HW#2 Solutions

1) a)

(i) a
$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$
 + b $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ + c $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
 $a+2b=0=0$ => $a=2b$
 $b+5c=0=> c=\sqrt{5}b$
 $3a+b+c=0=> 6b+b+\sqrt{5}b=0$
 $b=0$
 $b=0$

b)
$$T = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \quad A^2 = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$
$$2 \cdot T + A - A^2 = 0$$

The C-H Theorem 3007

a)
$$||\chi_{1}||_{1} = |2| + |-3| + |1| = 6$$

$$|(\chi_{1}||_{2} = \sqrt{2^{2} + (-3)^{2} + |2|} = \sqrt{|4|}$$

$$||\chi_{1}||_{0} = \max(|2|, |-3|, |1|) = 3$$

$$||x_{1}||_{\infty} = \max(|2|, |-3|, |1|)$$

$$||X_2||_1 = |+|+|=3$$

$$||X_2||_2 = \sqrt{|^2_{+1}|^2_{+1}|^2} = \sqrt{3}$$

6) Use Gram - Schmidt Normalization:
$$u_1 = x_1 \qquad g_1 = \frac{u_1}{\|u\|_2} = \frac{1}{\int u} \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$u_1 = x_1$$
 $u_2 = x_2 - (g_1^T x_2) g_1$

$$= \left[\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right) \right] \left[\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right]$$

$$q_2 = \frac{u_2}{||u_2||} = \frac{1}{53} \left[\frac{1}{1} \right]$$

Orthonormal basis:

Orthonormal basis:
$$g_1 = \frac{1}{514} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$g_2 = \frac{1}{53} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3)
$$A_{1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad A_{2} = \begin{bmatrix} 4 & 1 & -1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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Rank
$$(A_1) = 2$$
 $+ 4$

$$Ranh(A_z) = 3$$

Rank
$$(A_z) = 3$$

Nullity $(A_z) = 3 - 3 = 0$
Nullity $(A_z) = 3 - 3 = 0$
for A_1 (dumn) $= 2$ and $= 3$ are $\lim_{n \to \infty} \inf_{n \to \infty} \inf$

Range space =
$$\frac{2}{5} \left[\frac{1}{5} \right] \frac{3}{5}$$

Null space = $\frac{5}{5} \left[\frac{1}{5} \right] \frac{3}{5}$

for Az, all columns are lin-ind. (50')

Range space = $\frac{5}{5} \left[\frac{4}{3} \right] \left[\frac{1}{5} \right] \left[\frac{1}{5} \right] \frac{3}{5}$

Null space =
$$\frac{2[0]}{2[0]}$$
 drn=0

(a) $||A_{1}||_{1} = ||max|| (|+0+0|, 0+0+0|, 0+0+1|)$

$$||A_{1}||_{2} = \int ||max|| (|+0+0|, 0+0+0|, 0+0+1|)$$

(1Az1/20 = 6

(1A,1100 = 1

11Az11z= 8 11Az11z= 531.4 2 5.6

. Same process for Az:

$$e$$
-values one $1,2,3$

e-values are
$$1,2,3$$
upper triangular. 5

$$A_{7} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$(1 - A) v_1 = 0 = 0$$

2)
$$(2I-A)v_2=0 \Rightarrow v_2=\begin{bmatrix} 4\\ 0 \end{bmatrix}$$

3) $(3I-A)v_3=0 \Rightarrow v_3=\begin{bmatrix} 5\\ 0\\ 1 \end{bmatrix}$

$$(1I-A)V_1 - 0$$

$$(2I-A)V_2 = 0 - V_2$$

$$\frac{1}{2}$$
 $\frac{2}{50}$ $\frac{50}{3}$ $\frac{3}{3}$

determine e-vectors

()
$$(1I-A)V_1 = 0 = V_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_{2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -12 & 6 \end{bmatrix} \xrightarrow{control} control$$

$$|A-\lambda I| = 0$$

E-values:
$$|A-\lambda I| = 0$$

-2 V12 1 V1320

U= [2] only e-vector

$$7 = 2$$
E-vector:
$$(8-x_1)V_1 = 0 =) \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 8 & -12 & 4 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{13} \\ v_{13} \end{bmatrix} = 0$$

C.E. is

59-652+125-850

$$-2V_{11} + V_{12} = 0 \Rightarrow V_{12} = 2V_{11}$$

$$-2V_{12} + V_{13} = 0 \Rightarrow V_{13} = 2V_{12} = 4V_{11}$$

$$-2V_{12} + 4V_{13} = 0$$

$$8V_{11} - 12V_{12} + 4V_{13} = 0$$

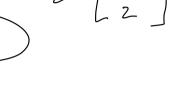
$$(8-XI)$$
 $v_z = v_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

$$V_{2} = \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 2 \end{bmatrix}$$

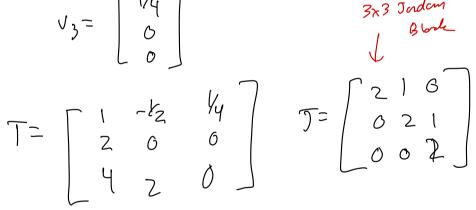
$$2v_{12} + v_{13} = 2$$

$$8v_{11} - 12v_{12} + 4v_{13} - 4$$

$$(B-\lambda T) V_3 = V_2 = \begin{bmatrix} -v_2 \\ 0 \\ 2 \end{bmatrix}$$







$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0 = 0$$
 $\lambda = 0, 1, 1$
a) $f(x) = \chi^{0}$

$$f(x) = \chi$$
Let $h(\lambda) = B_0 + B_1 \lambda + B_2 \lambda^2$

$$f(0) = h(0)$$

$$\beta = h'(1) : 10 = \beta_1 + 2\beta_2$$

 $\beta_8 = 0$, $\beta_1 = -8$, $\beta_2 = 9$

$$f'(i) = h'(i) : 10 = \beta_1 + 2\beta_2$$

$$f(0) = h(0)$$
: $0 = \beta_0$
 $f(1) = h(1)$: $1 = \beta_0 + \beta_1 + \beta_2$

C10= BoI + B1C+B2C2

 $= -8 \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 9 \begin{bmatrix} 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

b) Same process as above
$$\beta_0 = 0 \quad \beta_1 = -98 \quad \beta_2 = -98$$

$$\beta_{0} = 0$$
 $\beta_{1} = -98$ $\beta_{2} = 99$

$$c) = \begin{cases} 1 & 1 & 29 \\ 0 & 0 & 1 \end{cases}$$

$$c) = f(x) = e^{xt}$$

$$f(0) = h(0) : f_0 = 1$$

$$f(1) = h(1) : e^{t} = f_0 + f_1 + f_2$$

$$f'(1) = h'(1) : te^{t} = f_1 + 2f_2$$

$$f(1) = h(1)$$
 te = $\frac{1}{3}$ $\frac{1}{3} = \frac{1}{3} = \frac{1}{$

B, = 2et-tet-1

$$e^{ct} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + (2e^{t} - te^{t} - 1) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$+ (te^{t} - e^{t}) \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{ct} = \begin{bmatrix} e^{t} & e^{t} - 1 & te^{t} - e^{t} + 1 \\ 0 & 0 & e^{t} \end{bmatrix}$$

Contents

- Part a
- Part b
- Part c
- Part d

Part a

```
A = [-1,1,0; -4,-2,-5;5,2,6];
B = [0; -1;1];

%Find cofficients for CE
alpha = charpoly(A);
alpha2 = alpha(2);
alpha1 = alpha(3);

% Build Q
q_3 = B;
q_2 = A*q_3+alpha2*q_3;
q_1 = A*q_2+alpha1*B;
Q = horzcat(q_1,q_2,q_3);

%Determine transformation matrix
T = inv(Q)
```

T =

```
1 1 1
0 1 1
1 0 1
```

Part b

```
A_con = T*A*inv(T)
B_con = T*B
```

```
A_con =

0 1 0
0 0 1
1 2 3
```

B_con =

0
0
1

1 of 3

Part c

```
%Desired CE
syms s k1 k2 k3
a=s+2;
out = expand(a^3)

% Calculate the closed loop system
charpoly(A_con + B_con*[k1 k2 k3])

% Calculate the gains
k_3_num = -3-6;
k_2_num = -12-2;
k_1_num = -1-8;

%Double check
K_con = [k_1_num k_2_num k_3_num]
charpoly(A_con + B_con*K_con)
```

Part d

```
%Get actual gains
K = K_con*T
```

```
K = -18 -23 -32
```

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