(7a)

$$C = \int_0^{2\pi} \sqrt{ds^2}$$

$$= \int_0^{2\pi} \sqrt{g_{tt}\,dt^2 + g_{\phi\phi}\,d\phi^2}$$

$$dt = \sqrt{\left(\frac{r^3}{m}\right)} \cdot d\phi$$

$$= \int_0^{2\pi} \sqrt{\left(g_{tt}\frac{r^3}{m} + g_{\phi\phi}\right) d\phi^2}$$

$$= \int_0^{2\pi} \sqrt{\left(g_{tt}\frac{r^3}{m} + g_{\phi\phi}\right)}\, d\phi$$

$$= 2\pi \sqrt{-\frac{4}{5} \cdot 10^3 \cdot m^2 + 10^2 m^2}$$

$$= 2\pi m \cdot \sqrt{1000 \cdot -\frac{4}{5} + 100}$$

$$= 2\pi m \cdot \sqrt{700}$$

$$= 20\sqrt{7}\,\pi m$$

(b)

$$t = 2\pi \cdot \sqrt{\frac{r^3}{m}} = 20\sqrt{10}\,\pi m$$

(c) $\Delta\tau = \int_0^{2\pi} \sqrt{g_{tt}\frac{r^3}{M}}\cdot d\phi^2$

$= 2\pi \cdot 10 \cdot \sqrt{4}\, M$

$= 40\pi\sqrt{2}\, M$

(d) $t = 2\pi\sqrt{\frac{(6M)^3}{M}}$

$= 12\pi M\sqrt{6}$

(e) $\frac{\Delta\tau}{\Delta t} = \frac{2\pi\sqrt{\frac{r^3(1-3M/r)}{M}}}{2\pi\sqrt{\frac{r^3}{M}}}$

$= \sqrt{1-\frac{3M}{r}}$

$= \frac{\sqrt{2}}{2}$

$40\sqrt{2}$

(20a)

follow the first derivative of metric tensor.

only non vanished are:

1. $g_{tt,r}$ 2. $g_{rr,r}$ 3. $g_{\theta\theta,r}$ 4. $g_{\phi\phi,r}$ 5. $g_{\phi\phi,\theta}$

So only non vanish Christoffel symbols are

$\Gamma^0_{01}$ $\Gamma^1_{00}$ $\Gamma^1_{11}$ $\Gamma^1_{22}$ $\Gamma^1_{33}$ $\Gamma^2_{12}$ $\Gamma^2_{33}$ $\Gamma^3_{13}$

$\Gamma^3_{23}$.

(20b) for Riemann, 6 pair $\mu \neq \nu$, 6 pair $\alpha \neq \beta$

So 36 independent component

eliminate off diagonal, $5 \cdot \frac{6}{2} + 6 = 21$

by cyclic sum, eliminate one, so 20 independent component.

by this metric, only six non zero component.

$R_{trtr}$ $R_{t\theta t\theta}$ $R_{t\phi t\phi}$ $R_{r\theta r\theta}$ $R_{r\phi r\phi}$ $R_{\theta\phi\theta\phi}$

agree with question.