ME 555 - Spring 2025 Due Date: March 30^{th}
Midterm

Write and sign at the end of the exam: "I have neither given nor received outside aid during this exam, nor have I concealed any violation of the honor code."

Problem 1. *25 points (5 points each)*

Identify if each of these statements is true. Justify each statement accordingly. **Please be concise. One or AT MOST two sentences is enough**, other than for (e). Please don't put in 2-3 different explanations.

(a) (True/False) Given a square matrix, $A \in \mathbb{R}^{n\times n}$, the set $\{I, A^1, A^2, ..., A^k\}$ is a basis for a vector space of size $k = n^2 - 1$.

(b) (True/False) If the poles are in the left-half plane, a state-space representation of a dynamic system is unique.

(c) (True/False) Linearizing a system will always lead to an LTI solution.

(d) (True/False) Using the Lyuapunov Direct Method, then it is possible to show that an LTI system is stable in the sense of Lyuapunov, then it is always BIBO stable.

(e) (True/False) For the system described by $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = 0$ there is a unique transfer function, $G(s)$:

$$G(s) = \frac{1}{s^2 - 5s + 6}$$

Problem 2. *21 points (7 points each)*

Assume that $\{x_1, x_2, x_3\}$ is a basis for $\mathbb{R}^3$. Define a new set of vectors $\{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$, where

$$\hat{x}_1 = x_1, \qquad \hat{x}_2 = x_1 + 2x_2, \qquad \hat{x}_3 = 3x_2 + 2x_3$$

(a) Show that $\{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$ is also basis for $\mathbb{R}^3$.

(b) A vector can be represented as $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the $\{x_1, x_2, x_3\}$ basis. What is its representation, $\hat{b}$ in the $\{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$ basis? You can leave it as a product of matrix and vector, without calculating the matrix inverse or doing the matrix multiplication.

(c) A linear operator ($\mathbb{R}^3 \to \mathbb{R}^3$) can be represented as $C = \begin{bmatrix} c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ in the $\{x_1, x_2, x_3\}$ basis. What is its representation in the $\{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$ basis? You can leave it as a product of matrices, without calculating the inverse or doing the matrix multiplication.

Problem 3. *24 points (8 points each)*

Given the following state-transition matrix for a linear system $\dot{x}_t = A(t)x(t) + \begin{bmatrix} e^{t^2} \\ e^{t^2} \end{bmatrix} u(t)$, answer the following questions.

$$\Phi(t, t_0) = \begin{bmatrix} e^{t^2 - t_0^2} & 0 \\ 0 & e^{t^2 - t_0^2} \end{bmatrix}$$

(a) What is the corresponding A matrix?

(b) What is $x(1)$ if $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $u(t) = 0$?

(c) What is $x(t)$ if $x(0) = 0$ and $u(t) = 1$?

Problem 4. *30 points (a: 7 points, b: 9 points, c: 7 points, d: 7 points)*

Given the following for a state-space system: $A = \begin{bmatrix} c & 0 & 0 \\ 0 & -c & 2c \\ 0 & 0 & c \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$, where c, b are non-zero real numbers, i.e., $c, b \in \mathbb{R} \setminus \{0\}$.

(a) Can you tell if the system is stable? Why or why not?

(b) Use any method to determine the Jordan form (such as not using or using eigenvalues), $\hat{A}$, and transformation matrix, T, for the state dynamics matrix, where $\hat{A} = TAT^{-1}$?

(c) What is the $\hat{B}$ matrix for the similar system $\dot{x} = \hat{A}x + \hat{B}u$, where $\hat{A}$ is the Jordan form of the A matrix?

(d) What is the homogeneous solution for this system, given $x(0) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$?

Feel free to leave it as a multiplication of 4 matrices. You don't need to do a matrix inverse, but you do need to evaluate the terms inside each matrix. For example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} t & 2t \\ 3t & 4t \end{bmatrix}$.

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