	z_{hp}	$Tot.lin.contr.$
I_{k_h}	30.84	13.98	40.77	0.43	86.02
I_{k_m}	13.85	5.12	79.06	1.48	99.51
c_m	0.05	71.73	33.62	2.73	108.13
h_{m_1}	101.13	0.07	0.88	0.00	102.07
h_{m_2}	100.39	0.20	2.58	0.00	103.18
h_h	0.11	89.11	0.29	10.03	99.55

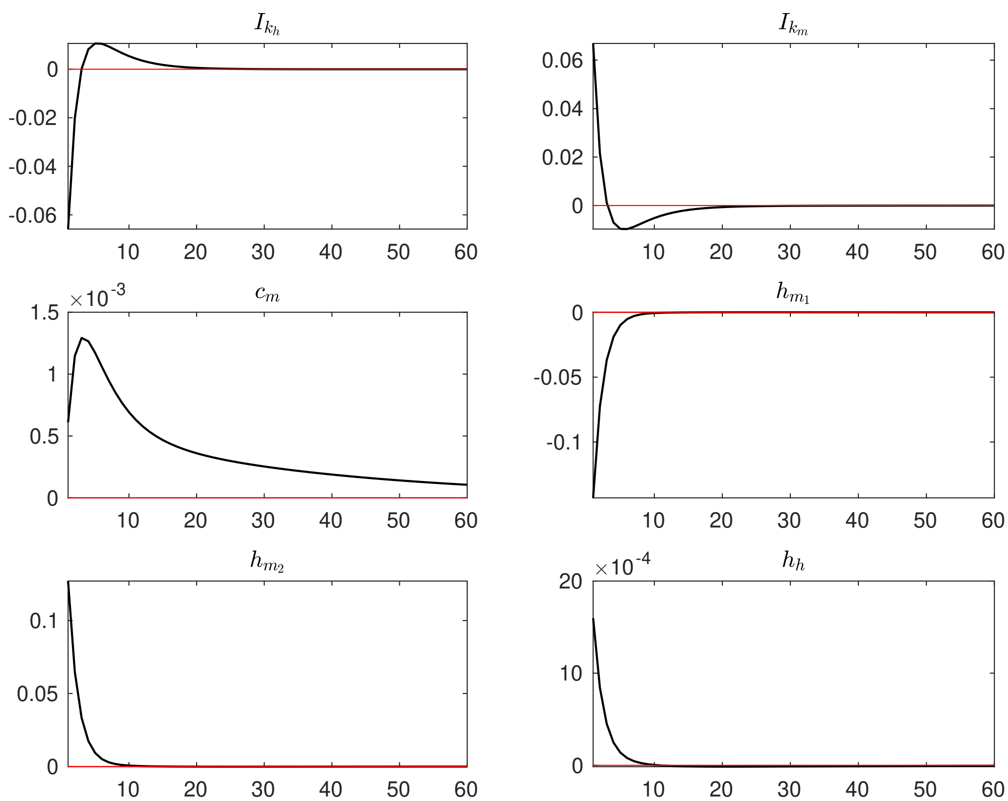

Figure 3: Impulse response functions (orthogonalized shock to z_l).

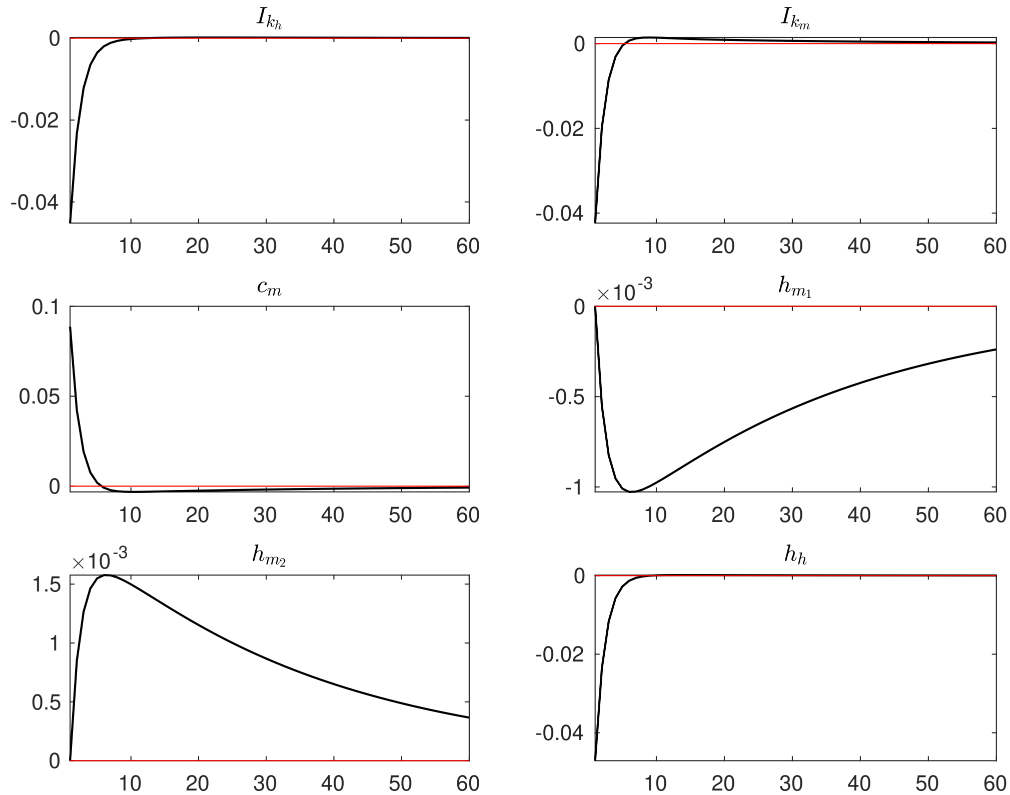

Figure 4: Impulse response functions (orthogonalized shock to ω).

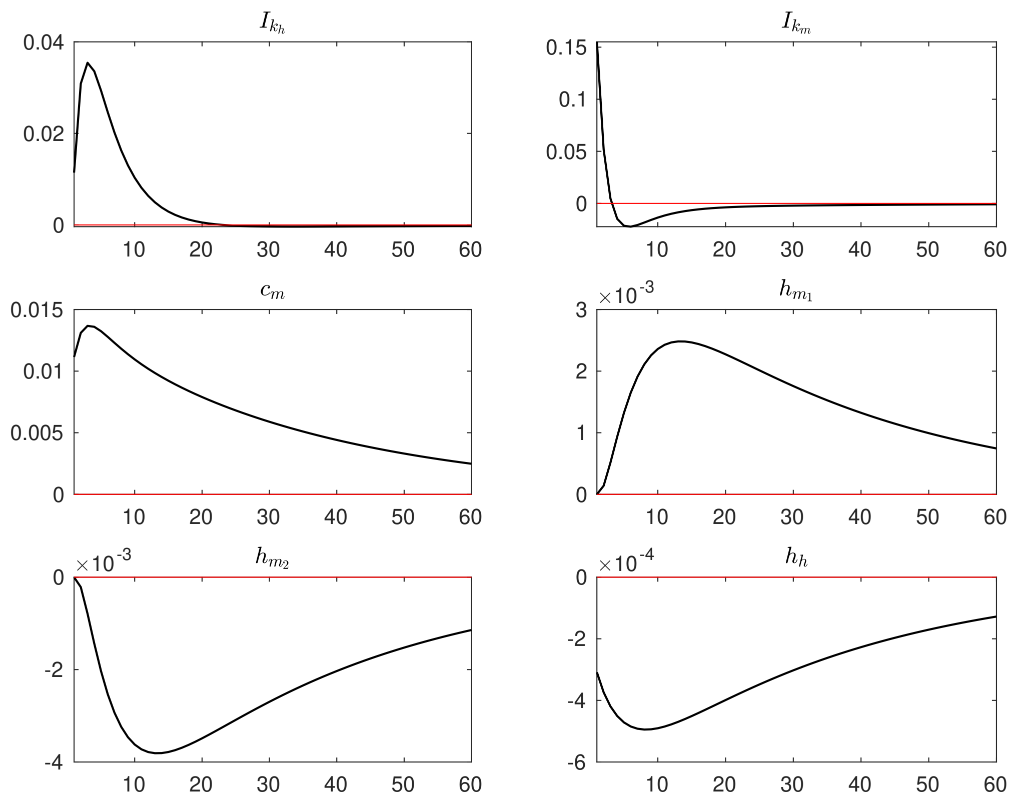

Figure 5: Impulse response functions (orthogonalized shock to z_{tfp}).

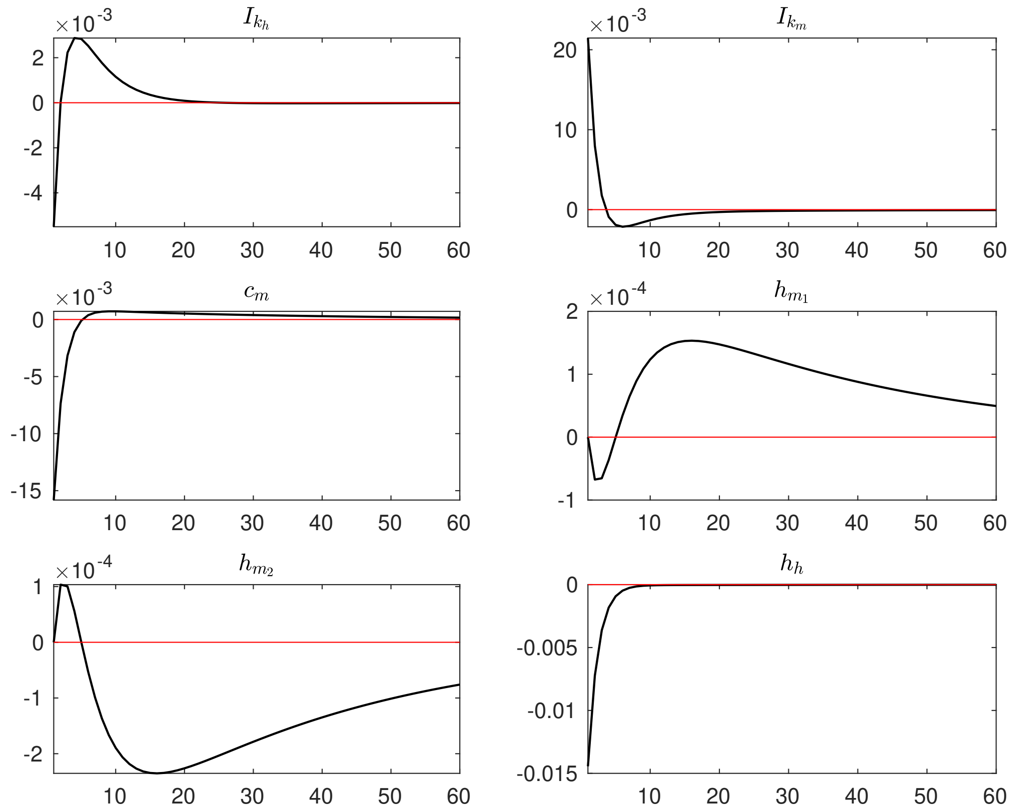

Figure 6: Impulse response functions (orthogonalized shock to z_{hp}).

3.3 Prior predictive analysis

4 Estimation

We use quarterly GDP, durable good consumption, nondurable good consumption, and hours spent working from home 2003:1–2021:4 to estimate the following parameters. ϵ_c, the parameter that governs the elasticity between market consumption and household production, and ϵ_l, the parameter that governs the elasticity between working from home and working at the office. α_m, the capital share in the market production function, α_h, the household capital share in the household production function, and α_l, the household capital share in "effective labor aggregator ", Equation 7. We also estimate four exogenous processes of TFP, discount rate, market consumption weight, and relative labor productivity at the office.

4.1 Data

Our durable and nondurable consumption data is from the real personal consumption expenditure on FRED. [1]. As we mentioned above, durable good includes four main categories, furniture and appliance, recreational, motor vehicles and parts, and other durables. In our model, c_m represents all nondurable market consumption. Map to the real data, we use

[1]FRED

the sum of nondurable goods consumption and services as our aggregate nondurable goods consumption, c_m. We obtain the time allocation data from the American time use survey (ATUS). If an activity is working and the place is the interviewee's home, then we take it as working from home, h_{m_1}; however, if the activity is performed at the working place, then we define it as h_{m_2}. All variables are quarterly; we use one side HP filter to obtain the cycle part of our data. The reason to use one side HP filter is that households make decisions only conditional on past information; however, HP-filter implicitly uses "future information" to decompose data.

4.2 Estimation

Bayesian estimation starts with prior distributions describing the information we have about the parameters before observing any real data. Then the observed data will be used to update the prior distribution via the Bayesian theorem to the posterior distributions of model parameters. Table 7 shows the prior specification. The first four rows are the prior distributions of the innovations in the relative productivity of working at the office, market consumption weight, TFP, and discount factor. The inverse gamma distribution is widely used as the conjugate prior distribution for the unknown variance of a normal distribution. We assume all four structural shocks follow the inverse gamma distribution with mean 0.05 and infinite standard deviation. We do not have much information on most of the persistence parameters, except for ρ_z; therefore, we set them to be beta distributions with a mean of 0.8 and standard deviation of 0.06 to cover the range between 0 and 1. The relative productivity process should not be too persistent, and we set the mean to be 0.3.

ϵ_c governs the elasticity of substitution between market consumption and household production; we set the prior to be a normal distribution with a mean of -0.3 and standard deviation of 1. This will cover a wide range of possible values between -2.3 and 1.7. We do not believe that ϵ_c could be greater than one because the final consumption aggregation function should be convex. ϵ_l governs the elasticity of substitution between hours spent working from home and at the office; we set the prior to be a beta distribution with a mean of 0.8 standard deviations of 0.1. We choose a prior with a relatively high mean for ϵ_l to reflect the prior information that people spend most of their time working at the office; therefore, we are not far away from a corner solution.

The capital share in the household production function, α_h, and the effective labor production function, α_l, are set to be beta distributions with a mean of 0.2 and standard deviation of 0.04. The reason is that α_h is in a range between 0.08 and 0.36 in the literature, Fisher (2007), Gomme et al. (2001), Benhabib et al. (1991). α_l in Fisher (2007) is 0.19, we set the prior to 0.2. The other reason we set the prior mean of α_h and α_l to be 0.2 is that the household capital is used in both the household production and effective labor; therefore, the sum of α_h and α_l should not be too large in our model.

Table 6: Prior information (parameters)

	Distribution	Mean	Mode	Std.dev.	Bounds*		90% HPDI	
					Lower	Upper	Lower	Upper
σ_{z_l}	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
σ_ω	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{tfp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{hp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
ϵ_c	Gaussian	-0.5000	-0.5000	0.5000	-3.6807	2.6807	-1.3224	0.3224
ϵ_l	Beta	0.8000	0.8462	0.1000	0.1025	0.9999	0.6146	0.9389
α_h	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
α_l	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
μ_z	Gaussian	0.1000	0.1000	0.1000	-0.5361	0.7361	-0.0645	0.2645
μ_ω	Gaussian	-0.5000	-0.5000	0.2000	-1.7723	0.7723	-0.8290	-0.1710
ρ_z	Beta	0.3000	0.2951	0.0500	0.0672	0.6432	0.2205	0.3850
ρ_ω	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{z_p}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{hp}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823

Note: Displayed bounds are after applying a prior truncation of options_.prior_trunc=1.00e-10

Table 7: Prior information (parameters)

	Distribution	Mean	Mode	Std.dev.	Bounds* Lower	Bounds* Upper	90% HPDI Lower	90% HPDI Upper
σ_{z_l}	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
σ_{ω}	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{tfp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{hp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
ϵ_c	Gaussian	-0.5000	-0.5000	0.5000	-3.6807	2.6807	-1.3224	0.3224
ϵ_l	Beta	0.8000	0.8462	0.1000	0.1025	0.9999	0.6146	0.9389
α_h	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
α_l	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
μ_z	Gaussian	0.1000	0.1000	0.1000	-0.5361	0.7361	-0.0645	0.2645
μ_ω	Gaussian	-0.5000	-0.5000	0.2000	-1.7723	0.7723	-0.8290	-0.1710
ρ_z	Beta	0.3000	0.2951	0.0500	0.0672	0.6432	0.2205	0.3850
ρ_ω	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{z_p}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{hp}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823

Note: Displayed bounds are after applying a prior truncation of options_.prior_trunc=1.00e-10

There are some parameters that we do not put into our estimation either because they have been extensively studied in the literature, such as α_m, the capital share in the market production function, or because they are weakly identified in our model, such as z_{hp}, the household production technology and z_l, the relative productivity of working at office. Graph 7 shows the identification strength in log scale. The upper panel shows the identification strength of the parameters based on the Fischer information matrix (stimulated) normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). However, if the prior mean is 0, then it will falsely indicate that the parameter is not identified; for example, in our case, μ_z and μ_{hp} are not identified because the prior mean is 0. We omit these two variables in our estimation because the identification is still very weak even if we change the prior mean.

The lower panel of figure 7 decomposes the effect that is shown in the upper panel. Generally speaking, there are two reasons for weak identification. First, the likelihood function could be relatively flat at some dimensions; therefore, the likelihood does not change with respect to these parameters, or the change is so tiny that we have a flat line in the log-posterior graph. If this is true, we will see minimal values in the lower panel, either standardized by the prior mean (blue bars) or by the prior standard deviation (red bars). However, we can see that this is not the case for our model; the lower panel shows it is unlikely that the likelihood function is flat in any dimension. The second case is that a parameter could linearly pick up the effect of other parameters; in other words, they have the same effect on the likelihood function. ϕ is collinear with other parameters; therefore, it is not identified in our model.

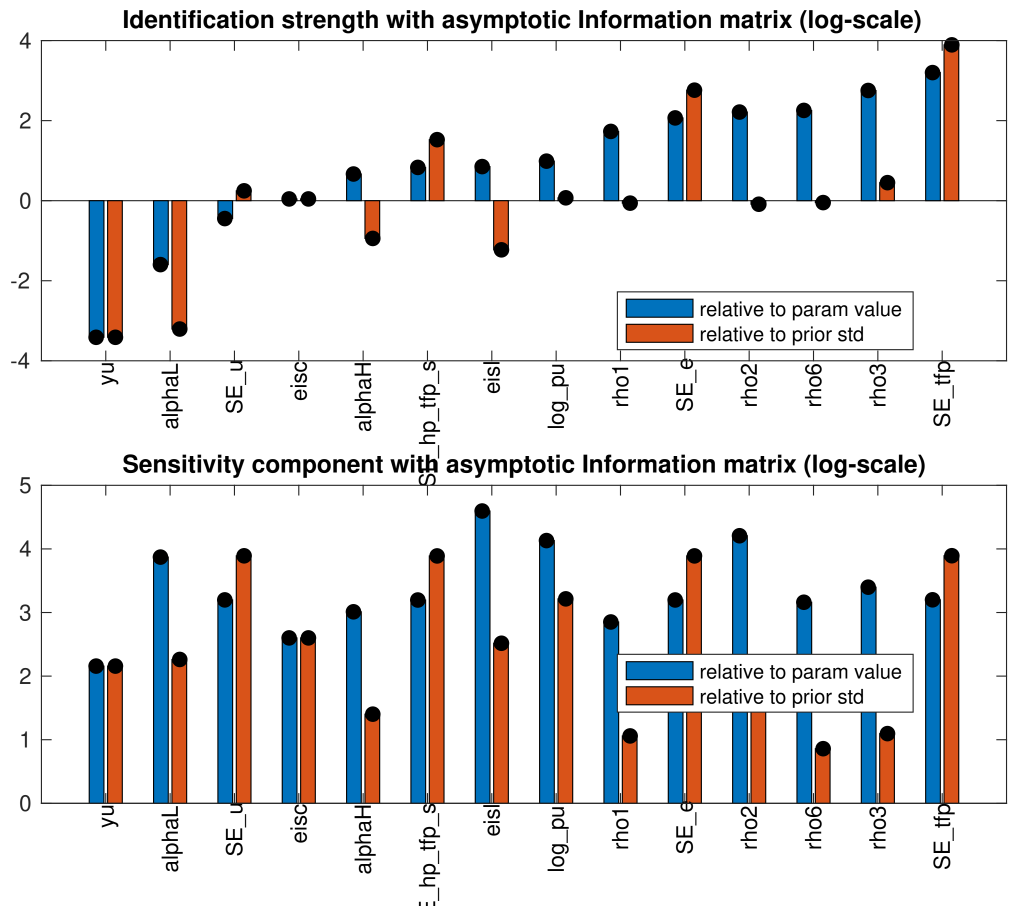

Figure 7: Prior mean - Identification using info from observables.

It is impossible to compute the posterior distribution analytically; the standard practice is to use Metropolis-Hastings sampling procedure to approximate the posterior distribution. cite here The number of draws should be large enough to achieve convergence of the MCMC and to compute the posterior distribution objects, such as mean, variance, etc. We set the number of replications of each MCMC chain to 50,000. The following multivariate/univariate diagnostics show that it is large enough to achieve convergence.

Figure 8 shows the Multivariate convergence diagnostics. This diagnostic is the same as the univariate convergence diagnostics but is aggregated by the posterior kernel. The univariate convergence diagnostics can be found in the appendix. The first panel shows the (Brooks and Gelman, 1998) convergence diagnostics for an 80% interval. The blue line is the 80% interval range computed using the pooled draws from all sequences. The red line is the mean interval range computed by the draws of the individual sequences. The second and third panels show the same statistics for the second and third moments. If the M-H algorithm has converged, then we expect the two lines to stabilize and be very close to each other.

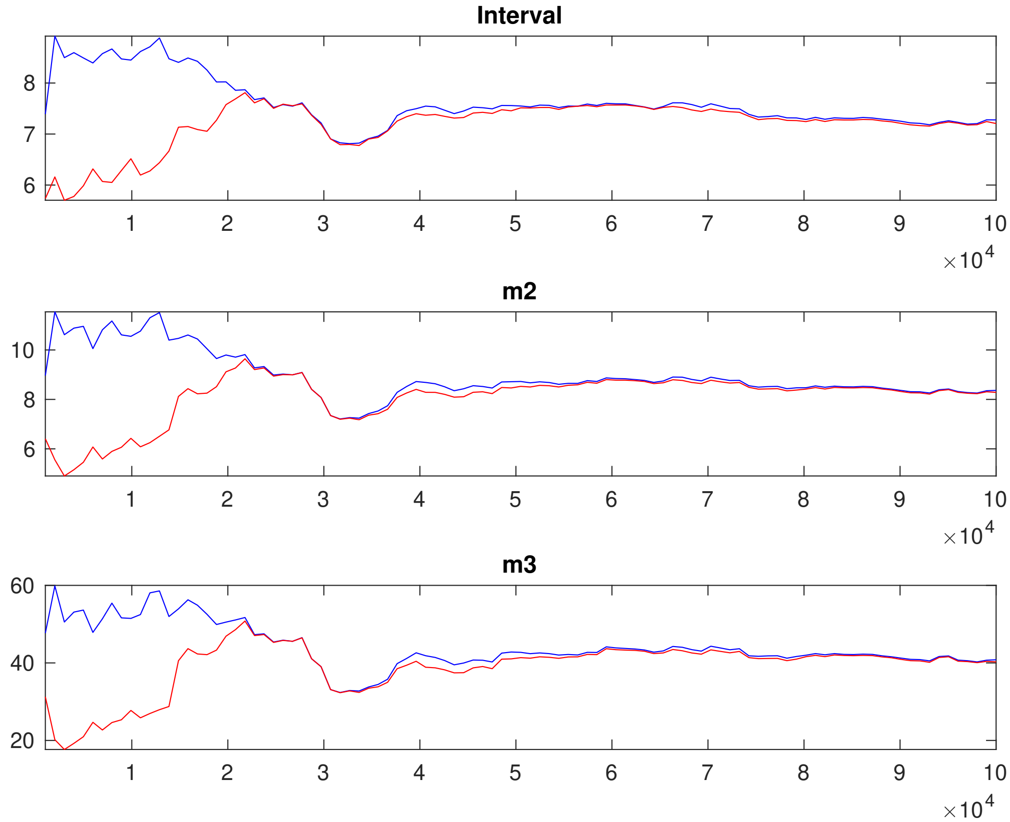

Figure 8: Multivariate convergence diagnostics for the Metropolis-Hastings. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

Table 8 and 9 show the Posterior distribution sampled by the Metropolis-Hastings algorithm. The left panel of table 8 shows the prior distribution type, mean, and standard deviation; the right panel shows the posterior mean, standard deviation, and the 90% highest posterior density interval. Figure 9 and 10 shows the prior distribution (gray), the posterior distribution (black), and the posterior mean value (green).

Table 8: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ϵ_c	norm	-0.500	0.5000	-0.382	0.0744	-0.5025	-0.2582
ϵ_l	beta	0.800	0.1000	0.885	0.0777	0.7696	0.9882
α_h	beta	0.200	0.0400	0.148	0.0334	0.0948	0.2024
α_l	beta	0.200	0.0400	0.116	0.0213	0.0814	0.1505
μ_z	norm	0.100	0.1000	0.325	0.0758	0.1968	0.4461
μ_ω	norm	-0.500	0.2000	-0.403	0.0973	-0.5517	-0.2389
ρ_z	beta	0.300	0.0500	0.316	0.0484	0.2366	0.3956
ρ_ω	beta	0.500	0.0500	0.565	0.0457	0.4910	0.6411
ρ_{z_p}	beta	0.500	0.0500	0.538	0.0450	0.4609	0.6089
ρ_{hp}	beta	0.500	0.0500	0.547	0.0429	0.4766	0.6178

Table 9: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
z_l	invg	0.050	0.1000	0.034	0.0122	0.0172	0.0531
ω	invg	0.050	0.1000	0.010	0.0013	0.0082	0.0124
z_{tfp}	invg	0.050	0.1000	0.014	0.0012	0.0121	0.0159
z_{hp}	invg	0.050	0.1000	0.127	0.0231	0.0905	0.1626

There are a few parameters we are particularly interested in. μ_z is the log mean value of the relative productivity of working at the office. $e^{\mu_z} \approx 1.47$. This implies that even household capital can improve labor productivity while working from home; workers are only 68% as productive as working at the office. This is consistent with empirical evidence. μ_ω is the log mean of the weight of market consumption in the CES aggregator function. $e^{\mu_\omega} \approx 0.65$, this implies that the share of market consumption good in the final consumption good aggregation function is 65% . α_h is the household capital share in household production; 0.148 is relatively low compared to the capital share in firms' production function, α_m, which is 0.36. In the literature, the capital share in firms' production is usually around one-third; however, the capital share in household production function is in a range between 0.08 and 0.36, Fisher (2007), Gomme et al. (2001), Benhabib et al. (1991). Our estimation of the capital share in household production lands in that range. α_l, the capital share in the effective labor production function is 0.096, which is lower than 0.19, the value used in Fisher (2007). ϵ_c governs the elasticity of substitution between market consumption and household production; we set the prior for this parameter to be a normal distribution with a mean of -0.3 and standard deviation of 1 to cover a wide range of values; the prior 90% HPDI is [-1.94,1.34]. The posterior distribution, however, is very concentrated around the posterior mean, -0.317, and the 90% HPDI is [-0.44,-0.20]

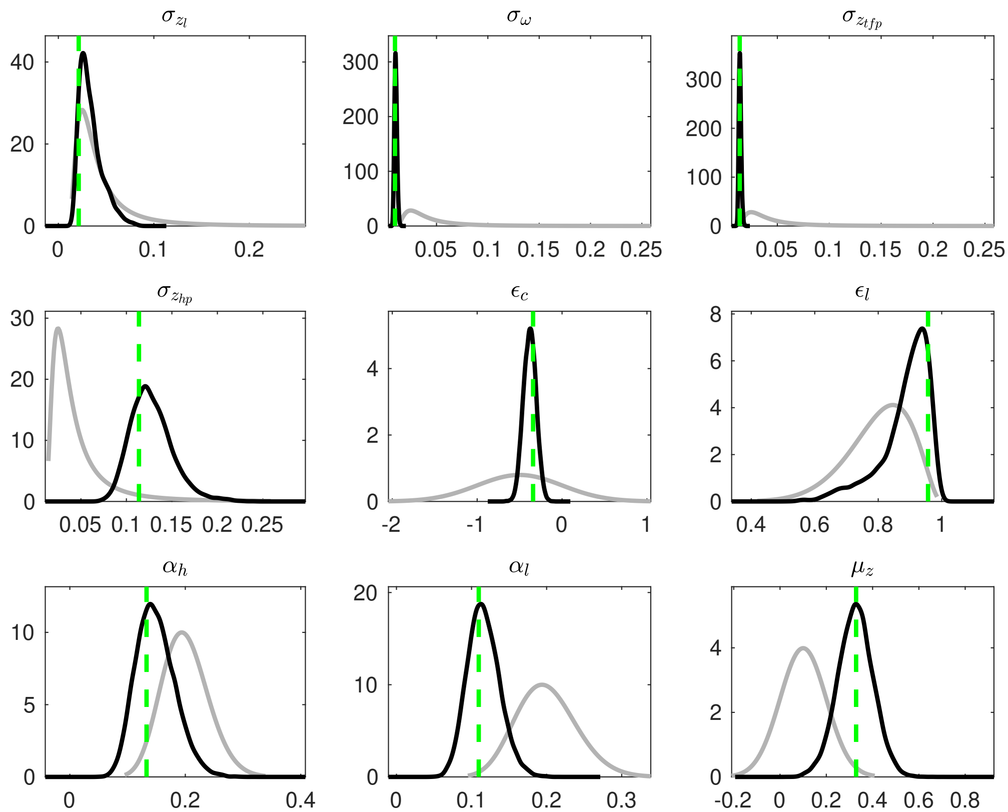

Figure 9: Priors and posteriors.

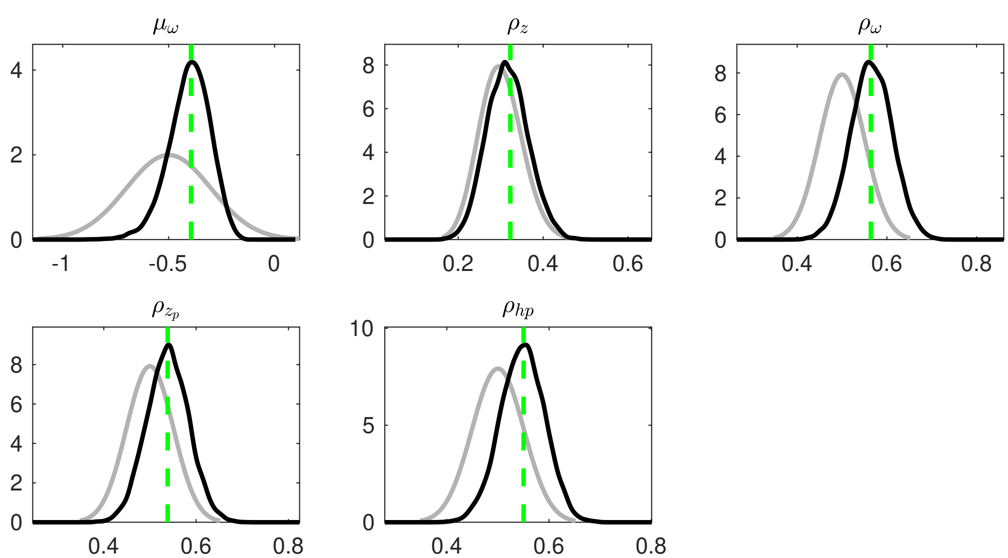

Figure 10: Priors and posteriors.

Figure 11 and 12 show the smoothed observed variables and smoothed shocks. As shown in the upper left panel of figure 11, there was a severe negative shock on labor productivity at the beginning of the pandemic. At the steady-state, see table 10, working at the office is more productive than working at home; combine with the high elasticity of substitution between working at home and working at the office, $\epsilon_l = 0.95$, we have that workers spend most of their time working at the office, $\frac{h_{m_1}}{h_{m_1}+h_{m_2}} = 15\%$, which is very close to what we have observed in ATUS. In ATUS, workers spend about 10% of their working time at home. We see a negative relative productivity shock hit the economy in 2020Q2 when the pandemic began, working at the office was only 50% as productive as working from home; this captures all features that could affect labor productivity while working at the office. For example, if one member of the household gets COVID, not only the infected member but the entire household may need to take a few days off from work to either recover from the infection or to take care of the sick one.

We also observe a negative TFP shock in the lower left panel of figure 11. This shock captures factors that negatively affect firms' productivity, such as the supply chain issue that lasted until now. The TFP shock could also capture the friction while transitioning to the working-from-home regime. We can see that the TFP quickly recovered and rebounded to a slightly higher level.

The top right panel of the figure shows the consumption weight variation. The consumption weight has two interpretations. It could be either part of the household's preference or the technology of final consumption good aggregation function. The consumption weight shock and the household production technology shock capture many aspects of the pandemic. First, these two shocks could have captured the effect of the lockdown. Agents reduced their market good consumption, c_{m_t}, not because their preferences have changed but because of access restrictions such as lockdown, social distancing, etc. More importantly, unemployment was high in 2020 Q2; however, we assumed constant EL_t, which will exaggerate the effect of TFP shock and household production technology shock.

Table 10: Steady state

Variable	**Value**
kh_i	0.157
cm	2.015
ch	0.512
k_m	19.304
k_h	6.284
h_{m_1}	0.104
h_{m_2}	0.562
h_h	0.333
$output$	2.66

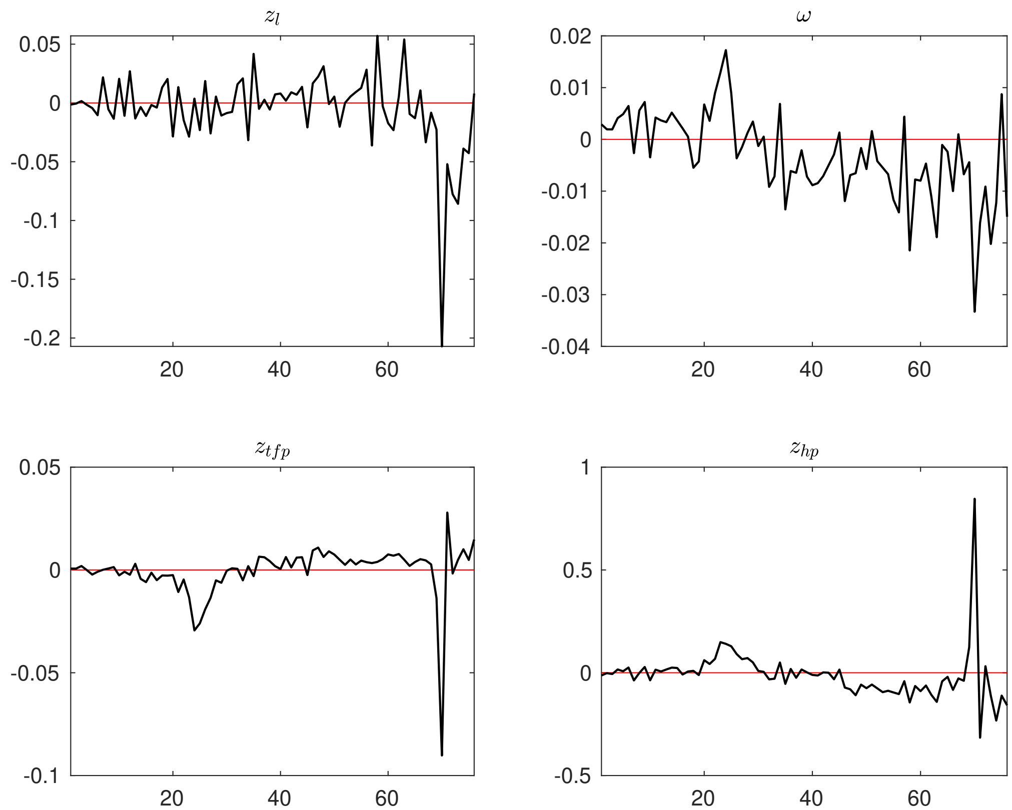

Figure 11: Smoothed shocks.

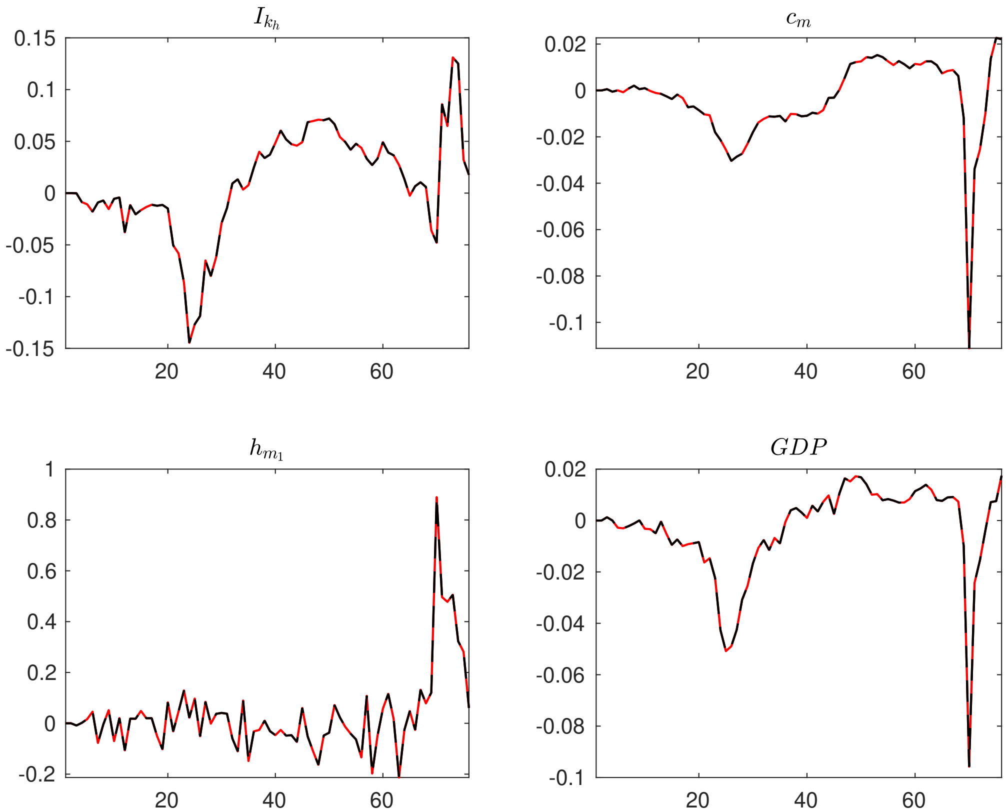

Figure 12: Historical and smoothed variables.

4.3 Shock Decomposition

In section 4.2, we estimated four shocks. In this section, we decompose the observed deviations of the durable good, nondurable good, and time allocations from their steady-state into contributions from the four shocks.

Figure 13 shows the effect of each shock on durable good consumptions. The blue bar represents the contribution of shocks to the relative productivity of working at the office. We can see that before 2020, the effect is insignificant; however, since the second quarter of 2020, it increased dramatically and accounts for about a third of the durable good consumption increase. Many workers started working from home at the end of 2020Q1, and working from home contributed greatly to the increase in durable goods consumption during 2020 Q2 and Q3. There are two reasons. First, there could be a lag between workers starting to work from home and deciding to upgrade their working environment. Second, there is an adjustment cost, which prevents households from making minor adjustments to their household capital stock.

The green and black bars represent the effect of shocks on market consumption weight in the final consumption good aggregate function and the household production technology. There are two interpretations of the weight parameter. First, it could be interpreted as part of the household's preference. Second, it could be a parameter of the household production function. The fluctuation of market consumption weight and household production technology captures the variation in household production technology and preference. For example, many market consumption goods could be inaccessible or undesirable due to precautionary measures such as social distancing or shelter in place at the early stage of the pandemic. These factors exogenously change the consumption composition of households and their ability to transform time, household capital, and nondurable consumption goods into final consumption goods.

Overall, there are two main driving forces behind the increase in durable consumption goods. First, the relative productivity of working from home is higher during the pandemic; therefore, many workers choose to work from home and upgrade their home office. Second, household substitutes from market consumption toward household production because many market consumption goods become inaccessible or undesirable due to the shelter-in-place order or the risk of getting covid.

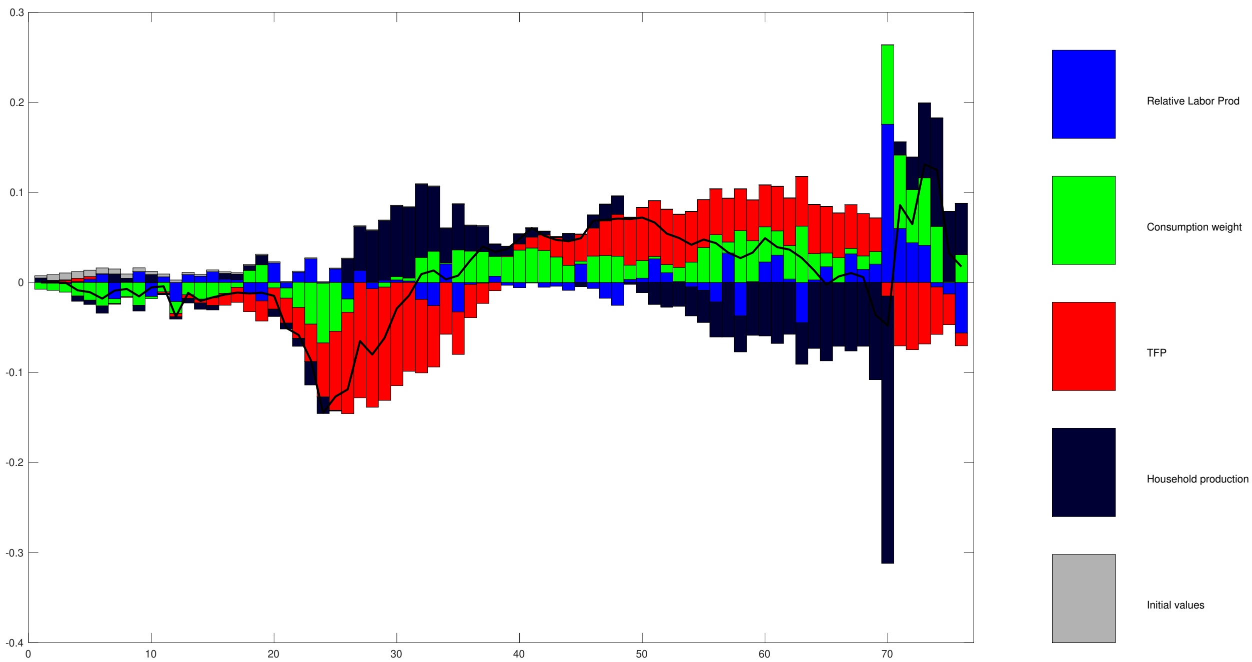

Figure 13: Historical shock decomposition: I_{k_h}.

Figure 14 shows the shock decomposition of the nondurable consumption,c_m. We can see that at the early stage of the pandemic, the effect of consumption weight shock on nondurable good consumption, the green bars, is small compare to the effect of household productivity shock, the black bar. This is because the decrease in nondurable goods consumption is much bigger than the increase in durable goods consumption, and to match the data, we need a particularly big shock on household productivity. The other side effect of this is that the time spent on household production increased way much more than what we observed in the real data, see figure18

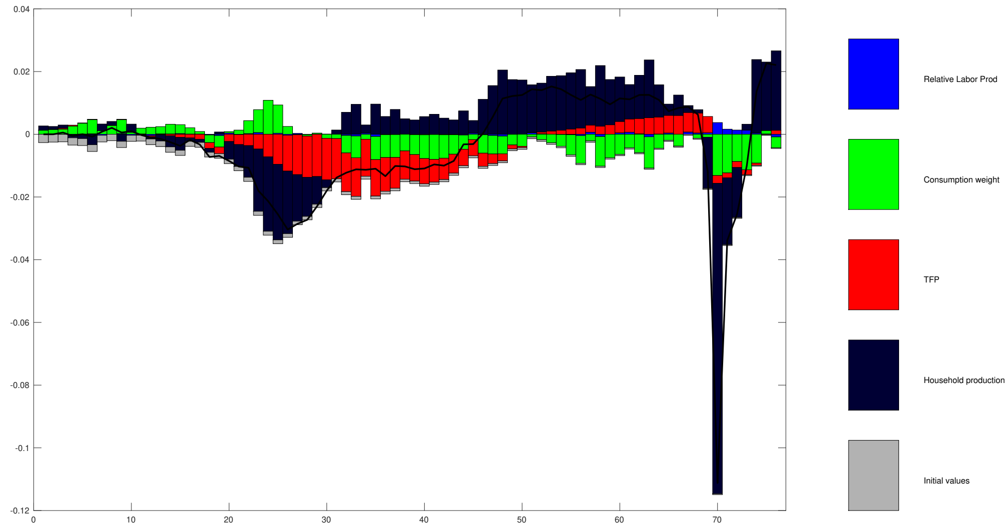

Figure 14: Historical shock decomposition: c_m.

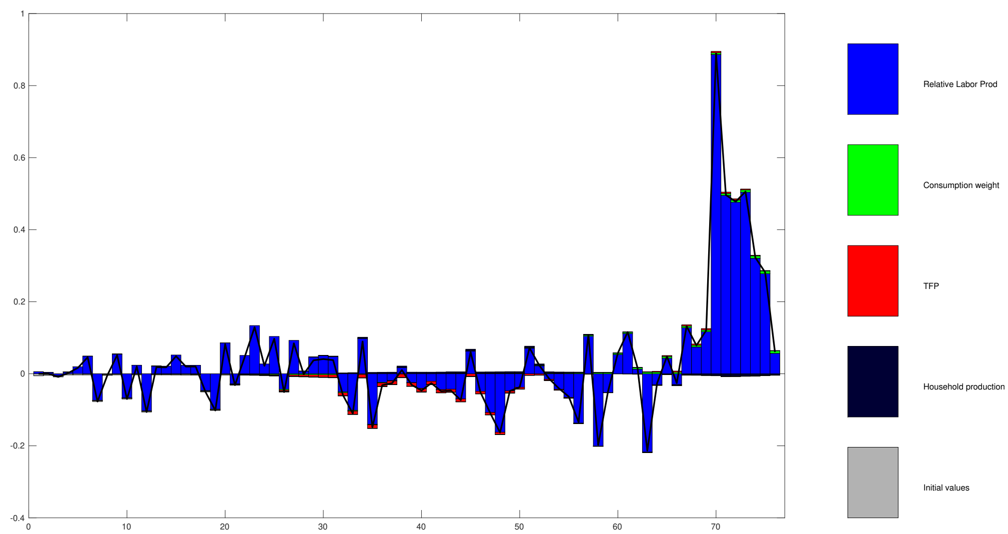

Figure 15: Historical shock decomposition: h_{m_1}.

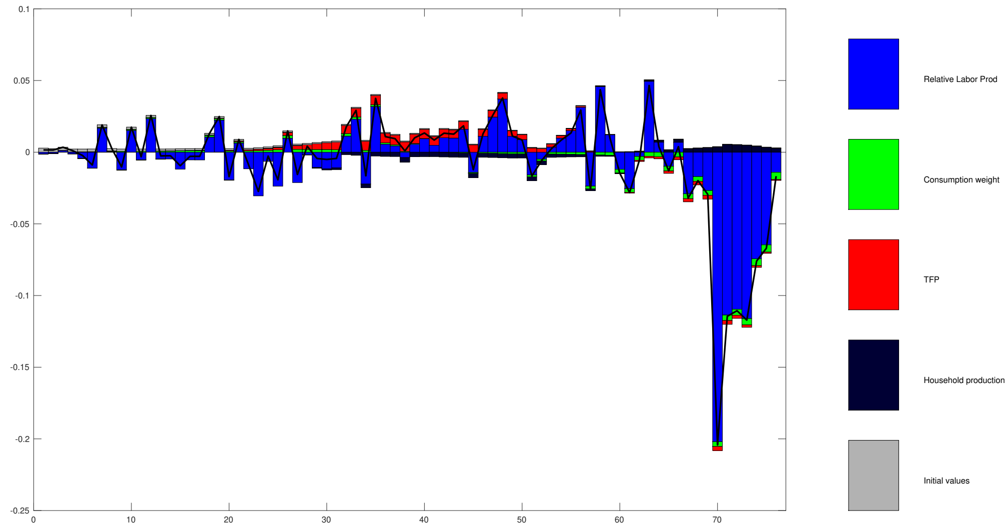

Figure 16: Historical shock decomposition: h_{m_2}.

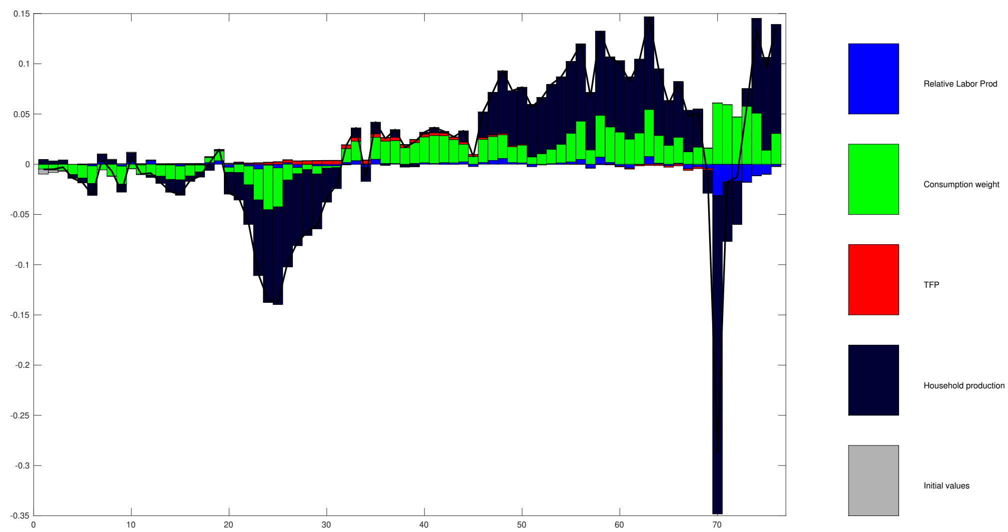

Figure 17: Historical shock decomposition: h_h.

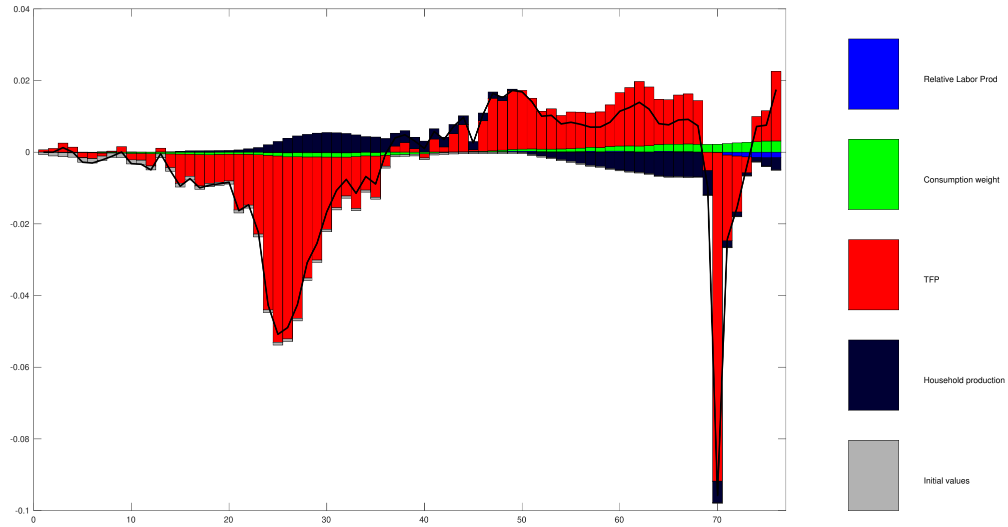

Figure 18: Historical shock decomposition: h_h.

5 Conclusion

In this paper, I showed that the unusually large increase in durable good consumption during and post the pandemic cannot be explained by the substitution between market nondurable consumption and household production because many nondurable consumption goods and services are highly specialized, most average household could not substitute them with household production. I proposed a new model that allows household to endogenously decide how to allocate their time. This model is able to capture the fact that working from home is an important factor that drives the durable goods consumption besides household production.

The COVID-19 pandemic could have forever changed our way of life and work. For example, many workers now have the option of working from home. Many companies have decided that their employees will continue to work from home until 2022. Some companies proposed a hybrid working schedule that employees can work from home for three or four days a week, and they only need to come to their office once or twice a week. Those changes are unlikely to be reversed soon. Hence, household capital will be more critical than ever before, the complementarity between household capital and market capital will become stronger. It will be important to keep this channel in mind when model household time allocation and consumption decisions.

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A Euler equations

$$
\begin{aligned}
&\mathbb{E}_t(C_t^{\frac{1-\gamma}{\epsilon_c}-1}\omega c_{m,t}^{\epsilon_c-1} - \lambda_t) = 0 \\
&\mathbb{E}_t(\lambda_t \left((k_{h,t}^{\alpha_l} h_{m_1,t}^{1-\alpha_l})^{\epsilon_l} + y h_{m_2,t}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1} (1-\alpha_l) k_{h,t}^{\alpha_l \epsilon_l} h_{m_1,t}^{(1-\alpha_l)\epsilon_l - 1} - H^{\phi}) = 0 \\
&\mathbb{E}_t(\lambda_t \left((k_{h,t}^{\alpha_l} h_{m_1,t}^{1-\alpha_l})^{\epsilon_l} + y h_{m_2,t}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1} y_{1,t} h_{m_2,t}^{\epsilon_l - 1} - H^{\phi}) = 0 \\
&\mathbb{E}_t(C_t^{\frac{1-\gamma}{\epsilon_c}-1}(1-\omega) c_{h,t}^{\epsilon_c-1} k_{h,t}^{\alpha_h} h_{h,t}^{-\alpha_h}(1-\alpha_h) - H^{\phi}) = 0 \\
&\mathbb{E}_t(\beta(1+r_{t+1})\lambda_{t+1} - \lambda_t) = 0 \\
&\mathbb{E}_t(-\lambda_t + \beta(1-\delta_h)\lambda_{t+1} = 0 \\
&+ \beta\lambda_{t+1} \left((k_{h,t+1}^{\alpha_l} h_{m_1,t+1}^{1-\alpha_l})^{\epsilon_l} + y h_{m_2,t+1}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1} \alpha_l k_{h,t+1}^{\alpha_l \epsilon_l - 1} h_{m1,t+1}^{(1-\alpha_l)\epsilon_l} \\
&+ \beta C_{t+1}^{\frac{1-\gamma}{\epsilon_c}-1}(1-\omega) c_{h,t+1}^{\epsilon_c-1} \alpha_h k_{h,t+1}^{\alpha_h-1} h_{h,t+1}^{1-\alpha_h}) \\
r_t &= \alpha_m k_{m,t}^{\alpha_m-1} EL_t^{1-\alpha_m} - \delta_m \\
w_t &= (1-\alpha_m) k_{m,t}^{\alpha_m} EL_t^{-\alpha_m}
\end{aligned}
$$

B convergence

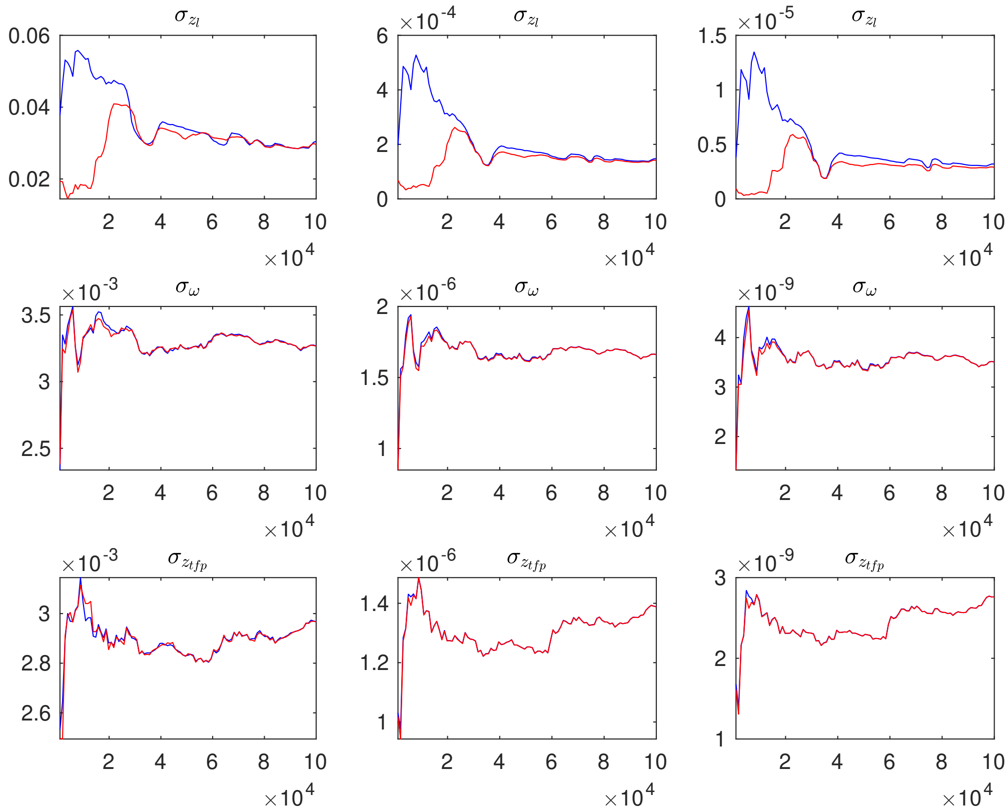

Figure 19: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

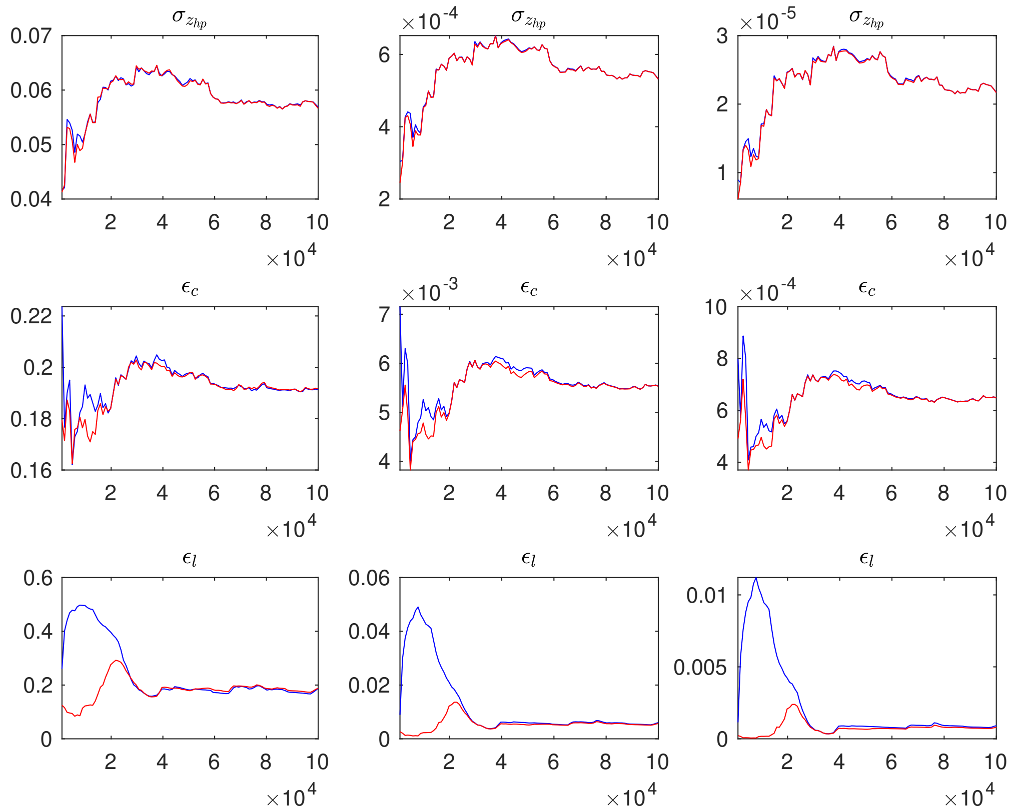

Figure 20: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

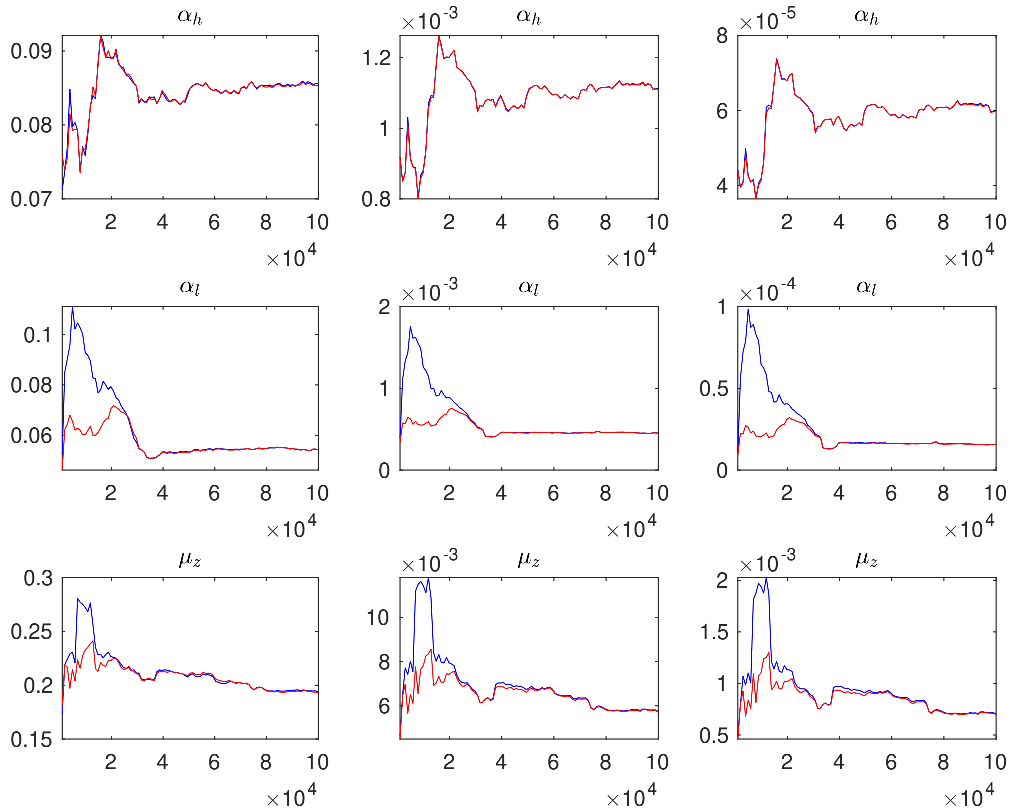

Figure 21: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

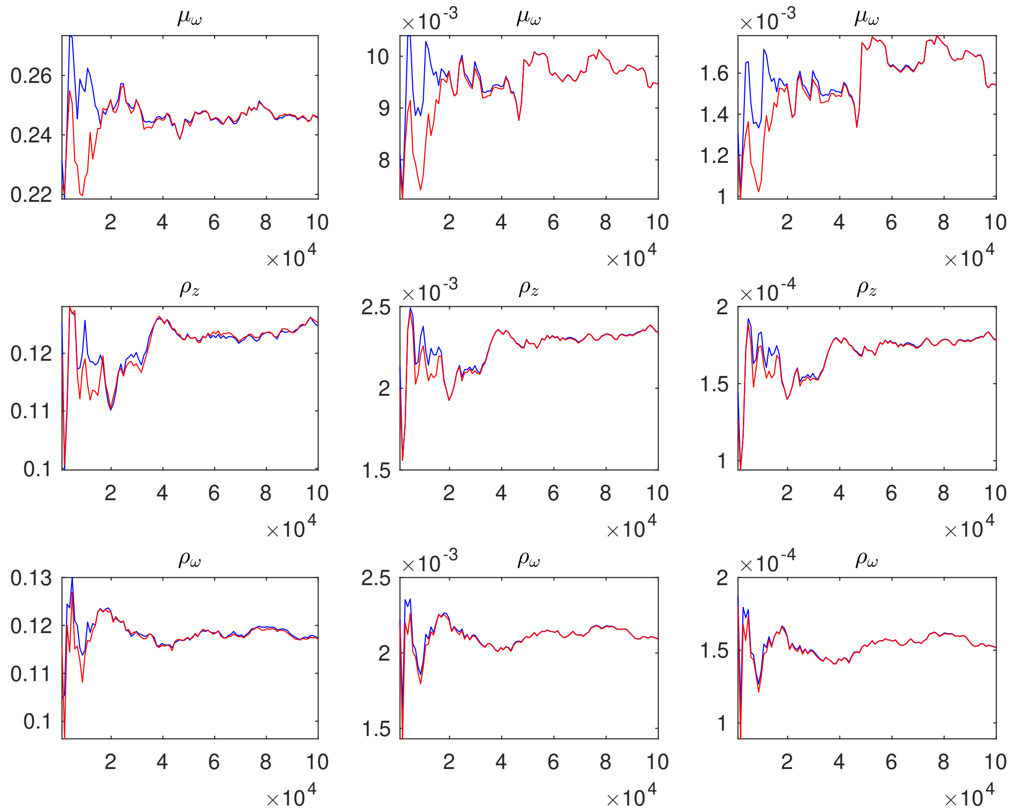

Figure 22: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

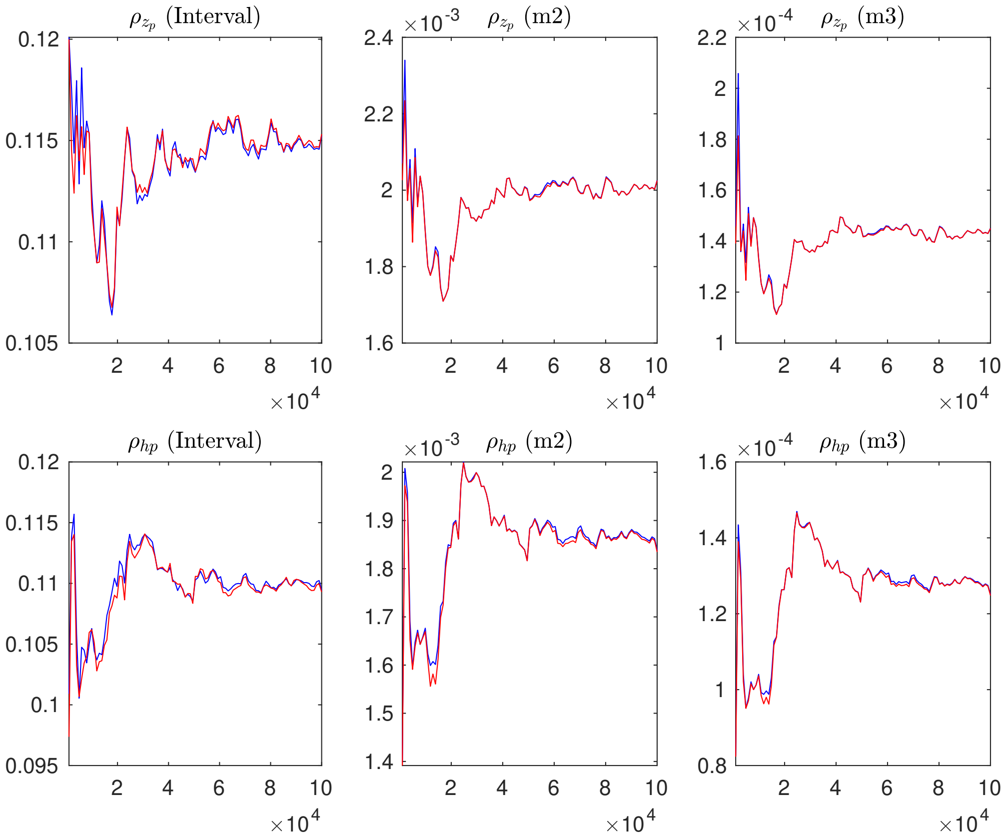

Figure 23: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.