

7a)

$$C = \int_0^{2\pi} \sqrt{ds^2}$$

$$= \int_0^{2\pi} \sqrt{g_{tt} dt^2 + g_{\phi\phi} d\phi^2}$$

$$\left| \quad dt = \sqrt{\left(\frac{r^3}{n}\right)} \cdot d\phi \right.$$

$$= \int_0^{2\pi} \sqrt{\left(g_{tt} \frac{r^3}{n} + g_{\phi\phi}\right) d\phi^2}$$

$$= \int_0^{2\pi} \sqrt{\left(g_{tt} \frac{r^3}{n} + g_{\phi\phi}\right)} d\phi$$

$$= 2\pi \sqrt{-\frac{4}{5} \cdot 10^3 M^2 + 10^3 M^2}$$

$$= 2\pi M \cdot \sqrt{1000 \cdot -\frac{4}{5} + 1000}$$

$$= 2\pi M \cdot \sqrt{700}$$

$$= 20\sqrt{7}\pi M$$

7b)

$$t = 2\pi \cdot \sqrt{\frac{r^3}{n}} = 20\sqrt{10}\pi M$$

①

$$\Delta \phi = \int_0^{2\pi} \sqrt{g_{tt} \frac{r^2}{n}} \cdot d\phi^2$$

$$= 2\pi \cdot 10 \cdot \sqrt{4} M$$

$$= 40\pi \sqrt{2} M$$

②

$$t = 2\pi \sqrt{\frac{(6M)^2}{n}}$$

$$= 12\pi M \sqrt{6}$$

③

$$\frac{\Delta \phi}{\Delta t} = \frac{2\pi \sqrt{\frac{r^2(1-3M/r)}{n}}}{2\pi \sqrt{\frac{r^2}{n}}}$$

$$= \sqrt{1 - \frac{3M}{r}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\frac{40\sqrt{2}}{2}$$