Hw2 - GANGYI Qu

Part I:

we want show:

$$\exp(-\tfrac{1}{2}(r+n\phi)(\theta-\tfrac{1}{r+n\phi}(r\theta_0+\phi\sum_{i=1}^{n}x_i))^2)$$

same form

as ~~$\exp(-\tfrac{1}{2}(r+n\phi)(\theta-\tfrac{1}{r+n\phi}(r\theta_0+\phi\sum_{i=1}^{n}x_i))^2)$~~

$$\exp[-\tfrac{1}{2}((\sum_{i=1}^{n}\phi(x_i-\theta)^2)+r(\theta-\theta_0)^2)]$$

Step we remove the exp() and $\frac{1}{2}$ we get:

① $(r+n\phi)(\theta-\frac{1}{r+n\phi}\underbrace{(r\theta_0+\phi\sum_{i=1}^{n}x_i)}_{\text{as "V"}})^2$

② $\sum_{i=1}^{n}\phi(x_i-\theta)^2)+r(\theta-\theta_0)^2)$

Simplify equation ①, we get

$(r+n\phi)\theta^2-2V\theta+\frac{V^2}{r+n\phi}$

$=(r+n\phi)(\theta^2-\frac{2V\theta}{r+n\phi}+(\frac{V}{r+n\phi})^2)$

$=(r+n\phi)(\theta-\frac{V}{r+n\phi})^2$

$=(r+n\phi)(\theta-\frac{(r\theta_0+\phi\sum_{i=1}^{n}x_i)}{r+n\phi})^2$

Simplify ② we get:

$\phi\sum_{i=1}^{n}(x_i-\theta)^2+r(\theta-\theta_0{}^2)$

$=(r+\phi)(\frac{\phi\sum_{i=1}^{n}(x_i-\theta)^2}{r+\phi}+\frac{r(\theta-\theta_0)^2}{r+\phi})$

~~remove "(r+φ)" for both, get:~~

~~$①(\theta-\frac{r\theta_0+\phi\sum_{i=1}^{n}x_i)^2}{r+n\phi})$~~

~~②~~ See next page.

we got ①, ② simplified

$$① \to (r+n\phi)\left(\theta - \frac{r\theta_0+\phi\sum_{i=1}^{n} x_i}{r+n\phi}\right)^2$$

Then canel r

$$② \to (r+\phi)\left(\frac{\phi\sum_{i=1}^{n}(x_i-\theta)^2+r(\theta-\theta_0)^2}{r+\phi}\right)$$

we could see their proportional.

So they could expreess each other.