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On-line Aerodynamic Model Identification on Small Fixed-Wing UAVs with Uncertain Flight Data

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Abstract— **This paper focuses on real-time estimation of the aerodynamic model parameters of small-scale fixed wing Unmanned Aerial Vehicles (UAVs) without the aid of wind-tunnel experiments, using exclusively flight data. The key tool of the following analysis centers around the principles of Total Least Squares estimation. Contrary to Ordinary Least Squares, this method accounts for errors in both explanatory data and variables to-be-explained. This is a highly desirable property for UAVs equipped with low-cost sensor systems. The proposed implementation combines both batch and real-time schemes, while deals efficiently with the problem of Insufficient System Excitation. On-line adaptation to model changes is performed by applying a Variable Forgetting Factor to the estimation data. Finally, a Monte Carlo approach is developed for uncertainty estimation regarding compound aerodynamic variables.**

I. INTRODUCTION

System Identification in the discipline of aerodynamics incorporates various Parameter Identification (PId) techniques to model the exerted forces and moments on an airframe. Core to this process is the determination of the stability and control derivatives [1], which govern the relationship between flight dynamics and fundamental aircraft variables.

The escalating complexity associated with Unmanned Aerial Vehicles (UAVs) renders the identification of the aerodynamic model pivotal to various cutting edge control systems (adaptive control, fault tolerant control, accurate flight simulators etc). For this reason, a considerable amount of research has been conducted in the field. Providing feasible solutions to such prerequisites is a non-trivial goal, especially in small-scale UAVs. Payload, cost and computational constraints are dominant factors in the design of such PId algorithms. Moreover, costly and time consuming wind tunnel tests cannot be considered a viable option for many research groups with limited resources [2]. Contrariwise, the characterization of the aerodynamic model by means of flight data can overcome such deficiencies while providing the respective confidence intervals for the parameter estimates.

Several attempts to identify the dynamics of UAVs have been made in the past years. Many former studies centered their analyses on Maximum Likelihood Estimation, using either output error [3],[4] or hybrid Neural Network (NN) approaches [5]. Nevertheless, output error techniques hinge on numerical methods, leading to convergence issues [1], while they do not account for process noise [3]. It is also well established that such methods rely on batch-processing and are not suitable for real-time implementations[6], without providing quantitative measures of the accuracy of the estimates.On the other hand, training of a NN can be time and resource consuming, while processes with equivocal internal structure are often undesirable. Other typical solutions propose the use of the Ordinary Least Squares (OLS) estimator, for both batch and real-time applications [1],[7], unfortunately without allowing for uncertainties in the explanatory variables, leading to biased estimates. This deficiency can be overcome by taking a unified approach to the state estimation/parameter identification problems and developing a non-linear Kalman Filter architecture (EKF, UKF) [8]. Arguably, this approach requires a significant amount of fine tunning, while dealing with one high dimensional estimation problem, instead of decomposing it in lower dimension sub-problems. Finally, filter convergence can be at stake without reliable a priori estimates of the parameters [1].

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This paper presents a fully developed, computationally efficient PId procedure based on the Total Least Squares (TLS) method [9],[10]. It is available both in batch and a novel sequential implementation, endowed with parameter tracking capabilities with the use of a Variable Forgetting Factor, while lack of Persistent Excitation of the input signals is handled by Principal Component Analysis. Finally, we devised a Monte Carlo method for accurate uncertainty quantification in synthetic aerodynamic variables, a crucial step for securing the robustness and reliability of the process.

In Section II of this work the non-linear model of the UAV is presented. In Section III the Total Least Squares estimator is introduced, while in Sections IV and V the Variable Forgetting Factor and uncertainty quantification approaches are presented, respectively. Simulation and experimental results can be found in Section VI.

II. AIRCRAFT AERODYNAMIC MODEL

A fixed wing UAV is modeled as a 6-DoF rigid body. In airborne aircraft, the applied forces and moments arise from aerodynamics, gravity and propulsion. Gravitational forces can be analytically calculated and propulsion forces and torques can be modeled through engine ground tests [1].

Hence, the system identification problem reduces to the determination of the aerodynamic forces ($\mathbf{F_A}$) and moments ($\mathbf{M_A}$). These forces are commonly expressed in terms of the non-dimensional aerodynamic coefficients:

$$\mathbf{F_A} = \bar{q}S\begin{bmatrix}-C_D & C_Y & -C_L\end{bmatrix}^{\mathsf{T}} \quad (1)$$

$$\mathbf{M_A} = \bar{q}S\begin{bmatrix}bC_l & \bar{c}C_m & bC_n\end{bmatrix}^{\mathsf{T}} \quad (2)$$

with S being the wing reference area, b the wingspan, $\bar{c}$ the mean aerodynamic wing chord and $\bar{q} = \frac{1}{2}\rho V_a^2$ the dynamic pressure, consisting of airspeed V_a and air density ρ. The non-dimensional aerodynamic force (C_L, C_D, C_Y) and moment (C_l, C_m, C_n) coefficients depend on fundamental aircraft variables [1]. Using a Taylor series approximation, a typical representation of the non-dimensional aerodynamic model is:

$$C_L = C_{L0} + C_{L\alpha}\alpha + C_{Lq_n}q_n + C_{L\delta\epsilon}\delta\epsilon \tag{3a}$$
$$C_D = C_{D0} + C_{D\alpha}\alpha + C_{D\alpha^2}\alpha^2 + C_{Dq_n}q_n + C_{D\delta\epsilon}\delta\epsilon \tag{3b}$$
$$C_Y = C_{Y\beta}\beta + C_{Yp_n}p_n + C_{Yr_n}r_n + C_{Y\delta r}\delta r \tag{3c}$$
$$C_l = C_{l\beta}\beta + C_{lp_n}p_n + C_{lr_n}r_n + C_{l\delta r}\delta r \tag{3d}$$
$$C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq_n}q_n + C_{m\delta\epsilon}\delta\epsilon \tag{3e}$$
$$C_n = C_{n\beta}\beta + C_{np_n}p_n + C_{nr_n}r_n + C_{n\delta r}\delta r \tag{3f}$$

where α is the angle-of-attack, β is the sideslip angle, $\{\delta_e, \delta_a, \delta_r\}$ are the control inputs (elevator, aileron and rudder deflections) and the terms p_n, q_n, r_n correspond to the non-dimensional angular rates.

III. Parameter Identification

A model structure characterized by equations (3a)-(3f) is a *linear-in-the-parameters* estimation problem. If the aircraft state is known, aerodynamic PId centers around the determination of the set of parameters $C_{\cdot,\cdot}$ on the right side of (3) which encapsulates the functional relationship between independent variables and aerodynamic derivatives.

In generic terms, the model of an aerodynamic coefficient y is given by:

$$y = X\theta \tag{4}$$

where $X \in \mathbb{R}^{N \times n_\theta}$ is the matrix of inputs -or explanatory variables-, $\theta \in \mathbb{R}^{n_\theta \times 1}$ is the vector of unknown parameters and y represents the model output.

The inputs and outputs of the system are measured or estimated up to a level of uncertainty such that:

$$X_m = X + \Delta X \tag{5}$$
$$z = y + \varepsilon \tag{6}$$

where ΔX is the perturbation matrix of the explanatory variables, ε is the measurement noise on the observed output, X_m is the matrix of inputs and z is the observation vector.

As long as uncertainty is present in X, aerodynamic estimation must be carried out by means of Errors-In-Variables models. Simpler parameter estimation techniques such as OLS fail to deliver unbiased estimates of the parameter vector and cannot be considered as a consistent solution [11].

A. Total Least Squares

The method of TLS, introduced by [12], derived a robust solution to overdetermined linear systems $AX = B$ where both A, B are perturbed by measurement errors. TLS postulates a unified approach to the problem by concatenating X_m and z into the augmented data matrix

$$D = \begin{bmatrix} X_m & z \end{bmatrix} = \begin{bmatrix} X & y \end{bmatrix} + \begin{bmatrix} \Delta X & \varepsilon \end{bmatrix} = D_0 + \begin{bmatrix} \Delta X & \varepsilon \end{bmatrix} \tag{7}$$

consisted of the true values (D_0) and the errors ($[\Delta X \;\; \varepsilon]$).

Additionally, an optimal TLS estimator in terms of *Maximum Likelihood (ML)*, requires the elements of $[\Delta X \;\; \varepsilon]$ to be independent, zero mean, normally distributed random variables of unitary variance [13]. Nevertheless, estimates of the aircraft state are usually constructed from multiple data sources with different precision. Consequently, measurement errors are unequally sized (heteroscedastic) and/or correlated. Thus, even if error statistics can be approximated by zero-mean normal random variables, the unitary variance assumption cannot be sustained.

For the purpose of this study, the augmented error matrix will be assigned a simple yet efficient model:

$$[\Delta X \;\; \varepsilon] = EC \tag{8}$$

E is a matrix of uncorrelated standard normal deviates and C is the Cholesky factor of the error covariance matrix W_T:

$$W_T = C^T C = E\left([\Delta X \;\; \varepsilon]^T [\Delta X \;\; \varepsilon] \right) \tag{9}$$

Generalized Total Least Squares (GTLS) is an extension of the TLS approach that allows for heteroscedasticity and/or correlation in the errors [14]. This includes the rescaling of data matrices, by inversing matrix C, so as to meet the ML prerequisites:

$$D^* = DC^{-1} = D_0^* + [\Delta X \;\; \varepsilon]^* \tag{10}$$

In the presence of measurement errors, D is of full rank ($\mathbf{rank}(D) = n_\theta + 1$). GTLS is defined as the following rank reduction problem which seeks for the closest rank-deficient approximation of D:

$$\min_{\hat{\theta}, \Delta\hat{X}, \hat{\varepsilon}} \left\| [\Delta X \;\; \varepsilon]^* \right\|_F$$
$$\text{subject to} : \left(D^* - \left[\Delta\hat{X} \;\; \hat{\varepsilon} \right]^* \right) C \left[\theta - 1 \right]^{\mathsf{T}} = \mathbf{0} \tag{11}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. If C is positive-definite, GTLS is proven to be a consistent estimator [15].

Singular Value Decomposition (SVD) [9] can be used to solve the above optimization. First, the scaled data matrix of eq. (10) is factorized as follows:

$$D^* = USV^T = \sum_{j=1}^{n_\theta+1} u_j \sigma_j v_j^T \tag{12}$$

where $U \in \mathbb{R}^{N \times N}$,$V \in \mathbb{R}^{n_\theta+1 \times n_\theta+1}$ are orthonormal and unitary matrices whose columns are denoted as u_j, v_j respectively. The non-negative diagonal matrix $S \in \mathbb{R}^{N \times n_\theta+1}$ contains the set of singular values of D^* in decreasing order:

$$S = \mathbf{diag}\left(\sigma_1, \ldots, \sigma_{n_\theta}, \sigma_{n_\theta+1} \right) \tag{13}$$

Under sufficient excitation conditions [16], D^* can be reduced to its closest rank-deficient approximation. By means of SVD, this reduction is accomplished by [12]:

$$\hat{D}_0^* = \sum_{j=1}^{n_\theta} u_j \sigma_j v_j^T \tag{14}$$
$$\left[\Delta\hat{X} \;\; \hat{\varepsilon} \right]^* = \sigma_{n_\theta+1} u_{n_\theta+1} v_{n_\theta+1}^T \tag{15}$$

where $\mathbf{rank}(\hat{D}_0^*) = n_\theta$. Subsequently, the estimated parameter vector is found as:

$$\left[\hat{\theta} \quad -1 \right]^{\mathsf{T}} = hC^{-1} v_{n_\theta+1} \tag{16}$$

where $v_{n_\theta+1}$ is the rightmost column vector of V and h is a scalar multiplier so that the last element of the right hand side of eq. (16) is -1.

The covariance of the estimated parameters can be approximated by [10]:

$$\mathbf{cov}(\hat{\theta}) \approx \frac{\sigma_{n_\theta+1}^2}{N}\left(1+\|\hat{\theta}\|\right)^2\left(X_m^T X_m - \sigma_{n_\theta+1}^2 I\right)^{-1} \quad (17)$$

B. The problem of Insufficient System Excitation

The performance of an online parameter estimator, relies heavily on the Persistent Excitation of input signals [11]. However, this prerequisite cannot be guaranteed under all flight conditions. In many instances, inputs do not exhibit sufficient temporal variation or there exists a near linear dependence between two or more explanatory variables.

In such cases of Insufficient System Excitation (ISE), estimators may assign biased parameters, exhibit arbitrary transients or even crash due to numerical problems [1],[6]. Problems with ISE are reflected in the near equal value of the smallest singular values of D^* where:

$$\sigma_1 > \sigma_2 > \cdots > \sigma_{k+1} \simeq \sigma_{k+2} \simeq \cdots \simeq \sigma_{n_\theta+1} \quad (18)$$

for some $k < n_\theta$. Note also that, due to its random nature, the error matrix contributes its pseudo-information evenly to the entire set of singular values by [6]:

$$\forall_j \left(\sigma_j^2\right)_{errors} = N - n_\theta \quad (19)$$

Principal Component Analysis (PCA) can be used to confine the estimation process in the space of sufficiently excited data which can be found along the first k spectral directions of D^*. This is equivalent to further rank-reduction of D^* and its approximation by:

$$\hat{D}_{SE}^* = \sum_{j=1}^{k} u_j \sigma_j v_j^T \quad (20)$$

The rank discrimination parameter k is equal to the cardinality of the set ($\mathbf{card}\{\}$):

$$S_{SE} = \{\sigma_j : \sigma_j \geq ({}^s\!/\!{}_n + 1)\sqrt{N - n_\theta},\ \ j \leq n_\theta\} \quad (21)$$

where ${}^s\!/\!{}_n$ denotes the minimal desired signal-to-noise ratio between σ_j and the expected levels of uncertainty defined by eq. (19). Under ISE, TLS has no unique solution as any arbitrary combination of the last $n_\theta + 1 - k$ right singular vectors v_j can minimize equally well the criterion of (11):

$$\begin{bmatrix} \hat{\theta}_{PCA} \\ -1 \end{bmatrix} = h_{k+1} C^{-1} v_{k+1} + \cdots + h_{n_\theta+1} C^{-1} v_{n_\theta+1} \quad (22)$$

On these terms, uniqueness of solution is inferred by applying a minimum Weighted Norm criterion:

$$\hat{\theta} = \min \|W\theta_{PCA}\|_2^2 \quad (23)$$

which can be easily computed by a Weighted Least Squares estimator [6].

C. On-line Implementation

A feasible real-time implementation of the method hinges on efficient recursive SVD updating (RSVD). In all other respects, a sequential approach to PCA-GTLS is straightforward, as summarized in Algorithm 1. Considering that the costly matrix U is not involved in the estimation process and V is of fixed dimensions, RSVD reduces to partial updates of V, S. One of the various RSVD approaches can be found in [17]. The function and use of the forgetting factor λ_F will be explained in the following section.

Algorithm 1 Recursive PCA-GTLS

```
Initialize C, W, s/n, λ_F, N_c ← 0
input: x_m(t),  z(t)
loop
    d* ← [x_m(t)  z(t)] C^{-1}
    [S_t, V_t] ← R_SVD (d*, S_{t-1}, V_{t-1}, λ_F)
    if N_c > n_θ then
        k ← card{σ_i(t) > (1 + s/n) sqrt(N_c − n_θ)}
        if k < n_θ then
            θ̂_t ← min‖W θ_PCA‖_2^2
        else
            θ̂_t ← h C^{-1} v_{n_θ+1}(t)
    N ← N + 1
    estimate cov θ̂ using eq. (17)
```

IV. Adaptation to Parameter Changes

Parameter tracking is integral to most real-time PId schemes. Fault aware implementations, adaptive controllers as well as systems with changing dynamics, all require timely detection of model changes. To this end, the performance of on-line estimators is governed by a forgetting factor ($0 \ll \lambda_F < 1$) whose purpose is to constrain the identification process in a weighted data window. Commonly, λ_F creates an exponential window, moving towards the most recent observations. However, this implies the fading of preceding information, which may lead to estimate divergence in cases of non-persistent excitation. On the other hand, if an abrupt change to the system occurs, convergence to the new parameter vector is slow.

Such deficiencies can be minimized by an adaptive approach to exponential forgetting, where fading is imposed only when a model change is detected. The development of a time-varying λ_F pivots on the evaluation of the *a priori* and *a posteriori* residual vectors $\nu(t), r(t)$ in conjunction with the expected uncertainty bounds of the measured quantity:

$$\nu(t) = z(t) - \hat{x}_0(t)\hat{\theta}(t-1) \quad (24)$$

$$r(t) = z(t) - \hat{x}_0(t)\hat{\theta}(t) \quad (25)$$

where $\hat{x}_0(t)$ is the TLS-corrected input vector at the time instant t. Under the multivariate normal assumption for $[\Delta X\ \ \varepsilon]^*$, $\hat{x}_0(t)$ is the unique ML estimate of the noiseless input $x_0(t)$ [10, p. 233],[18].

Ideally, the a posteriori residual vector of a successfully estimated model must recover the corrupting signal of the measurement. This objective can be met by imposing:

$$E\left\{r^2(t)\right\} = \sigma_{\varepsilon_0}^2 \quad (26)$$

where $\sigma_{\varepsilon_0}^2 = E\{\varepsilon^2(t)\}$ is the measurement noise variance and its selection will be discussed further in Section V.

Based on the findings of [19] and the ML assumption for $\hat{x}_0(t)$, eq. (26) is satisfied by applying a Variable Forgetting Factor (VFF) given by:

$$\lambda_F(t) = \frac{\sigma_q(t)\sigma_\varepsilon(t)}{\sigma_\nu(t) - \sigma_\varepsilon(t)} \quad (27)$$

The terms $\sigma_q, \sigma_\nu, \sigma_\varepsilon$ derive from the signal power estimates:

$$\sigma_q^2(t)=\gamma_1\sigma_q^2(t-1)+(1-\gamma_1)q^2(t) \tag{28a}$$
$$\sigma_\nu^2(t)=\gamma_1\sigma_\nu^2(t-1)+(1-\gamma_1)\nu^2(t) \tag{28b}$$
$$\sigma_\varepsilon^2(t)=\gamma_2\sigma_\varepsilon^2(t-1)+(1-\gamma_2)\nu^2(t) \tag{28c}$$

$$q(t)=\hat{x}_0^T(t)\Big[\Pi(t-1)\Big]^{-1}\hat{x}_0(t) \tag{29}$$
$$\Pi(t)=\lambda_F(t-1)\Pi(t-1)+\hat{x}_0(t)^T\hat{x}_0(t) \tag{30}$$

γ_1, γ_2 are user-defined weighting factors such that:

$$0 \ll \gamma_2 < \gamma_1 < 1 \tag{31}$$

In theory, it should hold that $\sigma_\nu(t) \geq \sigma_\varepsilon(t)$. However, the use of power estimates in eqs. (28a)-(28c) may cause fluctuations of $\sigma_\nu(t)$ in the vicinity of $\sigma_\varepsilon(t)$. Therefore, the robustness of the solution can be enhanced by imposing $\lambda_F(t)=1$ when:

$$\sigma_\nu(t) \leq \gamma_0\sigma_\varepsilon(t) \tag{32}$$

where $1 < \gamma_0 \leq 2$. Otherwise, λ_F is evaluated as:

$$\lambda_F(t)=\min\left\{\frac{\sigma_q(t)\sigma_\varepsilon(t)}{\varsigma+|\sigma_\nu(t)-\sigma_\varepsilon(t)|},\ 1\right\} \tag{33}$$

where the small positive constant ς prevents division by zero.

V. Uncertainty Estimation

For the reasons explained in Section III-A, W_T must be known up to a level of proportionality [13]. Otherwise, artificial ill-conditioning of the optimization problem and introduction of large variations to the estimated parameter vector may occur [10, ch.3] Uncertainty in most explanatory variables can easily be derived from sensor specifications or state estimation processes. Nevertheless, this does not apply to compound (synthetic) variables such as the aerodynamic coefficients because these are obtained by combining multiple sources of data pertaining to various errors.

Ordinarily, uncertainty quantification involves a Taylor series approximation of the measurand. However, this imposes numerous simplifying assumptions on all variables, restricting information content from the measurand. Nonlinearities, time varying uncertainties and non-gaussian input distributions may yield overpredicted estimates of the error variance [20],[21]. The Unscented Transform is popular for UE, but not suitable for our application, since we seek an offline algorithm which allows for arbitrary input PDFs.

We propose a Monte-Carlo (MC) method for the estimation of uncertainty of compound variables, as they provide an alternative, less restrictive framework for Uncertainty Estimation (UE) which overcomes the above limitations. Consider a measurand (Z), formed by a non-linear function of random-variables (RVs) Ξ_i such that $Z=f(\Xi_1,\ldots,\Xi_n)$, where all Ξ_j derive from analytical or empirical probability density functions (PDFs) $g_{\Xi_j}(\xi_j)$.

MC is based on the evaluation of f by repeatedly sampling the inputs (Ξ_j) from their respective PDFs. After a sufficient number of runs (M), a numerical Cumulative Distribution Function (CDF) can be obtained such that:

$$\hat{G}_Z(z) \approxeq P\left[Z \leq z\right] \tag{34}$$

Uncertainty is then quantified by inversing the CDF and determining the bounds that correspond to a specified coverage interval p (commonly $p=95\%$).

Prior to further analysis, a series of assumptions must be taken into account. A) Systematic errors are known or estimated and can be eliminated from the measurements. B) All input uncertainties ($\Delta\Xi$) are known with reasonable precision. C) Reliable estimates of variance-covariance matrices of inputs (Σ_Ξ) and input errors ($\Sigma_{\Delta\Xi}$) are available.

If the above conditions are met, the error can be quantified at each iteration as:

$$\varepsilon(\Xi)=Z-\tilde{Z}=f(\mathbf{\Xi})-f(\mathbf{\Xi}+\mathbf{\Delta\Xi}) \tag{35}$$

where $\Xi, \Delta\Xi$ are drawn from their respective (joint or independent) PDFs.

Overall, we assume that input variates lie in known ranges and we chose to draw values from a joint uniform distribution. It should be noted that synthetic measurements of the aerodynamic coefficients are constructed from quantities which are often correlated. Correlated multivariate input processes can be constructed by means of Sklar's theorem and a Gaussian Copula [22]. In the case of a joint rectangular distribution, inputs are constructed as follows:

$$C_G(u_1,\ldots,u_n)=\Phi_R\left(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_n)\right) \tag{36}$$
$$\Rightarrow (u_1,\ldots,u_n)\sim\mathcal{U}(0,1) \tag{37}$$

where $C_G : [0,1]^n \to [0,1]$ is the gaussian copula, Φ_R denotes the joint CDF of a zero mean multivariate normal distribution with covariance matrix Σ_Ξ and Φ^{-1} is the inverse CDF of a standard normal RV.

The generated uniform variates are then shifted and scaled so as to match a specified nominal range $\left[r_{(-)}, r_{(+)}\right]$:

$$\xi_j=r_{(-)}+\left(r_{(+)}-r_{(-)}\right)u_j \tag{38}$$

Finally, based on the assumptions of the previous section, errors can be generated from a multivariate gaussian PDF:

$$\mathbf{\Delta\Xi}\sim\mathcal{N}(\mathbf{0},\Sigma_{\Delta\Xi}) \tag{39}$$

After an adequate number of iterations ($M > 10^6$), uncertainty is quantified by the estimated CDF ($\hat{G}_\varepsilon$). Using a coverage interval of 95%, upper and lower bounds $\left[U_{(-)}, U_{(+)}\right]$ can be specified as:

$$\left[U_{(-)}, U_{(+)}\right]=\left[\hat{G}_\varepsilon^{-1}(0.025),\ \ \hat{G}_\varepsilon^{-1}(0.975)\right] \tag{40}$$

For the purposes of this particular implementation, the last diagonal entry of the error variance-covariance matrix (W_T, eq. 9) which corresponds to the synthetic aerodynamic coefficients is formed as:

$$W_T(n_\theta+1,n_\theta+1)=\hat{\sigma}_\varepsilon^2:=\left[\frac{\max\left(|U_{(-)}|,|U_{(+)}|\right)}{1.96}\right]^2 \tag{41}$$

VI. Results

Our proposed algorithm was implemented in interpreted `MATLAB` and ran on a machine with an i7 4-core CPU and 16GB DDR3 RAM. For data taken at 100Hz, the processing time per sample was 1.25ms, meaning that there is ample calculation overhead for this algorithm to run in real-time.

A. Simulation Results

The proposed PId method has been tested against simulation data. Our own *last_letter* simulator (github.com/Georacer/last_letter) was used to provide a detailed non-linear model of a small-scale fixed-wing UAV, whose parameters are pre-defined and known. Additionally, realistic wind disturbance in the form of Dryden gusts and steady wind was imposed on the UAV. Virtual sensors were sampled at 100 Hz and corrupted with white noise.

Figure 1 presents a characteristic response of the tracked parameters for our proposed PCA-GTLS method, against a typical sequential OLS implementation. The progress of the singular values (σ_i) is displayed at the last subplot. The trajectories of the estimated stability derivatives for the Coefficient of Drag (C_{D0}, $C_{D\alpha}$, $C_{D\alpha^2}$) are shown to converge quickly after the application of the longitudinal excitation maneuvers at t=250 s. More importantly, we verify that once all of the singular values rise above the excitation threshold, the parameters stabilize within their final regions. This makes the evolution of the singular values a valuable descriptor of the estimation process status. Contrariwise, the respective OLS estimates fail to converge due to high levels of noise.

Variable and Constant forgetting schemes with TLS are compared in Figure 2. At time t=50 s the C_m parameters are abruptly changed, to simulate aircraft damage. The simulated value for the $C_{m\alpha}$ coefficient can be seen in Figure 2a as a red dashed line. The variable λ_F detects the model mismatch, reduces its value (Figure 2b) and the estimate converges almost immediately to the new parameter, within 10 s. Conversely, the constant λ_F does not adjust promptly, causing parameter transients, as incompatible information before and after the model change are combined. Moreover, constant forgetting imposes perpetual loss of past information and drives the estimator in the state of insufficient excitation (t=460 s). Past that moment, the parameter estimate will not converge until new excitation maneuvers are applied.

B. Experimental Results

Flight-tests were carried out on the NTU Athens, 2.7m wingspan UnATRaP UAV (Figure 3).

IMU and GPS data were collected using COTS solutions, while servo commands, engine RPM and airdata quantities (airspeed, static pressure, angle-of-attack, angle-of-sideslip and air temperature) were captured with tailor-made sensors and systems. All data were recorded on-board in an embedded computer, using the ROS framework to interface with the various sensory peripherals.

The UAV was flown manually, to capture the full spectrum of its dynamics, through a series of doublets and sine sweeps for each control input. Flight data segments corresponding to longitudinal and lateral excitation were used to assess the convergence rate and the model fit to the measurements. Figures 4 and 5 show that PCA-GTLS converges quickly to the final solution and exhibits an almost excellent fit, in terms of the coefficient of determination (R^2).

To validate the parameter estimates, we reconstructed synthetically the wind angles (α, β) of a different flight segment, using the estimated models. These are plotted against the respective states from our sensor fusion solution which served as ground truth (Figures 4 and 5). The prediction of

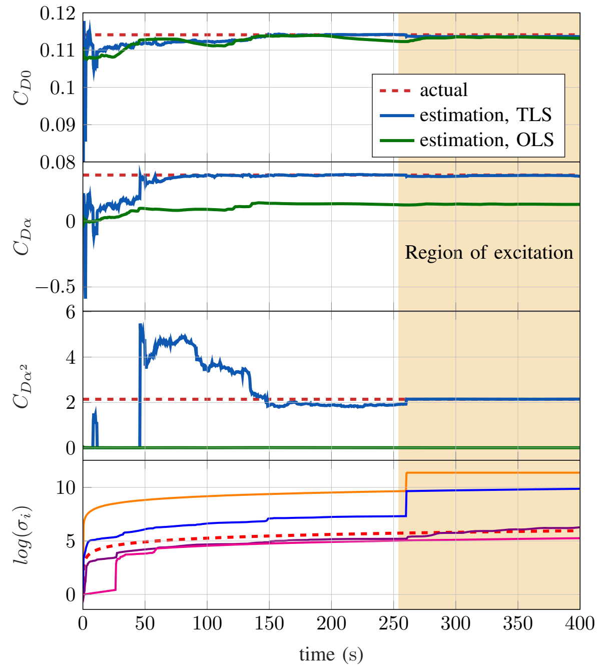
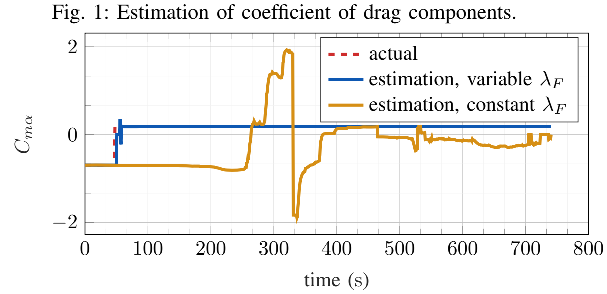
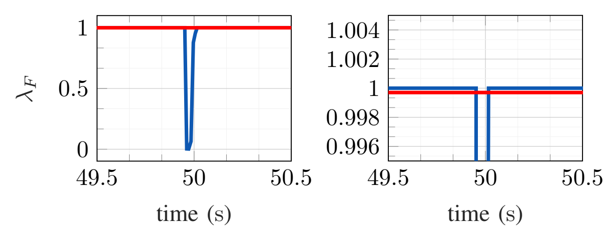


Fig. 1: Estimation of coefficient of drag components.



(a) Parameter trajectories.



(b) Forgetting factor trajectory. Detail on the right.

Fig. 2: Parameter estimation with constant vs. variable forgetting factor.

the model is in very good agreement with the reference and inside the error bounds (in gray). The mean error for α, β was calculated to $0.55, 0.77$ degrees respectively.

VII. Conclusions

In this work, the issue of aerodynamic parameters identification for small-scale fixed-wing UAVs, using uncertain flight data was discussed. Since the errors in the regressor measurements are not negligible, we advocated for the use of the Total Least Squares (TLS) method to obtain optimal estimates of the parameters. A novel sequential algorithm suitable for real-time parameter tracking was provided, which is endowed with a Variable Forgetting Factor and safeguarded against Insufficient System Excitation by employing Principal Component Analysis. Since accurate estimation of measurement uncertainty is required by the TLS method, a

Fig. 3: The UnATRaP experimental platform.

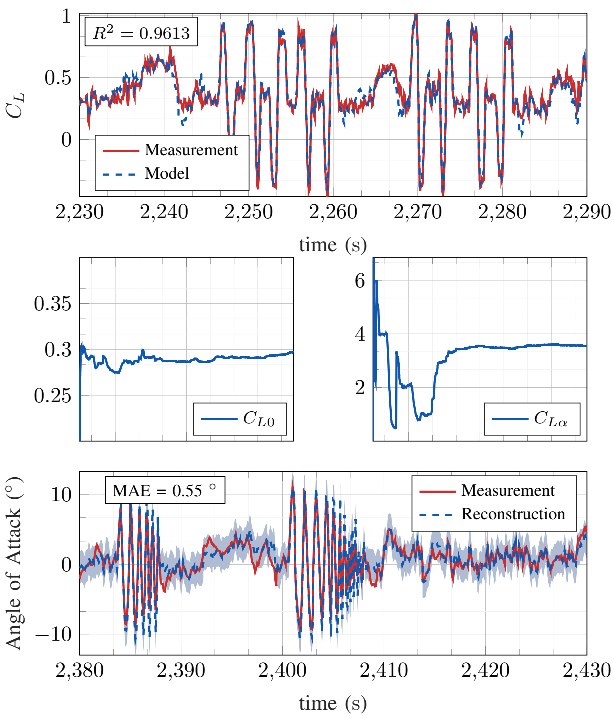


Fig. 4: Coefficient of lift flight data. Top: Model fit. Middle: Sequential parameter estimation. Bottom: Validation through reconstruction of α.

Monte-Carlo approach was employed for the calculation of the uncertainty of compound measurements, sampled from uniform random variable inputs. Our algorithm was tested against both simulated and real flight data with success.

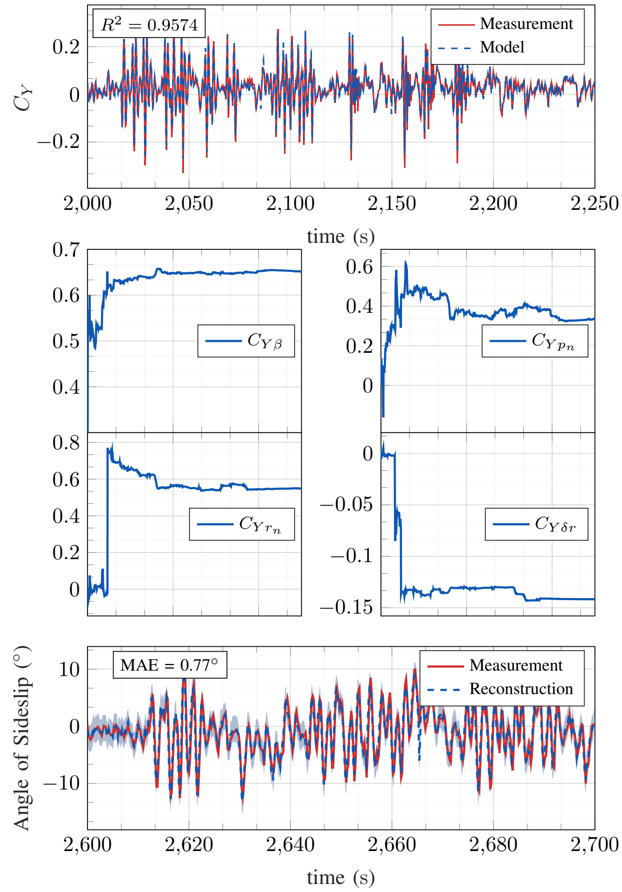


Fig. 5: Side force coefficient flight data. Top: Model fit. Middle: Sequential parameter estimation. Bottom: Validation through reconstruction of β.

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