①

ⓐ $M_{\alpha\alpha} A^{\alpha} B^{\alpha}$ compare to $M_{\alpha\beta} A^{\alpha\beta}$

missing the term that $\alpha \neq \beta$.

ⓑ because $\eta_{\alpha\beta} = \text{diagno}(-1, 1, 1, 1)$

$$A^{\alpha} B^{\beta} \eta_{\alpha\beta} = A^{\alpha} B^{\alpha} \eta_{\alpha\alpha}$$

$$= -A^0 B^0 + A^1 B^1 + A^2 B^2 + A^3 B^3$$

⑱

$\vec{A} = (-1, 0 -1, 0)$

$\vec{B} = (0, 1, 1, 0)$

$\vec{C} \to (1, 0, -1, 0)$

$\vec{B} \to$ same

ⓑ

$\vec{B} \to (-3, 0, -1, -1)$

$\vec{D} \to (-1, -1, 1, 1)$

$\vec{F} \to (0, -5, -1, 0)$

$\vec{S} \to (2, 1, 0, 0)$

(20)

$$\bar{A} = \Lambda \cdot A$$

$$\bar{D} = A^T \cdot B$$

(b)

$$\Lambda^k{}_i \Lambda^l{}_j \delta_{kl} = \Lambda^k{}_i \Lambda^k{}_j$$

$$= \Lambda^T \Lambda = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

(24)

$$M^{(\alpha\beta)} = \frac{1}{2}\left(M^{\alpha\beta} + M^{\beta\alpha}\right)$$

$$= \frac{1}{2}\left[\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 2 \\ 2 & 0 & 0 & 1 \\ 1 & 0 & -2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 2 & 1 & 0 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 2 & 2 & 1 \\ 2 & -2 & 0 & 2 \\ 2 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

$$M^{[\alpha\beta]} = \frac{1}{2}\begin{pmatrix} 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 3 \\ 1 & -2 & -3 & 0 \end{pmatrix}$$

(b) No

[illegible]

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(a) $A^{\alpha\beta} B_{\alpha\beta} = \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} A^{\alpha\beta} B_{\alpha\beta}$

$$\boxed{\begin{aligned} A^{\alpha\beta} &= -A^{\beta\alpha} \\ B_{\alpha\beta} &= B_{\beta\alpha} \end{aligned}}$$

$$= \underbrace{\sum_{\lambda=0}^{3} A^{\lambda\lambda} B_{\lambda\lambda}}_{=0} + \frac{1}{2}\left(A^{\alpha\beta} B_{\alpha\beta} + A^{\beta\alpha} B_{\alpha\beta}\right)$$

$$= 0$$

(b)

$$A^{\alpha\beta} C_{\alpha\beta} = -A^{\beta\alpha} C_{\alpha\beta}$$

$$\boxed{C_{\alpha\beta} = 2C_{[\alpha\beta]} + C_{\beta\alpha}}$$

$$C_{\beta\alpha} = 2C_{[\alpha\beta]} - C_{\alpha\beta}$$

$$= -A^{\beta\alpha}\left(2C_{[\alpha\beta]} + C_{\beta\alpha}\right)$$

$$= +A^{\beta\alpha} 2C_{[\beta\alpha]} - A^{\beta\alpha} C_{\beta\alpha}$$

$$= A^{\beta\alpha} 2C_{[\beta\alpha]} - A^{\beta\alpha}\left(2C_{[\alpha\beta]} - C_{\alpha\beta}\right)$$

$$A^{\alpha\beta} C_{[\alpha\beta]} = A^{\alpha\beta} \frac{1}{2}\left(C_{\alpha\beta} - C_{\beta\alpha}\right)$$

$$= \frac{1}{2}\left(A^{\alpha\beta} C_{\alpha\beta} + A^{\beta\alpha} C_{\beta\alpha}\right)$$

$$= \frac{1}{2}\left(A^{\alpha\beta}C_{\alpha\beta} + A^{\mu\nu}C_{\mu\nu}\right)$$

$$= A^{\alpha\beta}C_{\alpha\beta}$$

(c)

$$B_{\alpha\beta}D^{(\alpha\beta)} = B_{\alpha\beta}\frac{1}{2}\left(D^{\alpha\beta} + D^{\beta\alpha}\right)$$

$$= \frac{1}{2}\left(B_{\alpha\beta}D^{\alpha\beta} + B_{\beta\alpha}D^{\beta\alpha}\right)$$

$$= B_{\alpha\beta}D^{\alpha\beta}$$

(34)

$u = t - x$

$v = t + x$

$\vec{e}_\beta = (\vec{e}_u, \vec{e}_v, \vec{e}_y, \vec{e}_z)$

$$\vec{e}_\alpha = \Lambda^\beta{}_\alpha e_\beta$$

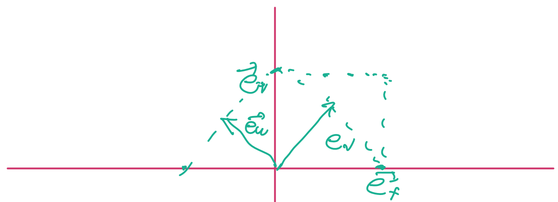
$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{e}_\beta$$

$$\vec{e}_t = \vec{e}_u + \vec{e}_v \qquad \vec{e}_v = \frac{\vec{e}_t + \vec{e}_x}{2}$$

$$\vec{e}_x = -\vec{e}_u + \vec{e}_v \qquad \vec{e}_u = \frac{\vec{e}_t - \vec{e}_x}{2}$$

$$\vec{e}_y = \vec{e}_y$$

$$\vec{e}_z = \vec{e}_z$$



linear combination of $\vec{e}_u$ and $\vec{e}_v$ span the $t-x$ plane and linear combination of $\vec{e}_\beta$ span the whole spacetime.

(c)

$$\vec{e}_u \cdot \vec{e}_v = g_{uv}$$

$$= \begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{e}_u \cdot \vec{e}_u$$

$$= \left\langle \frac{\vec{e}_t - \vec{e}_x}{2}, \frac{\vec{e}_t - \vec{e}_x}{2} \right\rangle$$

$$= \frac{1}{4}\left(\vec{e}_t \cdot \vec{e}_t + \vec{e}_t \cdot -\vec{e}_x + (-\vec{e}_x \cdot \vec{e}_t) + (-\vec{e}_x \cdot -\vec{e}_x)\right)$$

$$= \frac{1}{4}(-1 + 0 + 0 + 1) = 0$$

$\vec{e}_v \cdot \vec{e}_v$

$= 0$

$\vec{e}_u \cdot \vec{e}_v$

$= \frac{1}{4}\left(\vec{e}_t \vec{e}_t + \vec{e}_t(-\vec{e}_x) + \vec{e}_x \cdot \vec{e}_t + \vec{e}_x(-\vec{e}_x)\right)$

$= \frac{1}{4}(-1 + 0 + 0 + (-1))$

$= -\frac{1}{2}$

(d)

Δs^2 along $\vec{e}_u$

$= -(-1) - 1 = 0$

along $\vec{e}_v = -1 + 1 = 0$

$(\vec{e}_u, \vec{e}_v) = -\frac{1}{2} \neq 0$

(e) $\tilde{d}u = \tilde{d}t - \tilde{d}x$

$\tilde{d}v = \tilde{d}t + \tilde{d}x$

$g(\vec{e}_u, \quad) = -\frac{1}{2}\tilde{d}v$

$g(\vec{e}_v, \quad) = -\frac{1}{2}\tilde{d}u$