ME 555 - Spring 2025 Due Date: April 20$^{\text{th}}$
Homework #4

Problem 1. *Controllability*

(Girard and Kabamba, Linear System Theory) The linearized equations of motion of a satellite are given by the following:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u$$

x_1 and x_2 are perturbations in the radius and the radial velocity. x_3 and x_4 are perturbations in the angle and the angular velocity. The input vector consists of a radial thrust u_1 and a tangential thrust u_2.

(a) Show that the thesis is controllable from u.

(b) Can the system still be controlled if the radial thruster fails? What if the tangential thruster fails?

Problem 2. *Controllability (LTV)*

Is this system controllable and/or observable?

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ e^{-t} \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & e^{-t} \end{bmatrix} x$$

Problem 3. *Realizations*

$$\text{System 1: } \dot{x} = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x$$

$$\text{System 2: } \dot{x} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x$$

(a) Are these two realizations of the same system?

(b) Are the two transfer functions associated with the two state equations above the same?

Problem 4. *Realization*

Consider the transfer function: $G(s) = \frac{s^2+1}{s^3+2s+10}$.

(a) Transform the state equations into the controllability canonical form (CCF).

(b) Transform the state equations into the observability canonical form (OCF).

Problem 5. *Controllability, Observability, and Kalman Decomposition (LTI)*

(a) Is this system controllable and/or observable?

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} x$$

(b) Using similarity transformation matrices, what is the Kalman decomposition?

Problem 6. *Full State Feedback*

Consider the uncontrollable state equation:

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

(a) Is it possible to find a gain $\mathbf{k}$ so that the equation with state feedback $u = r - \mathbf{kx}$ has eigenvalues -2, -2, -1, -1? Is it possible to have eigenvalues -2, -2, -2, -1? How about -2, -2, -2, -2?

(b) Is this system stabilizable (i.e., are all uncontrollable states/modes already stable)?