ME 555 - Spring 2025 Due Date: February 9^{th}
Homework #1

Problem 1. *Classify the following systems (1) linear or nonlinear and (2) time-invariant or time-varying:*

(a) $y(t) = 0$ *for all* t

(b) $y(t) = \kappa$ *for all* t *for all* t, $\kappa \neq 0$

(c) $y(t) = x(t)$

(d) $y(t) = x(t) + 5$

(e) $y(t) = t^2 x(t)$

(f) $\ddot{y}(t) + x(t)y(t) = 0$

(g) $y(k+1) = y(k) + 3x(k)$, *where* $k \in \mathbb{Z}$

Problem 2. *Find state-space equations to describe the pendulum systems in Figure 1. The systems are useful to model one or two-link robot manipulators. If* θ_1, θ_2, *and* θ_3 *are very small, can you consider the two systems as linear?*

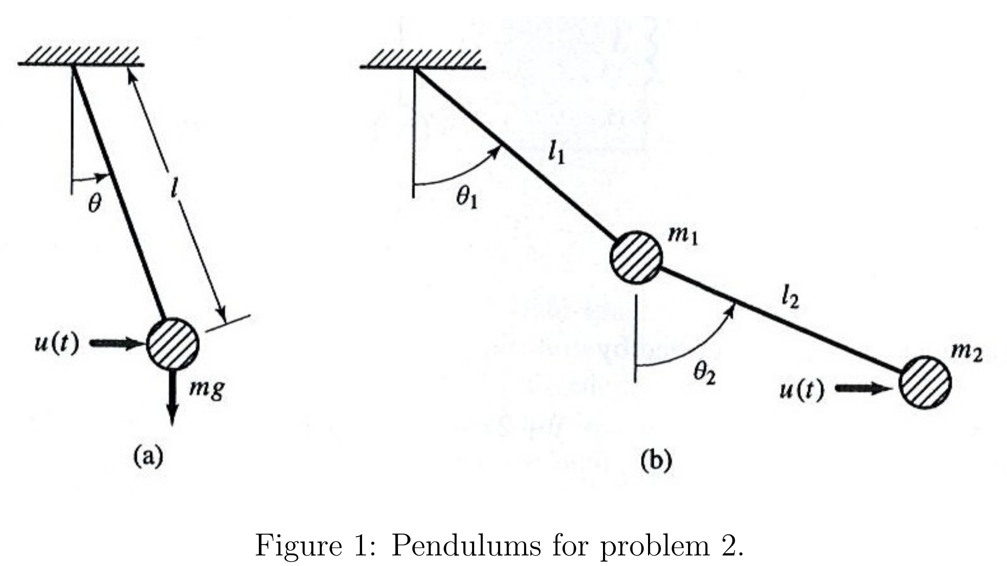

Figure 1: Pendulums for problem 2.

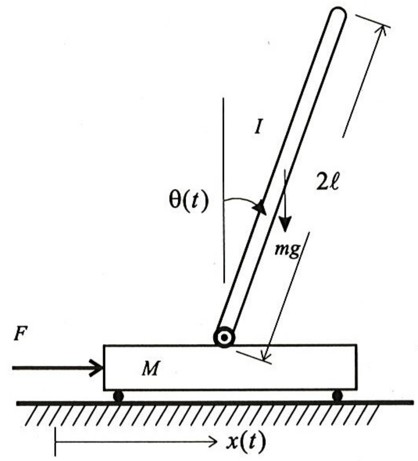

Figure 2: Inverted pendulum on a cart for problem 3.

Problem 3. *The motion of an inverted pendulum on a cart can be modeled as a set of differential equations below. For these equations, $\theta(t)$ is the angle of the pendulum clockwise with respect to the vertical, $x(t)$ is the horizontal position of the cart, $2l$ is the length of the pendulum, M is the mass of the cart, m is the mass of the pendulum, and I is the moment of inertia of the pendulum.*

$$(m+M)\ddot{x} + ml\ddot{\theta}cos(\theta) - ml\dot{\theta}^2 sin(\theta) = F \tag{1}$$

$$(I+ml^2)\ddot{\theta} + ml\ddot{x}cos(\theta) - mglsin(\theta) = 0 \tag{2}$$

(a) Linearize dynamic equations about the operating point, where $\theta_o = \dot{\theta}_o = \ddot{\theta}_o = 0$, $x_0 = \dot{x_0} = \ddot{x_0} = 0$, and $F = 0$. Write the linearized equations in state-space form.

Problem 4. *As discussed in Lecture 4, the unicycle model can be modeled as the following:*

$$\dot{p}_x = v\cos\theta \tag{3}$$

$$\dot{p}_y = v\sin\theta \tag{4}$$

$$\dot{\theta} = w \tag{5}$$

where v and w are your inputs. Linearize this system around the trajectory:

$$x_{sol} = \begin{bmatrix} \sin t \\ 1-\cos t \\ t \end{bmatrix} \qquad u_{sol} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Use the following to define the coordinate transformation

$$x \triangleq \begin{bmatrix} p_x\cos\theta + (p_y-1)\sin\theta \\ p_x\sin\theta + (p_y-1)\cos\theta \\ \theta \end{bmatrix} \qquad y = \begin{bmatrix} p_x \\ p_y \\ \theta \end{bmatrix}$$

(a) First, rewrite $\dot{x} = f(x,u)$ in the new coordinates by finding $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$.

(b) Then, rewrite y in the new coordinates (a bit tricky).

(c) Do a check to determine whether the trajectory is still a solution in the new coordinates by plugging in $p_x = \sin t$, $p_y = 1 - \cos t$, $\theta = t$, $v = 1$, $w = 1$ and checking whether $\dot{x} = f(x,y)$ in the new coordinates.

(d) Linearize.

Problem 5. *Given the single pendulum in 2(a) and parameters of $g = 10$, $l = 2$ and $m = 1$ (ignore units for now), you should get the following A and B matrices:*

$$A = \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$$

Using these matrices, do the following:

(a) Compute the transfer function by hand.

(b) Use Matlab to check your answer for (a) using: `ss2tf`

(c) Use Matlab to get back your state space `tf2ss`*. What are the states that are use in the output of the Matlab representation?*