Circuits RLC

- Réponse harmonique (module et phase)
- Réponse indicielle
- Exemple : Cas du filtre LC
- Application aux filtres

Plan du cours

- Rappels et suite ampli OP

- Rappel comportement des systèmes linéaires invariants (comportement fréquentiel, représentation complexe…)

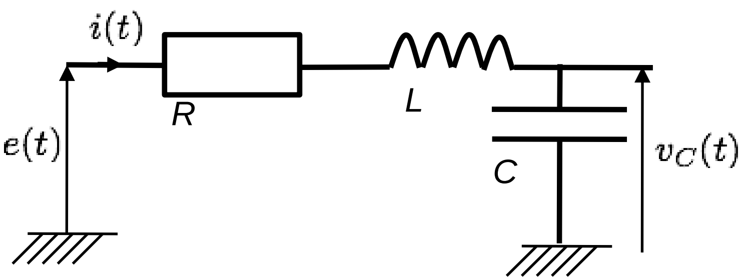
- Circuits RC
  - Réponse harmonique (module et phase)
  - Réponse indicielle

- Filtres actifs

- Circuits RLC
  - Réponse indicielle
  - Réponse harmonique (module et phase)

- Bilan du cours - A savoir

Circuit RLC: réponse indicielle



Source de tension: $e(t) = \begin{cases} = 0 \text{ si } t < 0 \\ = E \text{ si } t \geqslant 0 \end{cases}$

Condition initiale : $v_C(t) = 0$ Condensateur déchargé
$i(t) = 0$ Inductance déchargé

$$e(t) = Ri(t) + L\frac{di(t)}{dt} + v_C(t)$$
$$i(t) = C\frac{dv_C(t)}{dt}$$

$$e(t) = LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t)$$

La solution est la somme d'une solution particulière de l'équation complète et de la solution générale de l'équation sans second membre

Equation sans second membre:

$$LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$\frac{d^2v_c(t)}{dt^2} + 2z_0\omega_0\frac{dv_c(t)}{dt} + \omega_0^2 v_c(t) = 0$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ (pulsation propre)

$z_0 = \frac{R}{2}\sqrt{\frac{C}{L}}$ (amortissement)

$\lambda = z_0\omega_0$ (atténuation)

二阶常系数线性齐次微分方程

通解步骤： $$y''+py'+qy=0$$

(1) 写出方程对应的特征方程：$r^2+pr+q=0$ ；

(2) 求出特征方程的两个根： r_1 与 r_2 ；

(3) 根据两个特征根的不同情况，按照下表写出微分方程

特征方程 $r^2+pr+q=0$ 的根	方程 $y''+py'+qy=0$ 通解
两个不相等的实根 $r_1\neq r_2$	$y=C_1e^{r_1x}+C_2e^{r_2x}$
两个相等的实根 $r_1=r_2=r$	$y=(C_1+C_2x)e^{rx}$
一对共轭复根 $r_{1,2}=\alpha\pm\beta i$	$y=e^{\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$

Circuit RLC: réponse indicielle

$$LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$\frac{d^2v_c(t)}{dt^2} + 2\,\frac{R}{2}\,\frac{\sqrt{C}}{\sqrt{L}}\,\frac{1}{\sqrt{LC}}\,\frac{dv_c(t)}{dt} + \left(\frac{1}{\sqrt{LC}}\right)^2 v_c(t) = 0$$

$$\frac{d^2v_c(t)}{dt^2} + 2\,z_0\,\omega_0\,\frac{dv_c(t)}{dt} + \omega_0^2 v_c(t) = 0$$

Résolution de l'équation différentielle

$$\frac{d^2 v_c(t)}{dt^2} + 2z_0\omega_0 \frac{dv_c(t)}{dt} + \omega_0^2 v_c(t) = 0$$

On pose:

$v_c(t) = Ae^{\delta t}$ $\quad \frac{dv_c(t)}{dt} = \delta A e^{\delta t}$ $\quad \frac{d^2 v_c}{dt^2} = \delta^2 A e^{\delta t}$

$$\delta^2 A e^{\delta t} + 2z_0\omega_0 \delta A e^{\delta t} + \omega_0^2 A e^{\delta t} = 0 \quad \Big| : Ae^{\delta t}$$

Equation caractéristique:

$$\delta^2 + 2z_0\omega_0\delta + \omega_0^2 = 0$$

$$\delta_{1,2} = -\frac{2z_0\omega_0}{2} \pm \sqrt{\frac{4z_0^2\omega_0^2}{4} - \omega_0^2}$$

$$\delta_{1,2} = -\frac{2z_0\omega_0}{2} \pm \sqrt{\omega_0^2(z_0^2 - 1)}$$

Discriminant: $\Delta = \omega_0^2(z_0^2 - 1)$

Δ<0 si $z_0 < 1$

trois cas:

Δ>0		Δ<0
$z_0 > 1$	$z_0 = 1$	$z_0 < 1$
deux racines réelles δ_1 et δ_2	(cas non traité)	deux racines complexes conjuguées δ_1 et δ_2

Circuit RLC: réponse indicielle

La solution est donc: $v_c(t) = E + A_1 e^{\delta_1 t} + A_2 e^{\delta_2 t}$

On ajuste les constantes A_1 et A_2 avec les conditions initiales:

$v_C(t) = 0$ Condensateur déchargé

Pour $t = 0$ $v_C(t) = 0$

0=E+A_1+A_2

①

$i(t) = 0$ Inductance déchargée

Comme $i(t) = C\dfrac{dv_C(t)}{dt}$ $\left(\dfrac{dv_C(t)}{dt}\right)_{t=0} = 0$

$$\frac{dv_c(t)}{dt} = \delta_1 A_1 e^{\delta_1 t} + \delta_2 A_2 e^{\delta_2 t}$$

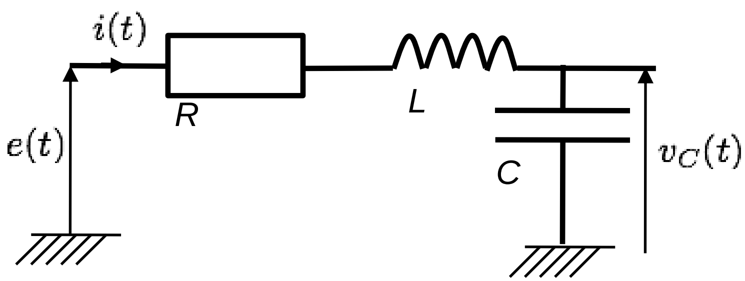
$\delta_1 A_1 + \delta_2 A_2 = 0$ $A_2 = -\dfrac{\delta_1}{\delta_2} A_1$

②

$$0 = E + A_1 - \frac{\delta_1}{\delta_2} A_1$$

$$\boxed{A_1 = E\frac{\delta_2}{\delta_1 - \delta_2} \qquad A_2 = -E\frac{\delta_1}{\delta_1 - \delta_2}}$$

Circuit RLC: réponse indicielle



$$v_c(t) = E + A_1 e^{\delta_1 t} + A_2 e^{\delta_2 t}$$

$$A_1 = E\frac{\delta_2}{\delta_1 - \delta_2} \qquad A_2 = -E\frac{\delta_1}{\delta_1 - \delta_2}$$

trois cas:

$z_0 > 1$

deux racines réelles δ_1 et δ_2

$$\delta_1 = -\lambda(1 - Z_0)$$

$$\delta_2 = -\lambda(1 + Z_0)$$

$$Z_0 = \sqrt{1 - \frac{1}{z_0^2}}$$

$$A_1 = -\frac{E}{2}\left(1 + \frac{1}{Z_0}\right))$$

$$A_2 = -\frac{E}{2}\left(1 - \frac{1}{Z_0}\right)$$

$z_0 = 1$

(cas non traité)

$z_0 < 1$

deux racines complexes conjuguées: δ_1 et δ_2

$$\delta_1 = -\lambda + jw_0\sqrt{1 - z_0^2}$$

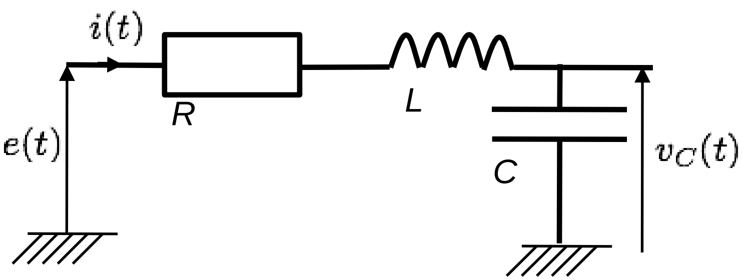
$$\delta_2 = -\lambda - jw_0\sqrt{1 - z_0^2}$$

A_1 et A_2 complexes conjugués:

$$A_1 = \frac{E}{2j\omega_0\sqrt{1 - z_0^2}}\left(-\lambda - j\omega_0\sqrt{1 - z_0^2}\right)$$

$$A_2 = \frac{E}{2j\omega_0\sqrt{1 - z_0^2}}\left(\lambda - j\omega_0\sqrt{1 - z_0^2}\right)$$

Circuit RLC: réponse indicielle



Source de tension:

$$e(t) = \begin{cases} = 0 \text{ si } t < 0 \\ = E \text{ si } t \geqslant 0 \end{cases}$$

Condition initiale :

$v_C(t) = 0$ Condensateur déchargé

$i(t) = 0$ Inductance déchargée

La solution est donc:

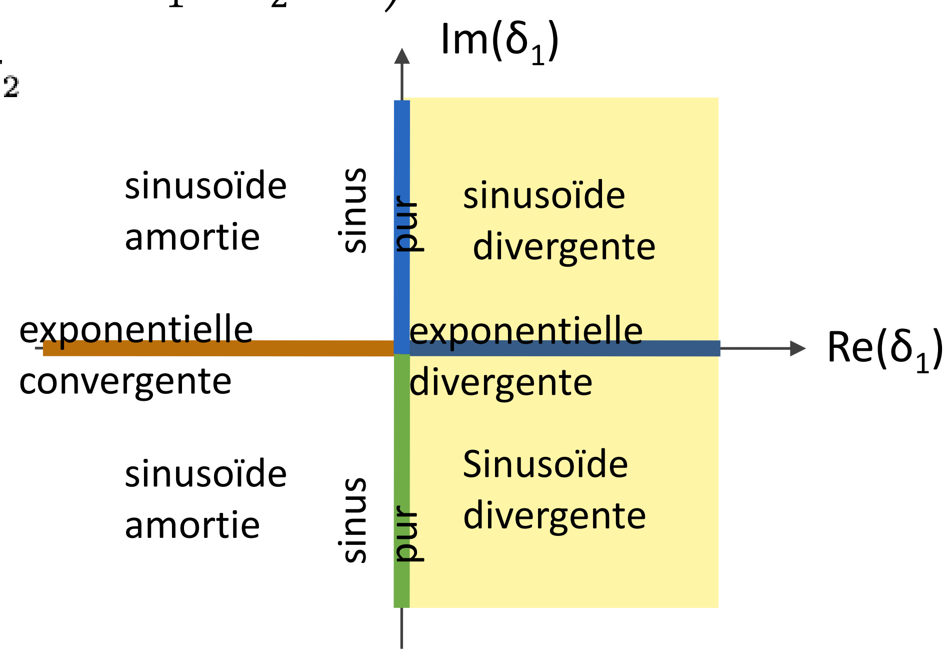
$$v_c(t) = E\left(1 + \frac{\delta_2}{\delta_1 - \delta_2} e^{\delta_1 t} - \frac{\delta_1}{\delta_1 - \delta_2} e^{\delta_2 t}\right)$$

La forme dépend des propriétés des δ_1 et δ_2 complexes

$$e^{\delta_1 . t} = e^{\mathrm{Re}(\delta_1).t} e^{j\,\mathrm{Im}(\delta_1).t}$$

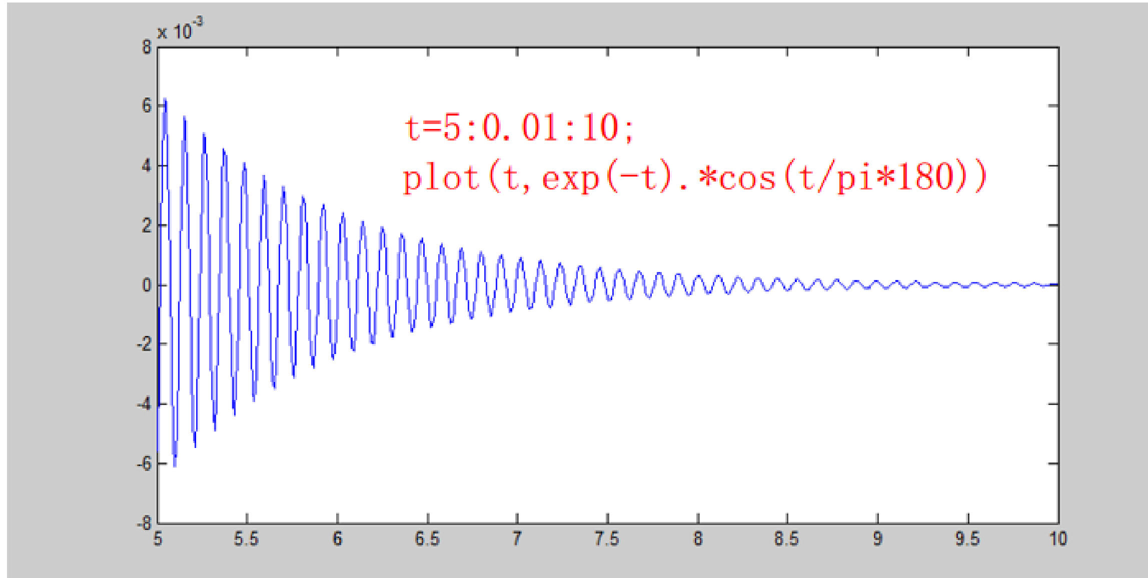
Terme exponentiel:
convergente si Re<0
divergente si Re>0

Terme sinusoïdal

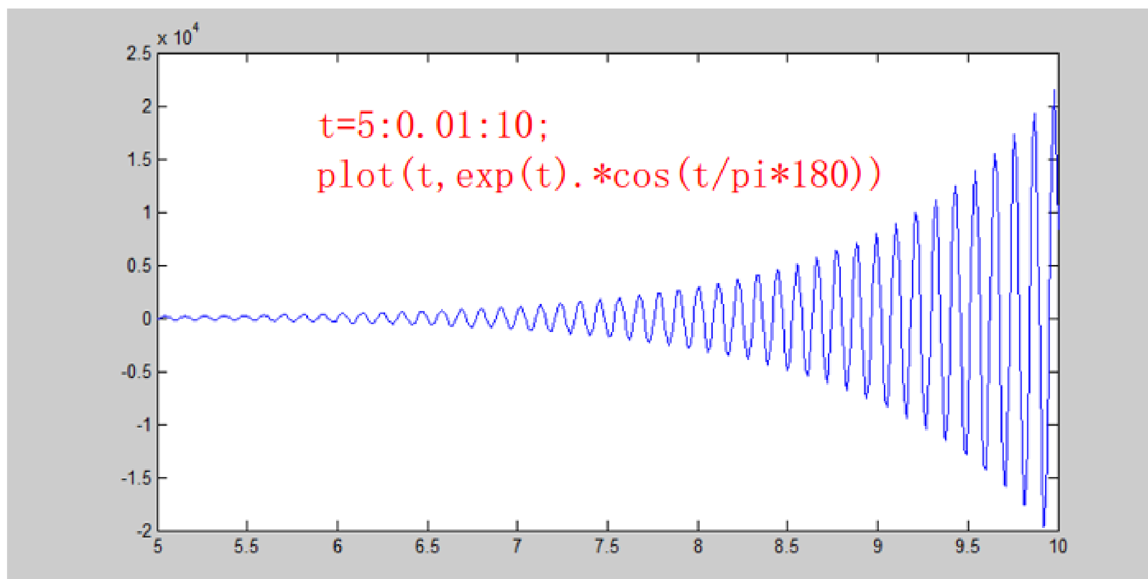


Circuit RLC: réponse indicielle

Convergent



Divergent



Circuit RLC: réponse indicielle

$\boxed{\text{Pour} \quad z_0 > 1}$

$$\delta_1 = -\lambda(1 - Z_0)$$
$$\delta_2 = -\lambda(1 + Z_0)$$

$$Z_0 = \sqrt{1 - \frac{1}{z_0^2}}$$

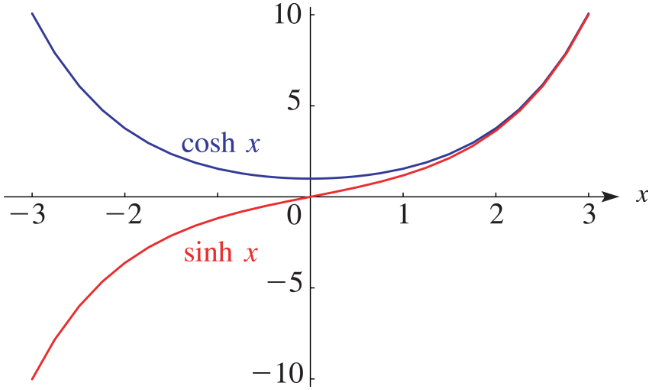
$$0 < Z_0 < 1$$

$$A_1 = -\frac{E}{2}\left(1 + \frac{1}{Z_0}\right)$$
$$A_2 = -\frac{E}{2}\left(1 - \frac{1}{Z_0}\right)$$

$$v_c(t) = E + A_1 e^{\delta_1 t} + A_2 e^{\delta_2 t}$$

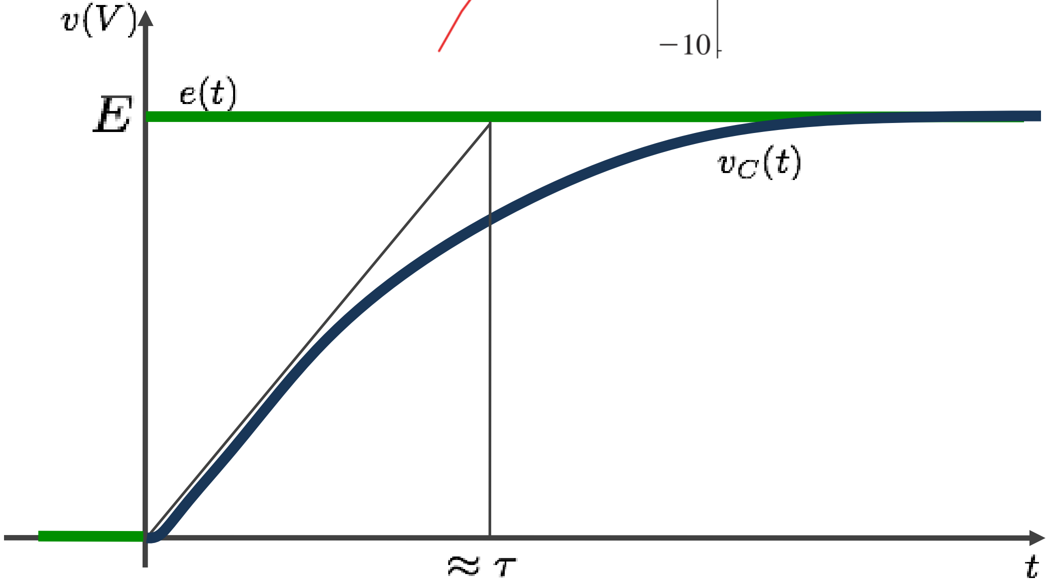
$$v_C(t) = E\left(1 - e^{-\lambda t}\left(\frac{e^{\lambda Z_0 t} + e^{-\lambda Z_0 t}}{2} + \frac{1}{Z_0}\frac{e^{\lambda Z_0 t} - e^{-\lambda Z_0 t}}{2}\right)\right)$$

$$v_C(t) = E\left(1 - e^{-\lambda t}\left(cosh(\lambda Z_0 t) + \frac{1}{Z_0} sinh(\lambda Z_0 t)\right)\right)$$



$$Re(\delta_1) < 0 \text{ et } Re(\delta_2) < 0$$
$$Im(\delta_1) = 0 \text{ et } Im(\delta_2) = 0$$

$$|\delta_1| < |\delta_2|$$
$$\tau \approx \max\left(\frac{1}{|\delta_1|}, \frac{1}{|\delta_2|}\right)$$
$$\tau \approx \frac{1}{|\delta_1|}$$



Circuit RLC: réponse indicielle

Pour $z_0 > 1$

Approximation pour $z_0 \gg 1$

$$Z_0 = \sqrt{1 - \frac{1}{z_0^2}} \approx 1 - \frac{1}{2z_0^2}$$

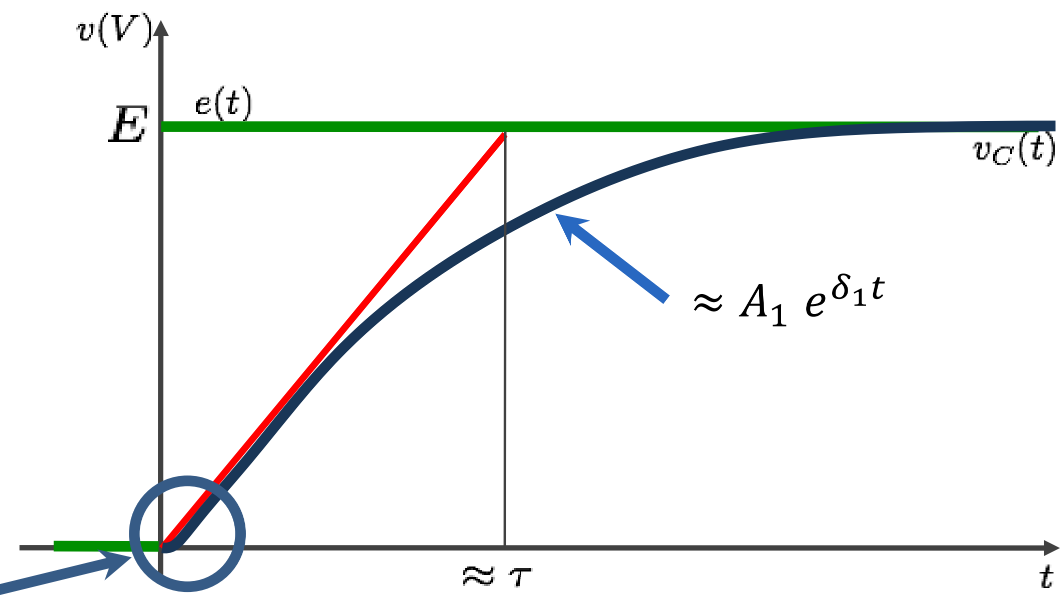
$$\begin{aligned}\delta_1 &= -\lambda(1 - Z_0)\\ &\approx -z_0 w_0 \left(1 - \left(1 - \frac{1}{2z_0^2}\right)\right)\\ &\approx -\frac{w_0}{2z_0} \quad \delta_1 \text{ petit} \rightarrow \text{réponse lente}\end{aligned}$$

$$v_c(t) = E + A_1 e^{\delta_1 t} + A_2 e^{\delta_2 t}$$

< 0 lent > 0 rapide

$$\begin{aligned}\delta_2 &= -\lambda(1 + Z_0)\\ &\approx -z_0 w_0 \left(1 + \left(1 - \frac{1}{2z_0^2}\right)\right)\\ &\approx -2z_0 w_0 + \frac{w_0}{2z_0}\end{aligned}$$

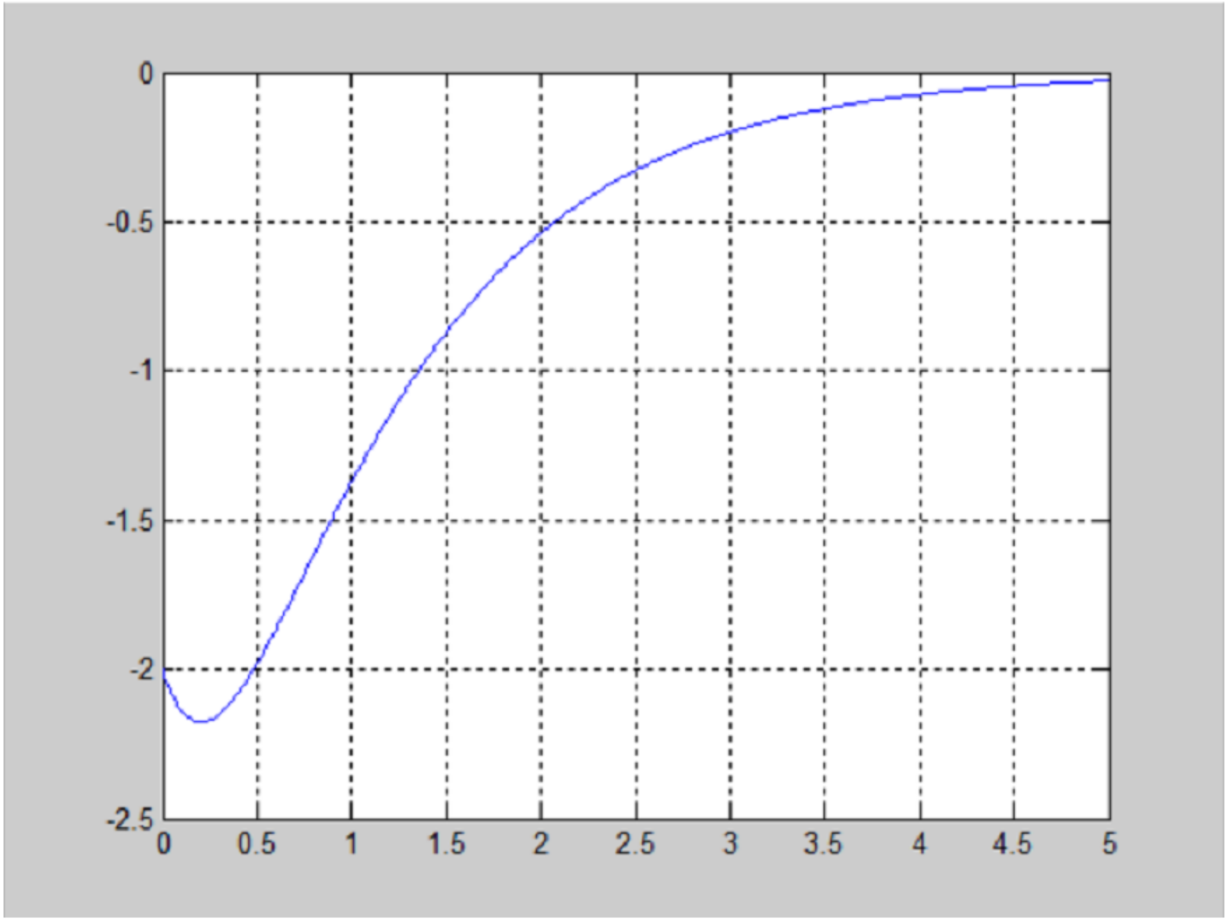
δ_2 grand $\rightarrow$ réponse rapide



Contribution de $A_2 e^{\delta_2 t}$

Circuit RLC: réponse indicielle

```
x=0:0.01:5;
plot(x,-4*exp(-1*x)+2*exp(-3*x)),grid on
```



Circuit RLC: réponse indicielle

Pour $z_0 > 1$

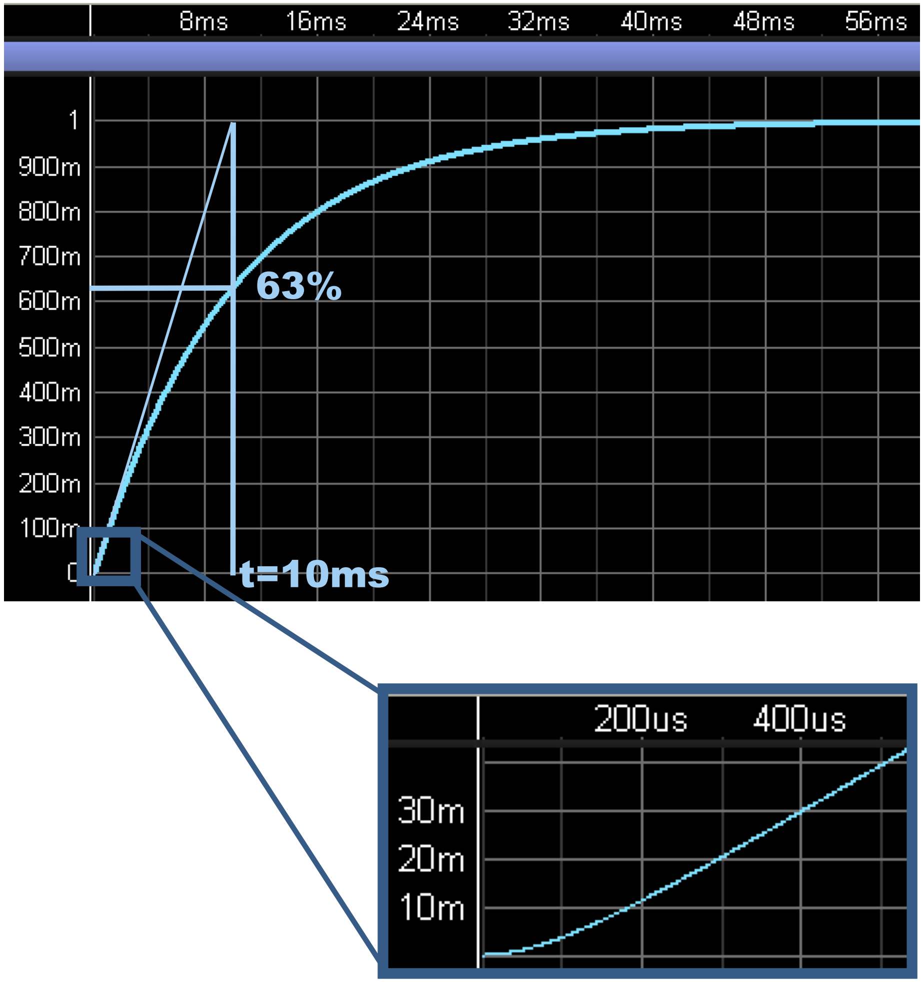
Simulation pour:

$$w_0 = 1\text{krad/s}$$

$$z_0 = 5$$

$$\delta_1 = -\frac{w_0}{2z_0} = -100 \text{ rad/s}$$

$$\delta_2 = -2z_0 w_0 + \frac{w_0}{2z_0} = -9.9 \text{ krad/s}$$



Fichier Spice:

```
* Systeme du second ordre
* Fonction de transfert definie par bloc Laplace
S1 RLC 0 IN 0
+ A0=1
+ B0=1 B1='2*Z0/WC' B2='1/(WC*WC)'

* Tension d'entrée:
VIN IN 0 PWL 0 0  1p 1

* Definition des parametres:
.PARAM WC=1k
.PARAM Z0=5
```

Circuit RLC: réponse indicielle

$w_0 = 1\text{krad/s}$

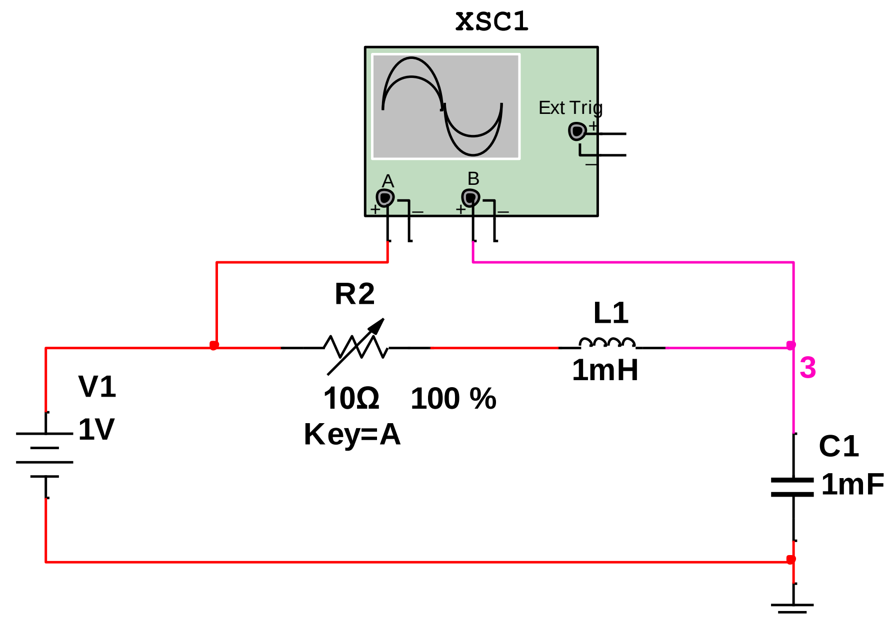
$z_0 = 5$

$$w_0 = \frac{1}{\sqrt{LC}} \rightarrow LC = \frac{1}{w_0^{\,2}} = 10^{-6}$$

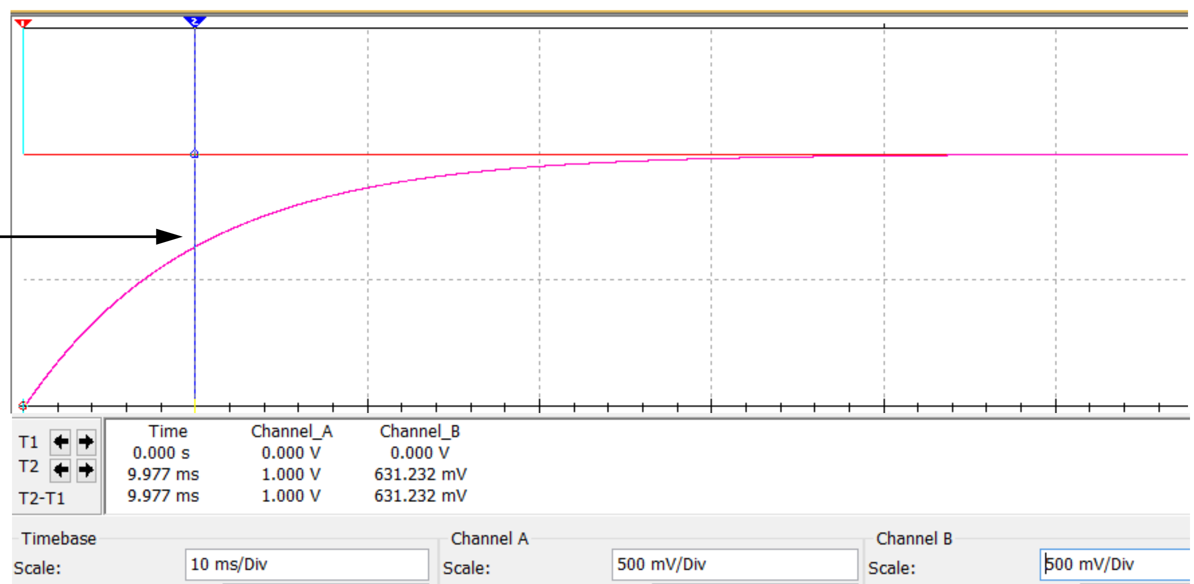
取 $L = 10^{-3}\mathrm{H}$，$\mathrm{C} = 10^{-3}F$

$$z_0 = \frac{R}{2}\sqrt{\frac{C}{L}} \rightarrow R = 2z_0\sqrt{\frac{L}{C}} = 10\Omega$$

$$\Gamma = \left|\frac{1}{\delta_1}\right| = 0.01s = 10ms$$



63.2%



Circuit RLC: réponse indicielle

Pour $z_0 < 1$

$$v_C(t) = E + Ae^{\delta_1 t} + Be^{\delta_2 t}$$

En posant:

$$\delta_1 = -\lambda + jw_0\sqrt{1 - z_0^2}$$

$$\delta_2 = -\lambda - jw_0\sqrt{1 - z_0^2}$$

$$A_1 = \frac{E}{2j\omega_0\sqrt{1 - z_0^2}}\left(-\lambda - j\omega_0\sqrt{1 - z_0^2}\right)$$

$$A_2 = \frac{E}{2j\omega_0\sqrt{1 - z_0^2}}\left(\lambda - j\omega_0\sqrt{1 - z_0^2}\right)$$

On peut montrer:

$$\boldsymbol{v_c(t) = E\left(1 - e^{-\lambda\, t}cos(\omega_n t + \Phi\,)\right)}$$

Pulsation naturelle: $w_n = w_0\sqrt{1 - z_0^2}$

Pulsation propre $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Atténuation $\lambda = z_0\omega_0$

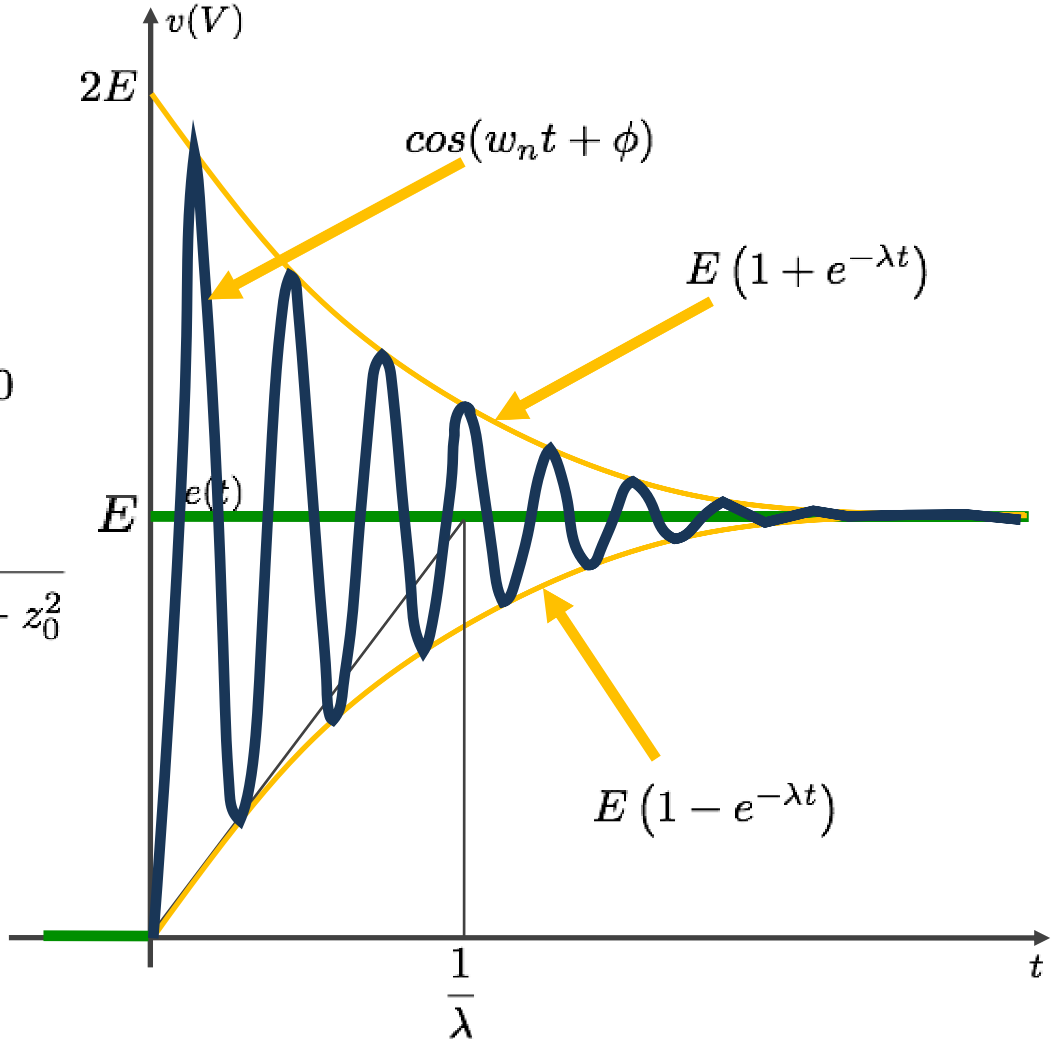
Circuit RLC: réponse indicielle

Pour $\boxed{z_0 < 1}$

$$v_C(t) = E\left(1 - e^{-\lambda t} cos(w_n t + \phi)\right)$$

Pour $Re(\delta_1) < 0$ et $Re(\delta_2) < 0$

$Im(\delta_1) \neq 0$ et $Im(\delta_2) \neq 0$

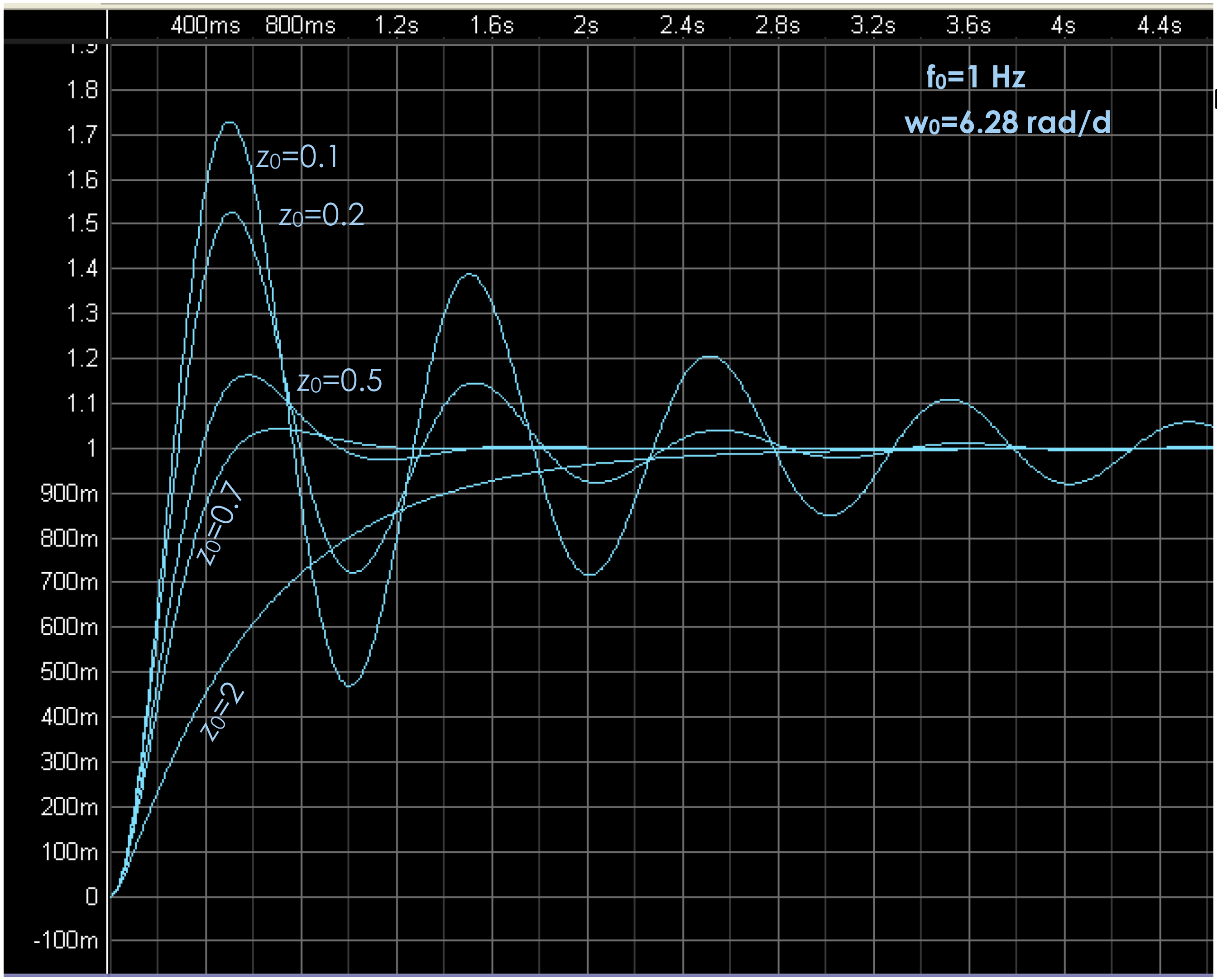


Pulsation naturelle: $w_n = w_0\sqrt{1 - z_0^2}$

Pulsation propre: $\omega_0 = \frac{1}{\sqrt{LC}}$

Atténuation: $\lambda = \frac{R}{2L}$

Circuit RLC: réponse indicielle



Fichier Spice:

```
* Fonction du second ordre
* Pulsation WC
* Amortissement Z0
* Bloc LAPLACE S
S1 RLC 0 IN 0
+ A0=1
+ B0=1 B1='2*Z0/WC' B2='1/(WC*WC)'
* Tension d'entrée:
VIN IN 0 PWL 0 0  1p 1

* Definition de paramètres (f=1Hz):
.PARAM PI=3.141592654
.PARAM WC='2*PI*1'
.PARAM Z0=0.5
```

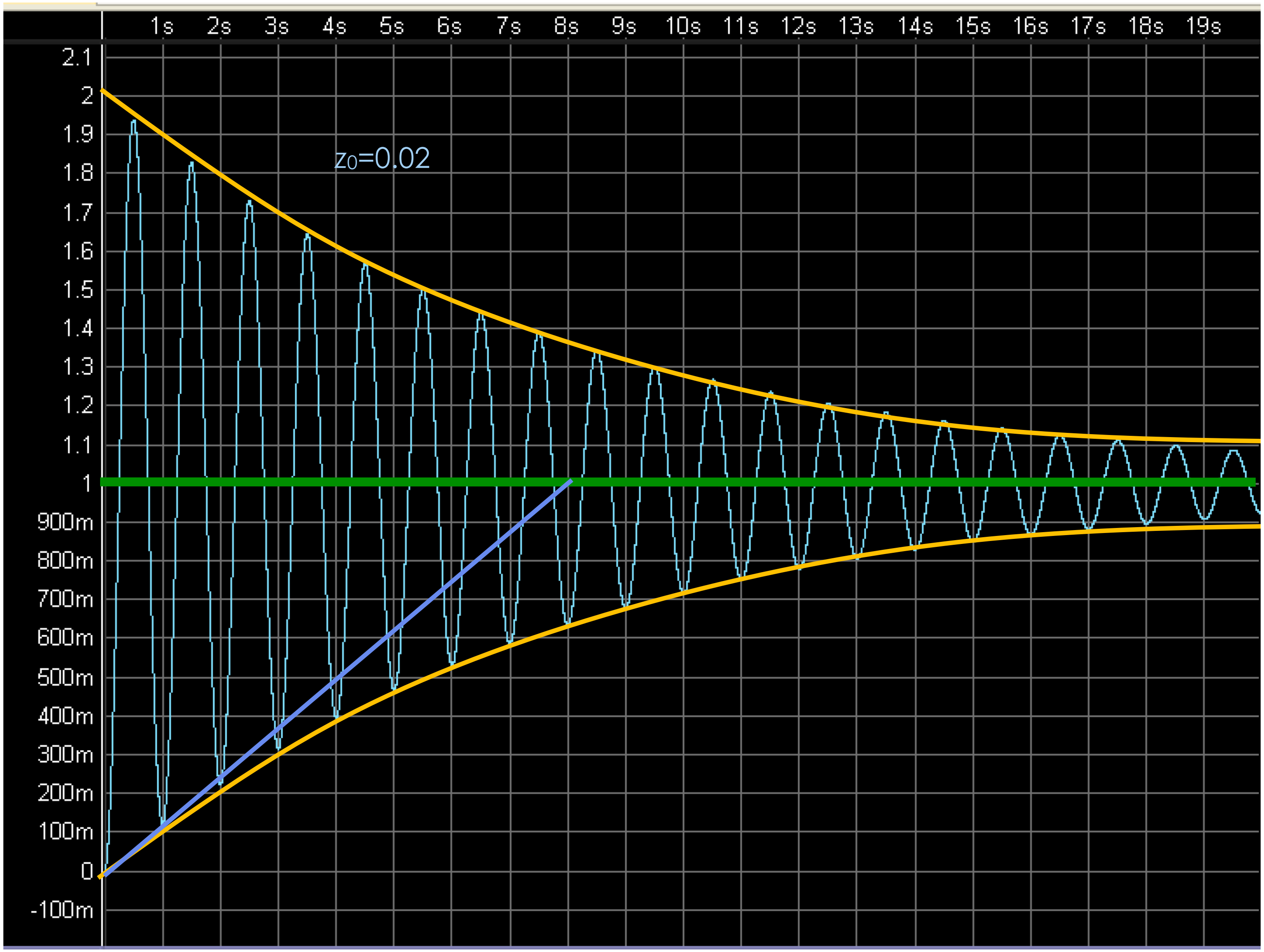
Analyse transient:

End time= 5

Print step= 1m

Time Step Control: Max=1m

Circuit RLC: réponse indicielle



$$\lambda = z_0 w_0$$

$$f_0 = 1$$

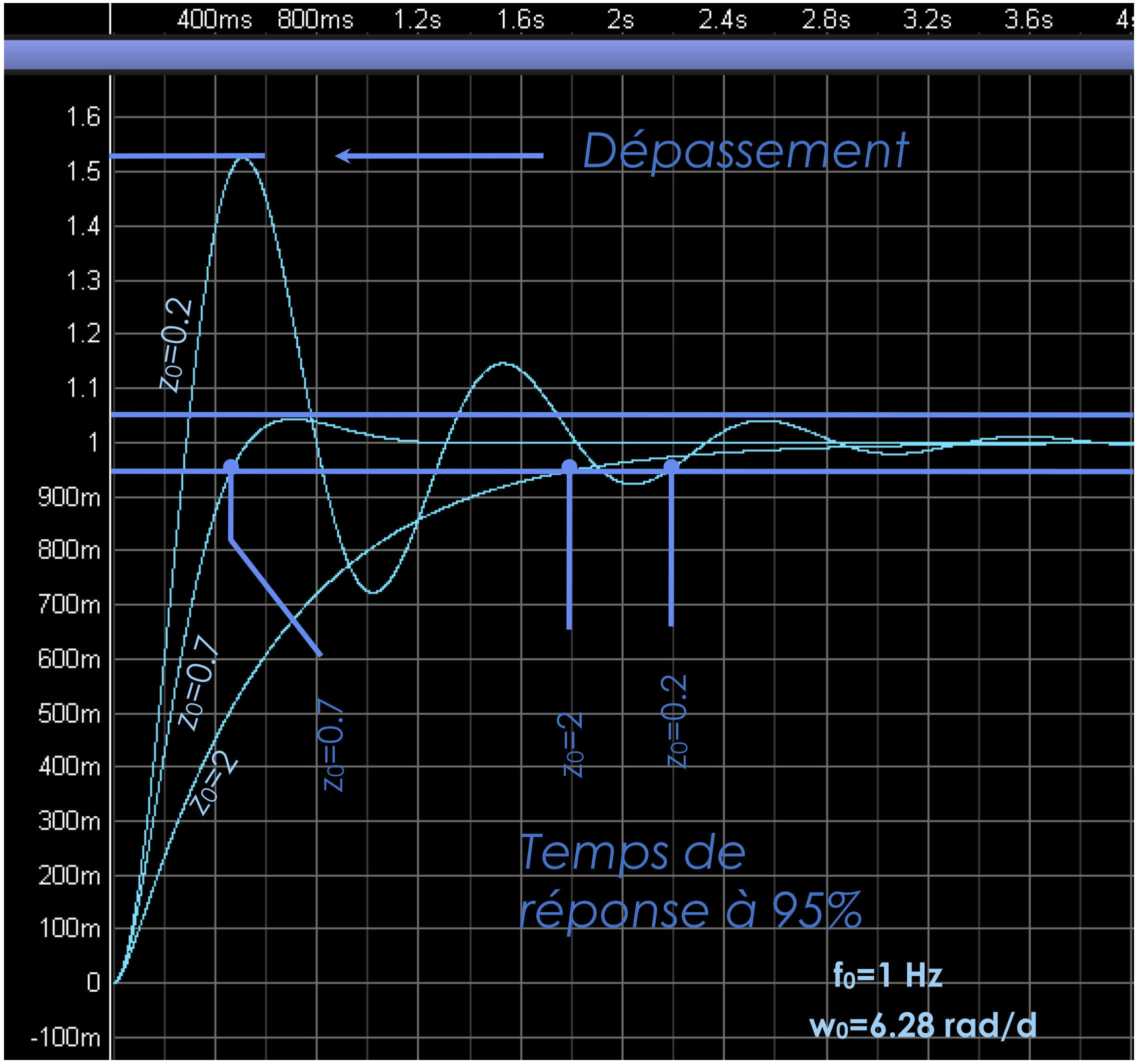
$$w_0 = 2\pi$$

$$z_0 = 0.02$$

$$\lambda = 0.125 \text{ rad/s}$$

$$\tau = \frac{1}{\lambda} = 7.96 \text{ s}$$

Circuit RLC: réponse indicielle



$$v_C(t) = E\left(1 - e^{-\lambda t} cos(w_n t + \phi)\right)$$

Dépassement :

$$D_{\%} = \frac{v_{max} - v_{\infty}}{v_{\infty}} \times 100$$

$$v_{max} - v_{\infty} \approx E e^{-\lambda \frac{\pi}{w_n}}$$

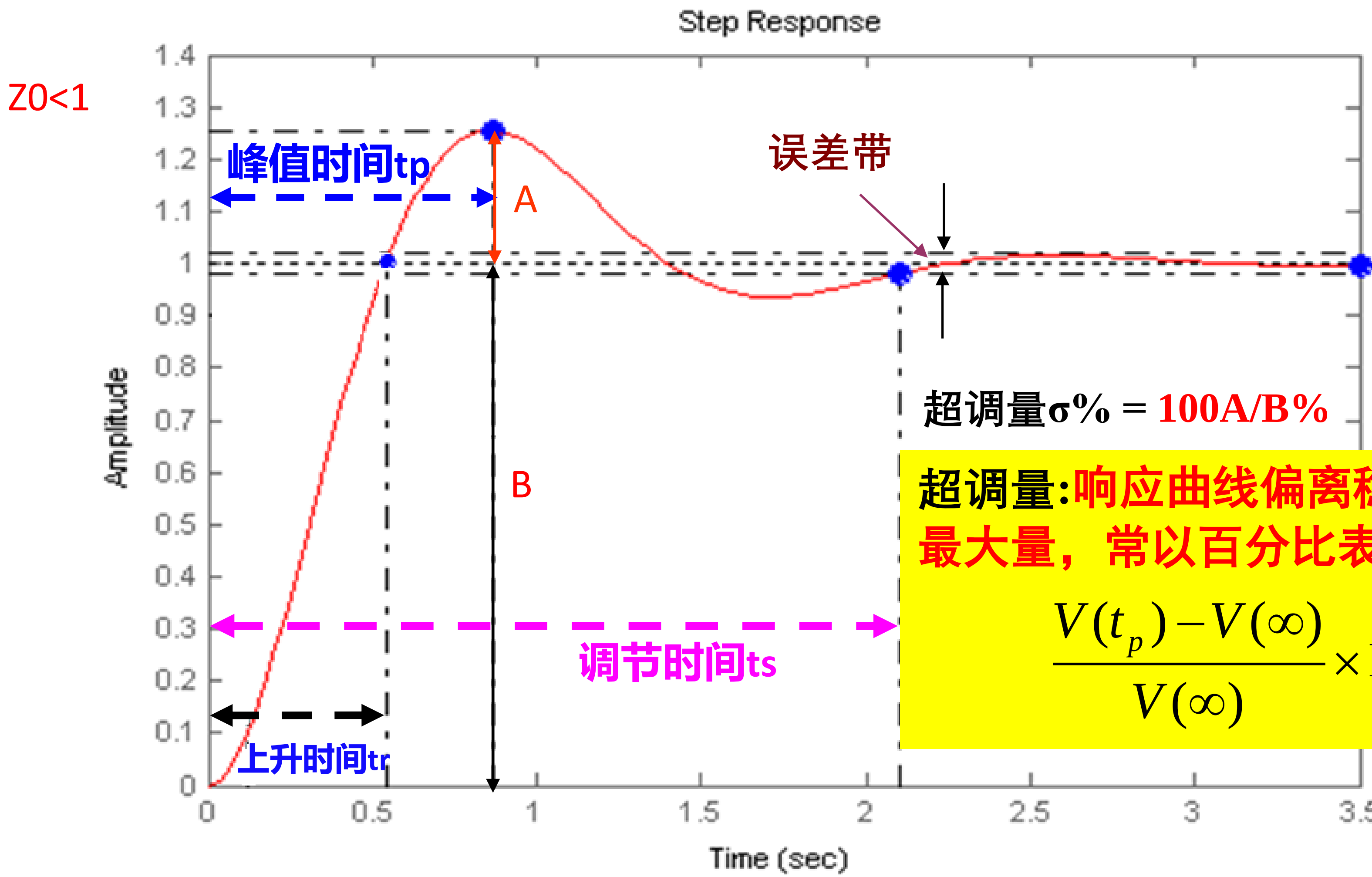
$$\approx E e^{-z_0 w_0 \frac{\pi}{w_0 \sqrt{1-z_0^2}}}$$

$$\approx E e^{-\pi \frac{z_0}{\sqrt{1-z_0^2}}}$$

$$D_{\%} \approx 100 e^{-\pi \frac{z_0}{\sqrt{1-z_0^2}}}$$

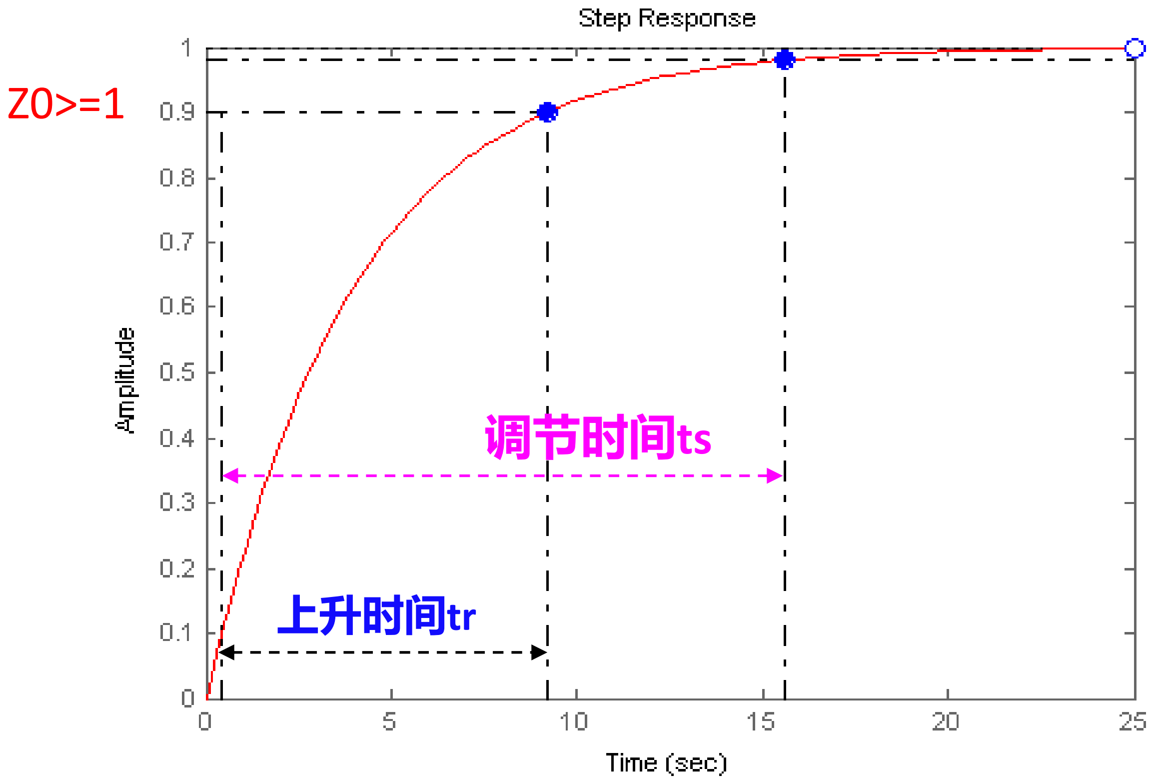
动态响应指标

- 描述稳定的系统在单位阶跃函数作用下，动态过程随时间的变化状况



超调量:响应曲线偏离稳态值的最大量，常以百分比表示，即

$$\frac{V(t_p)-V(\infty)}{V(\infty)}\times 100\%$$



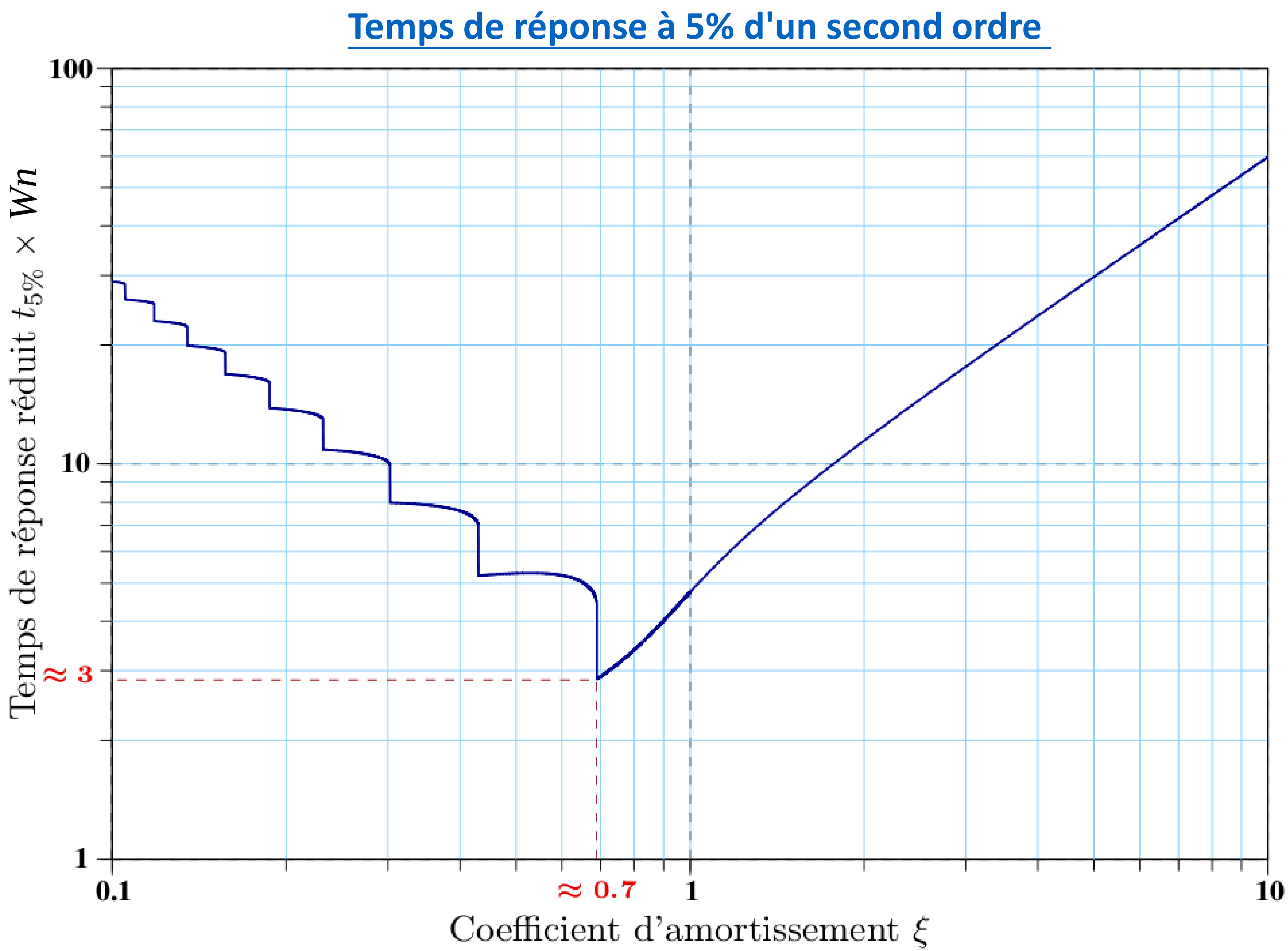
上升时间和峰值时间反映了系统的响应速度；超调量反映了系统的阻尼程度；调节时间同时反映系统响应速度和阻尼程度的综合性指标

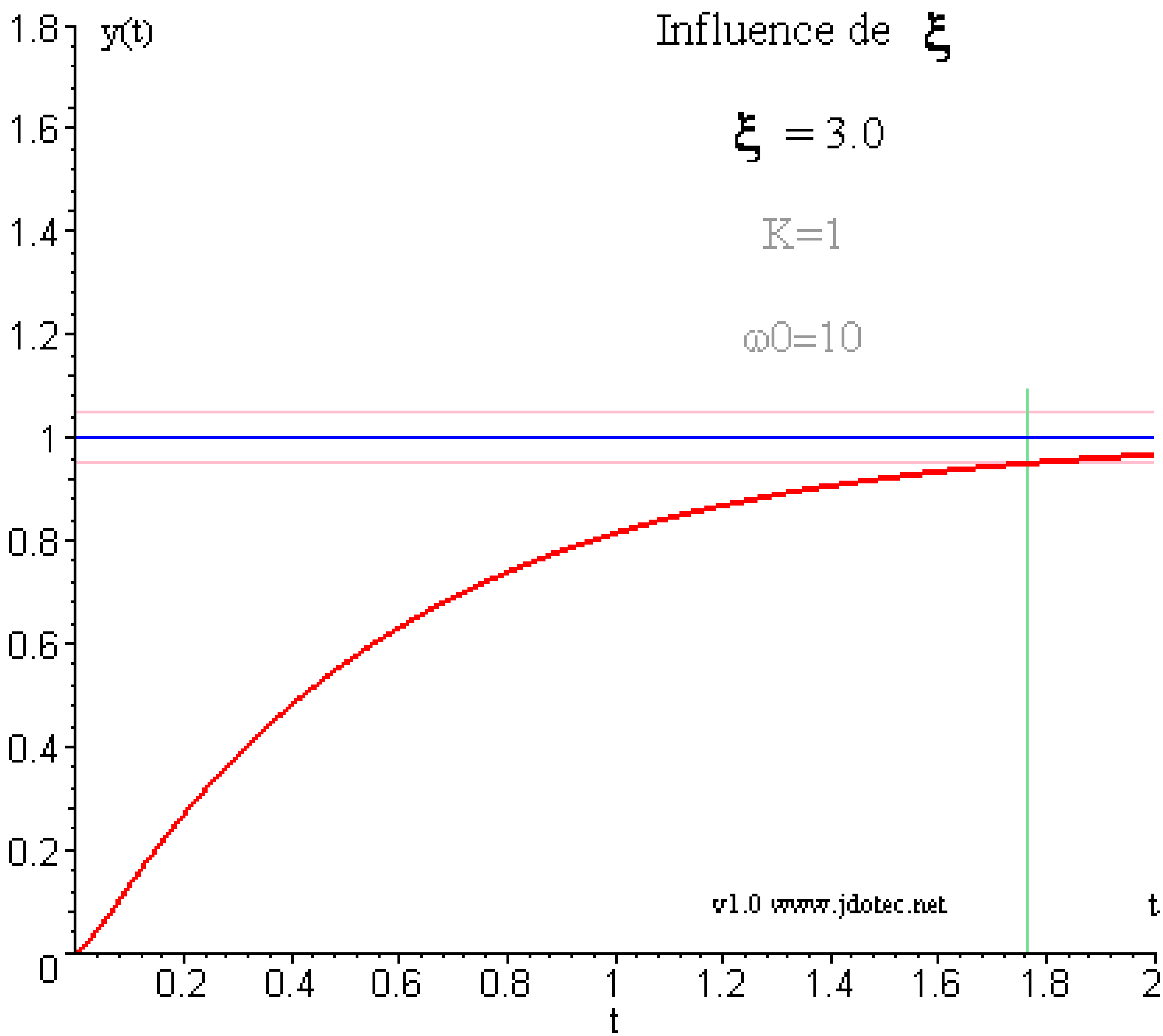
上升时间 t_r – rise time：响应曲线从零开始至第一次到达稳态值所需的时间。因为有些响应没有超调,理论上到达稳态值时间需要无穷大,因此,也将上升时间定义为响应曲线从稳态值的10%上升到稳态值90%所需的时间。

峰值时间 t_p – peak time：响应曲线到达第一个峰值所需的时间

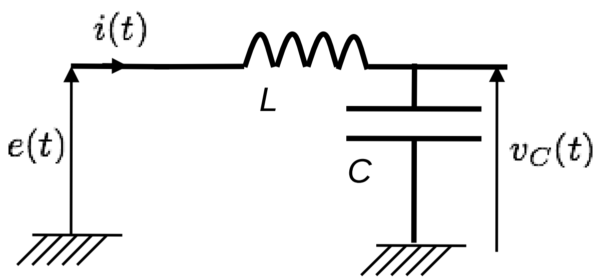
调节时间 t_s – settling time：**在响应曲线的稳态值附近，取±5%（或±2 %）作为误差带，响应曲线达到并不再超出该误差带的最小时间，定义为调整时间。从整体上反映了系统的快速性。**

Circuit RLC: réponse indicielle




Influence de ξ
ξ = 3.0
K=1
ω0=10
y(t)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t
v1.0 www.jdotec.net
t

Circuit RLC: exemple



Source de tension: $e(t) = Ecos(wt)$

Grandeurs complexes *e, i, vC*

Impédances complexes:

- Condensateur: $Z_C = \frac{1}{jCw}$
- Résistance: R
- Inductance: $Z_L = jLw$

$$\begin{aligned} e &= Z_L i + Z_C i \\ &= jLwi + \frac{1}{jCw} i \\ i &= e\frac{1}{jLw + \frac{1}{jCw}} \end{aligned}$$

$$\begin{aligned} v_C &= Z_C i \\ &= \frac{1}{jCw}\frac{1}{jLw + \frac{1}{jCw}} e \\ &= \frac{1}{1 - LCw^2} e \end{aligned}$$

Fonction de transfert:

$$H(jw) = \frac{1}{1 - LCw^2}$$

$$\begin{aligned} w_n &= w_0\sqrt{1 - z_0^2} \\ &= w_0 \end{aligned}$$

$$w_0^2 = \frac{1}{LC}$$ (pulsation naturelle)

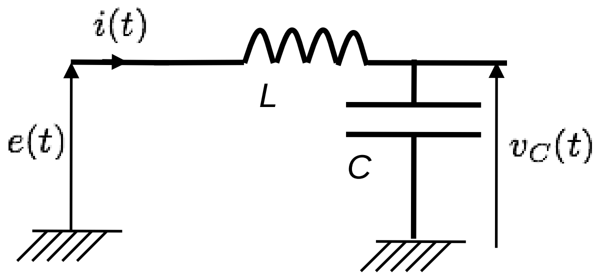
$$\lambda = \frac{R}{2L} = 0$$ (atténuation)

$$z_0 = \frac{R}{2}\sqrt{\frac{C}{L}} = 0$$ (amortissement)

$$v_C(t) = E\left(1 - e^{-\lambda t}\left(cos(jw_n t) + \frac{z_0}{\sqrt{1 - z_0^2}} sin(jw_n t)\right)\right)$$

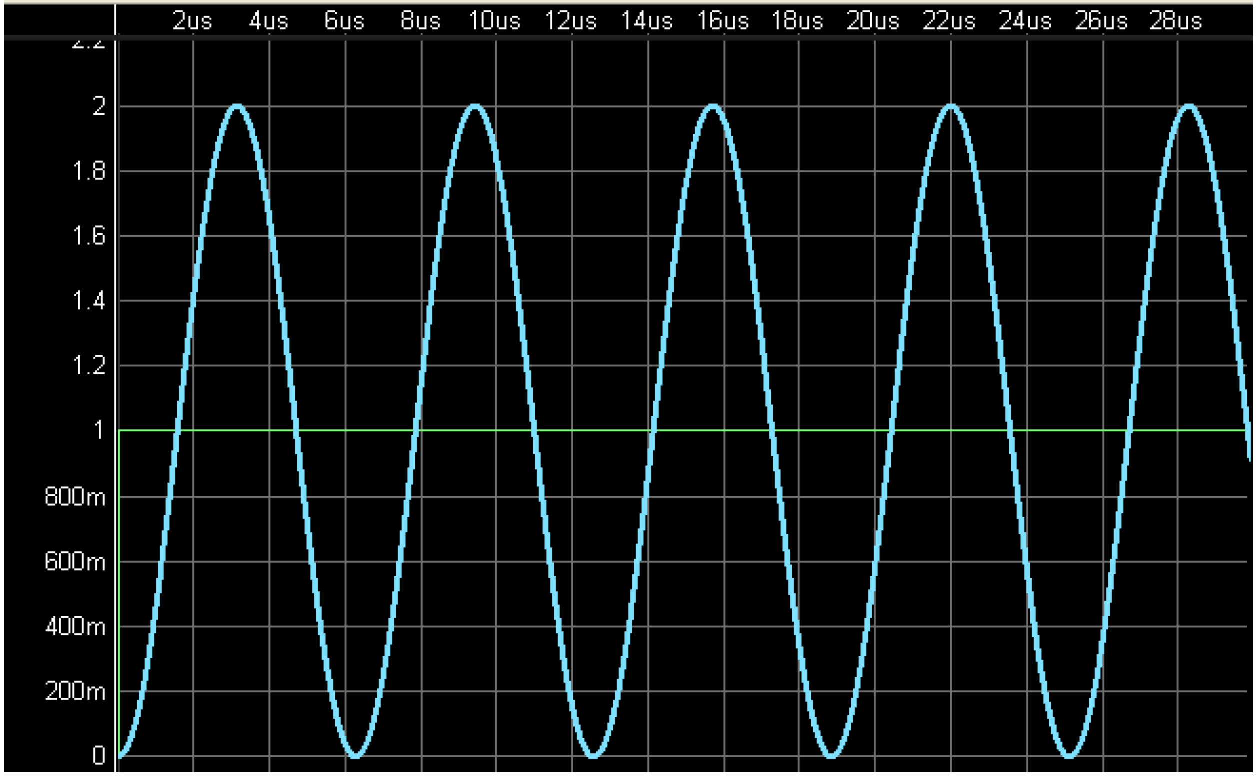
$$v_C(t) = E\left(1 - cos(w_0 t)\right)$$

Circuit RLC: exemple



C=100nF
L=10µH

$$w_0 = \frac{1}{\sqrt{LC}} \qquad T_0 = \frac{2\pi}{w_0}$$
$$= 1 \text{ Mrad/s} \qquad = 6.28\mu s$$
$$= 159 \text{ kHz}$$

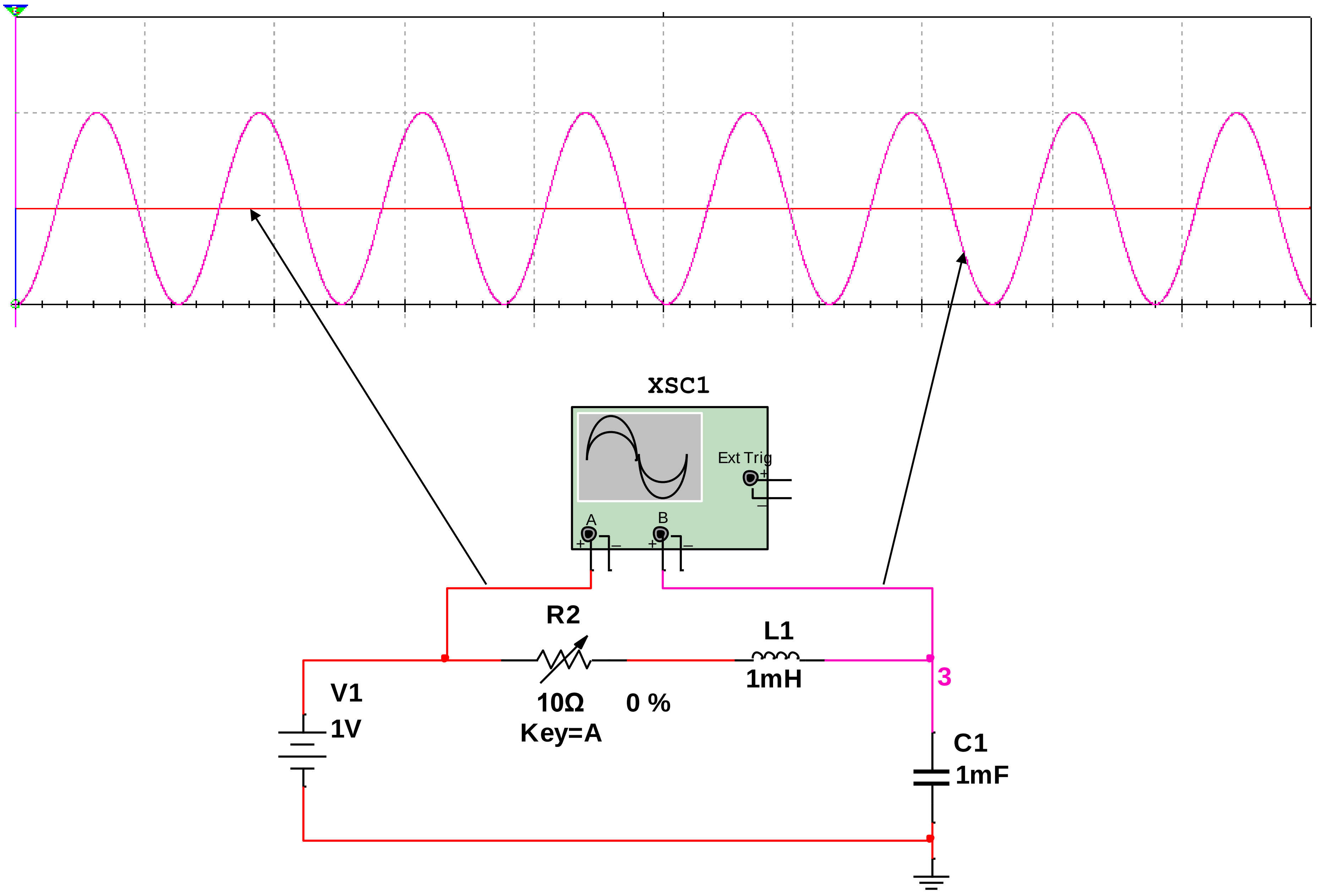


$$v_C(t) = E\left(1 - cos(w_0 t)\right)$$

Fichier Spice:

```
* Circuit LC
L1 IN OUT 10u
C1 OUT 0 100n

* Source de tension type echelon
VIN IN 0 PWL 0 0 1n 1
```

Analyse transient:
End time= 30u
Print step= 10n
Time Step Control: Max=10n


XSC1
Ext Trig
A
B
R2
10Ω
Key=A
0 %
L1
1mH
3
V1
1V
C1
1mF

Plan du cours

- ❑Rappels et suite ampli OP

- ❑Rappel comportement des systèmes linéaires invariants (comportement fréquentiel, représentation complexe…)

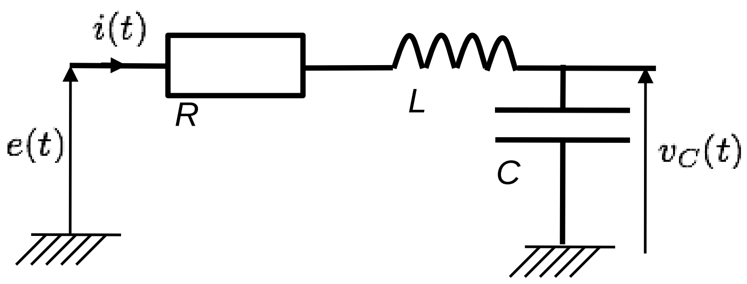
- ❑Circuits RC
  - Réponse harmonique (module et phase)
  - Réponse indicielle

- ❑Filtres actifs

- ❑Circuits RLC
  - Réponse indicielle
  - Réponse harmonique (module et phase)

- ❑Bilan du cours - A savoir

Circuit RLC: réponse harmonique



Source de tension: $e(t) = Ecos(wt)$

Grandeurs temporelles réelles

Grandeurs complexes *e, i, vC*

Impédances complexes:

Condensateur: $Z_C = \dfrac{1}{jCw}$

Résistance: R

Inductance: $Z_L = jLw$

$$e = Ri + Z_L i + Z_C i$$

$$e = Ri + jLwi + \frac{1}{jCw} i$$

$$i = e \frac{1}{R + jLw + \frac{1}{jCw}}$$

$$v_C = Z_C i$$

$$v_C = \frac{1}{jCw} \frac{1}{R + jLw + \frac{1}{jCw}} e$$

$$v_C = \frac{1}{1 + jRCw - LCw^2} e$$

Fonction de transfert:

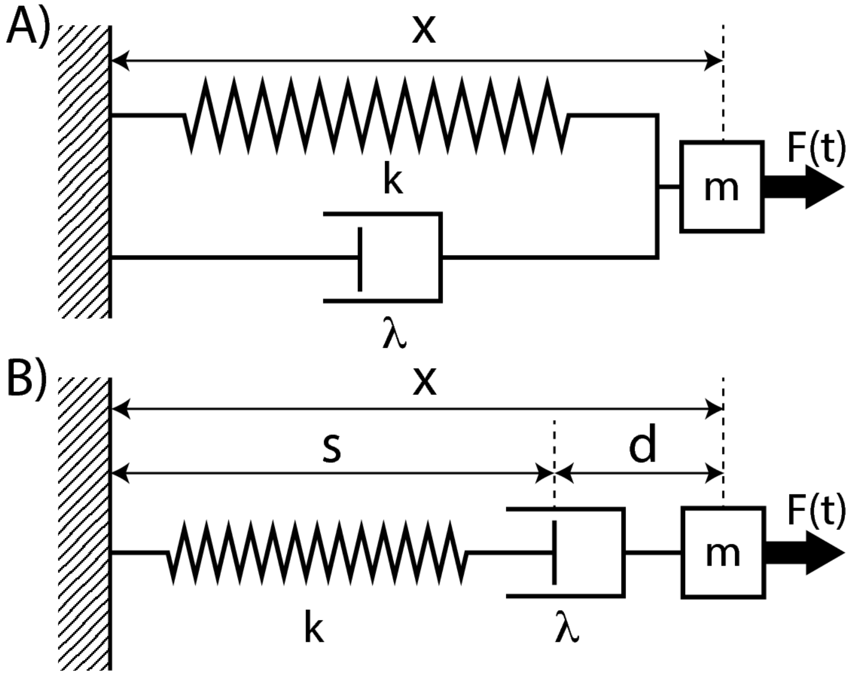
$$H(jw) = \frac{1}{1 + jRCw - LCw^2}$$

$$w_0 = \frac{1}{\sqrt{LC}} \qquad z_0 = \frac{1}{2} R \sqrt{\frac{C}{L}}$$

pulsation naturelle — amortissement

$$H(jw) = \frac{1}{1 + j2z_0 \frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

Système mécanique correspondant

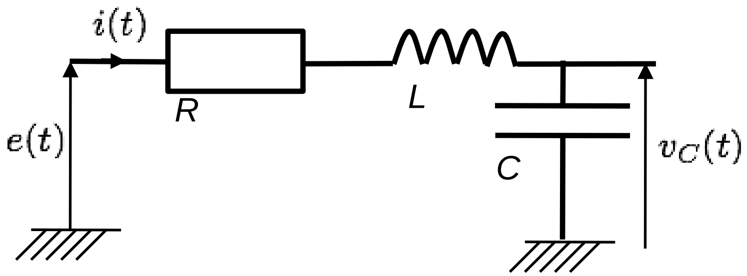


Force F(t)
Masse m
Raideur du ressort k
Frottement λ

$$m\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + kx = 0$$

Equation très similaire !

Circuit RLC: réponse harmonique



Source de tension: $e(t) = Ecos(wt)$

Grandeurs temporelles réelles

Fonction de transfert: $$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

$$H(jw) = \frac{\delta_1\delta_2}{(jw-\delta_1)(jw-\delta_2)}$$

δ1 et δ2 sont le pôles de la fonction de transfert:

La fonction de transfert est du second ordre (polynome de degré 2 au dénominateur)

Le dénominateur est nulle pour deux valeurs complexes de *w*: ce sont les **pôles**

$$LC(jw)^2 + RC(jw) + 1 = 0$$

$$(jw)^2 + \frac{R}{L}(jw) + \frac{1}{LC} = 0$$

La position des pôles détermine la fonction de transfert

$$\Delta = \omega_0^2(z_0^2 - 1)$$

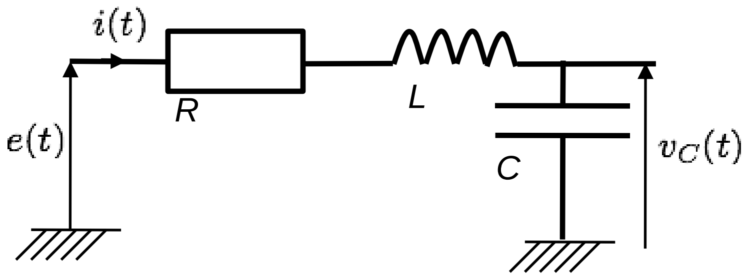
trois cas:

$z_0 > 1$	$z_0 = 1$	$z_0 < 1$
deux racines réelles δ_1 et δ_2	une racine réelle double δ_1	deux racines complexes δ_1 et δ_2

Circuit RLC: réponse harmonique

Source de tension: $e(t) = E cos(wt)$

Grandeurs temporelles réelles



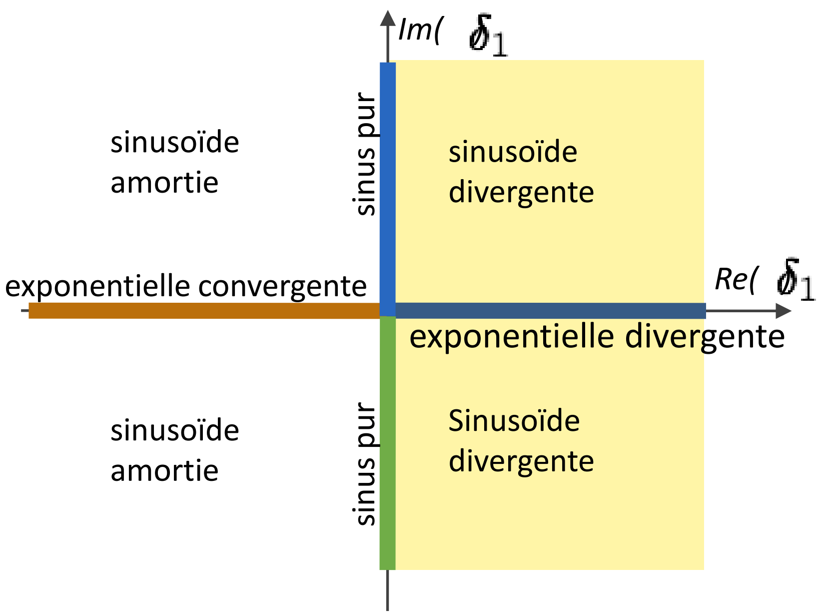
Fonction de transfert:

$$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

$$H(jw) = \frac{\delta_1\delta_2}{(jw - \delta_1)(jw - \delta_2)}$$

δ_1 et δ_2 sont le pôles de la fonction de transfert:

La position des pôles détermine la fonction de transfert



Circuit RLC: réponse harmonique

Pour $z_0 > 1$

$$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

$$= \frac{1}{(1 - j\frac{w}{\delta_1})(1 - j\frac{w}{\delta_2})}$$

$$H(jw) = \frac{1}{(1 + j\frac{w}{w_1})(1 + j\frac{w}{w_2})} \qquad \begin{aligned} w_1 &= -\delta_1 \\ w_2 &= -\delta_2 \end{aligned}$$

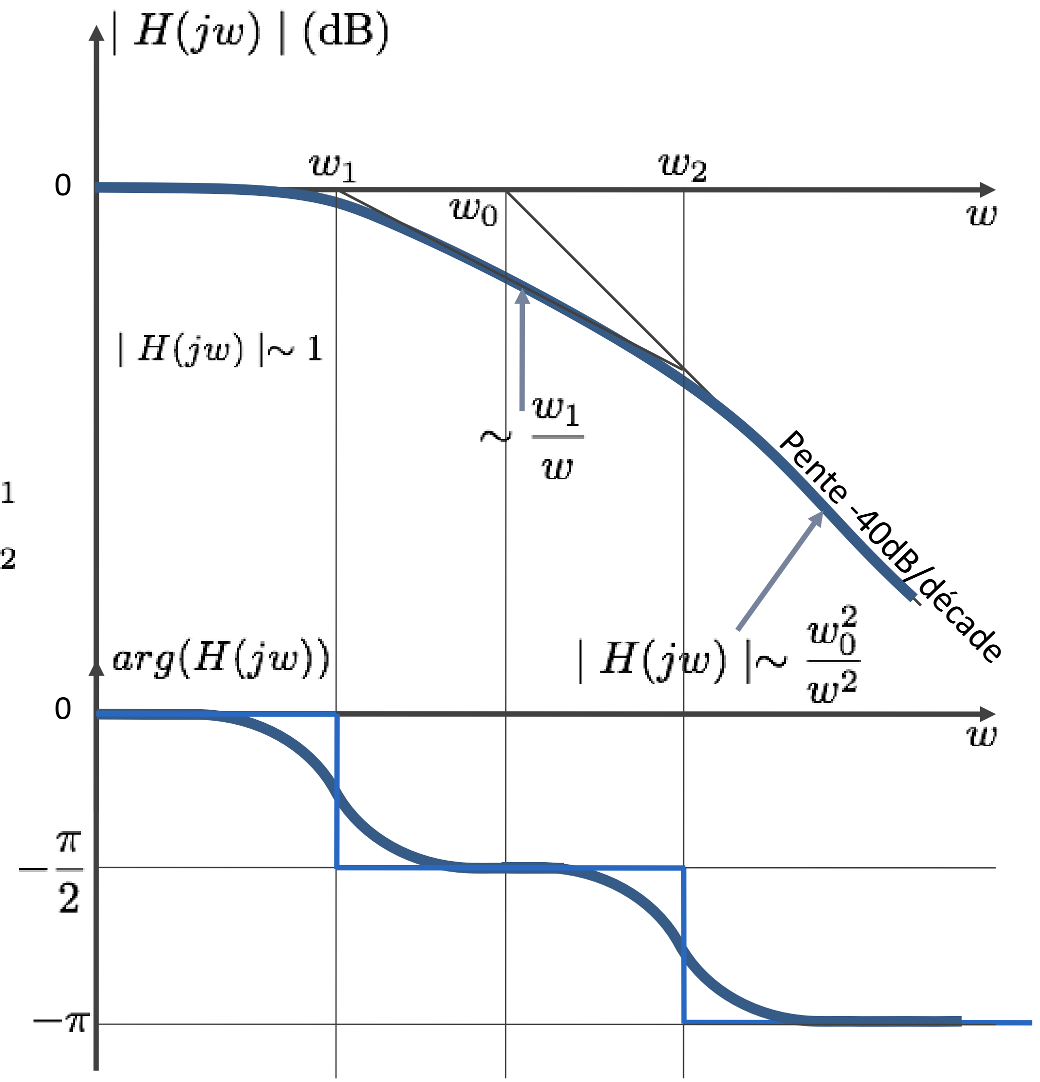


Diagramme asymptotique

$$w \ll w_1 \ll w_2 \rightarrow H(jw) \sim 1$$

$$w_1 \ll w \ll w_2 \rightarrow H(jw) \sim -j\frac{w_1}{w}$$

$$w_1 \ll w_2 \ll w \rightarrow H(jw) \sim -\frac{w_1 w_2}{w^2}$$

et $w_1 w_2 = w_0^2$

Circuit RLC: réponse harmonique

Pour $z_0 = 1$

$$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

$$= \frac{1}{(1 - j\frac{w}{\delta_1})(1 - j\frac{w}{\delta_1})}$$

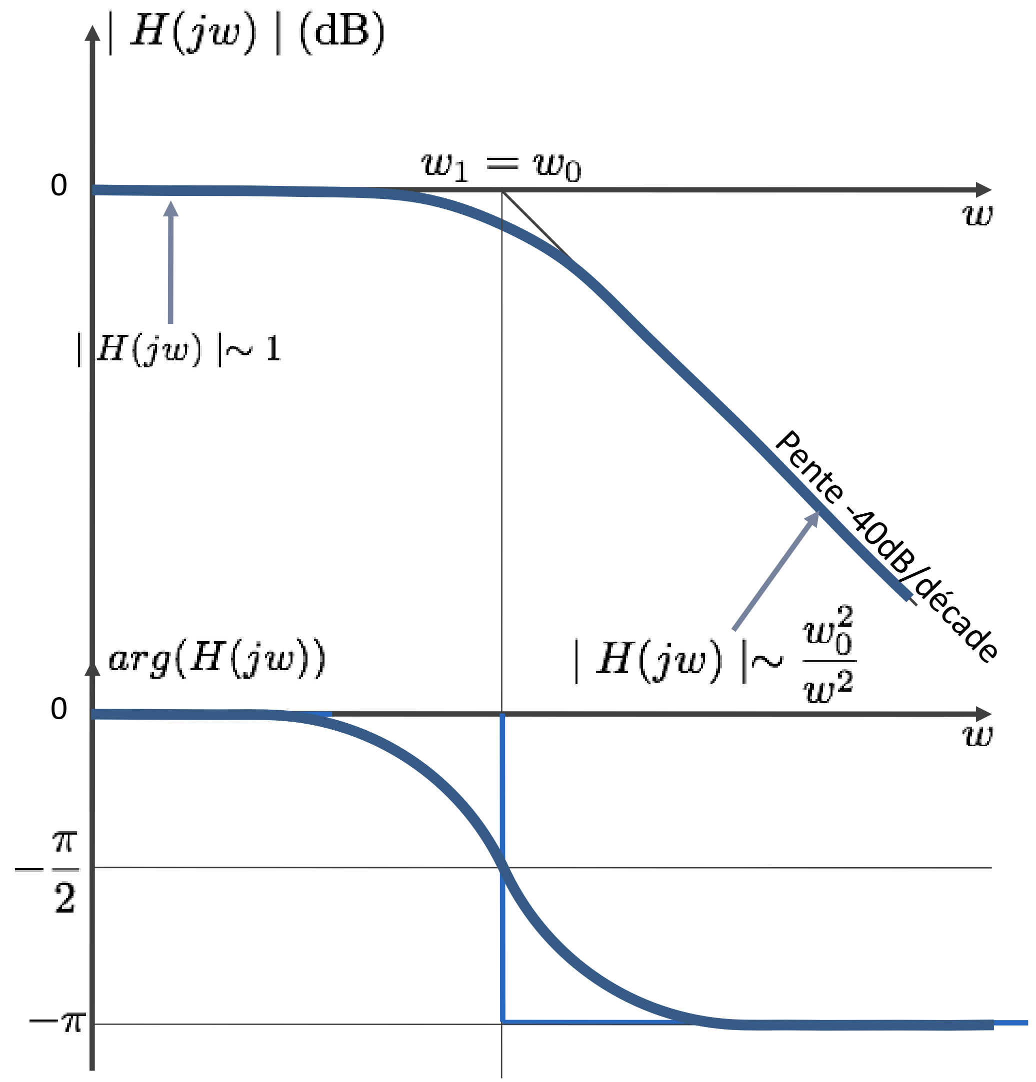
$$= \frac{1}{(1 + j\frac{w}{w_1})^2} \qquad w_1 = -\delta_1$$

Diagramme asymptotique

$$w \ll w_1 \rightarrow H(jw) \sim 1$$

$$w_1 \ll w \rightarrow H(jw) \sim -\frac{w_1^2}{w^2}$$

et $w_1 = w_0$



Circuit RLC: réponse harmonique

Pour $z_0 < 1$

Fonction de transfert:

$$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

Diagramme asymptotique

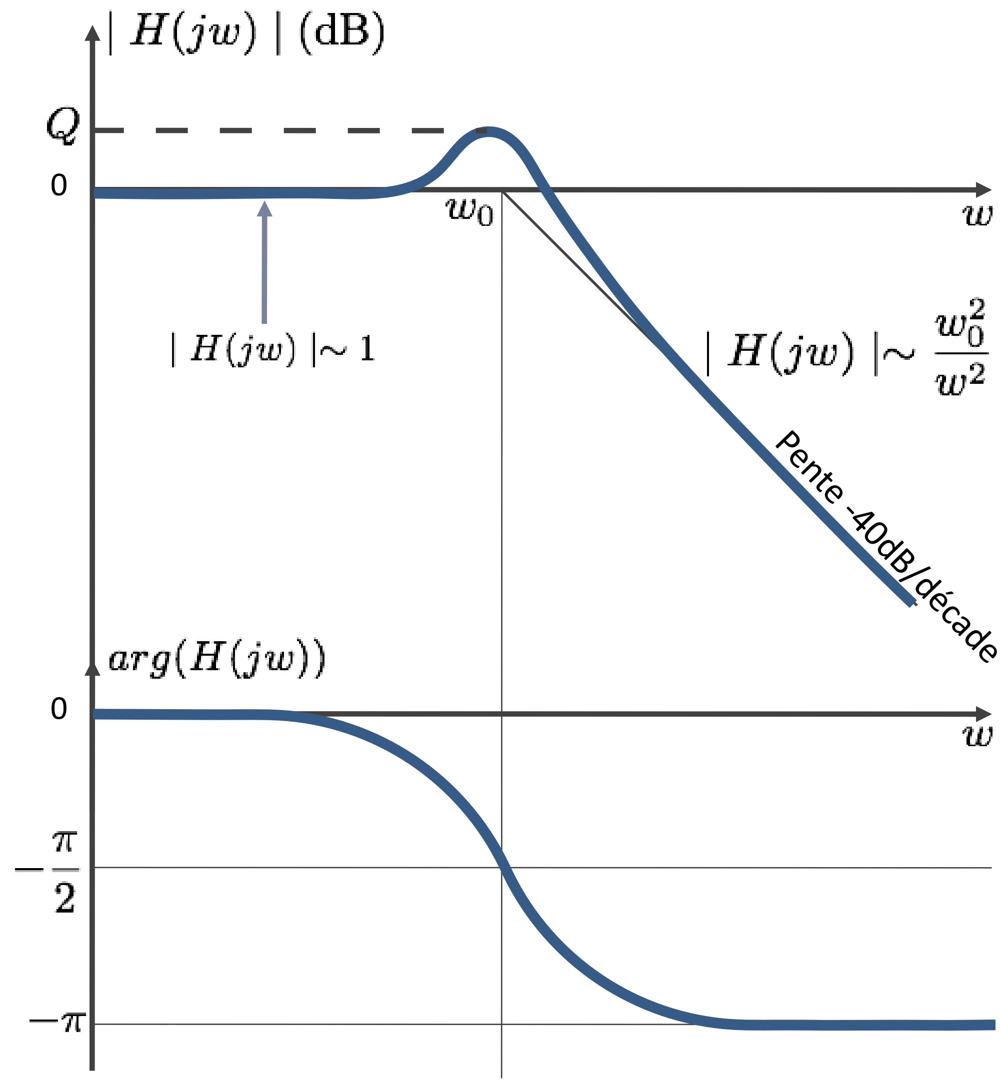
$$w \ll w_0 \rightarrow H(jw) \sim 1$$

$$w = w_0 \rightarrow H(jw) = \frac{1}{j2z_0}$$

$$w \gg w_0 \rightarrow H(jw) \sim -\frac{w_0^2}{w^2}$$

Facteur de qualité:

$Q = \dfrac{1}{2z_0}$ (pour z_0)$\ll 1$



Circuit RLC: réponse harmonique

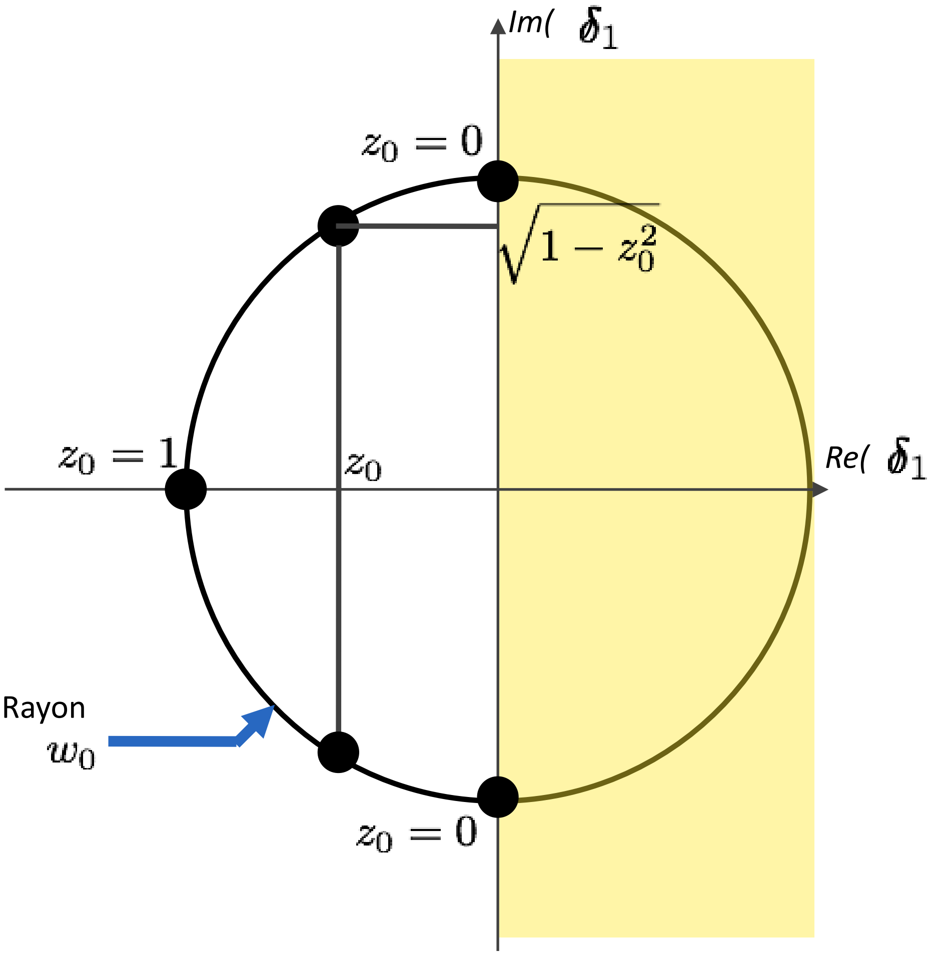
Pour $z_0 < 1$

Fonction de transfert:

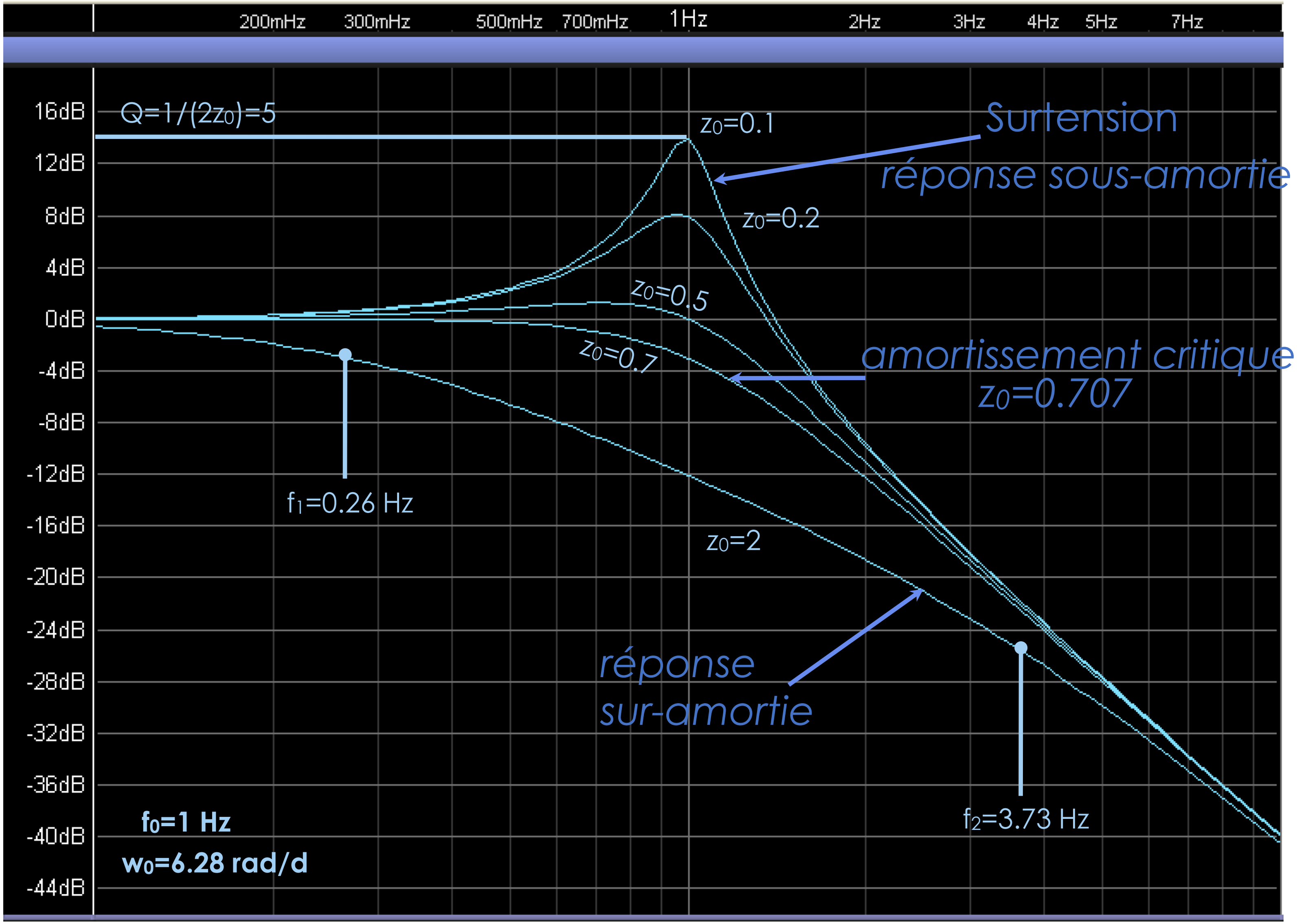
$$H(jw) = \frac{1}{1 + j2z_0\frac{w}{w_0} - \frac{w^2}{w_0^2}}$$

Pôles

$$\delta_1 = -\lambda + jw_0\sqrt{1-z_0^2}$$
$$= -z_0w_0 + jw_0\sqrt{1-z_0^2}$$
$$= w_0\left(-z_0 + j\sqrt{1-z_0^2}\right)$$
$$\delta_2 = w_0\left(-z_0 - j\sqrt{1-z_0^2}\right)$$



Circuit RLC: réponse harmonique



Circuit RLC: réponse harmonique

Pour $z_0 > 1$

Simulation pour:

$$w_0 = 1\text{krad/s}$$

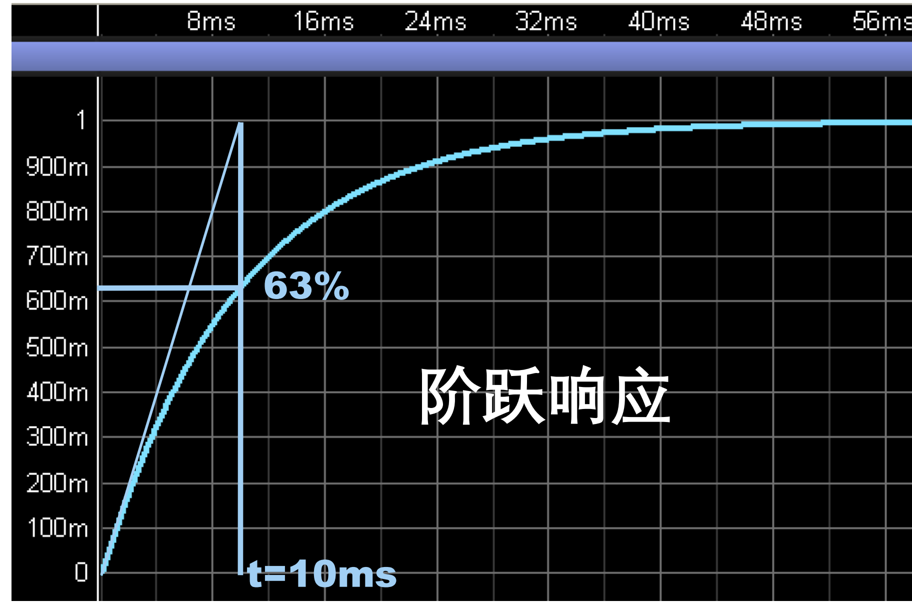
$$z_0 = 5$$

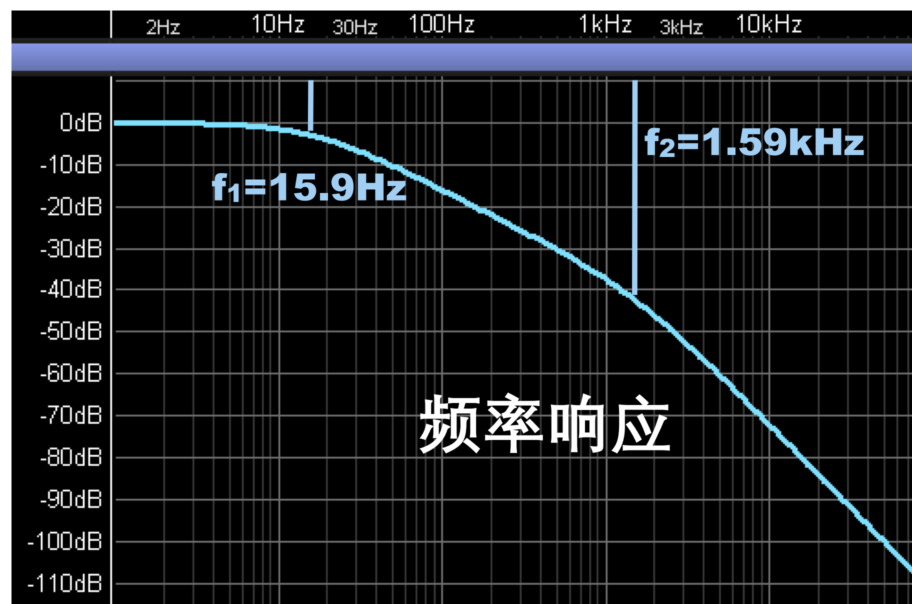
$$\delta_1 = -\frac{w_0}{2z_0} = -100 \text{ rad/s}$$

$$\delta_2 = -2z_0 w_0 - \frac{w_0}{2z_0} = -9.9 \text{ krad/s}$$

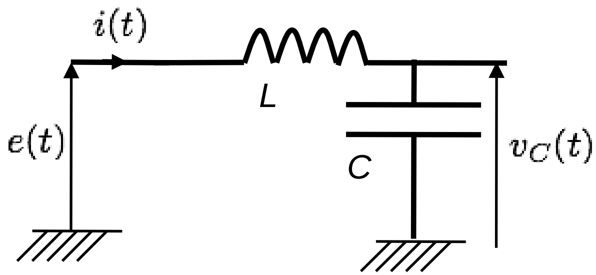
Fichier Spice:

```
* Systeme du second ordre
* Fonction de transfert definie par bloc Laplace
S1 RLC 0 IN 0
+ A0=1
+ B0=1 B1='2*Z0/WC' B2='1/(WC*WC)'

* Tension d'entrée:
VIN IN 0 PWL 0 0  1p 1 AC 1

* Definition des parametres:
.PARAM WC=1k
.PARAM Z0=5
```





Circuit RLC: exemple



Source de tension: $e(t) = E\cos(wt)$

Grandeurs complexes *e, i, vC*

Impédances complexes:

- Condensateur: $Z_C = \frac{1}{jCw}$
- Résistance: R
- Inductance: $Z_L = jLw$

Fonction de transfert:

$$H(jw) = \frac{1}{1 - LCw^2}$$

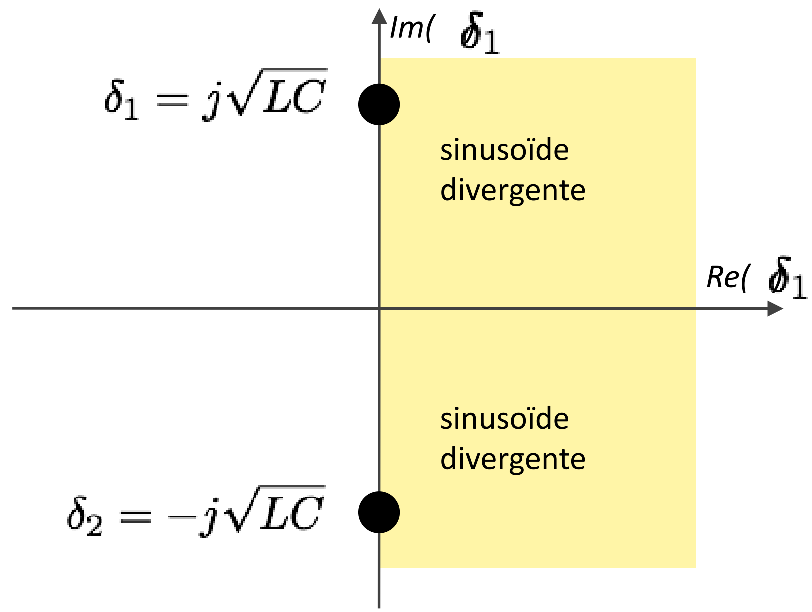
Détermination des pôles:

$$H(jw) = \frac{-LC}{\left(w + \sqrt{LC}\right)\left(w - \sqrt{LC}\right)}$$

$$= \frac{LC}{\left(jw + j\sqrt{LC}\right)\left(jw - j\sqrt{LC}\right)}$$

$$\delta_1 = j\sqrt{LC}$$

$$\delta_2 = -j\sqrt{LC}$$



Circuit RLC: réponse harmonique

Fonction de transfert:

$$H(jw) = \frac{1}{1 - LCw^2}$$

Diagramme asymptotique

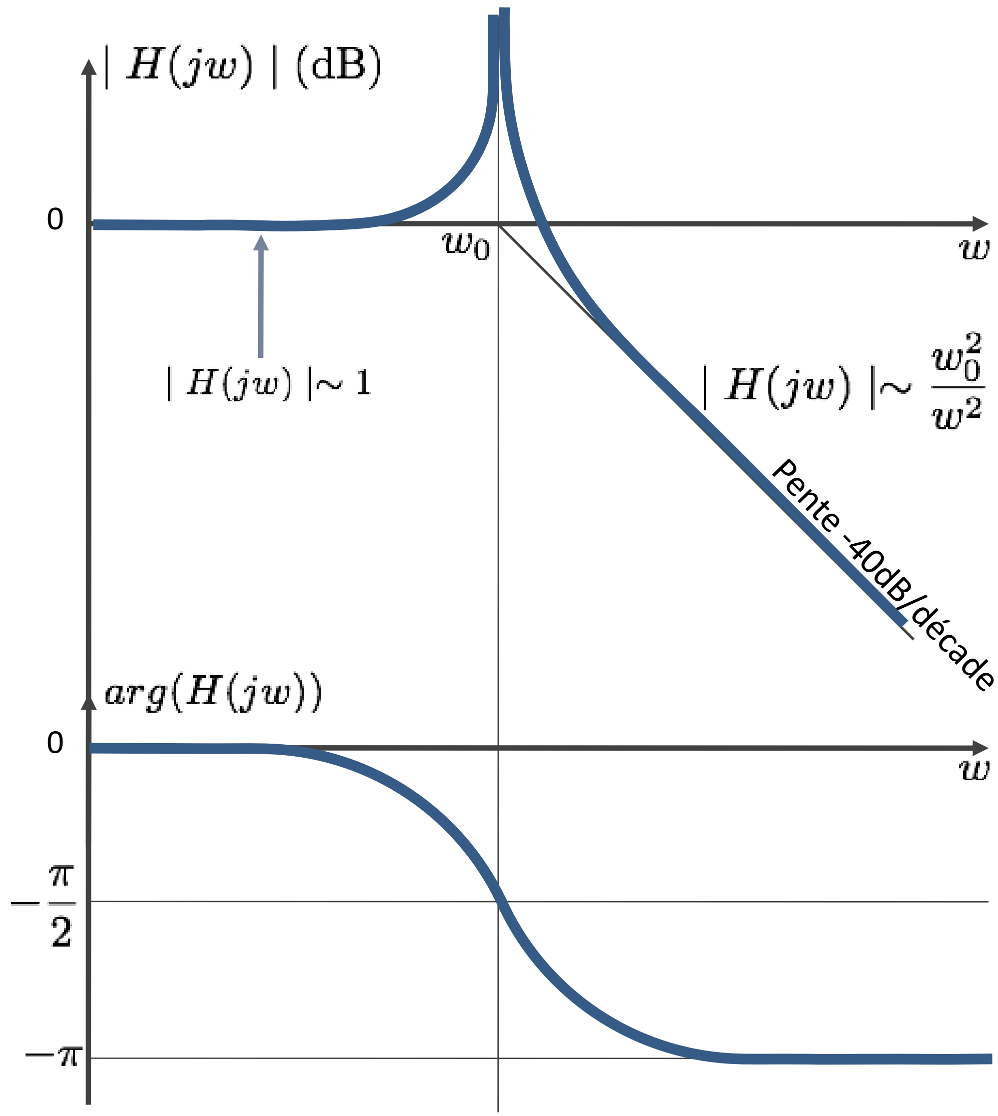
$$w \ll w_0 \rightarrow H(jw) \sim 1$$

$$w = w_0 \rightarrow H(jw) = \frac{1}{j2z_0}$$
$$= -j\infty$$

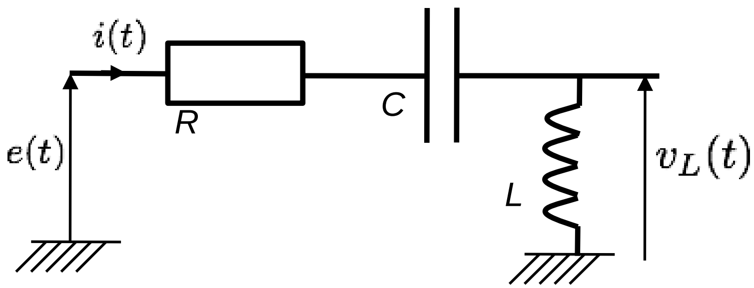
$$w \gg w_0 \rightarrow H(jw) \sim -\frac{w_0^2}{w^2}$$

Facteur de qualité:

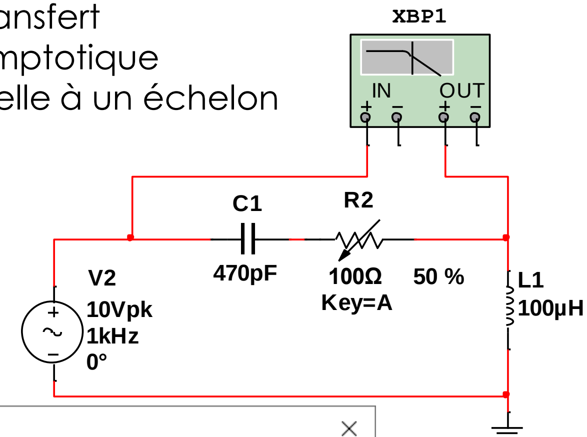
$$Q = \frac{1}{2z_0} = \infty$$



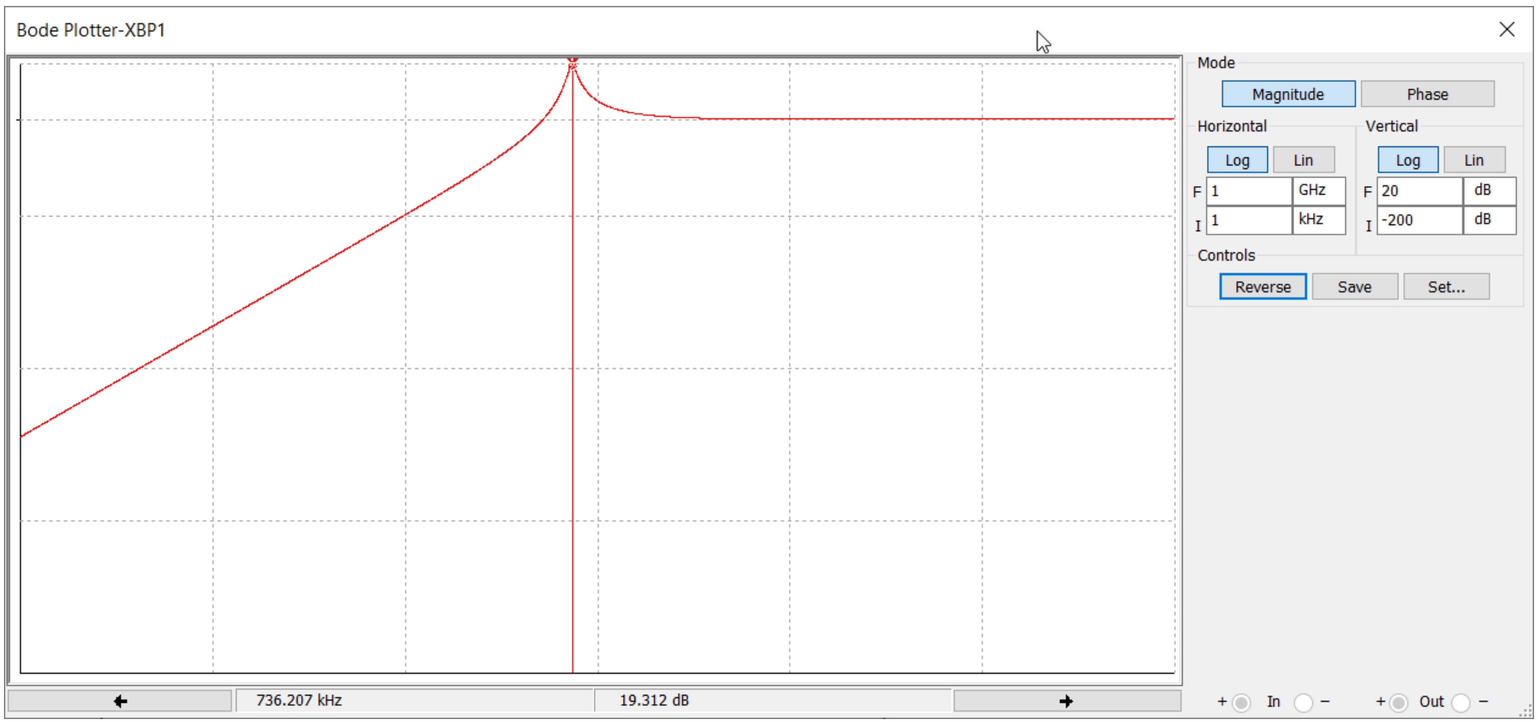
Applications aux filtres



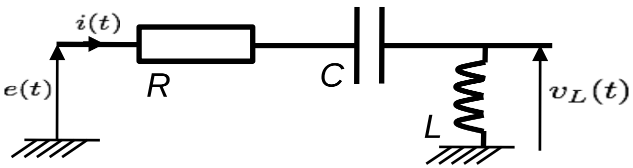
Calculer la fonction de transfert
Tracer le diagramme asymptotique
Tracer la réponse temporelle à un échelon



R1=50Ω
L1=100uH
C1= 470pF



Applications aux filtres

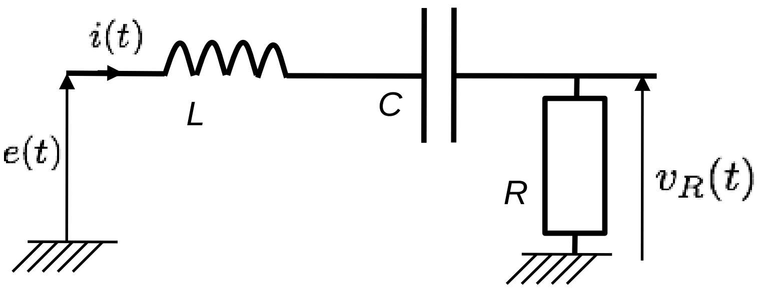


R1=50Ω
L1=100uH
C1= 470pF

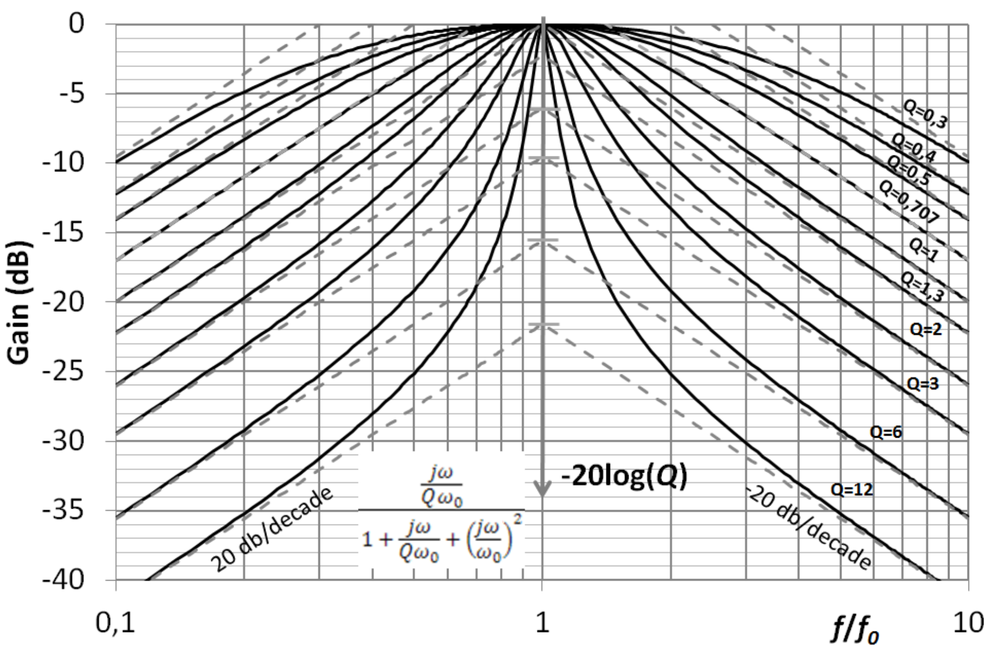
```
l=100e-6
c=472e-12
w0=1/sqrt(l*c)
f0=w0/(2*pi)
r=50
z0=r/2*sqrt(c/l)
Q=1/(2*z0)
QdB=20*log10(Q)
```

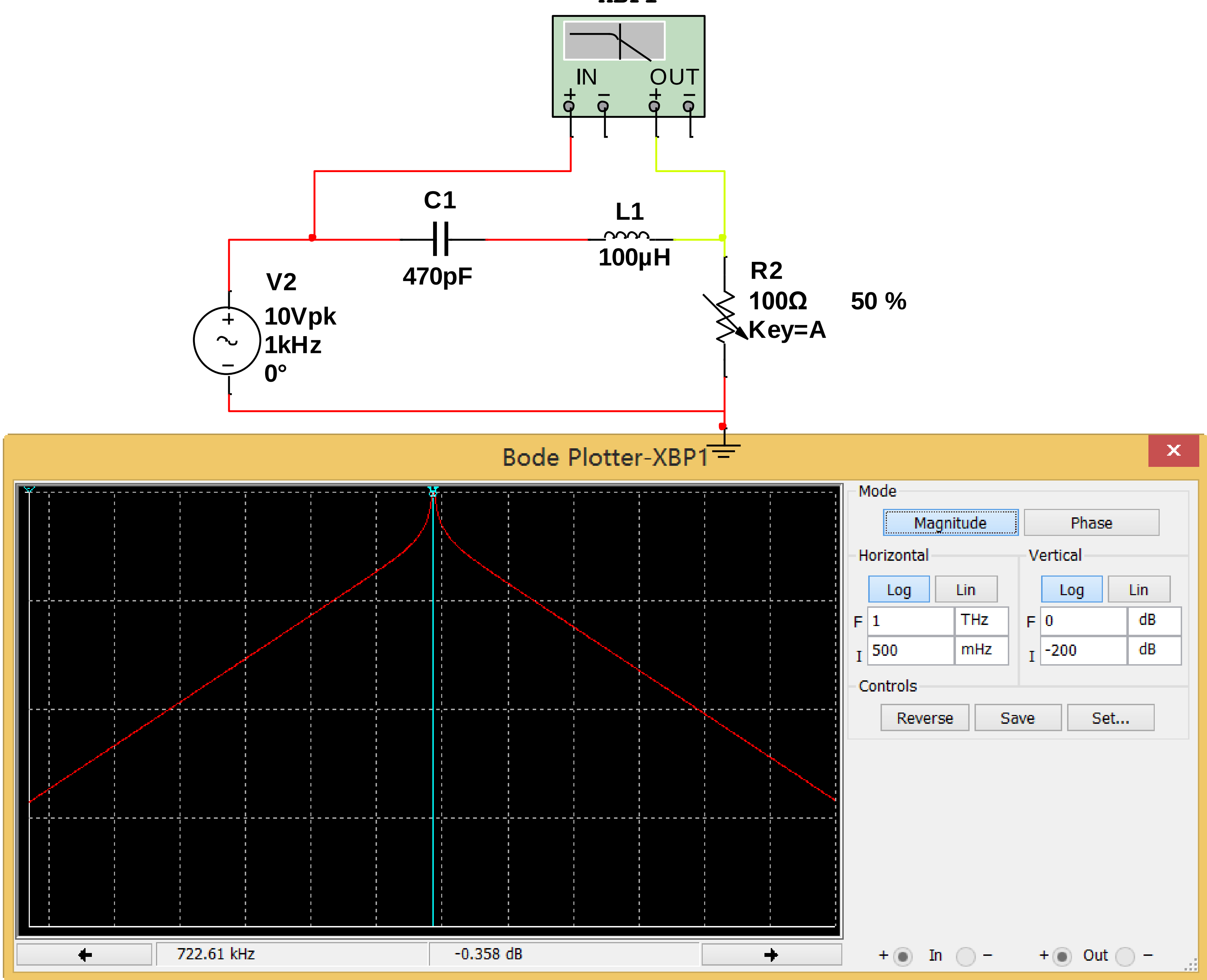
```
>> compute_RLC_w0_Q
l =
  1.0000e-04
c =
  4.7200e-10
w0 =
  4.6029e+06
f0 =
  7.3257e+05
r =
   50
z0 =
   0.0543
Q =
   9.2057
QdB =
  19.2812
```

Applications aux filtres



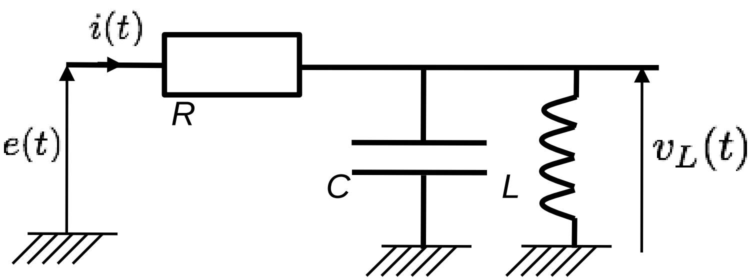
Calculer la fonction de transfert
Tracer le diagramme asymptotique
Tracer la réponse temporelle à un échelon



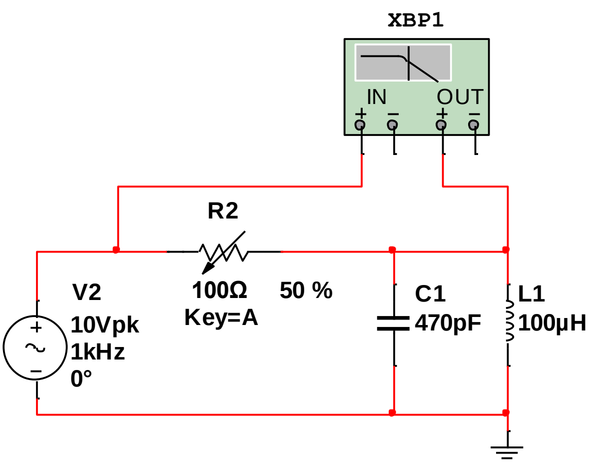


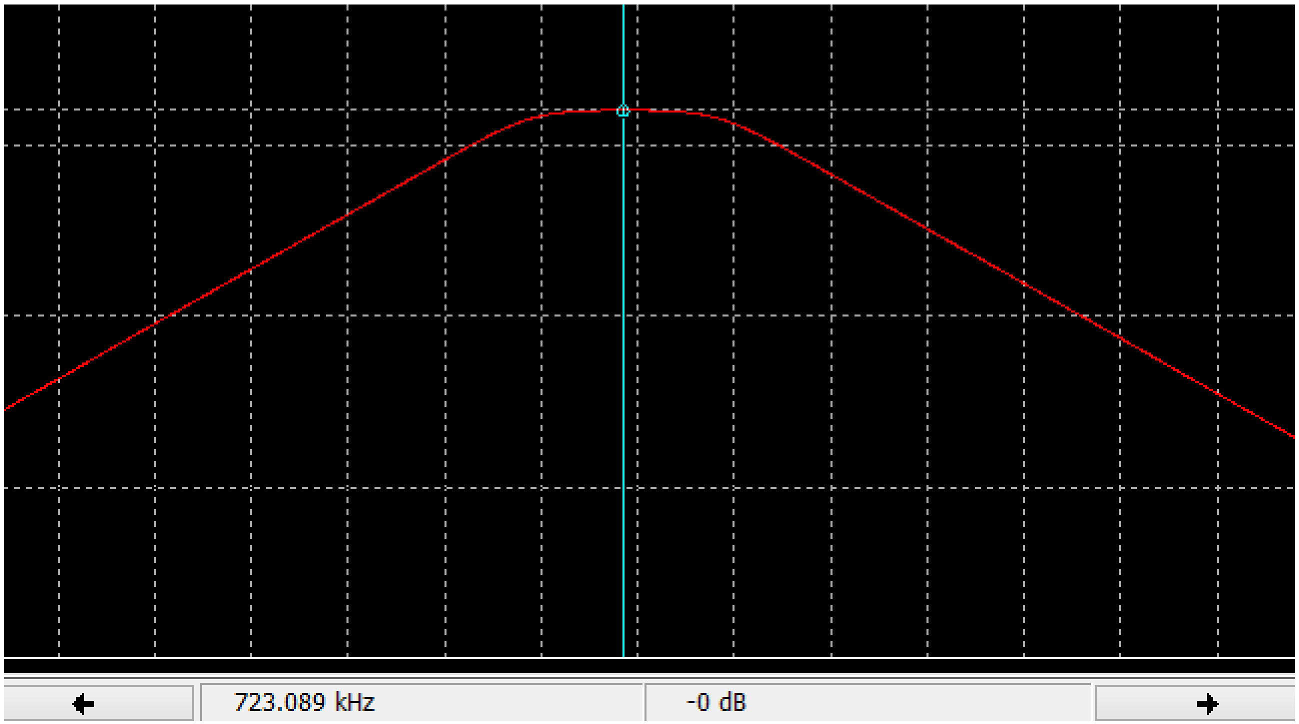
XBP1
IN
OUT
C1
470pF
L1
100µH
V2
10Vpk
1kHz
0°
R2
100Ω
50 %
Key=A
Bode Plotter-XBP1
Mode
Magnitude
Phase
Horizontal
Vertical
Log
Lin
F
1
THz
I
500
mHz
0
dB
-200
Controls
Reverse
Save
Set...
722.61 kHz
-0.358 dB
In
Out

Applications aux filtres



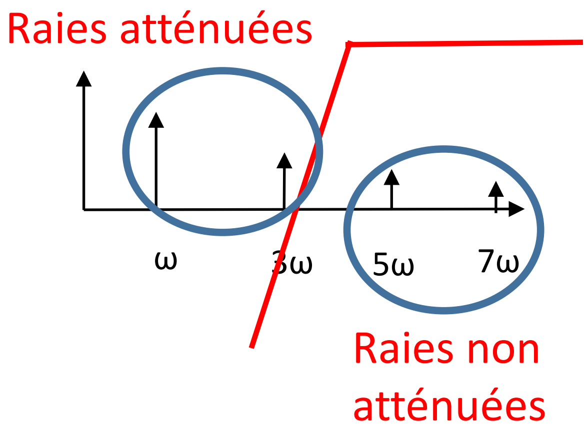
Calculer la fonction de transfert
Tracer le diagramme asymptotique



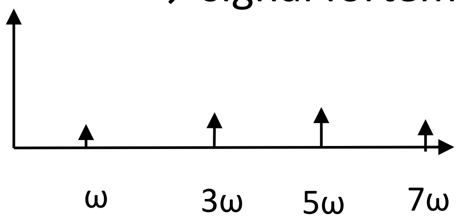


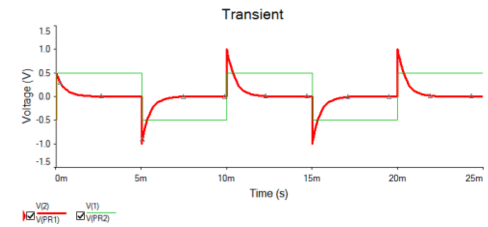
$\omega < \omega_c$

Filtre passe haut



Signal filtré
→ signal fortement modifié





$\omega > \omega_c$

Signal légèrement modifié