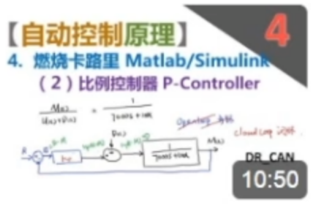
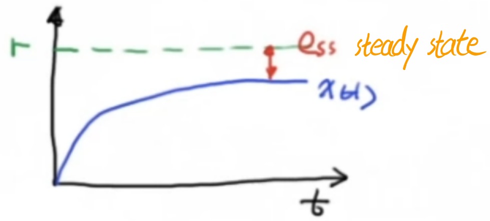
自动控制原理 5

终值定理和稳态误差

Final Value Theorem & Steady State Error





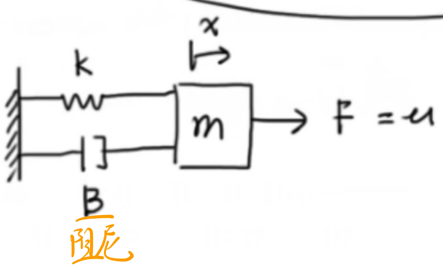
reference Value 参考　　Output 输出

$$e_{ss} = r - \lim_{t\to\infty} x(t)$$

求 $\lim_{t\to\infty} x(t)$ ， 终值定理 Final Value Theorem (无证明) FVT

$$\boxed{\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \qquad X(s) = \mathcal{L}[x(t)]}$$

条件？ $\lim_{t\to\infty} x(t)$ 存在　这样在终值的时候才有意义.



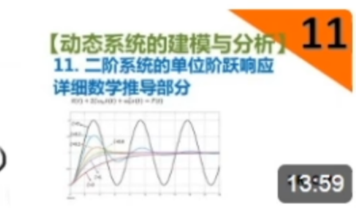
$$\mathcal{L}[m\ddot{x} + B\dot{x} + kx] = \mathcal{L}[u]$$

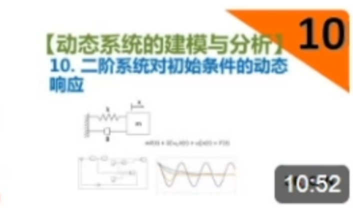
$$(ms^2 + Bs + k)X(s) = U(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + Bs + k}$$

$$U(s) \to \boxed{\frac{1}{ms^2+Bs+k}} \to X(s)$$

欠阻尼





狠狠踢一下

① $\delta(t)$ 冲激

$u(t) = \delta(t)$

$U(s) = \mathcal{L}[\delta(t)] = 1$

$$X(s) = 1 \cdot \frac{1}{ms^2 + Bs + k}$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{s}{ms^2 + Bs + k} = 0$$

$$\frac{\quad}{ms^2+Bs+k}$$

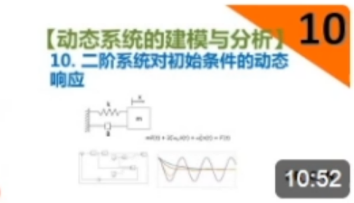
① $\delta(t)$ 冲激

$u(t)=\delta(t)$

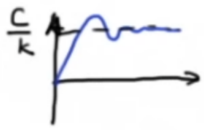
$U(s)=\mathcal{L}[\delta(t)]=1$

$$X(s)=1\cdot\frac{1}{ms^2+Bs+k}$$

$$\lim_{t\to\infty}x(t)=\lim_{s\to 0}sX(s)=\lim_{s\to 0}\frac{s}{ms^2+Bs+k}=0$$



② Step 阶跃



$$U(s)=\mathcal{L}[c]=\frac{c}{s}$$

$$X(s)=\frac{c}{s}\frac{1}{ms^2+Bs+k}$$

$$\lim_{t\to\infty}x(t)=\lim_{s\to 0}\cancel{s}\frac{c}{\cancel{s}}\frac{1}{ms^2+Bs+k}=\frac{c}{k}$$

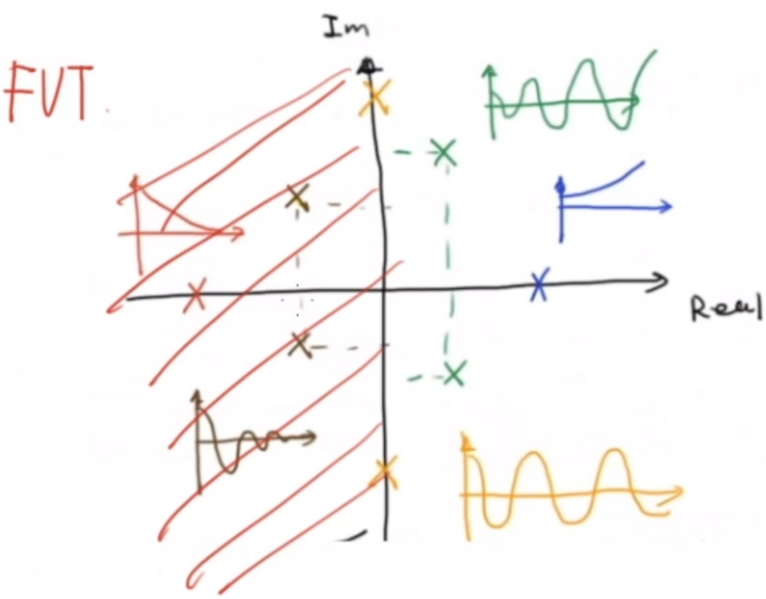
FVT works

③ $B=0$，无阻尼

$u(t)=\delta(t)$

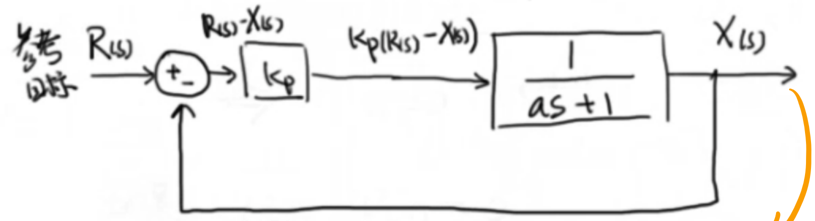
$$X(s)=1\cdot\frac{1}{ms^2+k}$$

$$\lim_{t\to\infty}x(t)=\lim_{s\to 0}s\frac{1}{ms^2+k}=0$$?? ✗



稳态误差 e_{ss}

比例控制



$$k_p(R(s)-X(s))\frac{1}{as+1}=X(s)$$

$$k_pR(s)-k_pX(s)=(as+1)X(s)$$

$$k_pR(s)=(as+1+k_p)X(s)$$

$$X(s)=\frac{k_pR(s)}{as+1+k_p}$$

$$as+1$$

$$k_pR(s)-k_pX(s)=(as+1)X(s)$$

$$k_pR(s)=(as+1+k_p)X(s)$$

$$X(s)=\frac{k_pR(s)}{as+1+k_p}$$

常数

$r(t)=r$

$R(s)=\mathcal{L}[r]=\frac{r}{s}$

$R(s)$: 稳定.

$as+1+k_p=0$

极点 $S=\frac{-1-k_p}{a}<0$

$\Rightarrow k_p>-1$

$$X(s)=\frac{k_p\frac{r}{s}}{as+1+k_p}$$

FVT: $\lim_{t\to\infty}x(t)=\lim_{s\to0}\cancel{s}\frac{k_p\frac{r}{\cancel{s}}}{\cancel{as}+1+k_p}=\frac{k_p}{1+k_p}r$

$$\boxed{e_{ss}=r-\frac{k_p}{1+k_p}r=\frac{1}{1+k_p}r}$$

$k_p\downarrow$ $e_{ss}\uparrow$

$k_p\to\infty$ $e_{ss}=0$

实际会造成输入过大

比例控制无法消除稳态误差.