1. $J = kg\ m^2 s^{-2}$

$c = 1, \quad 1s = 3\times10^8 m$

b) $1W = 1J/s = kg\cdot m^2 \cdot s^{-3}$

$100\ kg\cdot m^2\cdot s^{-3}$

$= 100\cdot kg\cdot m^2\cdot(3\times10^8 m)^{-3}$

$= 100\cdot kg\cdot m^{-1}\cdot(3\times10^8)^{-3}$

---

c) $1.05\times10^{-34}\ kg\cdot m^2\cdot s^{-2}\cdot s$

$= 1.05\times10^{-34}\ kg\cdot m^2\cdot(3\times10^8)^{-1}$

$= \frac{1.05\times10^{34}}{3\times10^8}\cdot kg\cdot m = 3.52\times10^{-43} kg\cdot m$

d) $30\ m\cdot s^{-1} = 30m\cdot(3\times10^{8}\cdot m)^{-1}$

$= 1\times10^{-7}$

e) $3\times10^{4}\ kg\ m\ s^{-1} = 3\times10^{4}\ kg\cdot m\cdot(3\times10^{8}m)^{-1}$

$= 1\times10^{-4}\ kg$

f) $N = kg\cdot m\cdot s^{-2}$

$10^{5}\ kg\cdot m\cdot s^{-2}\cdot m^{-2}$

$= 10^{5}\ kg\ m^{-1}\cdot(3\times10^{8}m)^{-2}$

$= \frac{10^{5}}{3\times10^{8}}\ kg\cdot m^{-3}$

g) $10^{3}\ kg\ m^{-3}$

2. a) $c = 1 = \frac{3\times10^{8}m}{s}$

$10^{-2}\cdot3\times10^{8}m/s = 3\times10^{6}m\cdot s^{-1}$

b) $P : kg \cdot s^{-2} \cdot m^{-1}$ $\boxed{m = (3 \times 10^{8})^{-1} s}$

$10^{19}\ kg\ m^{-3}$

$= 10^{19} \cdot kg \cdot m^{-1} \cdot s^{-2} \cdot (3 \times 10^{8})^{2}$

c) $t = 10^{16}\ m$

$= 10^{16} \cdot (3 \times 10^{8})^{-1} \cdot s$

$= \frac{1}{3} \times 10^{8} \cdot s$

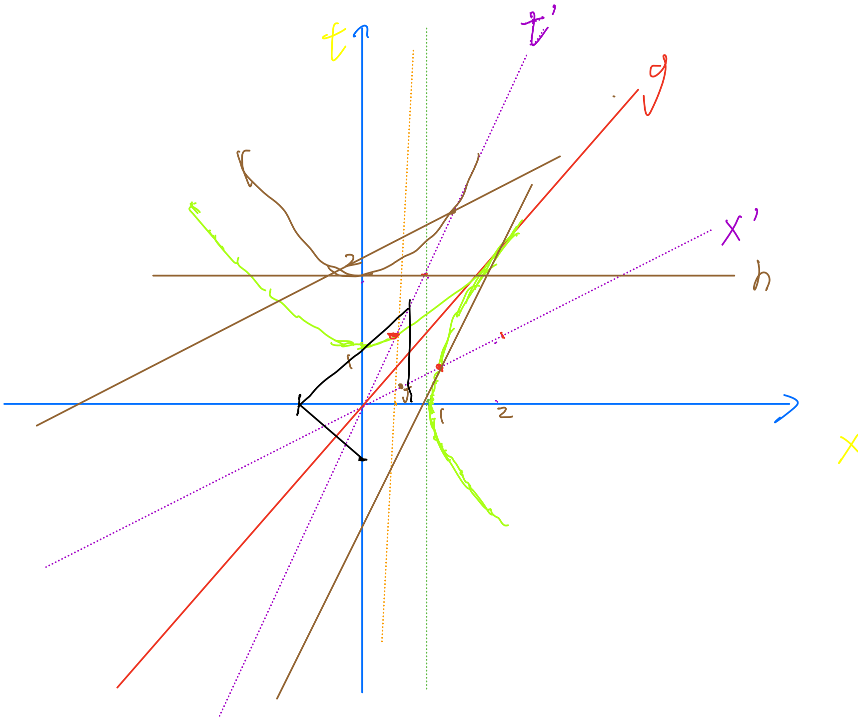
d) $\mu = kg\ m^{-3} = kg\ m^{-1}\ s^{-2} \cdot (3 \times 10^{8})^{2}$

e) $10\ m^{-1} \rightarrow m \cdot s^{-2}$

$10\ m \cdot s^{-2} \cdot (3 \times 10^{8})^{2}$

$9 \times 10^{17}\ m \cdot s^{-2}$

3)



$$\Delta s^2 = -\Delta t^2 + \Delta x^2 = -1$$

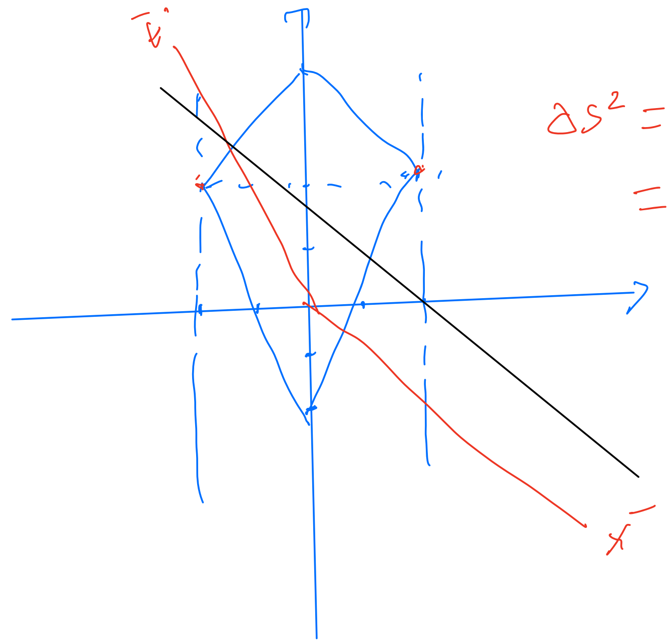
$$\Delta t^2 - \Delta x^2 = 1$$

4)

a) $$\sum_{\alpha=0}^{3} V_\alpha \Delta x^\alpha = V_0 \Delta x^0 + V_1 \Delta x^1 + V_2 \Delta x^2 + V_3 \Delta x^3$$

$$= V_0 \Delta t + V_1 \Delta x + V_2 \Delta y + V_3 \Delta z$$

5)



$$\Delta s^2 = -\Delta t^2 + \Delta x^2$$
$$= 4^2 = 16\ m^2$$

6)

$$\Delta \bar{s}^2 = \sum_{\alpha=0}^{3}\sum_{\beta=0}^{3} M_{\alpha\beta}(\Delta x^\alpha)(\Delta x^\beta)$$

$$= M_{00}(\Delta x^0)^2 + M_{01}(\Delta x^0)(\Delta x^1)$$
$$+ M_{02}(\Delta x^0)(\Delta x^2) + M_{03}(\Delta x^0)(\Delta x^1)$$
$$+ M_{10}(\Delta x^1)(\Delta x^0) + M_{11}(\Delta x^1)^2$$
$$+ M_{12}(\Delta x^1)(\Delta x^2) + M_{13}(\Delta x^1)(\Delta x^3)$$

7) ✓

$$= -(\Delta \bar{t})^2 + (\Delta \bar{x})^2 + (\Delta \bar{y})^2 + (\Delta \bar{z})^2$$
$$= -(\alpha t)^2 - 2(\alpha t \beta x) - (\beta x)^2 + u^2 t^2 + 2(utvx) + v^2 x^2$$
$$+ (ay)^2 + (bz)^2$$

$$= (-\alpha^2 + u^2) t^2 + (v^2 - \beta^2) x^2 + (2uv - 2\alpha\beta) x t$$

$$+ a^2 \cdot y^2 + b^2 \cdot z^2$$

$M_{00} = -\alpha^2 + u^2$ $\quad 2M_{10} = 2(uv - \alpha\beta)$

$M_{11} = v^2 - \beta^2$ $\quad M_{22} = a^2$ $\quad M_{33} = b^2$

other equal 0

8) a)

$$\Delta s^2 = M_{00} (\Delta r)^2 + 2M_{0i} \Delta x^i \Delta r + M_{ij} \Delta x^i \Delta x^j$$

$$\Delta r^2 = \cancel{\beta} \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$\Delta \bar{s}^2 = -\Delta \bar{t}^2 + \Delta \bar{x}^2 + \Delta \bar{y}^2 + \Delta \bar{z}^2$$

$$\Delta \bar{s}^2 = M_{00} \Delta t^2 + 2\sum_{1}^{3} M_{0i} \Delta t \Delta x^i + \sum_{ij}^{3} M_{ij} \Delta x^i \Delta x^j$$

for $\Delta \bar{s}^2 = 0$

$$\Delta t^2 = \Delta r^2$$

Sub this

b) $M_{00}(\Delta r)^2 + 2\left(\sum^{3} M_{0i}(-\Delta x^i)\right)\Delta r \; \cancel{\otimes} + \sum\sum M_{ij}\Delta x^i \Delta x^j$

only change   no change

$0 - 0 = 4 \cdot \sum^{3} M_{0i}(\Delta x^i)$

$M_{0i} = 0$

c) $0 = \Delta s^2 = M_{00}(\Delta r)^2 + 2\sum_i^{3} \underbrace{M_{0i}}_{0}(\Delta x^i) + \sum_i^{3}\sum_j^{3} M_{ij}\Delta x^i \Delta x^j$

$0 = M_{00}(\Delta r)^2 + \sum_i^3\sum_j^3 M_{ij}\Delta x^i \Delta x^j$

$-M_{00}(\Delta r)^2 = \sum_i^3 \sum_j^3 M_{ij}\Delta x^i \Delta x^j$

we know $M_{i0} = 0$

sub $\Delta r = \Delta x$

$\Delta r = \Delta y \quad \Delta r = \Delta z$

$-M_{00}\Delta x^2 = M_{11}\Delta x^2$

$-M_{00} = M_{22}$

$= M_{33}$

9) clock is stationary in $\bar{O}$ frame

10)

a) $\Delta s^2 = -(-1)^2 + (1)^2 + 0 + 0$

$= 0$ lightlike

b) $\Delta s^2 = -(-2)^2 + 0^2 + 1^2 + 2^2$

$= -4 + 1 + 4$

$= 1$ spacelike

c) $\Delta s^2 = -(-1)^2 + 0 + 0 + 0$

$= -1$ timelike

d) $\Delta s^2 = -(5)^2 + 0 + 0 + 5^2$

$= 0$ light like

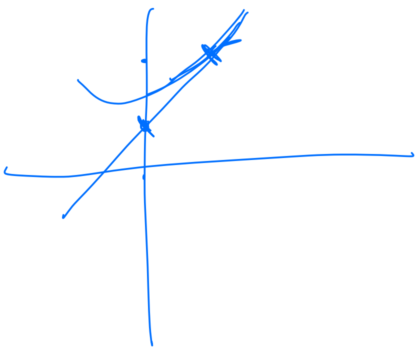
11)

for $-t^2+x^2=a^2$ $\quad x^2=a^2+t^2$

$\lim_{x\to\infty} x^2=t^2$ $\quad a^2$ neglect.

12)

a)



$AE=\Delta\tau$

$AB=\Delta\bar{t}=\dfrac{\Delta\tau}{\sqrt{1-v^2}}>\Delta\tau=AE$

b)

$(\Delta s^2)_{AC}=-(\Delta\tau)^2_{AC}$

$\Delta s^2_{AB}=-(\Delta\bar{t})^2_{AB}=-\dfrac{(\Delta\tau)^2}{1-v^2}=\Delta s^2_{AC}/1-v^2$

c)

$\Delta t_{AE}=\Delta\bar{t}_{AB}\sqrt{1-v^2}$

13) $\Delta \tau = 2.5 \times 10^{-8}\ s$

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-v^2}} = \frac{2.5 \times 10^{-8}\ s}{\sqrt{0.001}} = 5.6 \times 10^{-7}\ s$$

14) $|v| \ll 1$ $\quad (\sqrt{1-v^2})^{-1} = 1 + \frac{1}{2}v^2$

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-v^2}} = \Delta \tau \left(1 + \frac{1}{2}v^2\right)$$

$$t = \tau \sqrt{1-v^2} = \tau \left(1 - \frac{1}{2}v^2\right)$$

$$w' = \frac{w+v}{1+wv} = (w+v)(1+wv)^{-1} = (w+v)(1-wv)$$

15) $|v| = 1 - \varepsilon$

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-v^2}} = \Delta \tau / \sqrt{(1+v)(1-v)} = \Delta \tau / \sqrt{\varepsilon(2-\varepsilon)} = \Delta \tau / \sqrt{2\varepsilon}$$

$$t = \tau \sqrt{1-v^2} = \tau \sqrt{\varepsilon(2-\varepsilon)} = \tau \sqrt{2\varepsilon}$$

$$w' = \frac{w+v}{1+wv} = \frac{w+(1-\varepsilon)}{1+w(1-\varepsilon)} = (w+1-\varepsilon)(1+w-w\varepsilon)^{-1}$$

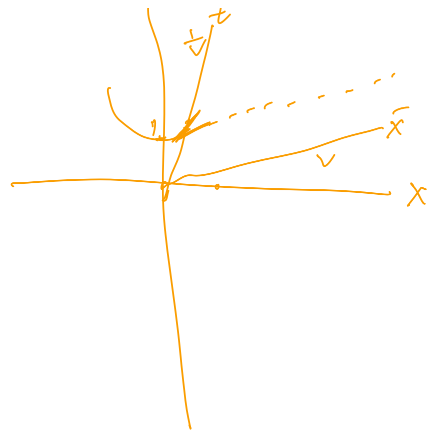
t =

16)

$$\bar{t} = \frac{t}{\sqrt{1-v^2}} - \frac{vx}{\sqrt{1-v^2}}$$

$$\bar{x} = \frac{-vt}{\sqrt{1-v^2}} + \frac{x}{\sqrt{1-v^2}}$$

use fig 1.14



$\bar{t}_B = \Delta\tau$

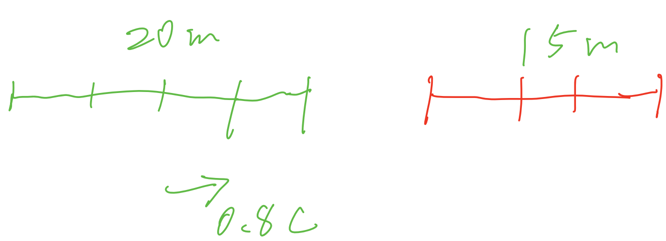
$t_B = \Delta t$

$$\bar{t}_B = \frac{t_B}{\sqrt{1-v^2}} - \frac{vx}{\sqrt{1-v^2}} \qquad \text{sub}(-v)$$

$$t_B = \frac{\bar{t}_B}{\sqrt{1-v^2}} + \frac{v\bar{x}_B}{\sqrt{1-v^2}} \qquad \bar{x}_B = 0$$

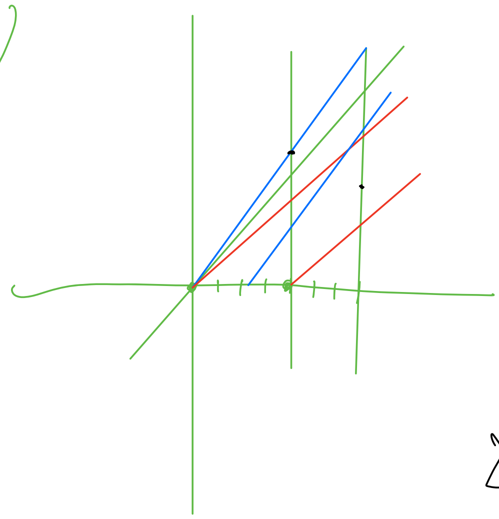
$$\Delta t = \frac{\Delta\tau}{\sqrt{1-v^2}}$$

17)



a) $l = \bar{l}\sqrt{1-v^2} = \bar{l}\cdot\sqrt{0.36} = 0.6\bar{l} = 12\text{m}$

b)



$20 \cdot \frac{1}{0.8} c = 25 = t_1$

$23 \cdot \frac{1}{0.8} = t_2 = 28.75$

$\Delta t = 3.75$

$\Delta x = 15$

$\Delta s^2 = 21 \, m^2$ spacelike

c) $15 \cdot 0.6 = 9m$ pole $= 20m$

d) No, spacelike event

e) pole can't act rigidly, front stop not mean end also stop.

f) above

18)

a) $w' = \frac{w+v}{1+wv} = \frac{\tanh V + \tanh u}{1 + \tanh V \cdot \tanh u}$

$\boxed{\tanh x = \frac{\sinh x}{\cosh x}}$

$$(\tanh U + \tanh u)\left[\frac{\sinh(U)\sinh(u)}{\sinh(U)\sinh(u)} + \frac{\sinh(U)\sinh(u)}{\cosh(U)\cosh(u)}\right]^{-1}$$

$$\times\left(\frac{\cosh(U)\cosh(u)}{\cosh(U)\cosh(u) + \sinh(U)\sinh(u)}\right)$$

$$(\tanh U + \tanh u)(\cosh U)(\cosh u)$$

$= \sinh U \cdot \cosh u + \cosh U \sinh u$ → numerator

$\cosh U \cosh u + \sinh U \cdot \sinh u$

$= \cosh(U+u)$ → denominator

so $w' = \dfrac{\sinh(U+u)}{\cosh(U+u)} = \tanh(U+u)$

b)

$w_N' = \tanh(U(N-1))$ $\quad\quad \tanh U = 0.9$

$= \dfrac{e^z - e^{-z}}{e^z + e^{-z}}$ $\quad z = U(N-1)$ $\quad U \approx 1.5$

for large N, $z \to$ large

$$W_h' = \left(1 - e^{-2z}\right)^2 \quad , \quad z = v \cdot (N-1)$$

1(a)

$$\bar{t} = \frac{t}{\sqrt{1-v^2}} - \frac{vx}{\sqrt{1-v^2}} \qquad \bar{y} = y$$

$$\bar{x} = \frac{-vt}{\sqrt{1-v^2}} + \frac{x}{\sqrt{1-v^2}} \qquad \bar{z} = z$$

$$v = \tanh(u)$$

$$\bar{t} = \frac{t}{\sqrt{1-\tanh^2 u}} - \frac{\tanh u \, x}{\sqrt{1-\tanh^2 u}} \qquad \sqrt{1-\tanh^2 u} = \operatorname{sech} u$$

$$= t \cdot \cosh u - \sinh u \cdot x$$

$$\bar{x} = \frac{-\tanh u \, t}{\sqrt{1-v^2}} + \frac{x}{\sqrt{1-v^2}}$$

$$= -\sinh u \cdot t + \cosh u \, x$$

b) $\bar{t}^2 = t^2\cosh^2 u - 2tx\cosh u\sinh u + x^2\sinh^2 u$

$\bar{x}^2 = t^2\sinh^2 u - 2tx\sinh u\cosh u + x^2\cosh^2 u$

$$\Delta\bar{s}^2 = -\Delta\bar{t}^2 + \Delta\bar{x}^2 = \Delta t^2(\sinh^2 u - \cosh^2 u) + \Delta x^2(\cosh^2 u - \sinh^2 u)$$

$$= -\Delta t^2 + \Delta x^2$$

c) analog of Δs^2 is the euclidian distance

$\Delta d^2 = \Delta x^2 + \Delta y^2$

$x^2 + y^2 = \text{constant}$, circle.

20) $\bar{x}^A = A \cdot x^A$ $\qquad \gamma = \frac{1}{\sqrt{1-v^2}}$

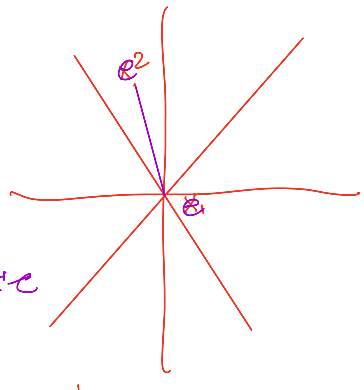
$$\begin{bmatrix} \bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{bmatrix} = A \cdot \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t\cdot\gamma - vx\cdot\gamma \\ -vt\gamma + x\gamma \\ y \\ z \end{bmatrix}$$

$$t\gamma - vx\cdot\gamma = t\cdot A_{11} + x\cdot A_{12}$$

$$-vt\gamma + x\gamma = t\cdot A_{21} + x\cdot A_{22}$$

$$A = \begin{bmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

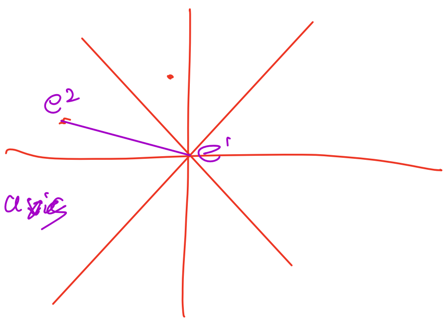
21) a) $\Delta s^2 < 0$



$\Delta t^2 > \Delta x^2$

connect two event as t axie

b) $\Delta x^2 > \Delta t^2$



conect the event as x axis