Assignment 1

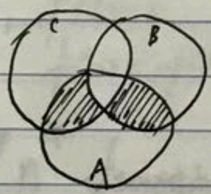
1. (a) $\gcd(288, 120) = \gcd(168, 120)$
$= \gcd(120, 48) = \gcd(72, 48)$
$= \gcd(48, 24) = 24$

(b) $\mathrm{lcm}(-91, 52)$ $\quad 13 \,\underline{\big|\, -91, 52}$
$\qquad\qquad\qquad\qquad\quad -7 \quad 4$
$\mathrm{lcm}(-91, 52) = |13 \times (-7) \times 4| = 364.$

(c) $\gcd(n, n+1)$ for $n \in N$ is always 1.

2. (a) $\mathrm{card}(Pow(Pow(\phi))) = \mathrm{card}(\{\phi, \{\phi\}\}) = 2$.

(b)



This venn diagram shows that $A \cap (B \oplus C)$ and $(A \cap B) \oplus (A \cap C)$ are the same area of this diagram. It is the shadow.

(c) For example, assume set $A = \{1.2.3.4.5.6\}$
$B = \{4\ 5\ 6\ 7\ 8\ 9\}$
$C = \{3\ 5\ 7\ 10\ 11\ 12\}$

for $A \oplus (B \cap C)$:
$B \oplus C = \{3.4.6.8.9.10.11.12\}$
$A \cap (B \oplus C) = \{3.4.6\}$

for $(A \oplus B) \cap (A \oplus C)$:
$A \oplus B = \{1.2.3.7.8.9\}$
$A \oplus C = \{1.2.4.6.7.10.11.12\}$
$(A \oplus B) \cap (A \oplus C) = \{1.2.7\}$

So that $A \cap (B \oplus C) = \{3.4.6\} \neq \{1.2.7\} = (A \oplus B) \cap (A \oplus C)$

3 (a) Lexcographic order:

ϕ, a, aa, aaa, aab, ab, aba, abb, b, ba, baa, bab, bb, bba, bbb

(b) Lenlex order: ϕ, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb

4. (a) 1. $f(x)=0$ $f(a)=f(b)=f(c)=0$
2. $f(a)=0$, $f(b)=0$, $f(c)=1$
3. $f(a)=0$, $f(b)=1$, $f(c)=0$
4. $f(a)=1$, $f(b)=0$, $f(c)=0$.
5. $f(a)=0$, $f(b)=1$, $f(c)=1$
6. $f(a)=1$, $f(b)=0$, $f(c)=1$
7. $f(a)=1$, $f(b)=1$, $f(c)=0$.
8. $f(x)=1$ $f(a)=f(b)=f(c)=1$.

(b) (i) n^m functions from A to B
(ii) 2^{mn} relations between A and B

(c) The number of functions in (a) is equal to card(Pow({a, b, c})).

5. (a) (i) (abab, baba)
and (v) (λ, bbb)

(b) R is an equivalence relation, iff it satisfies Reflexive, Symmetric, Transitive.

Proof: $\boxed{\text{Reflexive}}$: Assume (w, w) is a set, and wv$\in$L. Because Length(wv) must equal to length(wv) so for every w in set(w, w) (w, w)$\in$R. So it is Reflexive.

$\boxed{\text{Symmetric}}$: Assume (w, w')$\in$R then must exist v that wv$\in$L and w'v$\in$L, so that (w', w)$\in$R $\therefore$ It is Symmetric.

$\boxed{\text{Transitive}}$: Assume (w, w')$\in$R and (w', w'')$\in$R, so that exist v that wv$\in$L w'v$\in$L w''v$\in$L. As a result (w, w'') must $\in$R so it is Transitive.

(C) There are three equivalence classes for R.