ME 555 - Spring 2025 Due Date: March 23^{rd}
Homework #3

Problem 1. *State Transition Matrix - LTI*

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \qquad A_2 = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$$

(a) What is the state-transition matrix for the system described by $\dot{x}(t) = A_1 x(t)$? What is the zero input (non-zero initial condition, x_0) response of this system?

(b) What is the state-transition matrix for the system described by $\dot{x}(t) = A_2 x(t)$? What is the zero input (non-zero initial condition, x_0) response of this system?

(c) Are these systems stable in the sense of Lyapunov? Why or why not?

Problem 2. *State Transition Matrix - Properties*

(a) Show that $\delta\Phi(t_0, t)/\delta t = -\Phi(t_0, t)A(t)$.

(b) The state transition matrix from 0 to τ is $\Phi(t, 0)$. What is the general state transition matrix, $\Phi(t, \tau)$?

$$\Phi(t, 0) = \begin{bmatrix} \cos t^2 & \sin t^2 \\ -\sin t^2 & \cos t^2 \end{bmatrix}$$

Note: this is the state transition matrix for $B_1 = \begin{bmatrix} 0 & t \\ -2t & 0 \end{bmatrix}$.

Problem 3. *State Transition Matrix - LTV*

$$C_1 = \begin{bmatrix} 0 & t \\ 0 & 1 \end{bmatrix} \qquad C_2 = \begin{bmatrix} -\sin t & 0 \\ 0 & -\cos t \end{bmatrix}$$

(a) What is the fundamental matrix for C_1? What is the state transition matrix?

(b) What is the fundamental matrix for C_2? What is the state transition matrix?

Problem 4. *Lyapunov Stability*

$$\text{a.} \quad \dot{x} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x$$

$$\text{b.} \quad \dot{x} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x$$

(a) Is the system shown in (a) stable? asymptotically stable?

(b) Is the system shown in (b) stable? asymptotically stable?

Problem 5. *BIBO Stability*

Is the following system BIBO stable?

$$\dot{x} = \begin{bmatrix} -1 & 5 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} -2 & 4 \end{bmatrix} x - 2u$$

Problem 6. *Lyapunov Function*

Assume that there exist positive definite matrices P and Q and some $\lambda > 0$, such that:

$$A^T P + PA - 2\lambda P = -Q$$

What can you say about the eigenvalues of A?

Hint: (Eigenvalues of $(A - \lambda I)$) = (eigenvalues of A) - λ.