Working from home, household production, and durable good consumption dynamics*

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Abstract

I decompose the durable good consumption increase during 2020-2021 using a household production model with working-from-home. Durable good consumption is susceptible to business cycles; in past recessions, durable good consumption either decreased or slowed down. However, durable goods consumption was very robust during and after the COVID-19 pandemic. I build a household production model with working from home and estimate the model using a Bayesian approach. Using Kalman smoother, we can then decompose the increase in durable good consumption into different channels. Working from home can account for up to one-third of the durable good consumption increase, and substitution between nondurable and durable can account for another one-third of the increase.

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1 Introduction

revise intro and enrich the body. in the first three paras , i should introduce the research question, my approach, my finding, why it is important and in somewhere i should mention that the relationship between household investment and durable goods. . then write lit review and demonstrate why stimulus check is not important. change all we to i or this paper.

The consumption of durable goods is typically susceptible to business cycles. In every post-WWII recession, the durable goods consumption either decrease or slowdown, see Figure 1. However, during the recession caused by the COVID-19 pandemic, the durable goods consumption increased dramatically. In this paper, I propose a model that combines working from home and household production to explain the unusual increase of durable goods consumption during and after the COVID-19 pandemic caused recession and decompose the increase into different channels. I find that working from home could count for about one third of the increase in durable good consumption.

In the literature, durable goods are typically modeled as household capital. Household combines nondurable good, time, and durable goods to produce final consumption good. Therefore, through our this paper, I use durable good and household capital interchangeably.[1]

In the past, the role of working from home as an important channel driving durable goods consumption has been overlooked for two reasons. Firstly, durable goods are typically considered as household capital that is used in the production of final consumption goods, in combination with nondurable goods and time as inputs. Secondly, both the average amount of time and the proportion of workers who worked from home were relatively low. However, the COVID-19 pandemic has fundamentally changed how people allocate their time, and many firms now allow their workers to continue working from home or adopt hybrid work models. It is expected that working from home will persist even after the economy has reopened, as the average time people spend on remote work is still four times higher than pre-pandemic levels. Therefore, it is crucial to recognize that working from home will continue to play an important role in shaping the dynamics of durable goods consumption.

The amount of time spent working from home has gradually increased since the early 2000s, and in early 2020, it spiked dramatically, as shown in Figure 2. This is primarily due to many companies allowing their employees to work remotely in order to mitigate the risk of COVID-19. Even if we only consider the pre-pandemic era, both the percentage of full-time workers who exclusively work from home and the average amount of time that hybrid workers spend working from home have risen. In the United States, full-time workers spent around 4 hours per week working from home prior to the pandemic. Among those who exclusively work from home, the average number of hours spent working per day increased from 4 hours in 2003 to 6.4 hours in 2019. During the same period, the percentage of these workers increased from 3.7% to 7%, nearly doubling, and the percentage of hybrid workers rose from 10% to 12.5%.

According to (Fisher, 2007), the key to reconciling RBC theory with investment dynamics (e.g., household investment leading the business cycle) is making household capital comple-

[1]Tangible products that can be stored or inventoried and that have an average life of at least three years. BEA

mentary to business capital and labor in market production. (Fisher, 2007) argues that labor needs to be replenished by practices as maintenance to keep business capital functioning effectively. Workers need to rest, relax, and take personal care to supply labor effectively, and all of these activities require household capital/durable goods. Working from home could have strengthened this complementarity and, therefore, could at least partially explain the sharp increase in durable goods consumption during the pandemic.

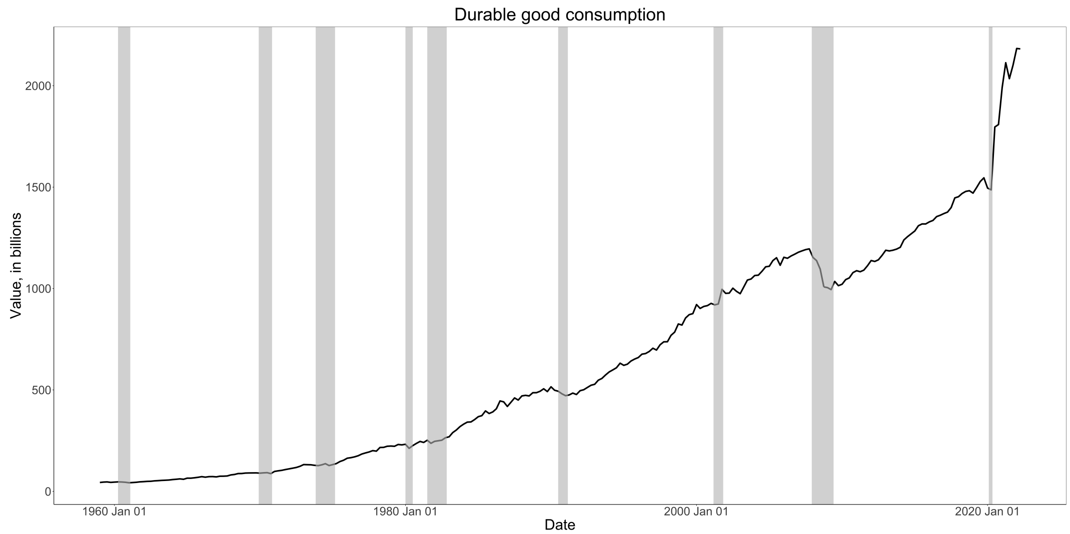

Figure 1: Aggregate household capital investment decomposition in terms of actual value

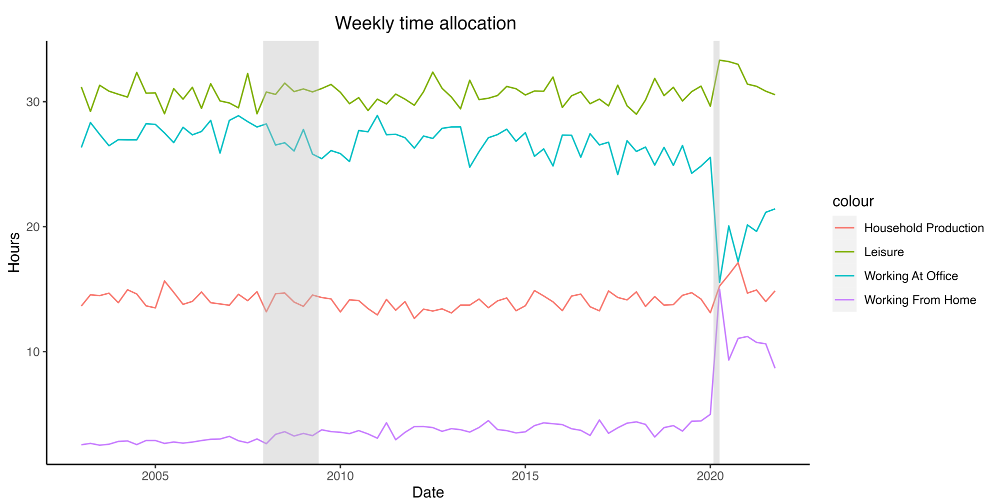

Figure 2: Time allocation

might need to move this to somewhere else. this is not very relevant in the intro.

There are two ways of modeling durable goods in the literature. The first is to assume that durable goods stock provides utility flows in each period, without requiring agents to put in any effort or time. This modeling choice is typically combined with an inelastic labor supply, in which agents are endowed with a unit of labor and do not value leisure.

The second approach is that agents use durable goods to produce homemade consumption goods. This is often seen when the labor supply is endogenous and agents value leisure. In this paper, we follow the second modeling choice because one of our assumptions is that the labor supply is endogenous. Working from home enhances the complementarity between household capital and labor productivity in two ways. First, household capital, such as home office equipment, directly affects labor productivity. Second, recreational durable goods can help agents produce "effective labor," which is used as an input for market production. This is because labor needs to be regenerated by activities that require household capital, such as rest and relaxation.

1.1 Literature review

Apart from working from home, there are several other factors that could potentially impact durable goods consumption. For instance, in the literature on household production, durable goods are often modeled as household capital, which serves as an input factor in the household production function for producing final consumption goods. During the COVID-19 pandemic, many nondurable goods became undesirable or infeasible to consume due to lockdown measures or the risk of contracting the virus. As a result, households may have reoptimized their consumption and time allocation, shifting away from market-based nondurable consumption towards household production. However, this alone may not fully explain the substantial increase in durable goods consumption observed during and after the pandemic. Firstly, many nondurable goods are services that cannot be easily replaced by household production. For example, research by (Been et al., 2020) indicates that only a maximum of 11% of nondurable consumption can be substituted by household production. Secondly, as demonstrated in Figure 2, the average time spent on household production did not show a significant increase. Therefore, it is unlikely that the substitution between nondurable goods and household production is the main driving force behind the increase in durable goods consumption during and after the pandemic.

Another potential factor contributing to the increase in durable goods consumption is the stimulus check sent out by the federal government. The impact of unanticipated income on both durable and nondurable goods has been extensively studied. In the past two decades, the US has experienced three recessions, and in response, the government has provided direct cash payments to households to mitigate the effects of economic downturns. For instance, the 2001 rebate was a payment that resulted from a new tax bracket, where the previous 15 percent tax bracket was reduced to 10 percent. Similarly, the 2008 rebate was a one-time stimulus payment, although it was administered through the tax system, and not related to any change in tax policy, similar to the 2020 stimulus. Many studies have shown that direct cash transfers tend to stimulate nondurable consumption, but may not have as significant an impact on durable goods consumption. This is because durable goods are often lumpy in nature and may involve adjustment costs, making them easier to postpone, particularly during recessions.

(Johnson et al., 2006) utilized the random timing of the arrival of the 2001 rebate to estimate how people spent their 2001 rebate. The timing of the sending of each rebate was determined by the second-to-last digit of the SSN. They found that households spent 20-40 percent of their rebates on non-durable goods during the three months they received their rebate. They also found no significant evidence of response in durable goods such as automobiles or large household equipment. In part, this may be due to the relatively small size of the average refund per household and the relatively high volatility of expenditure on durable goods. (Shapiro and Slemrod, 2003) report that only 21.8 percent of households raised their spending in response to the 2001 rebate. The amount spent on durable goods would be even smaller than the amount spent on non-durable goods. (Agarwal et al., 2007) find that households first improved their balance sheet by saving more or paying off credit card debt, then increased spending on non-durable goods in the following nine months. The increased amount on non-durable goods is $200 or about 40 percent of the average household rebate.

Recent research on the effect of the 2020 CARES Act stimulus has also found similar patterns: most of the stimulus payments are saved or used to pay off debt. (Coibion et al., 2020) report that only 15 percent of recipients say that they spent (or planned to spend) most of their stimulus payment, with the large majority of respondents saying instead that they either mostly saved it (33 percent) or used it to pay down debt (52 percent). When asked to provide a quantitative breakdown of how they used their checks, US households report having spent approximately 40 percent of their checks on average, with about 60 percent of the average check being saved or used to pay down debt. Little of the spending went to large durable goods.

(Baker et al., 2020) used transaction-level data to analyze how recipients spend their 2020 stimulus payments. Relative to the effects of previous economic stimulus programs in 2001 and 2008, they found faster effects, smaller increases in durables spending, more significant increases in spending on food, and substantial increases in payments like rents, mortgages, and credit cards. They also found substantially smaller impacts on durables spending and confirmed this in the survey of users.

Previous research found mixed results on the response of durable spending to stimulus payments. Most papers report either a negligible effect (Shapiro and Slemrod, 2003; Johnson et al., 2006; Graziani et al., 2016; Shapiro and Slemrod, 2009) or that auto and related services contribute to most of the durable spending responses (Parker et al., 2013; Kanishka Misra et al., 2014; Kueng, 2018).

Research has shown that durable spending is less responsive to economic stimulus, with households tending to save or spend stimulus payments on nondurables. However, the sharp increase in durable goods spending since early 2020 can be partially explained by shifting nondurable goods towards durable goods for household production, as precautions such as social distancing and lockdowns limited consumers' choices for consumption and services. As a result, households spent more time than usual at home, taking care of children, working or studying from home, engaging in home production, or enjoying leisure activities (Tauber and Van Zandweghe, 2021). The literature on household production, pioneered by (Becker, 1965), has extensively explored the allocation of capital and labor between market and household activities and their implications for business cycles (Heckman, 1974; McGrattan et al., 1997; Gomme et al., 2001; Greenwood, 1991; Benhabib et al., 1991; Boerma and Karabarbounis, 2021; Fang and Zhu, 2017; Chang, 2000). The widespread adoption of working from home during the pandemic has permanently changed households' consumption choices, time allocation, and housing choices, with residential housing often being modeled as durable goods in economic literature. Although there are studies exploring the impact of working from home on households' housing choices and on residential and commercial real estate (Delventhal et al., 2022; Brueckner et al., 2021; Stanton and Tiwari, 2021), there is a gap in the literature regarding the effect of working from home on the consumption of durable goods such as furniture, appliances, recreational vehicles, etc. This paper aims to fill that gap and demonstrates that the complementarity between household and business capital is an important factor in explaining durable goods consumption dynamics, particularly in the context of a sudden transition to working from home.

The remainder of this paper is organized as follows. Section 2 presents a model that incorporates household production and complementarity between household capital and market capital. Section 4 describes our estimation strategy, results and discusses their implication. Section 5 concludes.

2 Model

The extended household production model presented in this section consists of households and firms. We first describe the full model, and then we discuss the implication of our assumptions and the economic mechanism of the model. The key difference from the traditional household production model is that households now can choose how to allocate their time between working from home, working at the office, and household production. Following (Fisher, 2007), we allow labor productivity while working from home to be positively correlated with household capital stock.

2.1 Household

The representative household has preference over a consumption good that is purchased from the market; hereafter, we refer to it as market consumption, $c_{m,t}$, a consumption good that is produced at home, $c_{h,t}$, hours spend at working from home, $h_{m_1,t}$, hours spent working at the office, $h_{m_2,t}$, and hours spend on household production, $h_{h,t}$.

Households maximize the following expected utility take interest rate and wage as given.

$$Max\, \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \{U(C_j) - V(H_j)\} \tag{1}$$

Where C_j denotes the aggregate consumption at period j and H_j denote the total hours spent on working and household production. $U(\cdot)$ denotes the instantaneous utility flow from consumption, and $V(\cdot)$ denotes the disutility from working and household production. Household maximize their expected utility, Equation 2, conditional on time t information.

$$c_{h,t} = g(k_{h,t}, h_{h,j}) = z_{p,t} k_{h,t}^{\alpha_h} h_{h,t}^{1-\alpha_h} \tag{2}$$
$$C_t = (\omega_t c_{m,t}^{\epsilon_c} + (1-\omega_t) c_{h,t}^{\epsilon_c})^{\frac{1}{\epsilon_c}} \tag{3}$$
$$H_t = h_{m_1,t} + h_{m_2,t} + h_{h,t} \tag{4}$$
$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{5}$$
$$V(H_t) = \frac{H_t^{1+\phi}}{1+\phi} \tag{6}$$

Equation 2 and Equation 3 represent the household production technology and the final consumption good aggregator. $k_{h,t}$ is the household capital stock, and $h_{h,t}$ is the time spent on household production at period t. The aggregate consumption, C_t, is given by a CES aggregator function, in which ω_t is the weight on market consumption, and ϵ_c governs the elasticity of substitution between market consumption and home-produced good. ω_t follows a log AR(1) process such that $log(\omega_t) = (1-\rho_\omega)\mu_\omega + \rho_\omega log(\omega_{t-1}) + \epsilon_\omega$ where $\epsilon_\omega \sim \mathcal{N}(0, , \sigma_\omega^2)$. This process captures the variation of household production technology required to produce the final consumption good. For example, many consumption goods became either inaccessible or undesirable during the pandemic because of lockdowns or the risk of COVID. γ and ϕ are the relative risk aversion and Frisch labor supply elasticity.

Assumption 1 *Each period, household supply one unit of effective labor.*

$$1 = (k_{h,t}^{\alpha_l} h_{m_1,t}^{\epsilon_l} + z_t * h_{m_2,t}^{\epsilon_l})^{\frac{1}{\epsilon_l}} \tag{7}$$

ϵ_l governs the elasticity of substitution between working from home and at the office. z_t denotes the relative productivity of working at the office. It follows a log AR(1) process such that $log(z_t) = (1-\rho_z)\mu_z + \rho_z log(z_{t-1}) + \epsilon_z$ where $\epsilon_z \sim \mathcal{N}(0, \sigma_z^2)$. If $z_t > 1$ then agents are more productive at their office; however, if $z_t < 1$, then they are more productive working from home. This assumption implies that workers can endogenously decide where to work, home or office; however, they must complete a certain amount of tasks.

Assumption 2 *Household face a quadratic transaction cost when adjusting their household capital stock.*

$$A(k_{h,t+1}, k_{h,t}) = \frac{\Phi}{2}(\frac{k_{h,t+1}}{k_{h,t}} - 1)^2 k_{h,t} \tag{8}$$

This assumption implies that if the household decides to maintain its current level of household capital, then there is no transaction cost. However, if the household decides to adjust its household capital stock, either increase or decrease, there will be a transaction cost. For example, there is no transaction cost to maintain a car, but there is a transaction cost to replace a car with a new one. Transaction costs are needed in this model to prevent households from using durable goods as a means of saving, since adjustments in durable good stock are typically lumpy and come with a cost in reality.

$$c_{m,t} + k_{m,t+1} + k_{h,t+1} + A(k_{h,t+1}, k_{h,t}) = (1 + r_t)k_{m,t} + (1 - \delta_h)k_{h,t} + wage_t * EL_t \tag{9}$$

Equation 9 shows the budget constraint. $k_{m,t}$ is the saving vehicle, and $k_{h,t}$ is the stock of household capital. EL_t represents the effective labor supply; later, we will fix it to one.

2.2 Firms

The market is perfectly competitive, and firms produce consumer goods, durable goods, and investment goods using capital and effective labor to maximize their profits.

$$Y_t = z_{p,t} k_{m,t}^{\alpha_p} * EL_t^{1-\alpha_p} \tag{10}$$

$$r_t = \alpha_p z_{p,t} k_{m,t}^{\alpha_p - 1} EL_t^{1-\alpha_p} - \delta_m \tag{11}$$

$$wage_t = (1 - \alpha_p) z_{p,t} k_{m,t}^{\alpha_p} EL_t^{-\alpha_p} \tag{12}$$

Y_t is the total output of period t, r_t and $wage_t$ are given by the first order conditions. EL_t equals to one for all periods. z_p is the total factor productivity, it follows a log AR(1) process, $log(z_{p,t}) = (1 - \rho_{z_{p,t}})\mu_{z_p} + \rho_{z_p} log(z_{p,t-1}) + \epsilon_{z_p}$ where $\epsilon_{z_p} \sim \mathcal{N}(0,\, \sigma^2_{z_p})$.

$$k_{m,t+1} = (1 - \delta_m)k_{m,t} + I_{m,t} \tag{13}$$

$$k_{h,t+1} = (1 - \delta_h)k_{h,t} + I_{h,t} \tag{14}$$

Equation 13 and Equation 14 show the law of motion of business capital and household capital. $I_{m,t}$ is the business capital investment and $I_{h,t}$ is the durable good consumption.

2.3 Equilibrium

We first rewrite the household's objective function as follows.

$$\begin{aligned}\mathbf{V}(X, S) = & \max_{c_m, c_h, k'_m, k'_h, h_{m1}, h_{m2}, h_h} U(C) - V(h_h, h_{m1}, h_{m2}) \\ & + \beta\, \mathbb{E}_{|S}\{\mathbf{V}(X', S')\}\end{aligned}$$

where $X = \{k_m, k_h\}$ and $S = \{z, \omega, z_p, r, wage\}$. The optimal allocation of household's problem is given by the following conditions and budget constraints.

$$\beta\,\mathbb{E}_{|S}[(1+r')U_{C'} * C'_{c'_m}] = U_C * C_{c_m} \tag{15}$$

$$\beta\,\mathbb{E}_{|S}[U_{C'} * C'_{c'_h} * g_{k'_h} + (1-\delta_h - A'_2) * U_{C'} * C'_{c'_m} + \frac{V'_{h_{m_1}} EL'_{k'_h}}{EL'_{h'_{m_1}}}] = U_C * C_{c_m}(1+A_1) \tag{16}$$

$$EL_{h_{m_1}} = EL_{h_{m_2}} \tag{17}$$

$$U_C * C_{c_h} * g_{h_h} = H^{\phi} \tag{18}$$

Equation 15 is the Euler equation that governs intertemporal substitution if households use market capital as the saving vehicle. Equation 16 is the Euler equation that shows, at the optimal solution, how to allocate resources between current period market consumption and household capital in the next period. Note that there are three terms in the expectation operator of Equation 16 because household capital can be used not only for household production, but also for effective labor. Therefore, investing in next period's household capital/durable goods can improve marginal household production and labor productivity while working from home in the following period. Equation 17 implies that, at optimal allocation, the household is indifferent between working from home and working in the office because the marginal production of effective labor is the same in both places. Equation 18 implies that the household is indifferent between working from home or in the office, and engaging in household production.

A recursive equilibrium is given by household policy functions that solve the optimization problem : $f_{c_m}(X,S)$, $f_{c_h}(X,S)$, $f_{h_{m1}}(X,S)$, $f_{h_{m2}}(X,S)$, $f_{h_h}(X,S)$, $f_{k'_m}(X,S)$, $f_{k'_m}(X,S)$. r and $wage$ that maximize firms profit and clear the market.

3 Analysis of the model

Our model allows households to allocate their resources between market consumption, durable good consumption, and saving; and to allocate their time between working from home, working at the office, and household production. At the optimal solution, households allocate their resources such that the marginal utility from market consumption equals the discounted expected marginal utility of consumption in the next period, shown by Equation 15. Households can utilize their durable good stock/household capital in three ways. First, they can use it for household production. However, homemade goods are not tradable and cannot be stored across periods. Second, households can liquidize their durable goods stock for market consumption. Lastly, and most importantly, household capital can improve labor productivity while working from home. Some durable goods, such as desks, chairs, and computer monitors, can directly improve labor productivity. Other durable goods, like recreational equipment, can indirectly improve labor productivity by providing rest and relaxation for workers. The indirect channel improves overall labor productivity, but in our model, we only include the direct channel since we cannot identify these two channels separately. (Fisher, 2007) focuses on the indirect channel and does not distinguish between working from home and working at the office. In our model, the effective labor supply, EL_t, is exogenously determined. This is consistent with the literature, where households are endowed with one unit of time and do not value leisure. However, we allow households to endogenously determine how to allocate their time while the effective labor they have to supply is exogenously determined.

In this section, we discuss the mechanism by which the model generates an increase in durable consumption during the pandemic and how agents allocate their time between working from home, working at the office, and household production. The COVID-19 pandemic has affected the economy in many ways. Since March 2020, many firms have allowed their employees to work from home to mitigate the risk of getting COVID. This transition from working at the office to working from home is endogenous in our model; we treat this transition as a response to a negative relative labor productivity shock. In other words, where to work is an endogenous outcome that firms and their employees jointly determine. In the early stages of the pandemic, many market consumptions became unavailable or undesirable because of the shelter-in-place order, the risk of getting COVID, and many shops being closed or understaffed. Therefore, the pandemic will affect the consumption weight in the final consumption goods aggregate function, ω.

We model the process of ω as an AR(1); however, it not only captures the fluctuation of consumption but also the relative price change. To see this point, consider the following one-period static consumption choice model where agents allocate their resources between two consumption goods.

$$U(c_1, c_2) = \frac{[\omega c_1^{\epsilon} + (1-\omega)c_2^{\epsilon}]^{\frac{1-\gamma}{\epsilon}}}{1-\gamma} \tag{19}$$

$$p_1 c_1 + p_2 c_2 = w \tag{20}$$

The optimal allocation is that the agent allocate ω proportion of their resources to consumption good 1 and $1-\omega$ proportion of their resources to consumption good 2.

$$c_1 = \frac{\omega w}{p_1} \tag{21}$$

$$c_2 = \frac{(1-\omega)w}{p_2} \tag{22}$$

From the simplified example we can see that we couldn't separately identify the consumption weight ω and the relative price between two consumption goods from the consumption data only. The relative price between durable and nondurable goods actually increase, which implies if held eveything else constant, the durable good consumption should decrease. Therefore, ignoring the relative price change we will underestimate the effect of the pandemic on consumption weight and relative labor productivity.

3.1 Static Analysis of time allocation

The time allocation choice of households is a function of household capital stock. We can obtain the following policy functions using Assumption 1 and Equation 17. In our model, households can allocate their time between working from home, working at the office, and household production. h_{m_1} and h_{m_2} represent the time allocated to working from home and working at the office, respectively.

$$h_{m_1} = \frac{EL}{(k_h^{\frac{\epsilon\alpha}{\epsilon-1}} * z^{\frac{1}{1-\epsilon}} + k_h^{\alpha})^{\frac{1}{\epsilon}}} \tag{23}$$

$$h_{m_2} = \frac{EL}{(z^{\frac{\epsilon}{\epsilon-1}} * k_h^{\frac{\alpha}{1-\epsilon}} + z)^{\frac{1}{\epsilon}}} \tag{24}$$

$$\tag{25}$$

From the above two equations, we can see that h_{m_1} is a decreasing function of z, the relative productivity of working at the office. The intuition is that if the relative productivity of working at the office is higher, households will allocate less time to working from home. However, this does not necessarily mean that they will allocate more time to working at the office since h_{m_2} is not monotonically increasing in z.

Proposition 1 *There exist a z^* as function of k_h such that if $z \geq z^*$ then h_{m_2} is decreasing in z; if $z \leq z^*$ then h_{m_2} is increasing in z*

See the proof in the appendix.

The intuition behind Proposition 1 is that if workers are already very productive at their office, then increasing their relative productivity further will decrease the time they spend on both working from home and working at the office, because the effective labor they must supply is fixed. We will choose the prior of z based on Proposition 1. Details are provided in section 4. We model the COVID-19 pandemic as shocks on several variables, including relative labor productivity, TFP, consumption share in the final consumption good aggregator, and household productivity. Later on, we will also let the effective labor supply vary exogenously to capture fluctuations in unemployment.

4 Estimation

We estimate the model parameters using quarterly data on GDP, durable and nondurable good consumption, and hours spent working from home from 2003:Q1 to 2021:Q4. Specifically, we estimate ϵ_c, the elasticity parameter between market consumption and household production, and ϵ_l, the elasticity parameter between working from home and working at the office. We also estimate the capital shares in the household production function (α_h), and the "effective labor aggregator" function (α_l), as defined in Equation 7. Additionally, we estimate exogenous processes of TFP, household productivity, market consumption weight, and relative labor productivity at the office.

4.1 Data

Our durable and nondurable consumption data is from the real personal consumption expenditure on FRED. [2]. As we mentioned above, durable good includes four main categories,

[2] FRED

furniture and appliance, recreational, motor vehicles and parts, and other durables. In our model, c_m represents all nondurable market consumption. Map to the real data, we use the sum of nondurable goods consumption and services as our aggregate nondurable goods consumption, c_m. We obtain the time allocation data from the American time use survey (ATUS). If an activity is working and the place is the interviewee's home, then we take it as working from home, h_{m_1}; however, if the activity is performed at the working place, then we define it as h_{m_2}. All variables are quarterly; we use one side HP filter to obtain the cycle part of our data. The reason to use one side HP filter is that households make decisions only conditional on past information; however, the HP filter implicitly incorporates future data when decomposing the time series.

4.2 Estimation

Bayesian estimation starts with prior distributions describing the information we have about the parameters before observing any real data. Then the observed data will be used to update the prior distribution via the Bayesian theorem to the posterior distributions of model parameters. Table 1 shows the prior specification. The first four rows are the prior distributions of the innovations in the relative productivity of working at the office, market consumption weight, TFP, and household productivity. The inverse gamma distribution is widely used as the conjugate prior distribution for the unknown variance of a normal distribution. We assume the variance of all four structural shocks follow the inverse gamma distribution with mean 0.05 and infinite standard deviation.

We do not have much information on most of the persistence parameters, except for ρ_z; therefore, we set them to be beta distributions with a mean of 0.5 and a standard deviation of 0.05 to cover the range between 0 and 1. We set the mean of the beta distribution for the relative productivity process to be 0.3 because it should not be too persistent.

ϵ_c governs the elasticity of substitution between market consumption and household production; we set the prior to be a normal distribution with a mean of -0.5 and standard deviation of 0.5. This will cover a wide range of possible values between -1 and 0.5. Empirical evidence on the value of ϵ_c is mixed; it typically falls in the range of -0.5 to 0.5 Been et al. (2020); Rupert et al. (1995). We do not believe that ϵ_c could be greater than one because the final consumption aggregation function should be convex. ϵ_l governs the elasticity of substitution between hours spent working from home and at the office; we set the prior to be a beta distribution with a mean of 0.8 and standard deviation of 0.1. We choose a prior with a relatively high mean for ϵ_l to reflect the prior information that people spend most of their time working at the office; therefore, we are not far away from a corner solution.

The capital share in the household production function, α_h, and the effective labor production function, α_l, are set to beta distributions with a mean of 0.2 and a standard deviation of 0.04. The reason for this choice is that the literature provides a range for α_h between 0.08 and 0.36 Fisher (2007); Gomme et al. (2001); Benhabib et al. (1991), while α_l is estimated to be 0.19 in Fisher (2007), so a prior mean of 0.2 is reasonable. Another reason for setting the prior mean of α_h and α_l to 0.2 is that the household capital is used in both household production and effective labor, and the sum of α_h and α_l should not be too large in the model.

If $z_t > 1$ then agents are more productive at their office, however, if $z_t < 1$ then agents are more productive at home. The prior mean for $\mu_z = \mathbb{E}(log(z_t))$, should reflect the fact that, before the pandemic, even though the time workers spent working from home had been steadily increasing since 2010, it was still only a small percentage of the total working time because workers are generally more productive in their office than at home. We set the prior for μ_z to be a normal distribution with a mean of 0.1 and a standard deviation of 0.1 to reflect this. Because we use a CES aggregation function for the effective labor supply, the prior mean of μ_z should be greater than 0 so that people will allocate most of their working time to their office. However, it should not be too large, otherwise, when a negative shock hits the relative productivity parameter, workers would increase the time they spend working in their office, as indicated by Proposition 1.

Another important parameter in our model is the market consumption weight ω. We assume that household transfer a fixed ratio of market consumption goods,c_m, and household production, c_h, into final consumption goods using a CES aggregation function. The question is how much weight does households put on the market consumption. We set the prior of the log mean of ω to be a normal distribution with a mean of -0.5 and standard deviation of 0.2, so that the prior distribution will cover the range between -0.9 and -0.1. At the prior mean, $\omega = exp(-0.5) = 0.6$, which implies that households put more weight on market consumptions than on household production.

Table 1: Prior information (parameters)

	Distribution	Mean	Mode	Std.dev.	Bounds*		90% HPDI	
					Lower	Upper	Lower	Upper
σ_{z_l}	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
σ_ω	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{tfp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
$\sigma_{z_{hp}}$	Inv. Gamma	0.0500	0.0248	0.1000	0.0064	1337.9077	0.0176	0.1211
ϵ_c	Gaussian	-0.5000	-0.5000	0.5000	-3.6807	2.6807	-1.3224	0.3224
ϵ_l	Beta	0.8000	0.8462	0.1000	0.1025	0.9999	0.6146	0.9389
α_h	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
α_l	Beta	0.2000	0.1938	0.0400	0.0330	0.5080	0.1378	0.2691
μ_z	Gaussian	0.1000	0.1000	0.1000	-0.5361	0.7361	-0.0645	0.2645
μ_ω	Gaussian	-0.5000	-0.5000	0.2000	-1.7723	0.7723	-0.8290	-0.1710
ρ_z	Beta	0.3000	0.2951	0.0500	0.0672	0.6432	0.2205	0.3850
ρ_ω	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{z_p}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823
ρ_{hp}	Beta	0.5000	0.5000	0.0500	0.2098	0.7902	0.4177	0.5823

Note: Displayed bounds are after applying a prior truncation of options_.prior_trunc=1.00e-10

There are some parameters that we do not put into our estimation either because they have been extensively studied in the literature, such as α_m in our model, the capital share in the market production function, or because they are weakly identified in our model, such as z_{hp}, the household production technology. Figure 3 shows the identification strength in log scale. The upper panel shows the identification strength of the parameters based on the Fischer information matrix (stimulated) normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). However, if the prior mean is 0, then it will falsely indicate that the parameter is not identified; for example, in our case, μ_z and μ_{hp} are not identified because the prior mean is 0. We exclude these two variables from our estimation due to their weak identification even when the prior mean is changed

The lower panel of Figure 3 decomposes the effect that is shown in the upper panel. Generally speaking, there are two reasons for weak identification. First, the likelihood function could be relatively flat at some dimensions; therefore, the likelihood does not change with respect to these parameters, or the change is so tiny that we have a flat line in the log-posterior graph. If this is true, we will see minimal values in the lower panel, either normalized by the prior mean (blue bars) or by the prior standard deviation (red bars). However, we can see that this is not the case for our model; the lower panel shows it is unlikely that the likelihood function is flat in any dimension. The second case is that a parameter could linearly pick up the effect of other parameters; in other words, they have the same effect on the likelihood function. ϕ is collinear with other parameters; therefore, it is not identified in our model.

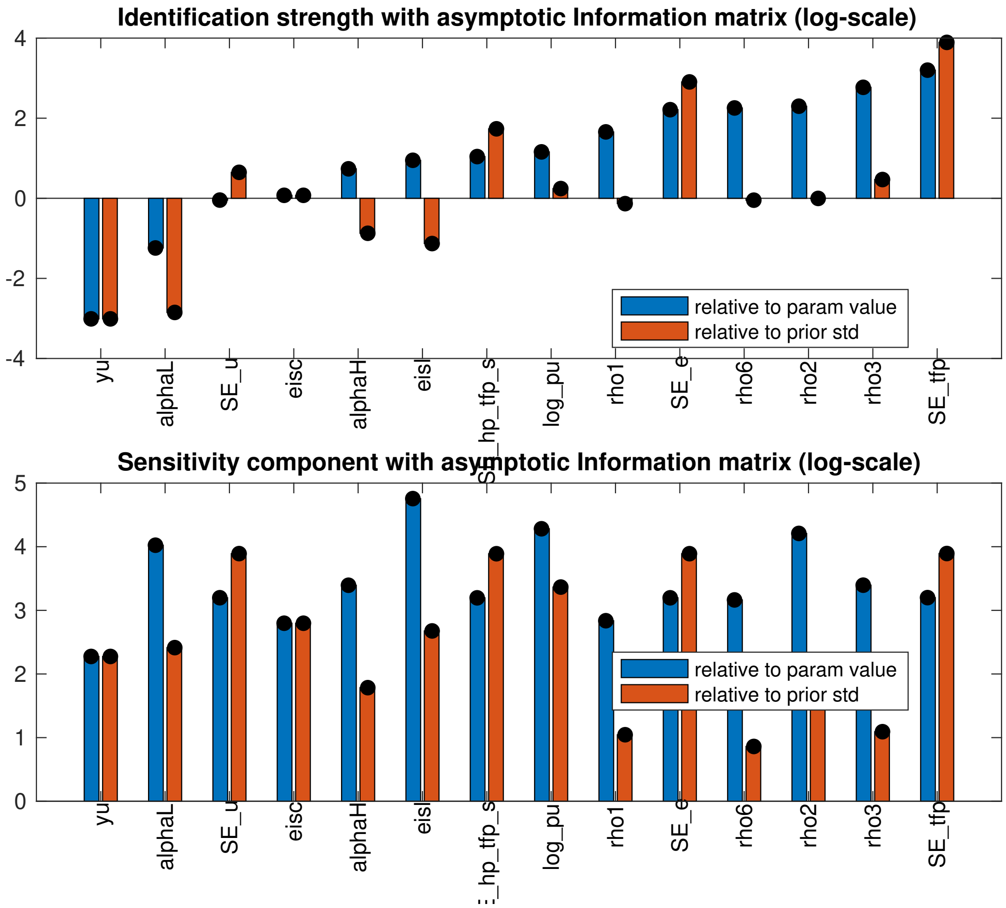

Figure 3: Prior mean - Identification using info from observables.

It is impossible to compute the posterior distribution analytically; the standard practice is to use a Metropolis-Hastings sampling procedure to approximate the posterior distribution. The number of draws should be large enough to achieve convergence of the MCMC and to compute the posterior distribution objects, such as mean and variance. We set the number of replications of each MCMC chain to 500,000. The following multivariate/univariate diagnostics show that this number is large enough to achieve convergence.

Figure 4 shows the multivariate convergence diagnostics. This diagnostic is the same as the univariate convergence diagnostics, but it is aggregated by the posterior kernel. The univariate convergence diagnostics can be found in the appendix. The first panel shows the (Brooks and Gelman, 1998) convergence diagnostics for an 80% interval. The blue line is the 80% interval range computed using the pooled draws from all sequences, and the red line is the mean interval range computed by the draws of the individual sequences. The second and third panels show the same statistics for the second and third moments. If the M-H algorithm has converged, then we expect the two lines to stabilize and be very close to each other.

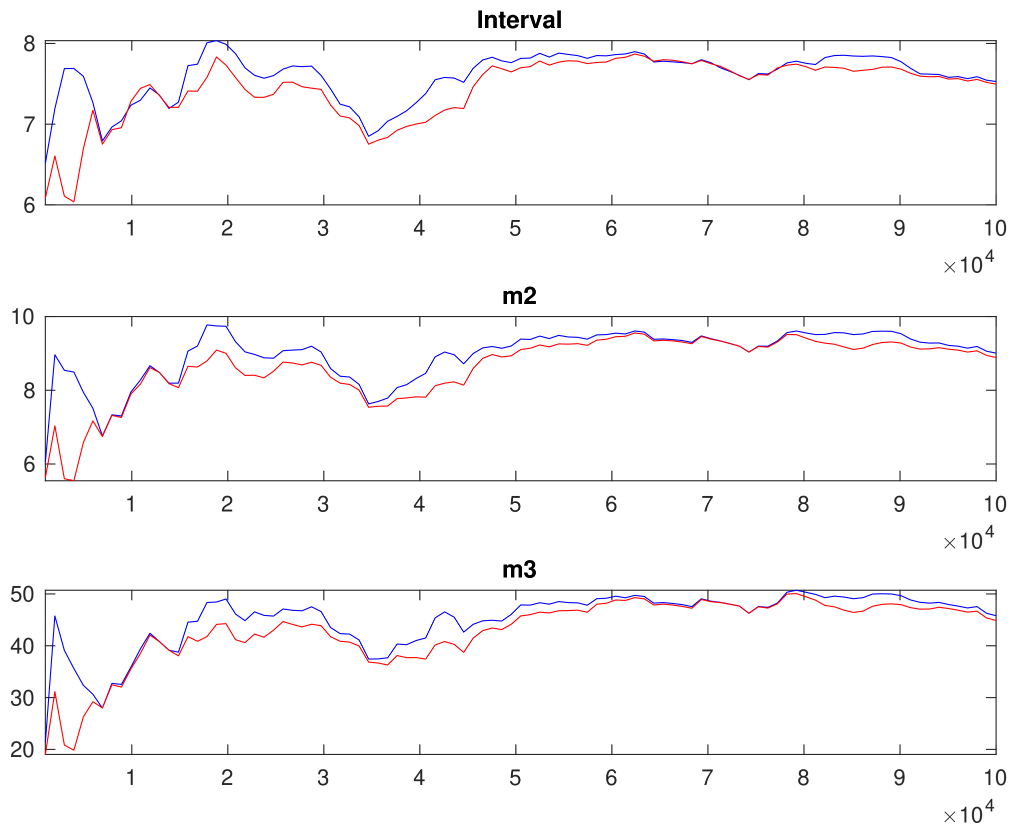

Figure 4: Multivariate convergence diagnostics for the Metropolis-Hastings. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

Table 5 and Table 3 show the Posterior distribution sampled by the Metropolis-Hastings algorithm. The left panel of Table 5 show the prior distribution type, mean, and standard deviation; the right panel shows the posterior mean, standard deviation, and the 90% highest posterior density interval. Figure 5 and Figure 6 shows the prior distribution (gray), the posterior distribution (black), and the posterior mean value (green).

Table 2: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ϵ_c	norm	-0.500	0.5000	-0.382	0.0747	-0.5033	-0.2588
ϵ_l	beta	0.800	0.1000	0.869	0.1111	0.7234	0.9912
α_h	beta	0.200	0.0400	0.149	0.0330	0.0922	0.1989
α_l	beta	0.200	0.0400	0.118	0.0247	0.0787	0.1568
μ_z	norm	0.100	0.1000	0.314	0.0867	0.1752	0.4582
μ_ω	norm	-0.500	0.2000	-0.408	0.1012	-0.5709	-0.2445
ρ_z	beta	0.300	0.0500	0.315	0.0476	0.2353	0.3910

(Continued on next page)

Table 2: (continued)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ρ_ω	beta	0.500	0.0500	0.565	0.0458	0.4867	0.6364
ρ_{z_p}	beta	0.500	0.0500	0.539	0.0443	0.4647	0.6103
ρ_{hp}	beta	0.500	0.0500	0.550	0.0432	0.4798	0.6204

Table 3: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
z_l	invg	0.050	0.1000	0.036	0.0156	0.0154	0.0613
ω	invg	0.050	0.1000	0.010	0.0013	0.0081	0.0124
z_{tfp}	invg	0.050	0.1000	0.014	0.0012	0.0122	0.0159
z_{hp}	invg	0.050	0.1000	0.127	0.0246	0.0862	0.1621

There are a few parameters we are particularly interested in. μ_z is the log mean value of the relative productivity of working at the office. $e^{\mu_z} \approx 1.47$. This implies that even household capital can improve labor productivity while working from home; workers are only 68% as productive as working at the office. This is consistent with empirical evidence. μ_ω is the log mean of the weight of market consumption in the CES aggregator function. $e^{\mu_\omega} \approx 0.65$, this implies that the share of market consumption good in the final consumption good aggregation function is 65% . α_h is the household capital share in household production; 0.148 is relatively low compared to the capital share in firms' production function, α_m, which is 0.36. In the literature, the capital share in firms' production is usually around one-third; however, the capital share in household production function is in a range between 0.08 and 0.36, Fisher (2007), Gomme et al. (2001), Benhabib et al. (1991). Our estimation of the capital share in household production lands in this range. α_l, the capital share in the effective labor production function is 0.096, which is lower than 0.19, the value used in Fisher (2007). ϵ_c governs the elasticity of substitution between market consumption and household production; we set the prior for this parameter to be a normal distribution with a mean of -0.3 and standard deviation of 1 to cover a wide range of values; the prior 90% HPDI is [-1.94,1.34]. The posterior distribution, however, is very concentrated around the posterior mean, -0.317, and the 90% HPDI is $[-0.44, -0.20]$.

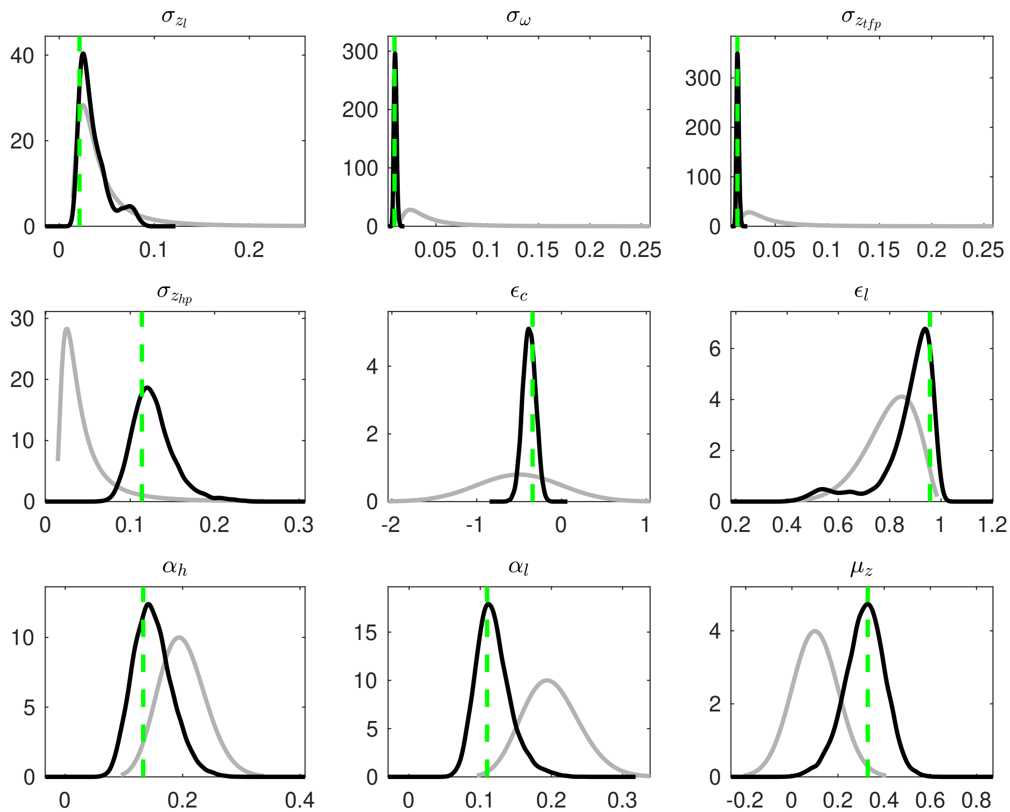

Figure 5: Priors and posteriors.

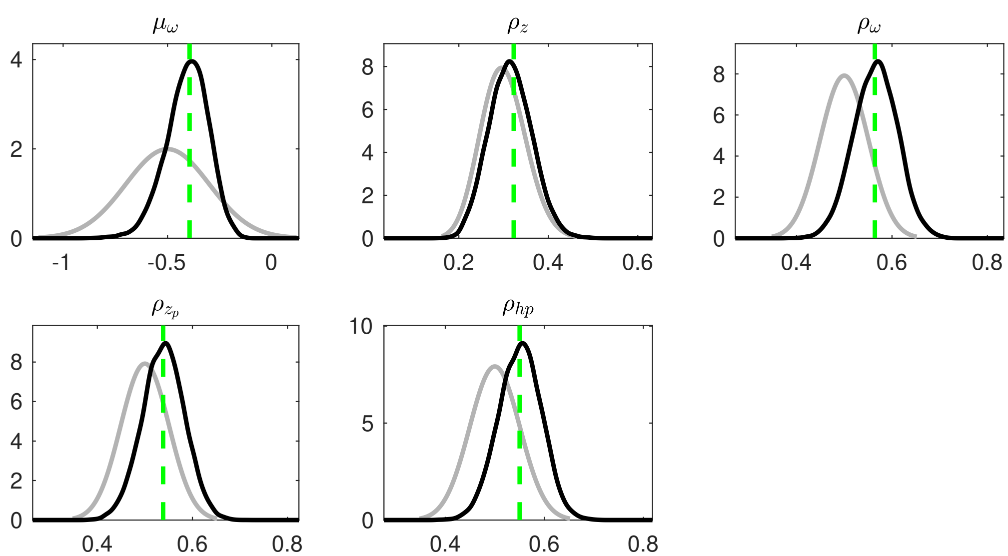

Figure 6: Priors and posteriors.

Figure 7 and Figure 8 show the smoothed observed variables and smoothed shocks. As shown in the upper left panel of Figure 7, there was a severe negative shock on labor productivity at the beginning of the pandemic. At the steady-state, as shown in table Table 6, working at the office is more productive than working at home. Combined with the high elasticity of substitution between working at home and working at the office, $\epsilon_l = 0.95$, we have that workers spend most of their time working at the office, with $\frac{h_{m_1}}{h_{m_1}+h_{m_2}} = 15\%$. This is very close to what we have observed in ATUS, where workers spend about 10% of their working time at home. We see a negative relative productivity shock hit the economy in 2020Q2 when the pandemic began, with working at the office being only 50% as productive as working from home. This captures all features that could affect labor productivity while working at the office, for example, if one member of the household gets COVID, not only the infected member but the entire household may need to take a few days off from work to either recover from the infection or to take care of the sick one.

We also observe a negative TFP shock in the lower left panel of Figure 7. This shock captures factors that negatively affect firms' productivity, such as the supply chain issue that has lasted until now. The TFP shock could also capture the friction while transitioning to the working-from-home regime. We can see that the TFP quickly recovered and rebounded to a slightly higher level.

The top right panel of the figure shows the consumption weight variation. The consumption weight has two interpretations. It could be either part of the household's preference or the technology of the final consumption good aggregation function. The consumption weight shock and the household production technology shock capture many aspects of the pandemic. First, these two shocks could have captured the effect of the lockdown. Agents reduced their market good consumption, c_{m_t}, not because their preferences have changed but because of access restrictions such as lockdown, social distancing, etc. More importantly, unemployment was high in 2020 Q2. However, we assumed constant EL_t, which will exaggerate the effect of the TFP shock and household production technology shock.

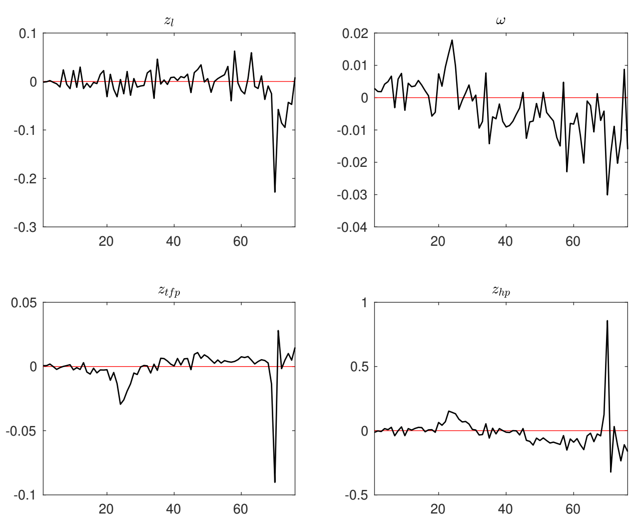

Figure 7: Smoothed shocks.

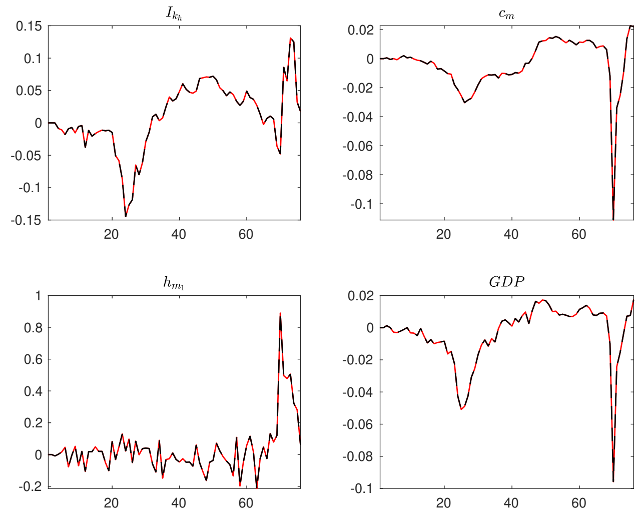

Figure 8: Historical and smoothed variables.

4.3 Shock Decomposition

In subsection 4.2, we estimated some key parameters and four exogenous shock processes in our model. In this section, we decompose the observed deviations of durable goods, nondurable goods, and time allocations from their steady-state into contributions from the four shocks.

Figure 9 shows the effect of each shock on durable goods consumption. The blue bar represents the contribution of shocks to the relative productivity of working at the office. We can see that before 2020, the effect was insignificant; however, since the second quarter of 2020, it increased dramatically and accounts for about a third of the increase in durable goods consumption. Many workers started working from home at the end of 2020 Q1, and working from home contributed greatly to the increase in durable goods consumption during 2020 Q2 and Q3. There are two reasons for this. First, there could be a lag between workers starting to work from home and deciding to upgrade their working environment. Second, there is an adjustment cost, which prevents households from making minor adjustments to their household capital stock.

The green and black bars represent the effect of shocks on the market consumption weight in the final consumption goods aggregate function and the household production technology. There are two interpretations of the weight parameter. First, it could be interpreted as part of the household's preference. Second, it could be a parameter of the household production function. The fluctuation of the market consumption weight and household production technology captures the variation in household production technology and preference. For example, many market consumption goods could be inaccessible or undesirable due to precautionary measures such as social distancing or shelter in place at the early stage of the pandemic. These factors exogenously change the consumption composition of households and their ability to transform time, household capital, and nondurable consumption goods into final consumption goods.

Overall, there are two main driving forces behind the increase in durable goods consumption. First, the relative productivity of working from home is higher during the pandemic;

therefore, many workers choose to work from home and upgrade their home office. Second, households substitute from market consumption toward household production because many market consumption goods become inaccessible or undesirable due to the shelter-in-place order or the risk of getting COVID.

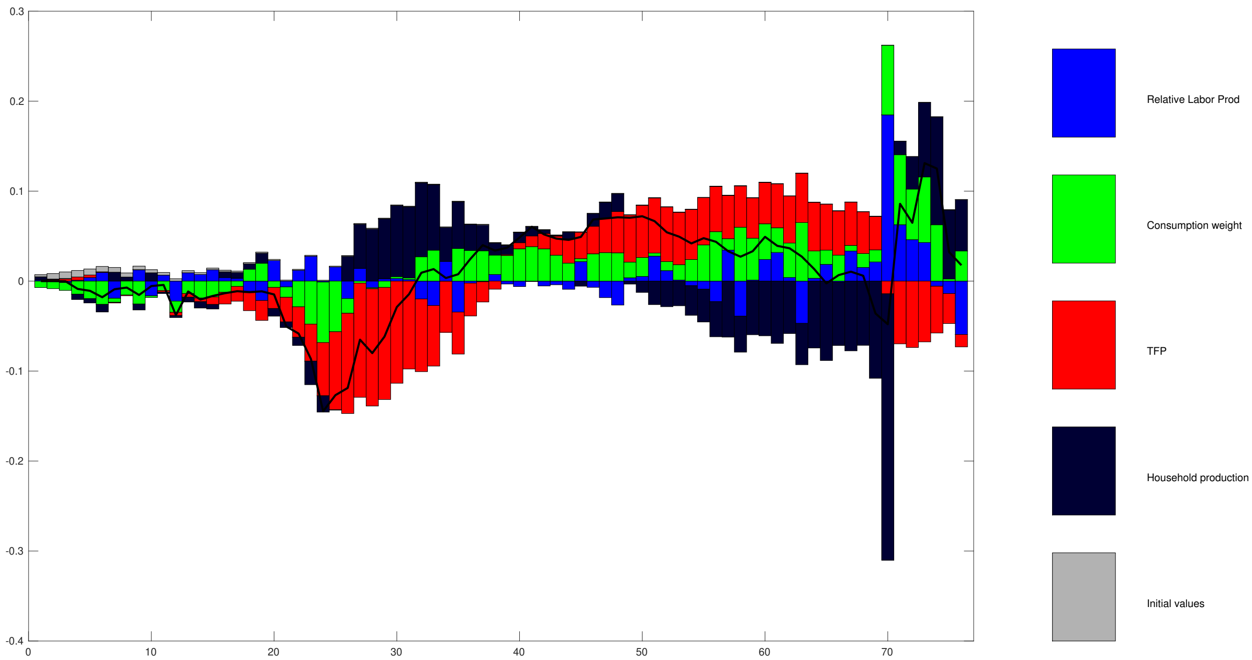

Figure 9: Historical shock decomposition: I_{k_h}.

Figure 10 shows the shock decomposition of the nondurable consumption,c_m. We can see that at the early stage of the pandemic, the effect of consumption weight shock on nondurable good consumption, the green bars, is small compare to the effect of household productivity shock, the black bar. This is because the decrease in nondurable goods consumption is much bigger than the increase in durable goods consumption, and to match the data, we need a particularly big shock on household productivity. The other side effect of this is that the time spent on household production increased way much more than what we observed in the real data, see Figure 18. [3]

Figure 11 shows that relative productivity is the only force that drives the allocation between working from home and working at the office. Before the COVID-19 pandemic, both the fraction of workers who worked from home full-time or in a hybrid model, and the average time people spent working from home, had been increasing. This could be seen as progress in technology that improves relative productivity while working at home.

[3]The decomposition graph of h_h, h_{m_2}, and y in Appendix E

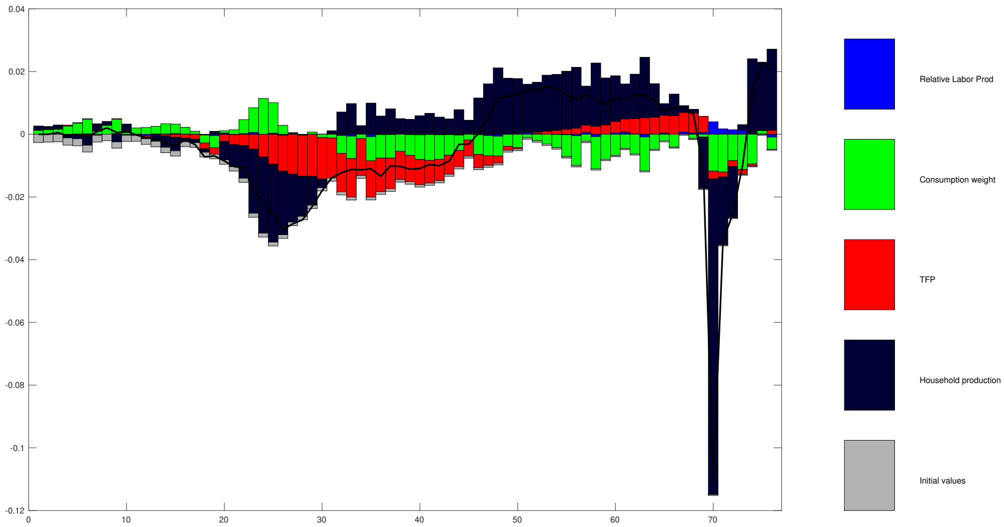

Figure 10: Historical shock decomposition: c_m.

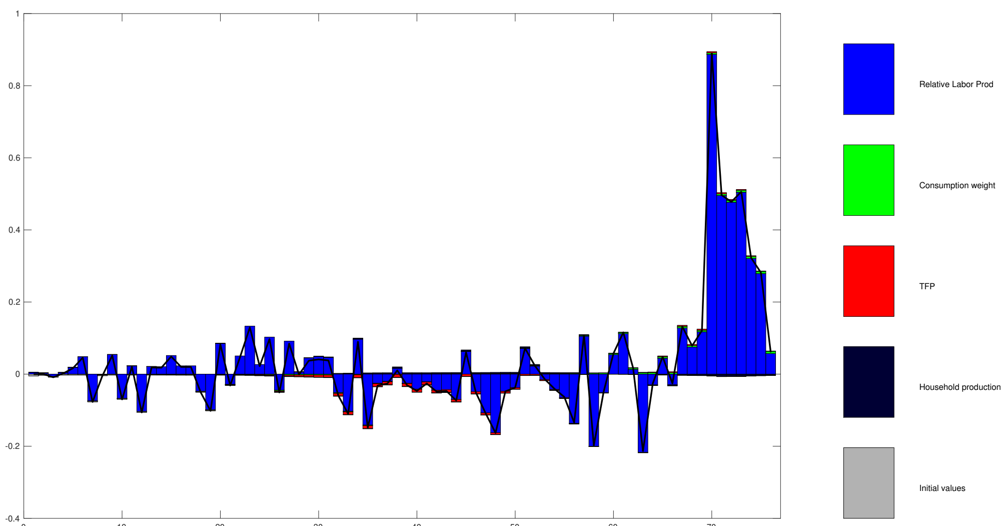

Figure 11: Historical shock decomposition: h_{m_1}.

4.4 Sensitivity Analysis

In this section, I demonstrate that our results are robust from two perspectives. Firstly, I show that using different priors results in slightly different posteriors, but nothing substantially different. Secondly, I demonstrate that when using data from only 2003:Q1 to 2019:Q4, we obtain different results, as expected.

One of the most critical parameters in our model is ϵ_c, which governs the elasticity of substitution between market consumption and household production. In the literature, ϵ_c is typically set to be between -0.54 and 0.5 (Been et al., 2020; Rupert et al., 1995). In our model, the final consumption good is aggregated by market goods and household productions using a CES function. We set the prior to be a normal distribution with a mean of -0.5 and standard deviation of 0.5 to cover a wide range of possible values between -1.5 and 0.5. We do this because we believe that household production and market consumption are more like complements than substitutes; in most cases, we transform a fixed ratio of c_m and c_h to the final consumption good. However, if there is a shortage of c_m, we cannot produce the same amount of final consumption good by simply increasing c_h. (Been et al., 2020) also shows that only about 11% of total consumption spending can potentially be replaced by home production. To ensure that our results are not solely driven by the prior, we set the prior to be a normal distribution with a mean of 0.2 and standard deviation of 0.4, so that the two standard deviations will cover a range between -0.6 and 1. The posterior mean of ϵ_c is -0.359 under this prior, and the full table can be found in Appendix C. It is only slightly different from the posterior mean we reported in Table 5, which is -0.365. In Appendix C, I also show that when I set the prior mean to 0.7 and keep the standard deviation the same, the posterior is still very similar to the result we report in subsection 4.2. Overall, our results are consistent with (Been et al., 2020) that households could not easily replace market consumption with household production. Different priors giving us similar results indicate that the data we used is very informative and the prior only plays a minor role in determining the posterior.

5 Conclusion

In this paper, I show that the unusually large increase in durable goods consumption during and post the pandemic cannot be explained solely by the substitution between market nondurable consumption and household production, as many nondurable consumption goods and services are highly specialized, making it difficult for the average household to substitute them with household production. I propose a new model that allows households to endogenously decide how to allocate their time, which captures the fact that working from home is an important factor that drives durable goods consumption, in addition to household production.

The COVID-19 pandemic may have forever changed our way of life and work, with many workers now having the option of working from home. Many companies have decided that their employees will continue to work from home until 2022, and some companies have proposed a hybrid working schedule that allows employees to work from home for three or four days a week, with only one or two days in the office. These changes are unlikely to be reversed soon, making household capital more critical than ever before, and the complementarity between household capital and market capital stronger. It will be important to keep this channel in mind when modeling household time allocation and consumption decisions.

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A Euler equations

$$
\begin{aligned}
&\mathbb{E}_t(C_t^{\frac{1-\gamma}{\epsilon_c}-1}\omega c_{m,t}^{\epsilon_c-1}-\lambda_t)=0\\
&\mathbb{E}_t(\lambda_t\left((k_{h,t}^{\alpha_l}h_{m_1,t}^{1-\alpha_l})^{\epsilon_l}+yh_{m_2,t}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1}(1-\alpha_l)k_{h,t}^{\alpha_l\epsilon_l}h_{m_1,t}^{(1-\alpha_l)\epsilon_l-1}-H^{\phi})=0\\
&\mathbb{E}_t(\lambda_t\left((k_{h,t}^{\alpha_l}h_{m_1,t}^{1-\alpha_l})^{\epsilon_l}+yh_{m_2,t}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1}y_{1,t}h_{m_2,t}^{\epsilon_l-1}-H^{\phi})=0\\
&\mathbb{E}_t(C_t^{\frac{1-\gamma}{\epsilon_c}-1}(1-\omega)c_{h,t}^{\epsilon_c-1}k_{h,t}^{\alpha_h}h_{h,t}^{-\alpha_h}(1-\alpha_h)-H^{\phi})=0\\
&\mathbb{E}_t(\beta(1+r_{t+1})\lambda_{t+1}-\lambda_t)=0\\
&\mathbb{E}_t(-\lambda_t+\beta(1-\delta_h)\lambda_{t+1}=0\\
&+\beta\lambda_{t+1}\left((k_{h,t+1}^{\alpha_l}h_{m_1,t+1}^{1-\alpha_l})^{\epsilon_l}+yh_{m_2,t+1}^{\epsilon_l}\right)^{\frac{1}{\epsilon_l}-1}\alpha_l k_{h,t+1}^{\alpha_l\epsilon_l-1}h_{m1,t+1}^{(1-\alpha_l)\epsilon_l}\\
&+\beta C_{t+1}^{\frac{1-\gamma}{\epsilon_c}-1}(1-\omega)c_{h,t+1}^{\epsilon_c-1}\alpha_h k_{h,t+1}^{\alpha_h-1}h_{h,t+1}^{1-\alpha_h})\\
r_t&=\alpha_m k_{m,t}^{\alpha_m-1}EL_t^{1-\alpha_m}-\delta_m\\
w_t&=(1-\alpha_m)k_{m,t}^{\alpha_m}EL_t^{-\alpha_m}
\end{aligned}
$$

B convergence

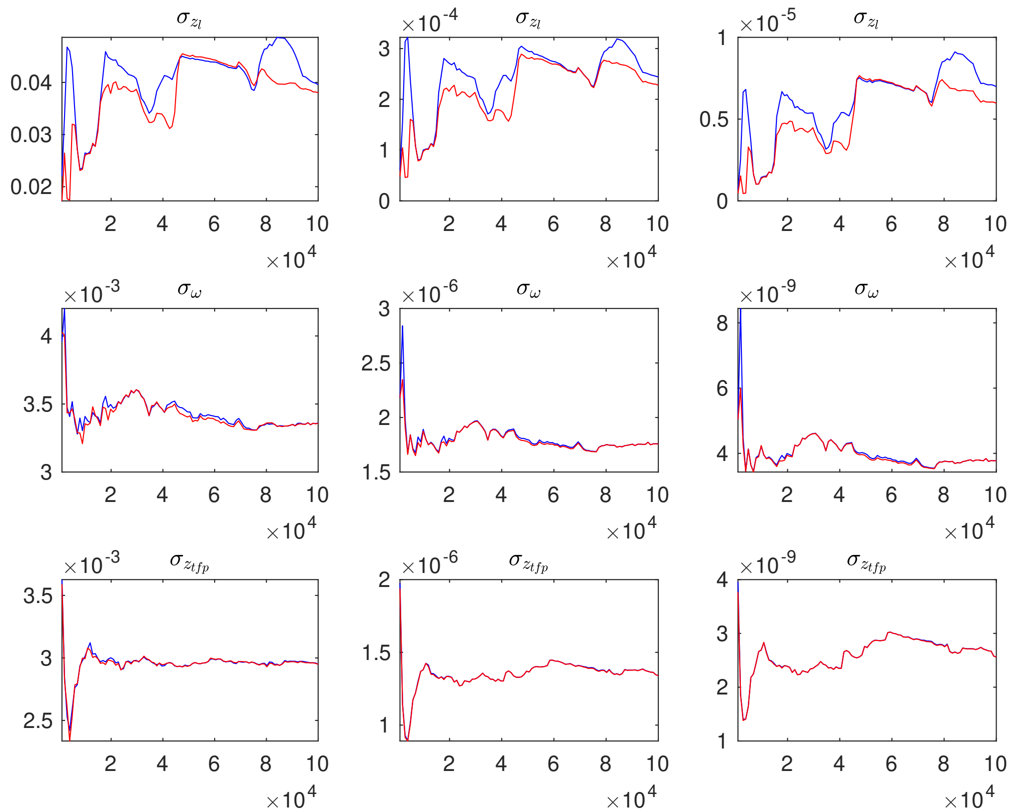

Figure 12: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

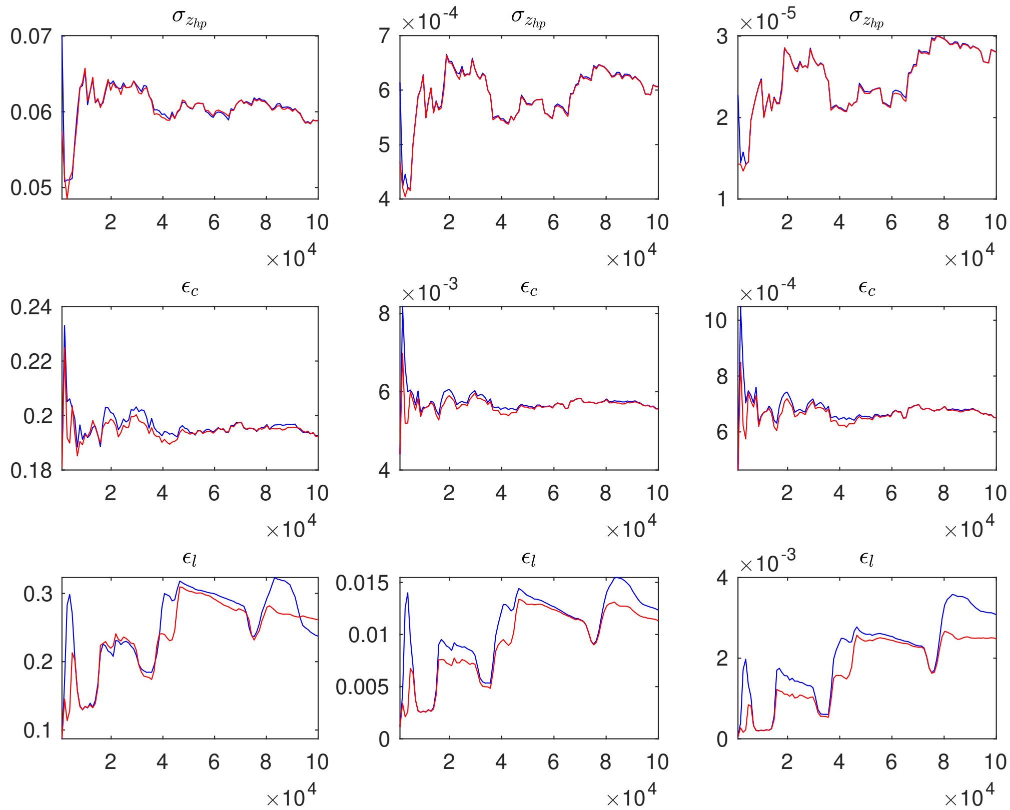

Figure 13: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

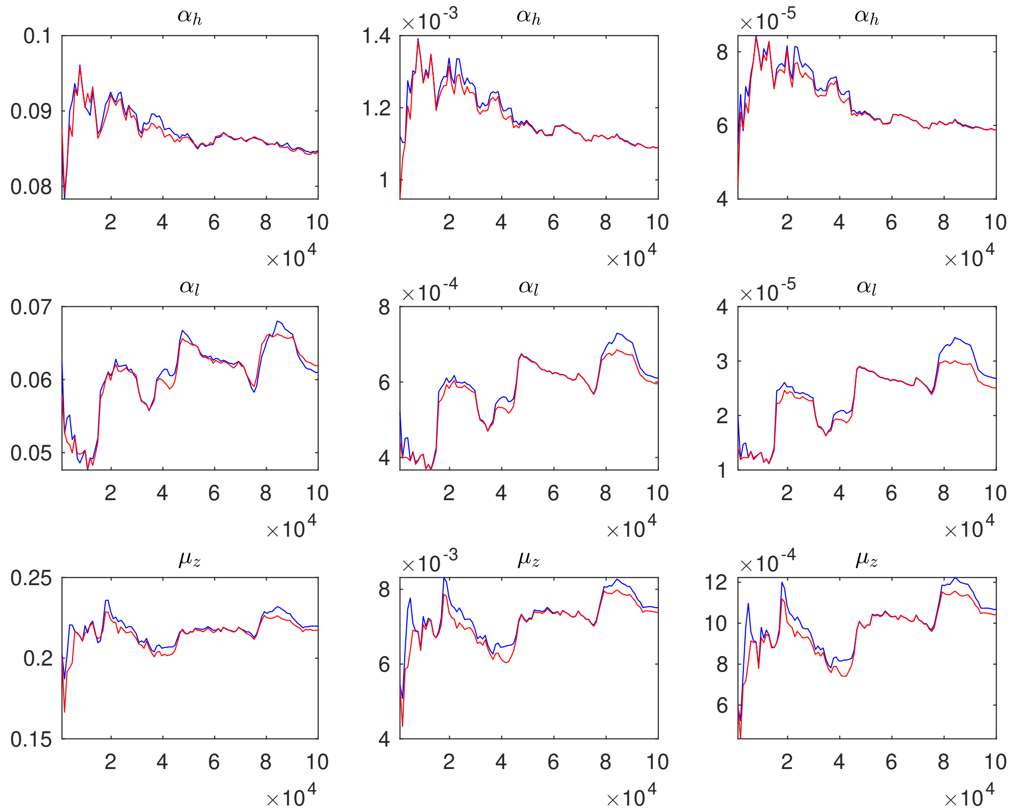

Figure 14: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

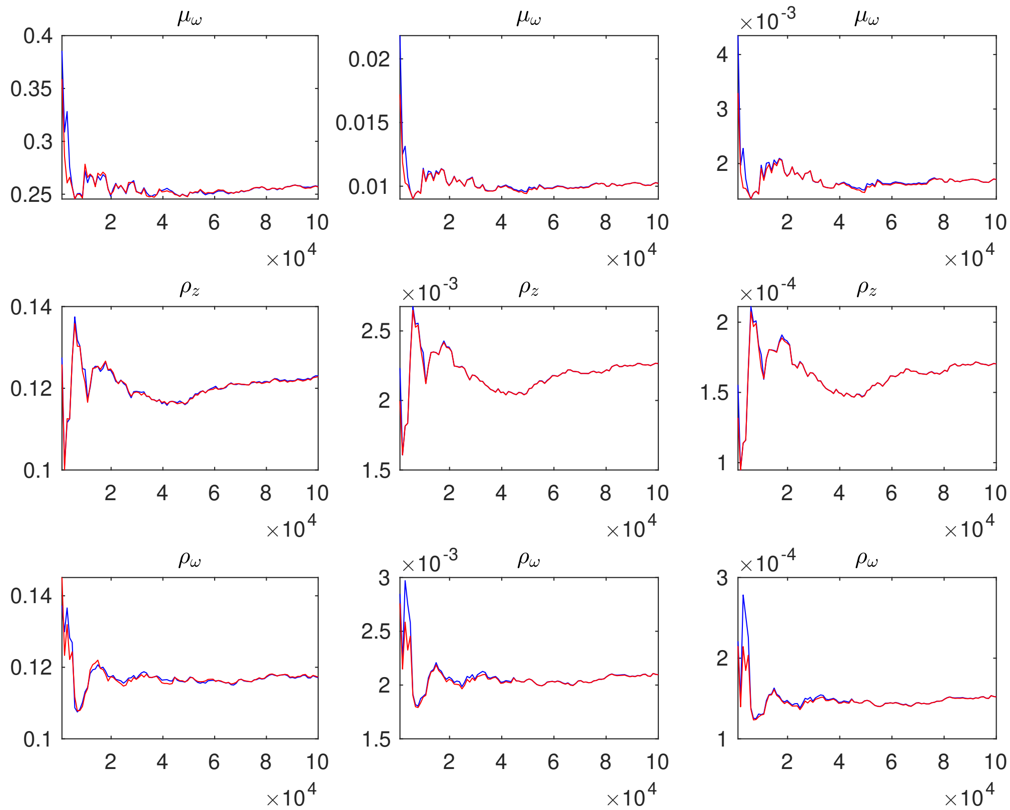

Figure 15: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

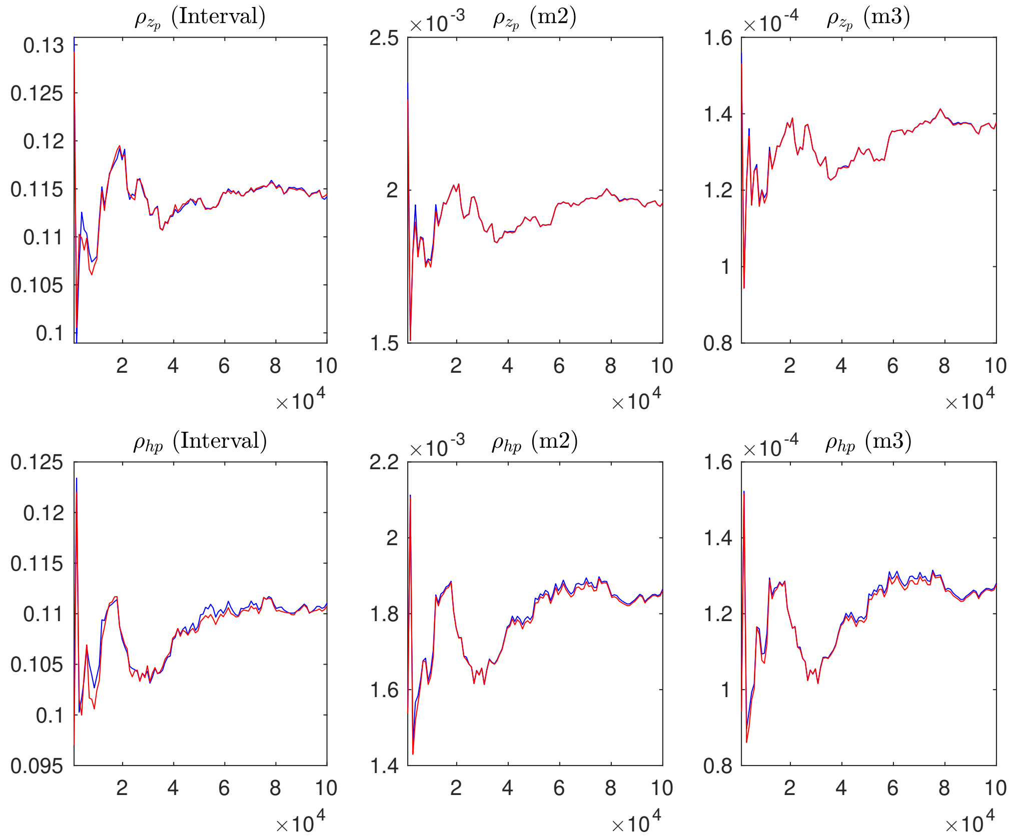

Figure 16: Univariate convergence diagnostics for the Metropolis-Hastings. The first, second and third columns are respectively the criteria based on the eighty percent interval, the second and third moments.

C Sensitivity Check

In this section, the estimation results for different priors or data are reported, including the result of using only pre-pandemic data (from 2003 Q1 to 2019 Q4). The Bayesian approach allows us to include the most recent data and deal with small samples, using a shorter span of data still gives us different results, as expected. The difference in results is driven by two anomalies, first is the elasticity of substitution between market consumption and household production (ϵ_c) and second is the relative productivity parameter (μ_z).

When only pre-pandemic data is used, the posterior mean of μ_z is not very different from the prior, but this is not because it is not identified, rather it is because the time spent on durable goods has been very stable, thus not much information can be extracted from the pre-pandemic era. On the other hand, the posterior mean of ϵ_c is negative, indicating that during the pandemic, market consumption and household production are more like complements than substitutes. This could be due to shortages and lockdowns. To make sure the results are not crazy in the pre-pandemic era, it is important to examine the results carefully.

The thing is that if we only use the pre pandemic data. cm and ch can replace each other very will. however, if we include the padenmcie, the eisc will be negative. the reason is that before pandmeic, severce is no severly affect, during the pandmeic, big chunk of the decase in nondurable consumpti8on is service, at least in my mnodel, that is how i definte nondurables. therefore, the lose in cm could not be replace by ch very well.

Table 4: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ϵ_c	norm	0.200	0.5000	-0.368	0.0755	-0.4911	-0.2435
ϵ_l	beta	0.800	0.1000	0.842	0.1466	0.5905	0.9938
α_h	beta	0.200	0.0400	0.150	0.0337	0.0952	0.2041
α_l	beta	0.200	0.0400	0.122	0.0295	0.0766	0.1650
μ_z	norm	0.100	0.1000	0.314	0.0869	0.1723	0.4567
μ_ω	norm	-0.500	0.2000	-0.403	0.0999	-0.5662	-0.2425
ρ_z	beta	0.300	0.0500	0.317	0.0482	0.2370	0.3953
ρ_ω	beta	0.500	0.0500	0.564	0.0465	0.4885	0.6414
ρ_{z_p}	beta	0.500	0.0500	0.537	0.0441	0.4643	0.6093
ρ_{hp}	beta	0.500	0.0500	0.552	0.0429	0.4810	0.6219

When only use data from 2003-2019

Table 5: Results from Metropolis-Hastings (parameters)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
ϵ_c	norm	-0.200	0.5000	0.624	0.1387	0.4038	0.8420
ϵ_l	beta	0.800	0.1000	0.775	0.0881	0.6431	0.9240
α_h	beta	0.200	0.0400	0.177	0.0332	0.1255	0.2342
α_l	beta	0.200	0.0400	0.210	0.0405	0.1407	0.2721
μ_z	norm	0.100	0.1000	0.043	0.0833	-0.0935	0.1794
μ_ω	norm	-0.500	0.2000	-0.346	0.0714	-0.4615	-0.2284
ρ_z	beta	0.300	0.0500	0.102	0.0155	0.0764	0.1264
ρ_ω	beta	0.500	0.0500	0.694	0.0407	0.6282	0.7596
ρ_{z_p}	beta	0.500	0.0500	0.625	0.0400	0.5620	0.6935
ρ_{hp}	beta	0.500	0.0500	0.645	0.0419	0.5774	0.7138

D Steady state at posterior mean

Table 6: Steady state

Variable	Value
k_{h_i}	0.157
c_m	2.015
c_h	0.512
k_m	19.304
k_h	6.284
h_{m_1}	0.104
h_{m_2}	0.562
h_h	0.333
output	2.66

E Shock Decomposition

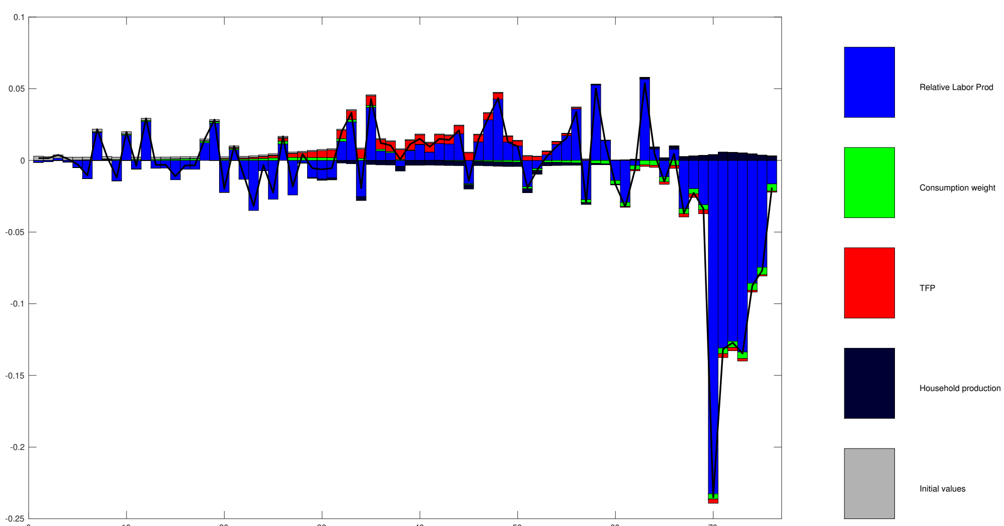

Figure 17: Historical shock decomposition: h_{m_2}.

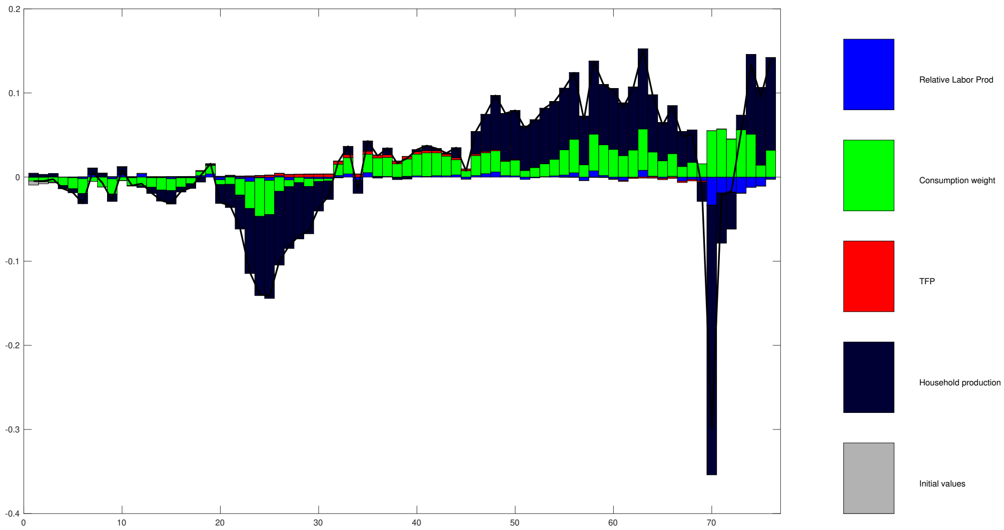

Figure 18: Historical shock decomposition: h_h.

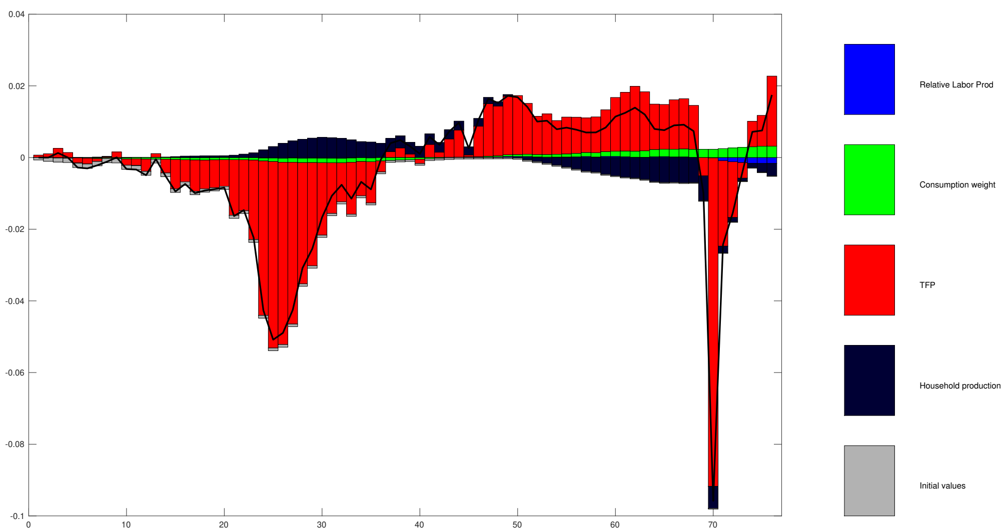

Figure 19: Historical shock decomposition: y.