480 PSet 1: Linear Algebra Self-Assessment

Question 1:

(a) Given points $\mathbf{p}_1 = (1, 6, 5)$ and $\mathbf{p}_2 = (5, 3, -7)$, solve for $\mathbf{v}_2$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_2$.

(b) Given a third point $\mathbf{p}_3 = (1, 6, 4)$, solve for $\mathbf{v}_3$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_3$.

(c) Find the values for the magnitudes of $\mathbf{v}_2$ and $\mathbf{v}_3$.

(d) Solve for the unit vectors in the directions of $\mathbf{v}_2$ and $\mathbf{v}_3$.

(a) My answer is :

The vector v_2 from p_1 to p_2 is p_2 - p_1 = (5-1, 3-6, -7-5) = (4, -3, -12)

(b) My answer is :

The vector v_3 from p_1 to p_3 is p_3 - p_1 = (1-1, 6-6, 4-5) = (0, 0, -1)

(c) My answer is :

Magnitudes of v_2 is $\sqrt[2]{4^2 + (-3)^2 + (-12)^2} = 13$

Magnitudes of v_3 is $\sqrt[2]{0^2 + 0^2 + (-1)^2} = 1$

(d) My answer is :

Unit vector of v_2 is v_2 / $|v_2|$ = (4/13, -3/13, -12/13)

Unit vector of v_3 is v_3 / $|v_3|$ = (0, 0, -1)

Question 2:

(a) Solve for the vector (cross) product $\mathbf{v}_2 \times \mathbf{v}_3$.

(b) Solve for $\mathbf{v}_3 \times \mathbf{v}_2$.

(c) Solve for the scalar (dot) product $\mathbf{v}_3 \cdot \mathbf{v}_2$.

(a) My answer is :

$v_2 \times v_3$

$= ((-3) \times (-1) - (-12) \times 0,\ (-12) \times 0 - 4 \times (-1),\ 4 \times 0 - (-3) \times 0)$

$= (3,\ 4,\ 0)$

(b) My answer is :

$v_3 \times v_2$

$= (0 \times (-12) - (-1)\ \times (-3),\ (-1) \times 4 - 0 \times (-12),\ 0 \times (-3) - 0 \times 4)$

$= (-3,\ -4,\ 0)$

(c) My answer is :

$v_2 \bullet v_3$

$= 4 \times 0 + (-3) \times 0 + (-12) \times (-1))$

$= 12$

Question 3:

If two vectors $\mathbf{u}, \mathbf{v} \in \Re^n$ are orthogonal, what is the value of their scalar (dot) product?

My answer is:

0, since they are orthogonal, which means that the angle between them is 90 degrees and the dot product of the vectors is zero.

Question 4:

Which of the following are unit vectors? (a) $(\frac{1}{2}, -\frac{1}{2}, 0)$ (b) $(0, -1, 0)$ (c) $\frac{1}{7}(-2, 3, 6)$

My answer is:

(b) and (c) are unit vectors.

For (a):

$$\sqrt[2]{(1/2)^2 + (-1/2)^2 + (0)^2} = \sqrt[2]{1/2} \neq 1$$

For (b):

$$\sqrt[2]{(0)^2 + (-1)^2 + (0)^2} = \sqrt[2]{1} = 1$$

For (c):

$$\sqrt[2]{(-2/7)^2 + (3/7)^2 + (6/7)^2} = \sqrt[2]{1} = 1$$

In conclusion, (b) and (c) are unit vectors.

Question 5:

We are given two non-zero vectors $\mathbf{u}, \mathbf{v} \in \Re^3$. Assume the angle between $\mathbf{u}$ and $\mathbf{v}$ satisfies $0 < \theta < \frac{\pi}{2}$. Use dot products and/or cross products of $\mathbf{u}$ and $\mathbf{v}$ to give expressions for:

(a) $\cos\theta$ (b) $\sin\theta$ (c) A vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$.

(a) My answer is:

$$\cos\theta = \frac{u \bullet v}{|u||v|}$$

(b) My answer is:

$$\sin\theta = \frac{|u \times v|}{|u||v|}$$

(c) My answer is:

$$u \times v$$

Since the vector resulting from the cross product u × v is a vector that is perpendicular to both u and v.

Question 6:

Given three square matrices $\mathbf{Q}, \mathbf{R}, \mathbf{S} \in \Re^{n \times n}$, which statements are true **in general**?

(a) $(\mathbf{QRS})^{-1} = \mathbf{Q}^{-1}\mathbf{R}^{-1}\mathbf{S}^{-1}$

(b) $\mathbf{QR} = \mathbf{RQ}$

(c) $(\mathbf{QRS})^{T} = \mathbf{S}^{T}\mathbf{R}^{T}\mathbf{Q}^{T}$

(d) $(\mathbf{R} + \mathbf{S})\mathbf{Q} = \mathbf{SQ} + \mathbf{RQ}$

(a) My answer is:

It’s false, since $(\mathrm{QRS})^{-1} = \mathrm{S}^{-1}\mathrm{R}^{-1}\mathrm{Q}^{-1}$.

(b) My answer is:

It’s false, unless they are special matrices.

(c) My answer is:

It’s true.

(d) My answer is:

It’s true.

Question 7:

Given a square matrix $\mathbf{A} \in \Re^{n\times n}$ whose columns form an orthonormal basis:

(a) What is the dot product of any pair of columns in $\mathbf{A}$?

(b) What is the inverse of $\mathbf{A}$?

(a) My answer is:

It's 1, since A's columns form an orthonormal basis so that the dot product of each pair of columns in A is 1.

(b) My answer is:

Since A's columns form an orthonormal basis, $\mathrm{A}^{-1} = \mathrm{A}^{\mathrm{T}}$