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Using Information Invariants to Compare Swarm Algorithms and General Multi-Robot Algorithms

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***Abstract*— Robotic swarms are decentralized multi-robot systems whose members use local information from proximal neighbors to execute simple reactive control laws that result in emergent collective behaviors. In contrast, members of a general multi-robot system may have access to global information, all-to-all communication or sophisticated deliberative collaboration. Some algorithms in the literature are applicable to robotic swarms. Others require the extra complexity of general multi-robot systems. Given an application domain, a system designer or supervisory operator must choose an appropriate system or algorithm respectively that will enable them to achieve their goals while satisfying mission constraints (e.g. bandwidth, energy, time limits). In this paper, we compare representative swarm and general multi-robot algorithms in two application domains — navigation and dynamic area coverage — with respect to several metrics (e.g. completion time, distance travelled). Our objective is to characterize each class of algorithms to inform offline system design decisions by engineers or online algorithm selection decisions by supervisory operators. Our contributions are (a) an empirical performance comparison of representative swarm and general multi-robot algorithms in two application domains, (b) a comparative analysis of the algorithms based on the theory of information invariants, which provides a theoretical characterization supported by our empirical results.**

I. INTRODUCTION

For more than the past decade, there has been significant growing interest in multi-robot systems (MRSs), which use a team of cooperating robots to accomplish tasks collaboratively. This enables multi-robot systems to complete tasks that cannot be completed by a single robot or would be less efficient or effective to complete with a single robot [1]. Multi-robot systems may be characterized along multiple dimensions [2] including but not limited to the mechanism for coordination among robots (e.g. communication vs. sensing only), centrality of coordination (e.g. centralized vs. decentralized), the extent of information available to team members (e.g. local information vs. global information), sophistication of the control logic executed by robots (e.g. reactive vs. deliberative collaboration), the structure of information propagation within the robotic network (e.g. neighbor-only connectivity vs. all-to-all communication), homogeneity or heterogeneity of the team members.

Existing multi-robot coordination algorithms [3], [4] combine different assumptions along these dimensions about the underlying multi-robot system to which they are applied. Conversely, the extensive behavior of the multi-robot system is dictated by the properties of the algorithm it uses for internal coordination. Now we consider a particular type of multi-robot system known as a *robotic swarm*. Robotic swarms are characterized by homogeneous robots executing a simple reactive control law using only local information from proximal swarm members and the environment within a limited spatial neighborhood. The collective behavior (e.g. flocking) of the swarm emerges as a result of all swarm members executing the same local control law and no individual swarm member ever necessarily becomes aware of the whole swarm. Swarm behaviors are often not goal-directed but may be combined through behavior composition to accomplish tasks for which no individual behavior was designed [5], [6]. Conventional wisdom in the literature has been that the simplicity of the local control laws executed by swarm members and the locality of the information required for their execution makes robotic swarms more scalable and robust than other types of multi-robot systems because members may be inserted and deleted with minimal system reconfiguration [4].

In this paper, we compare the performance of algorithms designed specifically for robotic swarms (i.e. systems whose members have access to local information, neighbor-only sensing and simple reactive control laws) to those designed for general multi-robot systems (i.e. systems whose members may have access to global information, all-to-all communication and sophisticated deliberative collaboration). Algorithms have been developed for several multi-robot applications [1] including navigation [7], [8], static area coverage [9], dynamic area coverage [10]–[13], patrolling [14] and many more. However, in this paper, we limit our comparison to representative algorithms in two application domains: (a) navigation and (b) dynamic area coverage.

Our objective in performing this comparison is to identify and highlight the strengths and weaknesses of swarm and general multi-robot algorithms in a manner that can inform offline design decisions made by engineers or online operational decisions made by supervisory operators of multi-robot systems. For example, for a given application domain, an engineer may decide a simple robotic swarm will suffice without the additional cost and complexity of a general multi-robot system. Conversely, given a multi-robot system, there may be situations where it is beneficial (e.g. reduced coordination complexity) for a supervisory operator to apply an algorithm designed for robotic swarms rather than incur the communication overhead of a general multi-robot coordination algorithm. Other examples exist and it is our hope that the results in this paper can inform such decisions.

We contribute the following: (a) an empirical evaluation of

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the performance of representative swarm and general multi-robot algorithms within two different application domains (navigation and dynamic coverage), (b) a comparative analysis of these algorithms based on the theory of information invariants, which provides a theoretical characterization supported by our empirical results.

II. Swarm and Multi-Robot Algorithm Selection

A. Algorithm Description and Implementation

As shown in Table I, five representative algorithms were implemented to study the relative performance of swarm and decentralized multi-robot algorithms. The algorithms were then analyzed in terms of their information invariants [15] and their empirical performance was compared with respect to multiple metrics.

1) Potential Fields (PF): This algorithm, as described in [16], is a gradient-based navigation approach with guaranteed goal convergence. Characteristics of this algorithm relevant to our comparison: a) no explicit communication between robots (sensing-only swarm algorithm), b) robots initially configured as a connected graph will converge to the goal as a connected graph, c) robots follow the gradient of a potential function with a global minima at the goal while avoiding obstacles and other robots.

2) Proportional Barrier Certificates (PBC): PBC uses a proportional controller and a barrier certificates reactive controller as described in [17], [18]. The barrier certificates algorithm guarantees forward invariance of the safe set, implying that inter-robot collision avoidance is guaranteed. Characteristics of this algorithm relevant to our comparison: a) no explicit communication between robots (sensing-only swarm algorithm), b) robots take action to avoid collisions only when sufficiently close (sense the proximity), c) robots follow a PC towards goal.

3) DMA-RRT: The DMA-RRT algorithm outlined in [7] embeds a closed-loop RRT [20] in each robot and introduces a merit-based token passing coordination mechanism. Agents with the largest incentive to replan will acquire the token and broadcast their updated plan to all other agents. Other agents then forward simulate this time parameterized trajectory and update their own constraints. Characteristics of this algorithm relevant to our comparison: a) explicit communication between the robots which is bounded above by a set of n waypoints, where n is the size of the created path, b) communication occurs through a token passing strategy, where only one robot gets to alter its plan at any given time, c) obstacle and inter-robot avoidance is done implicitly through the robots' creation of non-intersecting paths.

4) Individual Dynamic Coverage (IDC): Dynamic area coverage problems have been tackled by various stigmergic and frontier-based dynamic coverage algorithms such as [21], [22]. The IDC algorithm is a gradient-based dynamic coverage method outlined in [19]. The coverage error is guaranteed to be non-increasing but susceptible to local minima. Perturbations from local minima, in the computational space, involve the selection of a new point for each robot upon which to resume gradient descent. In the real world this corresponds to an individual robot moving to a new location and resuming application of the gradient-based control law. The algorithm was implemented in a decentralized way through a timer as in the provided implementation of [19].

5) Group Dynamic Coverage (GDC): This decentralized algorithm involves the multi-robot gradient based dynamic coverage strategy outlined in [13]. The algorithm is similar to IDC, but agents are now perturbed all at the same time once a global condition is met and a leader agent selects a new rendezvous point towards which the entire group moves. Once the multi-robot group is within a certain radius of the newly selected point, all robots shift into dynamic coverage mode and the state machine is reset. In the computational space, this corresponds to all agents reaching an overall local minima, and them all being perturbed by relocating themselves to the same region of the workspace, resuming the gradient descent from there. This method involves the communication of a new swarming point from the leader to every other agent in the group, which in the real world could be accomplished by wireless communication (for example).

III. Experimental Evaluation

A. Reproduction of Results and Scaling

Algorithms were implemented in Python and then integrated into the CMUSWARM Framework [23]. We first reproduced the results in the corresponding papers. For each algorithm, the number of robots were scaled for preliminary qualitative testing beyond the experiments in the original papers. As seen in Figure 1, the time to convergence for Potential Fields, PBC, and GDC generally increase with the number of robots, whereas Individual Dynamic Coverage decreases.

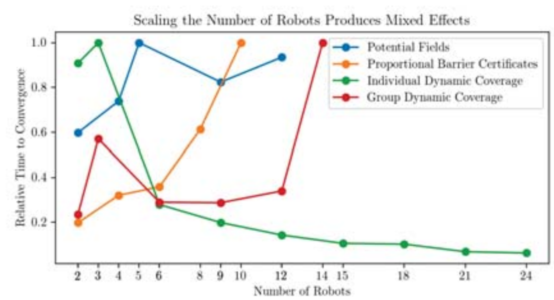

Fig. 1: Results of Preliminary Scaling

Figure 1 does not contain data on DMA-RRT due to the nature of the initial research. This algorithm was initially evaluated by having multiple robots cycle through a list of goals for 10 minutes. For DMA-RRT, we reproduced scenario A [7] of the paper which involved 10 agents cycling through 10 goals, and scenario B [7], with 4 agents cycling through 2 goals. We observed in our reproduction that more robots caused more collisions.

B. Experimental Setup

The algorithms mentioned above were benchmarked in the CMUSWARM Framework on ROS and Gazebo. Each algorithm was run for 20 trials on 5 different 20x20 meter maps, using 4, 8, and 16 holonomic 0.2 x 0.2 m iRobot Create vehicles totalling 300 trials. An experimental trial

TABLE I: Algorithms for Comparison

	Swarm	Multi-Robot
Navigation	Potential Fields [16] Proportional Barrier Certificates [17], [18]	DMA-RRT [7]
Dynamic Area Coverage	Individual Dynamic Coverage [19]	Group Dynamic Coverage [13]

is deemed successful when the algorithm converges and finishes the task within 10 minutes. We set a 10 minute cut-off for three reasons. Firstly, we wish to establish a mission-critical time frame, in which the task must be completed. Secondly, the successful trials took far less than 10 minutes, with more than 10 minutes passing when a robot was stuck on an obstacle or in some other observably unrecoverable state. Lastly, with a 10 minute cap on simulation time, with a real time factor making the actual trial take 14+ minutes, running 1000+ trials to cover all 5 algorithms took well over 150 hours.

If an algorithm fails to converge 20/20 times, we state it is intractable in the scenario, otherwise we will keep running trials until 20 successful trials are recorded. Parameter tuning was a manual process performed for each algorithm in the initial 4 robot case. Once a feasible set of parameters were found, they were used in the 4, 8, and 16 robot scenario. Manual re-tuning was done for each map.

Performance is measured by the metrics of convergence time (until task completion), distance travelled, area coverage (for navigation), sensor coverage (for dynamic coverage), and number of collisions. Our measures of convergence time and distance travelled correspond to the dynamic coverage Time and dynamic coverage Cost metrics outlined in [24], and are therefore classified as objective methods of comparison for dynamic coverage algorithms. For navigation, convergence time is the amount of time required for all robots to become within 3 meters of the goal location. For dynamic coverage algorithms, convergence time is measured as the amount of time to reach 100% coverage of the map.

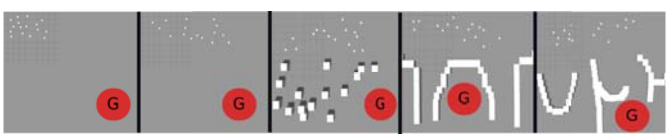

Fig. 2: The 5 maps in Gazebo used for our experiments. Maps From left to right 1 (Empty Map Dense with spawn region ([1-6], [1-8]), 2 (Empty Map Spread with spawn region ([1-6], [1-19])), 3 (Uniform Map), 4 (Corridor Map), 5 (Concave Map). The goal region is the red disk labeled G, with robots (White) spawning in the upper region. Robots spawn in the x in [1,6], y in [1,19] range for all maps except empty dense.

C. Experimental Results

Potential Fields Navigation: PF had 0 inter-robot collisions in obstacle free environments, and the fewest collisions in obstacle filled environments (map 3, Figure 2) among the navigation algorithms, however this algorithm becomes intractable as the number of robots surpasses roughly 8 in any scenario. We believe this is due to the connectivity constraints requiring all robots to be connected within some proximity radius. As more robots are introduced in a spread out scenario (maps 2-5) there is an increasing likelihood of breaking the initial connectivity based on initial spawn location. Furthermore as the number of robots or other static obstacles increases, the system becomes increasingly slow at maneuvering through obstacles (Figure 4). Thus, PF was intractable on the corridor and concave maps with our minimum scenario of 4 robots, as the purely reflexive behaviour results in deadlocks with obstacles. We refrained from scaling further on these maps as a result.

Proportional Barrier Certificates Navigation: The PBC controller was tractable on only the empty maps (1,2) but resulted in the least distance travelled, had the lowest convergence time, and covered the least area in its path among all navigation algorithms. The algorithm had few collisions in empty maps (1,2), slightly more than PF; however as the number of robots scaled to 16, it had the most collisions of all algorithms (Figures 3), likely due to the violation of initial safe set conditions. This algorithm requires robots to spawn at least some threshold (denoted by parameter Ds) away from each other at spawn time. In map 1 (Figure 2) where the robots are spawned in a dense region, this condition is violated in the 16 robot case. Furthermore, there is a chance of this condition being violated in the second map despite a wider initial spawn region. A collision is recorded when the collision disk of 2 robots intersect for every second. Thus, if the initial safe-set guarantee is violated then two or more robots may collide and incur many collisions, skewing the results.

This algorithm required less parameter tweaking than PF; but was very specific to scenario. We set our experimental evaluation to use the same set of parameters for all 3 cases of robots (4,8,16), and only changed parameters between different maps.

DMA-RRT Navigation: This algorithm scaled well as the number of robots increased, almost not increasing in convergence time on empty maps and the uniform map (maps 1,2,3) (Figure 2) as the robots had alternative routes to take. On more cluttered environments (maps 4, 5) (Figure 2) the convergence time increased more significantly as the robots must take turns navigating through dense regions (Figure 4). In every scenario, the amount of distance travelled increased by less than a factor of 2 despite the number of robots doubling, possibly due to faster token passing. With all robots acting at once some of them must take longer paths to avoid their neighbour's broadcasted trajectory. However, when there are more robots the convergence time increases because the robots must take turns; however now the robots may take better paths as they aren't all navigating to the goal at once, many are stationary. Despite the theoretical guarantee of collision avoidance, collisions were present in our evaluation due to latencies in the ROS system and token exchange. Using the extended algorithm, Cooperative DMA-RRT, which involves emergency stops, may reduce the number of collisions when multiple robots face the same goal as it allows an agent to request others to stop if in close proximity.

Individual Dynamic Coverage (IDC): The IDC algorithm had several parameters and required tuning. There were no collisions on empty maps (1,2) (Figure 2) without static obstacles, and the average number of collisions across all maps either remained the same or decreased as number of robots increased. One possible explanation is that with many robots in an environment, the coverage levels of the spawning region quickly increase which hinders many robots from exploring further. This results in less time for convergence, and less total distance travelled as only a few select robots scout the remaining portion of the environment (Figures 5, 6). Additionally, as the robots attract and repel each other by nature of the swarm algorithm, having more robots on the map may aid in preventing clustering of robots in obstacle dense regions.

Group Dynamic Coverage: GDC required more tuning than the swarm coverage algorithm. The behaviour was similar to that of the swarm dynamic coverage; however due to a leader robot guiding other robots to a specific region, significantly more distance was travelled as the whole group moved together and thus repeated coverage of each others area (Figure 6). This further resulted in a longer time for convergence.

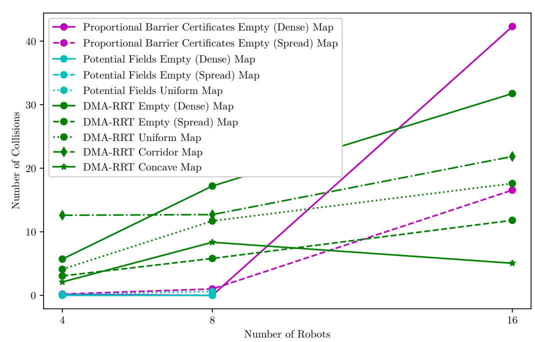

Fig. 3: Number of Collisions for Navigation Algorithms. Algorithms deemed infeasible in certain maps not plotted.

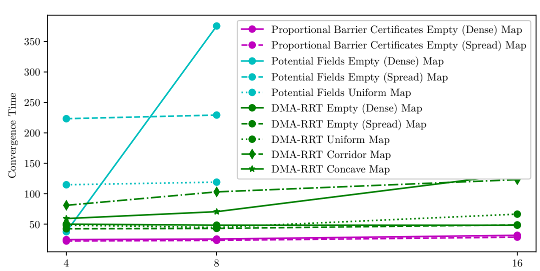

Fig. 4: Convergence Time for Navigation Algorithms

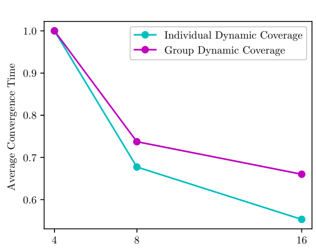

Fig. 5: Average Time to Convergence for the Dynamic Coverage Algorithms

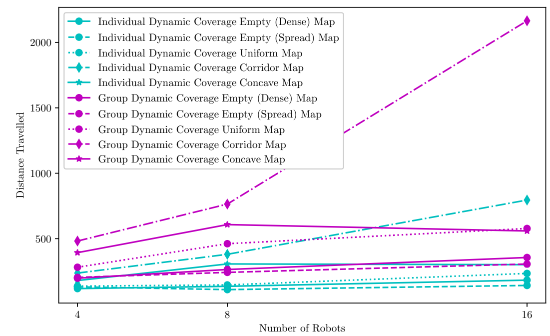

Fig. 6: Distance Travelled for both of the Dynamic Area Coverage algorithms on all map types.

In contrast with the navigation task, IDC overall performed better than multi-robot dynamic coverage. IDC required less distance travelled, and time to converge in every scenario. Furthermore, IDC had the lowest number of collisions among any algorithm on the empty maps (1,2) (Figure 2); while multi-robot dynamic coverage yielded less collisions on obstacle maps (3,4,5) (Figure 2).

D. Discussion

In general, swarm navigation had less collisions than multi-robot navigation on every map it converged on with less than 16 robots (1,2,3) (Figure 2); however the swarm algorithms quickly became intractable with obstacles and an increased number of robots. Furthermore, the swarm algorithms required significant tuning.

This means if all robots are closely grouped together, thus sensing each other, it would be equivalent in bandwidth to multi-robot dynamic coverage's $\sim$ 8.90 kb/sec because we have 8 robots communicating to each other. However, with swarm algorithms, the lower bound of bandwidth is actually still 0 kb/sec, the reason being if the robots are spread out far enough they won't sense each other, and thus won't receive the broadcast.

This highlights a fundamental contrast between the algorithms in our comparison: the swarm algorithms lack of explicit communication as opposed to multi-robot algorithms. Even with the case where sensing is treated as a form of communication, the swarm algorithms still do not have guaranteed communication due to limited sensing radius.

A final note of mention: despite each algorithm having theoretical guarantees for collision avoidance given some set of parameters, no such guarantee was satisfied during experimentation. We believe this is in part due to latency introduced by ROS, along with suboptimal parameter selection for the given scenario.

IV. Analysis

It was observed that, in the context of dynamic coverage, a swarm algorithm outperforms a multirobot algorithm in the metrics of distance travelled and time to convergence. In the context of navigation, however, this is not the case.

A. Information Invariants Concepts

The theory of Information Invariants for robotics was proposed in 1995 [15] and expands on the work of [25]. Since then, the theory has been used to analyze problems in robotic manipulation and control (ex: [26]).

The theory serves to quantify the tradeoff between sensing, actuation, calibration, and communication in multi-robot systems. As in [15], our groups of robots are modelled as circuit graphs. The following definitions will prove useful:

Definition 3.1 A set of sensors attached to computational and motor components able to alter their position in state space is termed a *Sensor System*. A system of robots is an example of a *Sensor System*.

Definition 3.2 *Circuit Graphs* are graphs representing the sensor system, $G = (V, E)$, where V are the sensori-motor components of the system (each individual robot) and the edges E are the sensory/explicit communication connections between sensors (robots).

Definition 4.1 [15] For two sensor systems S and Q we say Q simulates S if the *output* of Q is the same as the *output* of S. When viewing the sensors systems as dynamical systems, the *output* of a sensor system refers to the limit set of the sensor system, a term in Dynamical Systems Theory referring to the state a dynamical system reaches after an infinite amount of time has passed. We write $S \cong Q$, implying that S and Q *deliver equivalent information*.

A calibration of the system constrains the spatial relation between its various nodes, and a calibration required to install a sensor system (how it should be configured at the start of a mission to work) is termed an *installation calibration*.

Definition 5.1 [15] Consider two sensor systems S and Q. When S and Q require equivalent installation calibrations, and when the calibrations required to install Q are necessary to specify S, we say S dominates Q in calibration complexity.

According to **Definition 6.2** in [15], for two sensor systems J and Q we write $J \leqslant Q$ when:

1) J simulates Q ($Q \cong J$)
2) J dominates Q in calibration complexity
3) mb(Q) is bounded above by mb(J)

where mb(.) denotes the maximum bandwidth of the system as in Definition 6.1 of [15]:

Definition 6.1. [15] We define the internal (resp. external) bandwidth of a sensor system S to be the greatest bandwidth of any internal (resp. external) edge in S (an edge represents explicit communication between two robots). We define the maximum bandwidth mb(S) to be the greater of the internal bandwidth, external bandwidth, and the output size of S.

B. Dynamic Area Coverage

In this section, we try to establish an information reduction between IDC and GDC, so as to better explain our findings that multi-robot dynamic coverage does not always perform better than swarm dynamic coverage. Recall that IDC involves no explicit communication of a perturbation point, whereas GDC does.

Due to the gradient based nature of the two algorithms, the only information available to each robot (without explicit communication) are the own robots positions, and the discretized map with information of the coverage levels at every discretized point.

The discretized map used for multi-robot dynamic coverage tasks as in [13], [19] has the limitation that the coverage value of a cell is capped causing loss of information in the system. It is also necessary for the task in order for the gradients to not flow towards already explored areas. In the computational space, this refers to the fact that no additional information can be stored in the map's cell, and an agent acting upon this cell will cause no change. In the real world, this could correspond to robots marking the covered ground with paint, where the painted ground at some point saturates with so much paint that no change is visible.

Denote the system used to solve the multi-robot dynamic coverage task as $M = (V, E)$, and the system used to solve the IDC task as $S = (V', E')$. Following from definition 4.1 in [15] S and M simulate each other because their limit sets are the same, therefore $S \cong M$.

The multi-robot dynamic coverage system M possesses the initial calibration requirement of guaranteeing that all robots are within a proximity radius for communication. This requires the installation of n $2DOF(x, y)$ robots such that they are all in communication proximity. The IDC system S, however, requires no initial calibrations, and therefore no installation requirements. Thus, S and M do not require equivalent installation calibrations.

Following these definitions, we propose:

Proposition 1: The state $\mathbf{q}$ of neighbouring robots is not always deducible from the coverage levels of the map. See [27] for proof.

Proposition 2:

$$H_g + k \cdot comm(new\_point) \cong H_g + \sum_{i \in N} k \cdot comm(\mathbf{q_i})$$

where H_g is a swarm gradient dynamic coverage system, N is the set of all robots whose state is necessary by the rule for the new point calculation, and k is the number of total robots in the system -1, because the leader communicates to all other robots in the system.

This proposition suggests that a swarm gradient dynamic coverage system with the leader communication of a new target point simulates a system with the communication of the states of all robots involved in the rule for new point selection. The minimum information communication required by the leader to achieve the same result are the states of all robots involved in the calculation of the new swarming location. With this proposition we are attempting to quantify exactly how much information is inherent in the GDC algorithm, and show that this is more than that present in IDC. See [27] for proof.

Theorem 1: GDC where the coverage levels of the map are bounded above is $|N| \cdot k$-wire reducible to IDC using the same map model. In other words,

$$M \leqslant S + \sum_{i \in N} k \cdot comm(\mathbf{q_i})$$

where M is the multi-robot dynamic coverage system, S is the IDC system, k is the total number of agents -1, and N is the number of agents involved in the new point selection rule. See [27] for proof.

This result, together with our experimental findings demonstrating that individual dynamic coverage covered less distance and converged faster in every map scenario, shows that our swarm system S with less information complexity than our multi-robot dynamic coverage system M is able to constantly outperform the latter in these two metrics. A conclusion can be drawn that adding information complexity in the form of communication to a multiple robot dynamic coverage system does not always increase its performance with regards to convergence time and distance travelled.

C. Navigation

Experimental results demonstrated that DMA-RRT converges faster and travels less distance in cluttered environments, meanwhile PBC performs best overall on empty maps, and PF yields the smallest number of collisions on scenarios in which it converges. We will attempt to establish a reduction between these three algorithms in order to offer a better understanding of the experimental findings.

The circuit models for the three navigation algorithms *PF*, *Barrier Certificates*, and *DMA-RRT* will be denoted as P, B, and DMA for short. Since all three navigation algorithms are guaranteed to achieve the same limit sets, they *simulate* each other as computational circuits: $B \cong P \cong DMA$, where $\cong$ (simulation) is elementary transitive [15].

Despite the above result, we show in [27] that both swarm navigation algorithms are not provably reducible to the multirobot navigation algorithms and vice versa, suggesting that the experimental results relating the performances of the navigation algorithms, differently from those of the dynamic coverage algorithms, do not relate simply through an information theoretic reduction. It appears that the information complexity of DMA-RRT is fundamentally different from that of PBC and PF, offering possible explanations for the advantages of each algorithm in different scenarios.

V. Conclusion

Our research explored the pros and cons of swarm and general multi-robot algorithms in a manner that can inform offline design decisions made by engineers or online operational decisions made by supervisory operators of multi-robot systems. We first conducted an evaluation on the ROS platform using five algorithms in two application domains: navigation and dynamic area coverage. We then present several insights based on our collision, coverage, distance, and convergence metrics. Lastly, we extend our results to a comparative analysis of the algorithms based on the theory of information invariants, which provided a theoretical characterization supported by our empirical results.

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