Question1—a and b

Question 1.

(a) Utalization of CPU: $U(c) = \frac{4729s}{90mins}$

$= \frac{4729s}{5400s} = 0.8757$

Utalization of Disk: $U(d)$

$= \frac{2565s}{5400s} = 0.475$

Throughput of system: $X(0) = \frac{C(c)}{T}$

$= \frac{676}{5400s} = 0.125$

Service demand of CPU: $D(c) = \frac{U(c)}{X(0)}$

$= \frac{0.8757}{0.125} = 7$

Service demend of Disk: $D(d) = \frac{U(d)}{X(0)} = \frac{0.475}{0.125}$

$= 3.8$

(b) I think it is impossible to determine the bottleneck of the system without calculating the service demends. Because the service demands determine the highest demands of whole system. Then requests are more likely stuck in the highest demands device. Then it could be the bottle neck.

Question1-c and d

(c) $X(0) \le \min\left[\frac{1}{\max D_j}, \frac{N}{\sum_{i=1}^{K} D_j}\right]$

$\max D_j = D(c) = 7 \quad \frac{1}{\max D} = \frac{1}{7} = 0.143$

$\frac{N}{\sum_{i=1}^{K} D_j} = \frac{2}{7+3.8} = \frac{2}{10.8} = \frac{1}{5.4} = 0.185$

As a result $X(0) \le \min[0.143 \quad 0.185]$

The bound $= X(0) \le 0.143$

(d) $M = X_0 \times (Z+R)$

$= X_0 \times (31+R)$

$= 0.143 \times (31+10) \times 30$

$\approx 172 \quad \Sigma$

Question 2 a and b

Question2:

(a) $U = \frac{B}{T}$

Because $\mu_1 = 10$ So, time to process a request in system 1 is $\frac{1}{\mu_1} = \frac{1}{10}$, The same for system 2: $\frac{1}{\mu_2} = \frac{1}{15}$

Assume total amount of request that system1 process is N_1, for system 2 is N_2

Then $B_1 = N_1 \times \frac{1}{10}$ $B_2 = N_2 \times \frac{1}{15}$

Because $U = \frac{B}{T}$ T remain the same for system 1.2.

If make $U_1 = U_2$ then $B_1 = B_2$. So $\frac{N_1}{10} = \frac{N_2}{15}$

$N_1 : N_2 = 2:3$

So the $P : 1-P$ should also be $2:3$

As a result $P = 0.4$.

(b) Mean respose time t for M/M/1

$\lambda_1' = 0.4 \times 20 = 8$

for system1. $T = \frac{1}{\mu(1-\rho)} = \frac{1}{10-8} = 0.5 = \frac{1}{2}$

for system2. $T = \frac{1}{\mu(1-\rho)} = \frac{1}{15-12} = \frac{1}{3}$

$T_m = \frac{1}{2} \times 0.4 + \frac{1}{3} \times 0.6 = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} = 0.4$

Question 2 c

(c) For system 1 $\lambda_1 = P \times 20$ $\quad R_1 = \frac{1}{10-20P}$

$\mu_1 = 10$

system 2 $\lambda_2 = (1-P)\times 20$ $\quad R_2 = \frac{1}{15-(1-P)\times 20}$

$\mu_2 = 15$

$$T = \frac{P}{10-20P} + \frac{1-P}{15-(1-P)\times 20}$$

Now we want to minimize $T = \frac{P}{10-20P} + \frac{1-P}{15-20+20P}$

$$= \frac{P}{10-20P} + \frac{1-P}{20P-5}$$

$$= \frac{1}{10}\,\frac{4P^2-2P+1}{-8P^2+6P-1} \text{ where } 0 \le P \le 1$$

As a result P should be 0.3876

then the equation is minimized.

Question 3 a and b and c

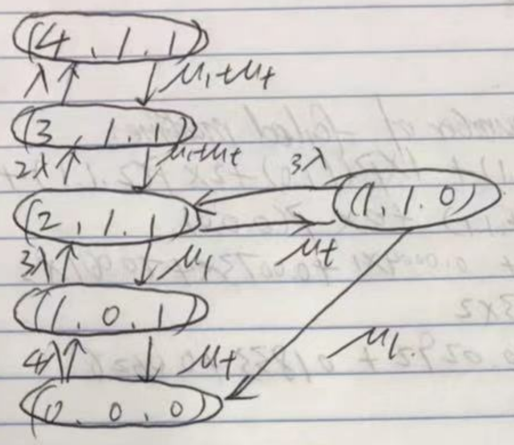
Question 3

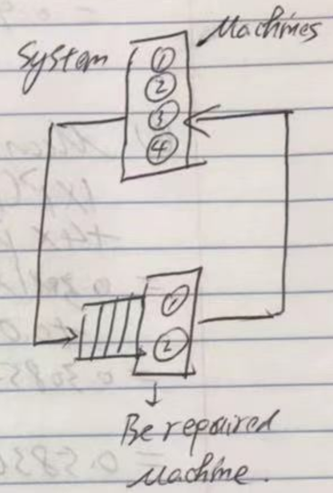
(a) $\lambda = \frac{1}{600}$ $\mu_1 = \frac{1}{60}$ $\mu_t = \frac{1}{90}$

μ_1 = Exponential Service time for leader

μ_t = Exponential Service time for trainee

State transition diagram for the Markov Chain





First of all list all the states which are (0.0.0), (1.0.1), (1.1.0), (2.1.1) (3.1.1), (4.1.1)
Then using → symbol to indicate the transition between states and add the λ, μ_1, μ_t (transition rates)

(b) 1. $4\lambda P(0.0.0) - \mu_t P(1.0.1) - \mu_1 P(1.1.0) = 0$

2. $(3\lambda+\mu_t) P(1.0.1) - 4\lambda P(0.0.0) - \mu_1 P(2.1.1) = 0$

3. $(2\lambda+\mu_1+\mu_t) P(2.1.1) - 3\lambda P(1.0.1) - (\mu_1+\mu_t) P(3.1.1) - 3\lambda P(1.1.0) = 0$

4. $(\lambda+\mu_1+\mu_t) P(3.1.1) - (\mu_1+\mu_t) P(4.1.1) - 2\lambda P(2.1.1) = 0$

5. $(\mu_1+\mu_t) P(4.1.1) - \lambda P(3.1.1) = 0$

6. $(\mu_1+3\lambda) P(1.1.0) - \mu_t P(2.1.1) = 0$

7. $P(0.0.0) + P(1.0.1) + P(2.1.1) + P(1.1.0) + P(3.1.1) + P(4.1.1) = 1$

(c) $P(0.0.0) = 0.5918$ $P(4.1.1) = 0.0073$
$P(1.0.1) = 0.3081$ $P(1.1.0) = 0.0004$
$P(2.1.1) = 0.0313$
$P(3.1.1) = 0.0611$

Question 3 d , e and f

d) P(at least three available)

$= P(0.0.0) + P(1.0.1) + P(1.1.0)$

$= 0.004 + 0.5918 + 0.3081$

$= 0.9003$

e) Mean number of failed machine:

$1 \times P(1.0.1) + 1 \times P(1.1.0) + 2 \times P(2.1.1) + 3 \times P(3.1.1)$

$+ 4 \times P(4.1.1) + 0 \times P(0.0.0)$

$= 0.3081 \times 1 + 0.0004 \times 1 + 0.0073 \times 4 + 0.0611 \times 3$

$+ 0.0313 \times 2$

$= 0.3085 + 0.0292 + 0.1833 + 0.0626$

$= 0.5836.$

f) Mean time to repair (MTTR)

= Queueing time + Actual repair time

$= 0.3081 \times 90 + 0.0313 \times (60+90) + 0.061 \times (90 \times 2 + 60)$

$+ 0.0073 \times (60+90) \times 2 + 0.0004 \times 60$

= 82.2 mins