ME 555 - Spring 2025 Due Date: February 9^{th}
Homework #2

Problem 1. *Checking linear independence*

(a) Determine whether the following are linearly dependent or linearly independent $\mathbb{R}^3$:

$$\begin{bmatrix}1\\0\\3\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}, \begin{bmatrix}0\\5\\1\end{bmatrix}$$

$$\begin{bmatrix}4\\5\\1\end{bmatrix}, \begin{bmatrix}3\\1\\2\end{bmatrix}, \begin{bmatrix}2\\8\\-2\end{bmatrix}$$

(b) Let $A = \begin{bmatrix}0 & 2\\1 & 1\end{bmatrix}$, then is the set $\{I, A, A^2\}$ a linearly independent set in $\mathbb{R}^{2x2}$? Can we use the Cayley-Hamilton Theorem to show this?

Problem 2. *Norms and orthormalization*

$$x_1 = \begin{bmatrix}2\\-3\\1\end{bmatrix} \qquad z_2 = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

(a) What are the 1-norm, 2-norm, and ∞-norm for x_1 and x_2?

(b) Determine an orthonormal basis for the space spanned by x_1 and x_2.

Problem 3. *Matrix properties*

$$A_1 = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix} \qquad A_2 = \begin{bmatrix}4 & 1 & -1\\3 & 2 & 0\\1 & 1 & 0\end{bmatrix}$$

(a) Find the ranks and nullities of A_1 and A_2.

(b) Find bases for the range space and null spaces of these matrices.

(c) What are the 1-norm, 2-norm, and ∞-norm for these matrices?

Problem 4. *Jordan Form*

$$B_1 = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -12 & 6 \end{bmatrix}$$

(a) Determine the Jordan Form for B_1. What is the corresponding transformation matrix, T, to achieve the similarity transformation?

Hint: T has the eigenvectors as columns.

(b) (More challenging) Determine the Jordan Form for B_2. What is the corresponding transformation matrix, T, to achieve the similarity transformation?

Hint: A few ways to do this. One way is to find the eigenvector, v_1. Then, you need to find the "generalized eigenvectors" where $(B - \lambda I)v_2 = v_1$ and $(B - \lambda I)v_3 = v_2$

Problem 5. *Matrix Exponential*

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Use Cayley-Hamilton to determine the following:

(a) C^{10}

(b) C^{100}

(c) C^{At}

Problem 6. *Control Canonical Form. Please feel free to use Matlab and/or other tools for this computation.*

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ -4 & -2 & -5 \\ 5 & 2 & 6 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} u$$

(a) Determine the transformation matrix, T, for the control canonical form (CCF).

(b) Write out the A and B matrices for CCF.

(c) Using CCF, design a full-state feedback controller to place the poles at -2.

(d) Determine the control law for the original system.