GR notes

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June 21, 2022

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Conventions

1. Greek index (e.g. α, β, μ, ν) take value from $\{0, 1, 2, 3\}$.
2. Events denoted by cursive capitals (e.g. $\mathscr{A}, \mathscr{B}, \mathscr{E}$).
3. $(x^0, x^1, x^2, x^3) \equiv (t, x, y, z) \equiv x^\alpha$
4. Latin index (e.g.i, j, k) take value from $\{1, 2, 3\}$.
5. New unit that speed of light $c = 1$

1 Special Relativity

1.1 4-Dimensional Spacetime

Definition 1.1. Inertial coordinate
The coordinate system must satisfy three property to be consider inertial coordinat:

1. The distance between two points are independent of time.
2. The clocks at every points ticking off time coordinate t at same rate.
3. The geometry of space is always Euclidean (flat).

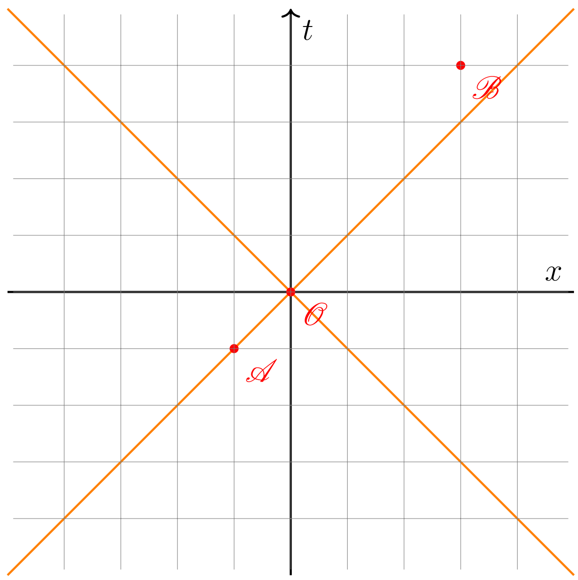

Figure 1: two events with coordinate $(-1,-1,0,0)$ and $(4,3,0,0)$. Orange line is light's worldline.

The event in 4-D spacetime is defined by a set of coordinate (t,x,y,z). For simplicity, we assume those events have $y=0, z=0$ so that we can draw a 2D graph to represent them.
Analog to Euclidean geometry, just like the euclidean distance $\Delta l^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$, we define the **spacetime interval** $\Delta s^2 = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$.

Remark. There are a lot different conventions to define the sign of interval, here we just use the popular one $(-,+,+,+)$.

Example.
Interval for the two events in Figure 1 is $\Delta s^2 = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -9$.

Due to universal speed of light, interval is invariant change of inertial coordinate, this means that $\Delta s^2 = \Delta \bar{s}^2$ When the interval Δs^2 is less than 0, we call it **timelike**; When the interval Δs^2 is equal to 0, we call it **lightlike** or null; When the interval Δs^2 is greater than 0, we call it **spacelike**. The $x^\mu = \{x^0, x^1, x^2, x^3\}$
$ds^2 = g_{\mu\nu}x^\mu x^\nu = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}x^\mu x^\nu$