

GR notes

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Conventions

1. Greek index (e.g. α, β, μ, ν) take value from $\{0, 1, 2, 3\}$.
2. Events denoted by cursive capitals (e.g. $\mathcal{A}, \mathcal{B}, \mathcal{E}$).
3. $(x^0, x^1, x^2, x^3) \equiv (t, x, y, z) \equiv x^\alpha$
4. Latin index (e.g. i, j, k) take value from $\{1, 2, 3\}$.
5. New unit that speed of light $c = 1$
6. Einstein summation convention $ds^2 = g_{\mu\nu}x^\mu x^\nu = \sum_{\mu=0}^3 \sum_{\nu=0}^3 g_{\mu\nu}x^\mu x^\nu$

1 Special Relativity

1.1 4-Dimensional Spacetime

Definition 1.1. Inertial coordinate

The coordinate system must satisfy three property to be consider inertial coordinat:

1. The distance between two points are independent of time.
2. The clocks at every points ticking off time coordinate t at same rate.
3. The geometry of space is always Euclidean (flat).

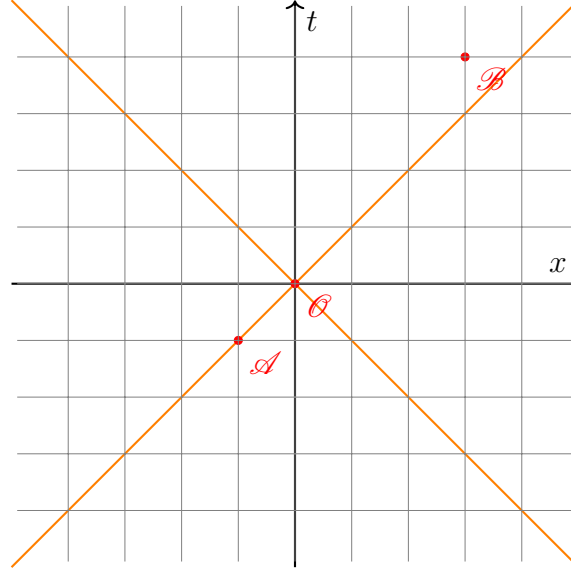


Figure 1: two events with coordinate $(-1, -1, 0, 0)$ and $(4, 3, 0, 0)$. Orange line is light's worldline.

The event in 4-D spacetime is defined by a set of coordinate (t, x, y, z) . For simplicity, we assume those events have $y = 0, z = 0$ so that we can draw a 2D graph to represent them.

Analog to Euclidean geometry, just like the euclidean distance $\Delta l^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$, we define the **spacetime interval** $\Delta s^2 = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$.

Remark. There are a lot different conventions to define the sign of interval, here we just use the popular one $(-, +, +, +)$.

Example.

Interval for the two events in Figure 1 is $\Delta s^2 = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 = -9$.

Due to universal speed of light, interval is invariant change of inertial coordinate, this means that $\Delta s^2 = \Delta \bar{s}^2$ When the interval Δs^2 is less than 0, we call it **timelike**; When the interval Δs^2 is equal to 0, we call it **lightlike** or null; When the interval Δs^2 is greater than 0, we call it **spacelike**. The $x^\mu = \{x^0, x^1, x^2, x^3\} = \{t, x, y, z\}$ is a set of coordinate.