CS70 - Lecture 16 Notes

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Set Notation Review

- Set A, Complement $\bar{A}$
- Union (In either: or): $A \cup B$
- Intersection (In both: and): $A \cap B$
- Difference (In A, not B) $A \setminus B$
- Symmetric Difference (In only one: xor) $A \Delta B$

Probability

- event E = subset of outcome: $E \subset \Omega$
- Any Sample Space: $\Pr[E] = \sum_{\omega \in E} \Pr[\omega]$
- Uniform Space: $\Pr[E] = \frac{|E|}{|\Omega|}$
- $p_n := \Pr[E_n] = \frac{|E_n|}{|\Omega|}$
  - $p_n := \frac{\binom{n}{k}}{|\omega|^n}$ if $E = n$ coin tosses with exactly k heads

Stirling Formula: (for large n)

- $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$
- $\Pr[E] = \frac{|E|}{|\Omega|}$
  - Can apply Stirling Formula because $|E|$ and $|\Omega|$ are defined by combinations (factorials)

Probability is Additive

- If events A and B are disjoint, then sum probabilities
- Non-disjoint sets, use Inclusion/exclusion property: $\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B]$
- **Union bound**: $\Pr[A_1 \cup \cdots \cup A_n] \leq \Pr[A_1] + \cdots + \Pr[A_n]$
- If $A_1, ..., A_N$ are a pairwise disjoint partition of Ω and $\cup_{m=1}^{N} A_m = \Omega$, then $\Pr[B] = \Pr[B \cap A_1] + \cdots + \Pr[B \cap A_N]$

Inclusion/Exclusion Property:
$\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B]$
Union Bound:
$\Pr[A_1 \cup \cdots \cup A_n] \leq \Pr[A_1] + \cdots + \Pr[A_n]$
Law of Total Probability:
If $A_1, ..., A_N$ are a pairwise disjoint partition of Ω and $\cup_{m=1}^{N} A_m = \Omega$ then,
$\Pr[B] = \Pr[B \cap A_1] + \cdots + \Pr[B \cap A_N]$

Conditional Probability

- Probability of A given B
- $\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$

Product Rule

$$\Pr[A_1 \cap \cdots \cap A_n] = \Pr[A_1]\Pr[A_2|A_1] \cdots \Pr[A_n|A_1 \cap \cdots \cap A_{n-1}] \tag{1}$$

Total Probabability $\times$ **Product Rule**

$$\Pr[B] = \Pr[A_1]\Pr[B|A_1] + \cdots + \Pr[A_N]\Pr[B|A_N] \tag{2}$$

Causality vs. Correlation

- Events A and B are **positively correlated** if $\Pr[A \cap B] > \Pr[A]\Pr[B]$, but this does not imply causation
- Eliminate external/common causes to test causality

Bayes Rule

- Let $m =$ number of situations where A and B occurred, and $n =$ number of situations where $\bar{A}$ and B occurred.
- Therefore: $\Pr[A|B] = \frac{m}{m+n}$

Bayes Rule (Simplified using Law of Total Probability)

$$\Pr[A_n|B] = \frac{\Pr[A_n]\Pr[B|A_n]}{\sum_m \Pr[A_m]\Pr[B|A_m]} = \frac{\Pr[A_n]\Pr[B|A_n]}{\Pr[B]} \tag{3}$$

Independence

- Two events A and B are independent if $\Pr[A \cap B] = \Pr[A]\Pr[B]$
- Two events A and B are independent if and only if $\Pr[A|B] = \Pr[A]$
- $\Pr[A]$ decreases/increases given B

If A and B are **independent** sets:

$$\Pr[\bar{A} \cap \bar{B}] = 1 - A \cup B \tag{4}$$

$$\Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A \cup B] \tag{5}$$