Efficient Hub Discovery in Relational Graphs

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ABSTRACT

Relational graphs encapsulate non-trivial interactions between entities from heterogeneous data sources. Identifying hubs in relational graphs is essential in many real-world applications such as protein complex discovery and social influence analysis. However, the substantial cost of constructing relational graphs from heterogeneous data sources impedes efficient hub discovery. In this paper, we propose a novel *sketch propagation* framework that approximately identifies hubs in an induced relational graph, *without* explicitly materializing it. Our framework provides provable guarantees for effective hub discovery under the notion of ϵ-separable sets. It specifically supports the identification of hubs based on *degree centrality* and *h-index*. We further devise efficient pruning techniques to improve the efficiency of our framework for *personalized* hub queries, which ask whether a node q is a hub, under both measures. Extensive experimentation confirms the efficacy and efficiency of our proposals, which achieve orders of magnitude speedups compared to exact methods while consistently showing accuracy beyond 90%.

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1 INTRODUCTION

Finding important nodes, or *hubs*, in graphs according to specific centrality measures [10, 28] has broad applications in different domains, such as evaluating network robustness [31, 33, 42, 49] and information propagation [9, 11, 30]. Most prior studies on hub queries assume that a materialized data graph is available and accessible with low latency. Still, in many real-world scenarios, relational graphs [6, 9, 30, 47] that capture complex relationships between entities should be meticulously extracted from heterogeneous data sources, such as knowledge graphs (KGs) [23, 24] and relational databases (RDBMS) [1, 40]. For example, using an academic KG, e.g., DBLP, as the data source, one can construct a relational graph that connects researchers who have co-authored one or more papers.

Applications. The need to find hubs in relational graphs arises in various real-world applications, including:

- **Protein Complex Discovery:** Protein-protein interaction (PPI) networks describe the collective interactions between proteins during physiological processes, formally expressed as "$(protein) \longrightarrow (process) \longleftarrow (protein)$"; hubs in a PPI network are pivotal in the discovery of protein complexes [38, 48].
- **Fraud Detection:** Financial data consisting of payment records reveal connections between transactions sharing the same billing accounts in the form "$(transaction) \longrightarrow (billing) \longleftarrow (transaction)$"; hubs in such a graph indicate potential fraudulent activities [6].
- **Influence Analysis:** Academic KGs record publications and induce collaboration networks embedding the coauthorship "$(author) \longrightarrow (paper) \longleftarrow (author)$" [30]; hubs in such graphs denote highly impactful researchers. Further restricted relationships [27, 36] such as "$(author) \longrightarrow (paper, venue=\text{`DB'}) \longleftarrow (author)$" can yield impactful database researchers.

The relationships encapsulated by the aforementioned relational graphs can be captured by *meta-path* semantics [14, 41, 55], one of the most widely used approaches for modeling complex relationships between entities in heterogeneous data sources. Thus, in this paper, we focus on the *meta-path* semantics and investigate the fundamental problem, HUB, of identifying hubs, i.e., nodes with high *degree* [10] or *h-index* [28] centrality in a relational graph induced by a meta-path $\mathcal{M}$.

Challenges. A naive solution to the HUB problem is to enumerate all instances of $\mathcal{M}$ to construct a relational graph H, compute the degree or h-index of each node in H, and return the nodes with the highest degrees or h-indexes as hubs. However, such explicit relational graph materialization is often time- and memory-intensive, especially when the data sources are of huge volume [51]. An alternative method is to traverse the meta-path instances and maintain a node's neighbors in the relational graph without explicit materialization. Nevertheless, to find the hubs exactly, we should access the neighborhood information of all nodes and potentially traverse any edge involved in each meta-path instance up to $O(|V_{\mathcal{M}}|)$ times, where $|V_{\mathcal{M}}|$ is the number of nodes in the relational graph. Thus, this approach can also falter when the relational graph is large. Furthermore, as the number of meta-paths extracted from a data source with various entity and relationship types using existing methods can be huge (up to several thousands in our experiments), it is inefficient to identify the exact hubs for every relational graph.

Our Contributions. We find that in many real-world scenarios, exactly enumerating all hubs is usually unnecessary, as it suffices to find an approximate set of hubs accurately and efficiently. Inspired by cardinality sketches [2, 12, 13, 16, 22], we propose a novel *sketch propagation* framework for approximate HUB queries. We construct a *neighborhood cardinality* sketch for each node to estimate its degree in the relational graph by iteratively propagating sketches along edges in matching instances of the given meta-path $\mathcal{M}$. Thereby, we jointly estimate the degrees of all nodes in the relational graph in a *single* propagation process and significantly enhance the efficiency of HUB queries under degree centrality.

We then extend the sketch propagation framework to approximate HUB queries with the *h-index* centrality measure [28]. A key challenge is that the h-index of a node is determined not only by its own degree but also by the degrees of its neighbors, while cardinality sketches provide information only on a node's own degree. To fill this gap, we propose a *pivot-based* algorithm built on the sketch propagation framework. Rather than directly estimating the h-index, this algorithm recursively finds nodes whose h-indexes are no less than that of a random pivot node based on the sketches, as quicksort does. Hence, it effectively skips nodes that cannot be hubs and quickly answers HUB queries based on the h-index.

Furthermore, we provide efficient methods to answer *personalized* HUB queries, i.e., to determine whether a query node is a hub. We maintain a lower bound on the number of nodes with centrality scores higher than the query node and terminate the sketch propagation when the lower bound exceeds the threshold. This early termination mechanism applies to personalized HUB queries based on both degree and h-index measures.

Under the notation of ϵ-sepearable sets, we prove that the sketch propagation framework provides unbiased and efficient approximations for HUB queries based on both measures. Our experiments reveal that the estimations can converge much faster than the worst-case bound on the number of propagations, achieving orders of magnitude speedups compared to exact methods.

Our main contributions are summarized as follows:

- We introduce a novel problem of querying hubs (HUB) based on degree and h-index measures over relational graphs induced by meta-paths. (Section 2)
- We propose a *sketch propagation* framework for approximate degree-based HUB queries and extend it to personalized HUB queries with early termination. (Section 3)
- We further devise a *pivot-based* algorithm, also with early termination, for approximate (personalized) HUB queries based on h-index. (Section 4)
- Through extensive experimentation, we demonstrate that our proposals scale well to large relational graphs where exact methods fail to terminate in a reasonable time and achieve speedups of orders of magnitude over exact methods in most cases while yielding accuracy beyond 90% with default parameter settings. A case study on influence maximization shows that HUB methods achieve comparable influences to the greedy framework over relational graphs. (Section 5)

2 PRELIMINARIES

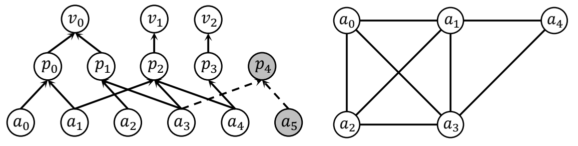
In this section, we introduce the basic notations, formulate the problem of finding hubs over relational graphs (HUB), and present the exact algorithms for HUB query processing.

2.1 Notations and Problem Formulation

We model a heterogeneous data source as a knowledge graph (KG). Specifically, a KG is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where $\mathcal{V}$ and $\mathcal{E}$ represent the sets of node and edge instances. An edge instance $e \in \mathcal{E}$ connects two node instances $u, v \in \mathcal{V}$. The function $\mathcal{L}$ maps each node or edge instance, v or e, to its type, $\mathcal{L}(v)$ or $\mathcal{L}(e)$. Note that our formulation and methods are orthogonal to the format of the heterogeneous data and can be extended to support other data sources such as RDBMS.

Meta-paths are commonly used to extract relational graphs from KGs [14, 52]. We provide the formal definitions of *meta-path* and its *matching instances* on the KG as follows:

Definition 1 (Meta-path). *An L-hop meta-path is a sequence of node and edge types denoted as $\mathcal{M}(x_0, y_0, x_1, \dots, x_{L-1}, y_{L-1}, x_L)$, where x_i is the type of the i-th node and y_i is the type of the i-th edge. The inverse of $\mathcal{M}$ is denoted as $\mathcal{M}^{-1}$ $(x_L, y_{L-1}^{-1}, x_{L-1}, \dots, x_1, y_0^{-1}, x_0)$, where y_i^{-1} is the inverse type of y_i.*



(a) Exemplary KG $\mathcal{G}$ (b) Relational graph H

Figure 1: Exemplary KG $\mathcal{G}$ and relational graph H derived from $\mathcal{G}$ using meta-path $\mathcal{M}(A,$ 'write'$, P,$ 'publish'$, V)$. The unshaded nodes and solid edges in (a) denote the matching graph of $\mathcal{M}$.

Definition 2 (Matching Instance). *A matching instance M to a meta-path $\mathcal{M}$ is a sequence of node and edge instances in $\mathcal{G}$ denoted by $M(v_0, e_0, v_1, \dots, v_{L-1}, e_{L-1}, v_L) \triangleright \mathcal{M}$ satisfying:*

(1) $\forall i \in \{0, \dots, L\}$, $\mathcal{L}(v_i) = x_i$.
(2) $\forall i \in \{0, \dots, L-1\}$, $e_i = (v_i, v_{i+1}) \in \mathcal{E}$ and $\mathcal{L}(e_i) = y_i$.

When the context is clear, we use $M(v_0, v_1, \dots, v_L)$ or simply M to denote a matching instance and use $M(x_i)$ to denote the node v_i in M. Although our methods support any meta-path $\mathcal{M}$, following the established convention [14, 25, 52], we concatenate $\mathcal{M}$ and its inverse $\mathcal{M}^{-1}$ to induce a homogeneous relational graph $H_{\mathcal{M}}$.

Definition 3 (Relational Graph). *For a given KG $\mathcal{G}$, a meta-path $\mathcal{M}$ induces a relational graph $H_{\mathcal{M}} = (V_{\mathcal{M}}, E_{\mathcal{M}})$ from $\mathcal{G}$ with node set $V_{\mathcal{M}}$ containing all the nodes in $\mathcal{V}$ that match x_0 in any instance of $\mathcal{M}$ and edge set $E_{\mathcal{M}}$ containing each edge $e = (u, v)$ for any two nodes $u, v \in V_{\mathcal{M}}$ such that there are $M \triangleright \mathcal{M}$ and $M' \triangleright \mathcal{M}^{-1}$ in $\mathcal{G}$ with (1) $M(x_0) = u$, (2) $M'(x_0) = v$, and (3) $M(x_L) = M'(x_L)$.*

We denote the set of neighbors of u in $H_{\mathcal{M}}$ as $\mathcal{N}(u)$ and the *degree* of u in $H_{\mathcal{M}}$ as $N(u) = |\mathcal{N}(u)|$. Furthermore, $n = \max_{u \in H} N(u)$ denotes the maximum degree in $H_{\mathcal{M}}$. When the context is clear, we drop $\mathcal{M}$ and use $H = (V, E)$ to represent a relational graph for ease of presentation.

Example 1. *Fig. 1 presents an exemplary academic KG with three node types A ('author'), P ('paper'), and V ('venue') and two edge types $A \to P$ ('write') and $P \to V$ ('publish'). A sequence $(A,$ 'write'$, P,$ 'publish'$, V)$ denotes a meta-path $\mathcal{M}$, which induces the relational graph H in Fig. 1(b). For instance, two nodes a_0 and a_2 are neighbors in H because there exist an instance $M = (a_0, p_0, v_0)$ of $\mathcal{M}$ and another instance $M' = (v_0, p_1, a_2)$ of the inverse $\mathcal{M}^{-1}$ of $\mathcal{M}$. More intuitively, two authors a_0 and a_2 are connected since they ever published papers in the same venue v_0.*

Problem Statement. In this paper, we study the problem of finding hubs in relational graphs. Hubs are nodes that rank higher than most other nodes in the network according to a function $c : V \to \mathbb{R}_{\geq 0}$ that measures node importance. Specifically, we use degree centrality [10], i.e., the number of nodes connected to a node, and h-index [28], i.e., the largest integer h such that a node has h neighbors each of degree at least h, as the measures of node importance. The HUB query we consider is formalized as the following:

Problem 1 (HUB Query). *Given a relational graph H and a parameter $\lambda \in (0, 1)$, find every node $u \in V$ such that $c(u) \geq c(v^{\lambda})$, where v^{λ} is the $(1-\lambda)$-quantile node under a centrality measure $c(\cdot)$.*

We also consider *personalized* HUB queries, which inquire whether a query node q is a hub of H.

Problem 2 (Personalized HUB Query). *Given a relational graph H, a node q, and $\lambda \in (0, 1)$, decide if $c(q) \geq c(v^\lambda)$.*

Processing HUB queries on large-scale relational graphs is computationally expensive. Thus, we consider how to answer HUB queries approximately and efficiently using randomized algorithms based on the notion of ϵ-separable sets [18].

Definition 4 (ϵ-Separable Set). *Given any node $u \in V$ and $\epsilon \in (0, 1)$, the two ϵ-separable sets of u, denoted as $S^+_\epsilon(u)$ and $S^-_\epsilon(u)$, are the sets of nodes in H with centrality larger and smaller than $c(u)$ by a relative ratio of ϵ, respectively, i.e., $S^+_\epsilon(u) = \{v \in V | c(v) > (1+\epsilon)\cdot c(u)\}$ and $S^-_\epsilon(u) = \{v \in V | c(v) < (1-\epsilon)\cdot c(u)\}$.*

Consequently, we define the approximate versions of HUB and personalized HUB queries as follows:

Problem 3 (ϵ-Approximate HUB Query). *Given a relational graph H and $\epsilon, \lambda \in (0, 1)$, return a set S of nodes in H such that $S^+_\epsilon(v^\lambda) \subseteq S$ and $S \cap S^-_\epsilon(v^\lambda) = \varnothing$.*

Problem 4 (ϵ-Approximate Personalized HUB Query). *Given a relational graph H, a node q, and $\epsilon, \lambda \in (0, 1)$, return* `TRUE` *if $q \in S^+_\epsilon(v^\lambda)$ and* `FALSE` *if $q \in S^-_\epsilon(v^\lambda)$.*

2.2 The Exact Algorithms

The materialization of large relational graphs can often be memory-prohibitive [51]. To address this issue, we propose a memory-efficient baseline method for exact HUB queries, which computes the exact centrality of each node in H using a generic worst-case optimal join algorithm [3, 15, 32, 44] on the *matching graph* of $\mathcal{M}$ over $\mathcal{G}$. We first introduce the notion of *matching graph* of a meta-path and the successors and predecessors of a node in the matching graph as follows:

Definition 5 (Matching Graph). *Given a KG $\mathcal{G}$ and a meta-path $\mathcal{M}$, the* matching graph *of $\mathcal{M}$ over $\mathcal{G}$ is a* multi-level *graph $\mathcal{G}^*(\mathcal{V}^*, \mathcal{E}^*)$ with a node set $\mathcal{V}^* = \bigcup_{0\leq i\leq L} \mathcal{V}_i$ and an edge set $\mathcal{E}^* = \bigcup_{0\leq i<L} \mathcal{E}_i$, where $\mathcal{V}_i$ is the set of all node instances of type x_i contained in any matching instance of $\mathcal{M}$ and $\mathcal{E}_i$ consists of all edges of type y_i that connects two nodes between $\mathcal{V}_i$ and $\mathcal{V}_{i+1}$.*

Definition 6 (Successors & Predecessors). *For each node $u \in \mathcal{V}_i$, we use $\mathcal{V}^+(u)$ to denote the nodes in $\mathcal{V}_{i+1}$ connected with u through edge type y_i, i.e., the* successors *of u in $\mathcal{G}^*$; and use $\mathcal{V}^-(u)$ to denote the nodes in $\mathcal{V}_{i-1}$ connected with u through edge type y_{i-1}, i.e., the* predecessors *of u in $\mathcal{G}^*$.*

Example 2. *In Fig. 1(a), the unshaded nodes and solid edges denote the matching graph $\mathcal{G}^*$ of meta-path $\mathcal{M}(A, \text{'write'}, P, \text{'publish'}, V)$. Nodes a_5 and p_4 are not in $\mathcal{G}^*$ since they are not included in any instance of $\mathcal{M}$. The node set $\mathcal{V}_0 = \{a_0, a_1, a_2, a_3, a_4\}$ consists of 'authors' contained in a matching instance of $\mathcal{M}$. Similarly, we have $\mathcal{V}_1 = \{p_0, p_1, p_2, p_3\}$ and $\mathcal{V}_2 = \{v_0, v_1, v_2\}$. The set of predecessors of p_2 is $\mathcal{V}^-(p_2) = \{a_1, a_3, a_4\}$, which contains in-neighbors of p_2 in $\mathcal{G}^*$ along edge type 'write.' The set of successors of p_2 is $\mathcal{V}^+(p_2) = \{v_1\}$, which contains out-neighbors of p_2 in $\mathcal{G}^*$ along edge type 'publish.'*

The matching graph $\mathcal{G}^*$ can be extracted from $\mathcal{G}$ efficiently by a *forward* and *backward* breadth-first search (BFS). At each level $i \in [0, \ldots, L-1]$, the forward BFS iteratively visits all neighbors of each candidate matching node of x_i via edge type y_i as the candidates for x_{i+1}. Subsequently, the backward BFS visits all neighbors of the candidates for x_{i+1} via edge type y_i^{-1} as the candidates for x_i at each level $i \in [L-1, \ldots, 0]$. Only the candidate nodes and their adjacent edges visited by both forward and backward BFS are identified as the matching nodes and edges. Based on our experimental findings (see Table 2), the cost of extracting $\mathcal{G}^*$ from $\mathcal{G}$ is negligible compared to the total computational cost for HUB queries.

To avoid generating excessive intermediate results, a generic worst-case optimal join is employed on the matching graph to compute the neighbors of each node $u \in V$ in the relational graph. This process starts with a depth-first search (DFS) from each node u and traverses the matching instances of $\mathcal{M}$ and $\mathcal{M}^{-1}$. Specifically, DFS$(u, u, +)$ (Procedure DFS) is executed for each node $u \in V$. Starting from u, it recursively traverses the successors (Lines 4–5) of the current node. Once it visits a node in $\mathcal{V}_L$, the direction of DFS will be switched from traversing the instances of $\mathcal{M}$ to those of $\mathcal{M}^{-1}$ (Line 2). If the traversal reaches $\mathcal{V}_0$ via any instance of $\mathcal{M}^{-1}$, the node v will be added to $\mathcal{N}(u)$ (Line 3). Note that after performing a DFS for each node, the set $\mathcal{N}(u)$ is discarded and only the degree $|\mathcal{N}(u)|$ is kept to ensure a small memory footprint. To compute the h-indexes of all nodes without *materializing* the relational graph, the worst-case optimal join is performed in two rounds. The first round also computes and stores the degree of each node $v \in V$. Then, the second round computes the h-index of each node v by combining its neighborhood $\mathcal{N}(v)$ with the degrees kept in the first round.

Procedure DFS$(u, v, \circ)$

```
  Input: The matching graph $\mathcal{G}^*$ and the direction $\circ \in \{+, -\}$
1 Mark $(v, \circ)$ as visited;
2 if $v \in \mathcal{V}_L$ and $\circ = +$ then DFS$(u, v, -)$;
3 if $v \in \mathcal{V}_0$ and $\circ = -$ then Add $v$ to $\mathcal{N}(u)$;
4 for $w \in \mathcal{V}^\circ(v)$ do
5     if $(w, \circ)$ is not visited then DFS$(u, w, \circ)$;
```

Although the memory consumption is bounded by $O(|\mathcal{V}^*| + |\mathcal{E}^*|)$ and thus is not a bottleneck, exact algorithms are still very inefficient due to repeated edge traversal. Each edge in $\mathcal{E}^*$ is visited up to $O(|V|)$ times to compute the neighborhood of each node in V. Consequently, the time complexity of the exact algorithms is $O(|V|\cdot|\mathcal{E}^*|)$, which can be prohibitively expensive when processing large relational graphs.

3 THE SKETCH PROPAGATION FRAMEWORK

In this section, we introduce a novel *sketch propagation* framework to efficiently estimate the degree of each node in H by constructing cardinality sketches for their neighbors. As a result, it effectively answers HUB queries based on degree centrality. In addition, we propose an early termination technique to further accelerate personalized HUB query processing.

3.1 Sketch Propagation for HUB

Our basic idea is to approximate the degree of each node in H by constructing KMV sketches for the neighborhood $\mathcal{N}(v)$ of each node $v \in V$ without materializing H. The KMV sketch was initially proposed for distinct-value estimation [5] and defined as follows.

Definition 7 (KMV Sketch). *Given a collection $C = \{o_0, o_1, ..., o_{|C|}\}$ of items and an integer $k > 0$, the KMV (k-th Minimum Value) sketch $\mathcal{K}(C)$ is built by independently drawing a number uniformly at random from $(0,1)$ for each item in C and maintaining the k smallest random numbers. The set of random numbers generated for each item in C is called the* basis *for the KMV sketch. The cardinality of C is estimated as $|\widetilde{C}| = \mathbb{E}[(k-1)/\zeta]$, where ζ is a random variable by taking the largest number in $\mathcal{K}(C)$.*

There are two key challenges in applying KMV sketches and their corresponding estimators to HUB queries. First, the KMV sketch is designed primarily for stream processing [2, 12, 13, 16, 22], where a one-pass scan over the collection C is required. However, for HUB queries, scanning the neighborhood of each node is quite expensive, as discussed in the description of exact algorithms. Second, previous work only provides asymptotic error bounds for the estimation when $|C| \longrightarrow \infty$. Although assuming that a large number of items will appear is reasonable for stream processing, the neighborhood size of each node in H can be relatively small. Thus, the original estimator for the KMV sketch does not provide meaningful error bounds for degree estimations in HUB queries.

To address the above two challenges, we propose the *sketch propagation* framework that constructs a KMV sketch for $\mathcal{N}(v)$ of each $v \in H$ without explicitly generating and scanning $\mathcal{N}(v)$ and offers a different estimator with a tight error bound for the case where $\mathcal{N}(v)$ is relatively small. Our framework is based on the notion of *images.*

Definition 8 (Images). *The forward image of a matching node $u \in \mathcal{V}_i$, denoted as $\mathcal{I}_f(u)$, contains a node $v \in \mathcal{V}_0$ if there exists an instance M of $\mathcal{M}$ including both u and v, i.e.,*

$$\mathcal{I}_f(u) = \{v \in \mathcal{V}_0 \mid \exists M \triangleright \mathcal{M}, M(x_0) = v \wedge M(x_i) = u\}.$$

The backward image of $u \in \mathcal{V}_i$, denoted as $\mathcal{I}_b(u)$, contains a node $v \in \mathcal{V}_0$ if there exists two concatenated instances M and M' of $\mathcal{M}$ and $\mathcal{M}^{-1}$ that include u and v, respectively, i.e.,

$$\mathcal{I}_b(u) = \{v \in \mathcal{V}_0 \mid \exists M \triangleright \mathcal{M}, M' \triangleright \mathcal{M}^{-1}, M(x_i) = u \wedge M(x_L) = M'(x_L) \wedge M'(x_0) = v\}.$$

Example 3. *In Fig. 1(a), the forward image $\mathcal{I}_f(p_1)$ contains two nodes a_2 and a_3, as they are connected with p_1 through $M(a_2, p_1, v_0)$ and $M(a_3, p_1, v_0)$, respectively. The backward image $\mathcal{I}_b(p_1)$ contains nodes a_0, a_1, a_2 and a_3. For example, node a_0 belongs to $\mathcal{I}_b(p_1)$ as it is connected to p_1 through $M(a_2, p_1, v_0)$ and $M'(v_0, p_0, a_0)$.*

An important insight is that $\mathcal{N}(u) = \mathcal{I}_b(u)$ for each node $u \in V$ in the relational graph due to its definition.

Forward and Backward Propagation. We build a KMV sketch for $\mathcal{N}(u)$ of each $u \in V$ by iteratively maintaining the sketches for the forward and backward images of nodes from $\mathcal{V}_0$ to $\mathcal{V}_L$. First, a random number $r(u)$ is generated for each node $u \in \mathcal{V}_0$. Since $\mathcal{I}_f(u) = \{u\}$, the sketch is initialized as $\mathcal{K}(\mathcal{I}_f(u)) = \{r(u)\}$. The sketches for the forward images of the nodes in $\mathcal{V}_i$ are then constructed by iteratively propagating the sketches of the nodes in $\mathcal{V}_{i-1}$. Specifically, each node in $\mathcal{V}_{i-1}$ propagates its sketch to all its successor nodes in $\mathcal{V}_i$ and a node in $\mathcal{V}_i$ builds its sketch by retaining the k smallest random numbers among all sketches it receives. After building the sketches for the forward images of the nodes in $\mathcal{V}_L$ (which is equal to the sketches w.r.t. their backward images), the algorithm propagates the sketches back from the nodes in $\mathcal{V}_L$ to the nodes in $\mathcal{V}_0$ in the same way as for forward propagation so as to build the sketches for the backward images of the nodes in $\mathcal{V}_0$ to estimate the degree of each node in H.

The following lemma ensures the correctness of the *sketch propagation* framework.

Lemma 1. *Given a meta-path $\mathcal{M}$, for each node $u \in \mathcal{V}_i$*

$$\mathcal{K}(\mathcal{I}_f(u)) = \oplus_{v \in \mathcal{V}^-(u)} \mathcal{K}(\mathcal{I}_f(v)) \quad \text{and} \tag{1}$$

$$\mathcal{K}(\mathcal{I}_b(u)) = \oplus_{v \in \mathcal{V}^+(u)} \mathcal{K}(\mathcal{I}_b(v)), \tag{2}$$

hold if all KMV sketches are constructed from the same basis on $\mathcal{V}_0$. The operator $\oplus$ extracts the k smallest random numbers from the union of KMV sketches.

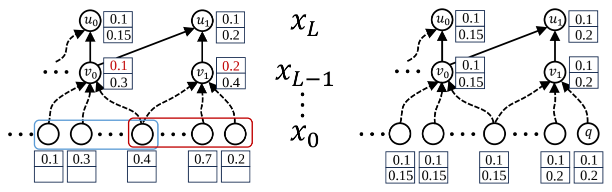
Proof. According to [5, Thm. 5], $\mathcal{K}(A) \oplus \mathcal{K}(B)$ is a KMV sketch for the collection $A \cup B$ for two collections A and B. Therefore, we only need to prove $\mathcal{I}_f(u) = \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$ and $\mathcal{I}_b(u) = \bigcup_{v \in \mathcal{V}^+} \mathcal{I}_b(v)$ for each node $u \in \mathcal{V}_i$.

On the one hand, for any node $u' \in \mathcal{I}_f(u)$, there exists $M \triangleright \mathcal{M}$ such that $M(x_i) = u$ and $M(x_0) = u'$. Suppose that $v = M(x_{i-1})$, we have $u' \in \mathcal{I}_f(v)$ and $v \in \mathcal{V}^-(u)$. Thus, $u' \in \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$, i.e., $\mathcal{I}_f(u) \subseteq \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$. On the other hand, for any node $u' \in \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$, there exists a node $v \in \mathcal{V}^-(u)$ such that $u' \in \mathcal{I}_f(v)$, i.e., there exists an instance $M' \triangleright \mathcal{M}$ such that $M'(x_0) = u'$ and $M'(x_{i-1}) = v$. Moreover, since $v \in \mathcal{V}^-(u)$, there exists an instance $M'' \triangleright \mathcal{M}$ such that $M''(x_{i-1}) = v$ and $M''(x_i) = u$. By concatenating $M'(x_0) = u'$ to $M'(x_{i-1}) = v$ with $M''(x_{i-1}) = v$ to $M''(x_L)$, we construct an instance $M \triangleright \mathcal{M}$ such that $M(x_0) = u'$ and $M(x_i) = u$. Thus, $u' \in \mathcal{I}_f(u)$, i.e., $\bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v) \subseteq \mathcal{I}_f(u)$. Therefore, we have $\mathcal{I}_f(u) = \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$ for each node $u \in \mathcal{V}_i$. By symmetry, we can also prove that $\mathcal{I}_b(u) = \bigcup_{v \in \mathcal{V}^+} \mathcal{I}_b(v)$. □

Algorithm 1 details the *sketch propagation* framework. It receives a KG $\mathcal{G}$, a meta-path $\mathcal{M}$, a ratio $\lambda \in (0,1)$, the number of propagations $\theta \in \mathbb{Z}^+$, and the sketch size $k \in \mathbb{Z}^+$ as input, and returns a set of nodes S for the (approximate) HUB query. It first initializes two arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ to store θ KMV sketches for the respective forward and backward images of each node $u \in \mathcal{V}^*$ (Lines 1–4). Next, it calls the function Propagate, which constructs KMV sketches within $\mathcal{K}_f$ and $\mathcal{K}_b$ (Lines 9–13) and estimates the degree of each node u in H using $\mathcal{K}_b$ (Lines 14–21). Specifically, the function Receive depicts the step for KMV sketch construction when each node receives the sketches propagated from other nodes. Given a node u, the array $\mathcal{K}$ to store sketches, and a parameter $\circ$ that indicates the propagation direction, it scans the random numbers in the sketches of all nodes in $\mathcal{V}^+(u)$ or $\mathcal{V}^-(u)$ and keeps the k smallest numbers in $\mathcal{K}[u][t]$ with a min-heap of size k (Lines 24 and 25). Finally, it returns top-$(\lambda \cdot |V|)$ nodes with the highest estimated degrees as S (Lines 6 & 7). The ties are broken arbitrarily.

Algorithm 1: Sketch Propagation

```
Input: KG $\mathcal{G}$, meta-path $\mathcal{M}$, quantile parameter $\lambda$, number of
       propagations $\theta$, KMV sketch size $k$
Output: A set of nodes $S$ for (approximate) HUB query
forall $u \in \mathcal{V}^*$ do
    for $t = 1$ to $\theta$ do
        $\mathcal{K}_f[u][t] \leftarrow \emptyset$ and $\mathcal{K}_b[u][t] \leftarrow \emptyset$;
        if $u \in \mathcal{V}_0$ then add $r(u) = \mathtt{Rand}(0,1)$ to $\mathcal{K}_f[u][t]$;
$\widetilde{N} \leftarrow \mathtt{Propagate}(\mathcal{K}_f, \mathcal{K}_b)$;
$S \leftarrow$ top-$(\lambda \cdot |V|)$ nodes with highest degrees in $H$ w.r.t. $\widetilde{N}$;
return $S$;
Function Propagate($\mathcal{K}_f, \mathcal{K}_b$):
    for $i = 1$ to $L$ do
        forall $u \in \mathcal{V}_i$ do Receive($u, \mathcal{K}_f, -$);
    forall $u \in \mathcal{V}_L$ do $\mathcal{K}_b[u] \leftarrow \mathcal{K}_f[u]$;
    for $i = L-1$ to $0$ do
        forall $u \in \mathcal{V}_i$ do Receive($u, \mathcal{K}_b, +$);
    forall $u \in \mathcal{V}_0$ do
        $\widetilde{\mu} \leftarrow 0$;
        for $t = 1$ to $\theta$ do
            if $|\mathcal{K}_b[u][t]| = k$ then
                $\widetilde{\mu} \leftarrow \widetilde{\mu} + \frac{\max(\mathcal{K}_b[u][t])}{\theta}$;
            else
                $\widetilde{\mu} \leftarrow \widetilde{\mu} + \frac{k}{(|\mathcal{K}_b[u][t]|+1)\cdot\theta}$;
        $\widetilde{N}[u] \leftarrow k/\widetilde{\mu} - 1$;
    return $\widetilde{N}$;
Function Receive($u, \mathcal{K}, \circ$):
    for $t = 1$ to $\theta$ do
        $\mathcal{K}[u][t] \leftarrow \oplus_{v \in \mathcal{V}^{\circ}(u)} \mathcal{K}[v][t]$;
```



(a) Forward propagation (b) Backward propagation

Figure 2: Illustration of forward and backward propagation with $k = 2$ and $\theta = 1$. Nodes in the blue and red boxes represent the forward images $\mathcal{I}_f(v_0)$ and $\mathcal{I}_f(v_1)$, respectively.

Example 4. *Fig. 2 illustrates the sketch propagation framework with $k = 2$ and $\theta = 1$. The forward image $\mathcal{I}_f(u_1)$ is the union of the forward images of nodes in its predecessors $\mathcal{V}^-(u_1) = \{v_0, v_1\}$, that is, the union of nodes in the blue and red boxes in the layer $\mathcal{X}_0$. The sketch $\mathcal{K}(\mathcal{I}_f(u_1))$ is constructed by taking the two smallest numbers from the union of sketches of the forward images of nodes in $\mathcal{V}^-(u_1)$. Thus, $\mathcal{K}(\mathcal{I}_f(u_1)) = \{0.1, 0.3\} \oplus \{0.2, 0.4\} = \{0.1, 0.2\}$. During the backward propagation, the sketch $\mathcal{K}(\mathcal{I}_b(v_0))$ is constructed similarly but in the reverse direction by taking the smallest two numbers from the union of sketches of backward images of nodes in $\mathcal{V}^+(v_0)$. Thus, $\mathcal{K}(\mathcal{I}_b(v_0)) = \{0.1, 0.15\} \oplus \{0.1, 0.2\} = \{0.1, 0.15\}$.*

Theoretical Analysis. Next, we prove that *sketch propagation* approximates HUB queries with high probability (w.h.p.) (cf. Theorem 2). For the proof, we first establish the following two lemmas: (1) Lemma 2 indicates the unbiasedness of sketch propagation in estimating the image sizes for each node in the matching graph; (2) Lemma 3 demonstrates the concentration properties that guarantee tight approximations around the true values.

For any node $u \in \mathcal{V}^*$, we use $I_f(u)$ and $I_b(u)$ to denote the sizes of $\mathcal{I}_f(u)$ and $\mathcal{I}_b(u)$. Moreover, we use ζ to denote the random variable that takes the maximum random number in the sketch of either u's forward or backward image and μ to denote the expectation of ζ. When the context is clear, we abbreviate $I_b(u)$ and $I_f(u)$ as I. We also use $\widetilde{I}$ and $\widetilde{\mu}$ to denote the estimation of I and μ provided by sketch propagation.

Lemma 2. *For any $u \in \mathcal{V}^*$, $I = k/\mu - 1$ if $I \geq k$.*

Proof. According to [5], if $I \geq k$, the probability density function of ζ will be $f_\zeta(t) = \frac{t^{k-1}(1-t)^{I-k}}{B(k, I-k+1)}$, where $B(a, b)$ denotes the beta function. Thus, we have

$$\begin{aligned} \mu &= \int_0^1 t f_\zeta(t)\,dt = \int_0^1 \frac{t^k(1-t)^{I-k}}{B(k, I-k+1)}\,dt \\ &= \frac{B(k+1, I-k+1)}{B(k, I-k+1)} = \frac{k \cdot \binom{I}{k}}{(k+1)\cdot\binom{I+1}{k+1}} = \frac{k}{I+1}, \end{aligned} \tag{3}$$

which indicates that $I = k/\mu - 1$. □

Remark. When $I < k$, a KMV sketch directly provides the true value of I with the number of random numbers in the sketch. Furthermore, the original estimator $\mathbb{E}[(k-1)/\zeta]$ in Definition 7 is unbiased, but with only asymptotic error bounds. In contrast, our proposed estimator in Lemma 2 utilizes Bernstein's inequality to establish a tighter confidence bound for arbitrary image sizes.

Theorem 1 (Bernstein Inequality [4]). *Let $X_1, X_2, \ldots, X_\theta$ be real-valued i.i.d. random variables such that $X_i \in [0, r]$, $\mathbb{E}[X_i] = \mu$, and $\mathrm{Var}[X_i] = \sigma^2$. Let $\overline{X}_\theta = \frac{1}{\theta}\sum_{i=1}^{\theta} X_i$ denote the empirical mean. With probability at least $1 - p$, we have*

$$|\overline{X} - \mu| \leq \sqrt{\frac{\sigma^2 \log(2/p)}{\theta}} + \frac{2r\log(1/p)}{\theta}.$$

Lemma 3. *For any $\delta, p \in (0, 1)$ and $u \in \mathcal{V}^*$, if $\theta = \Theta(\frac{n}{\delta k}\log\frac{1}{p})$, then $(1-\delta)\cdot I \leq \widetilde{I} \leq (1+\delta)\cdot I$ holds with probability at least $1 - p$.*

Proof. When $I < k$, the exact value of I will always be obtained, i.e., $\widetilde{I} = I$, and the lemma holds trivially. When $I \geq k$, let $\Delta\mu = |\widetilde{\mu} - \mu|$ and $\Delta I = |\widetilde{I} - I|$. According to Lemma 2, $I = \frac{k}{\mu} - 1$ and thus

$$\Delta I = \begin{cases} \frac{k}{\mu} - \frac{k}{\mu + \Delta\mu}, & \text{if } \mu < \widetilde{\mu}; \\ \frac{k}{\mu - \Delta\mu} - \frac{k}{\mu}, & \text{if } \mu \geq \widetilde{\mu}. \end{cases}$$

For any $\delta \in (0, 1)$, it follows that $\Delta I \leq \delta I$ if $\Delta\mu \leq \frac{\delta I k}{(I+1)(I+1+\delta I)}$. Furthermore, the variance of ζ is

$$\sigma^2 = \mathbb{E}[\zeta^2] - \mu^2 = \frac{k\cdot(k+1)}{(I+1)(I+2)} - \left(\frac{k}{I+1}\right)^2 = \frac{k(I-k+1)}{(I+1)^2(I+2)}.$$

According to Bernstein's inequality, for any $p \in (0, 1)$,

$$\Delta\mu \leq \sqrt{\frac{\sigma^2 \log(2/p)}{\theta}} + \frac{2\log(1/p)}{\theta}$$

holds with probability at least $1 - p$. Thus, we ensure $\Delta I \leq \delta I$ with probability at least $1 - p$ by setting θ such that

$$\sqrt{\frac{\sigma^2 \log(2/p)}{\theta}} + \frac{2\log(1/p)}{\theta} \leq \frac{\delta I k}{(I+1)(I+1+\delta I)}.$$

By solving the above inequality as a quadratic inequality in terms of $\sqrt{\theta}$, $\Delta I \leq \delta I$ holds with probability at least $1 - p$ if

$$\theta \geq \left(\sqrt{\tfrac{I-k+1}{I+2} \log \tfrac{2}{p}} + \sqrt{\tfrac{I-k+1}{I+2} \log \tfrac{2}{p} + 8\tfrac{\delta I(I+1)}{I+1+\delta I} \log \tfrac{1}{p}}\right)^2 \cdot \tfrac{(I+1+\delta I)^2}{4\delta^2 I^2 k}.$$

By simplifying the above inequality, we have $\theta = \Theta(\frac{I}{\delta k} \log \frac{1}{p})$. Since $I \leq n$ for any image, by setting $\theta = \Theta(\frac{n}{\delta k} \log \frac{1}{p})$, $\Delta I \leq \delta \cdot I$ holds with probability at least $1 - p$. □

Theorem 2. *Let $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$ for any $\epsilon \in (0,1)$, Algorithm 1 correctly answers an ϵ-approximate HUB query under degree centrality with probability at least $1 - p$.*

Proof. Let $\widetilde{N}(u)$ denote the estimated degree of node v through sketch propagation. Given any nodes $u \in S_0^+(v^\lambda)$ and $v \in S_\epsilon^-(v^\lambda)$,

$$\begin{aligned}\widetilde{N}(v) &\leq (1+\delta) \cdot N(v) \leq (1+\delta)(1-\epsilon) \cdot N(v^\lambda) \\ &\leq (1+\delta)(1-\epsilon) \cdot N(u) \leq \frac{(1+\delta)(1-\epsilon)}{1-\delta} \cdot \widetilde{N}(u)\end{aligned}$$

hold due to Lemma 3 and the definitions of $S_\epsilon^-(v^\lambda)$ and $S_0^+(v^\lambda)$. By setting $\delta < \frac{\epsilon}{2-\epsilon}$, we have $\widetilde{N}(v) < \widetilde{N}(u)$, i.e., the sketches rank v lower than u, with probability at least $1-p$. Taking the union bound over all nodes in $S_0^+(v^\lambda)$ and setting $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$, the sketches rank all nodes in $S_0^+(v^\lambda)$ higher than $v \in S_\epsilon^-(v^\lambda)$. Since there are at least $\lambda|V|$ nodes in $S_0^+(v^\lambda)$, v will not be included in the result S. Similarly, given any nodes $u \in S_0^-(v^\lambda)$ and $v \in S_\epsilon^+(v^\lambda)$, we have $\widetilde{N}(v) \geq (1-\delta) \cdot N(v) \geq (1-\delta)(1+\epsilon) \cdot N(v^\lambda) \geq (1-\delta)(1+\epsilon) \cdot N(u) \geq \frac{(1-\delta)(1+\epsilon)}{1+\delta} \cdot \widetilde{N}(u)$. By setting $\delta < \frac{\epsilon}{2+\epsilon}$, it holds that $\widetilde{N}(v) > \widetilde{N}(u)$, i.e. the sketches rank v higher than u, with probability at least $1-p$. Also taking the union bound over all nodes in $S_0^-(v^\lambda)$ and setting $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$, the sketches rank all nodes in $S_0^-(v^\lambda)$ lower than $v \in S_\epsilon^+(v^\lambda)$. Since there are at least $(1-\lambda)|V|$ nodes in $S_0^-(v^\lambda)$, v will be included in S. Finally, by taking the union bound over all nodes in $S_\epsilon^+(v^\lambda)$ and $S_\epsilon^-(v^\lambda)$ and setting $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|^2}{p}) = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$, all nodes in $S_\epsilon^+(v^\lambda)$ are included in S, whereas all nodes in $S_\epsilon^-(v^\lambda)$ are excluded from S, with probability at least $1 - p$. □

Finally, the time complexity of *sketch propagation* is $O(\frac{n|\mathcal{E}^*|}{\epsilon} \log \frac{|V|}{p})$. The bottleneck of *sketch propagation* lies in the construction of KMV sketches, which propagates θ KMV sketches of sizes at most k over the matching graph with $|\mathcal{E}^*|$ edges. The space complexity of *sketch propagation* is $O(k\theta|\mathcal{V}^*| + |\mathcal{E}^*|)$. Compared to exact algorithms, the additional space is used to store KMV sketches.

Algorithm 2: `OptimizedReceive`

```
   Input: Node $u$, arrays of KMV sketches $\mathcal{K}_f, \mathcal{K}_b$
   Output: If sketch propagation can be terminated
1  Receive($u, \mathcal{K}_b, +$);
2  $\widetilde{I}_f \leftarrow 0$ and $\widetilde{I}_b \leftarrow 0$;
3  for $t = 1$ to $\theta$ do
4      if $|\mathcal{K}_f[u][t]| = k$ then $\widetilde{I}_f \leftarrow \widetilde{I}_f + \frac{\max(\mathcal{K}_f[u][t])}{\theta}$;
5      else $\widetilde{I}_f \leftarrow \widetilde{I}_f + \frac{k}{(|\mathcal{K}_f[u][t]|+1)\cdot\theta}$;
6      if $|\mathcal{K}_b[u][t]| = k$ then $\widetilde{I}_b \leftarrow \widetilde{I}_b + \frac{\max(\mathcal{K}_b[u][t])}{\theta}$;
7      else $\widetilde{I}_b \leftarrow \widetilde{I}_b + \frac{k}{(|\mathcal{K}_b[u][t]|+1)\cdot\theta}$;
8  $\widetilde{I}_f \leftarrow k/\widetilde{I}_f - 1$ and $\widetilde{I}_b \leftarrow k/\widetilde{I}_b - 1$;
9  if $\widetilde{I}_f \geq \lambda|V|$ and $\widetilde{I}_b \geq N(q)$ then return TRUE;
10 return FALSE;
```

3.2 Early Termination for Personalized HUB

Given a query node q, a personalized HUB query can be answered directly using the estimated node degrees through *sketch propagation*. Specifically, if q is ranked higher than the $(1-\lambda)$-quantile node in terms of estimated degree, it returns TRUE for the personalized query, and FALSE otherwise. The following corollary is a direct extension of Theorem 2 for personalized queries.

Corollary 1. *Let $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$ for any $\epsilon \in (0,1)$, the sketch propagation framework correctly answers any personalized ϵ-approximate HUB query under degree centrality with probability at least $1 - p$.*

Nevertheless, we can further optimize personalized HUB query processing by terminating the *sketch propagation* early once we have determined that q is not a hub with high confidence. First, we give the following lemma to inspire the design of the early termination optimization.

Lemma 4. *Given a query node q, if there exists a node $u \in \mathcal{V}^*$ such that $I_f(u) \geq \lambda|V|$ and $I_b(u) > N(q)$, then the answer to the personalized HUB query is* FALSE.

Proof. For any node $u \in \mathcal{V}^*$, it is easy to see that any $v_1 \in \mathcal{I}_f(u)$ and $v_2 \in \mathcal{I}_b(u)$ must be neighbors in H. Hence, each node in $\mathcal{I}_f(u)$ has at least $I_b(u)$ neighbors. Since $I_b(u) > N(q)$ and $I_f(u) \geq \lambda|V|$, there are at least $\lambda|V|$ nodes with degrees higher than $N(q)$. □

Based on Lemma 4, we can terminate the *sketch propagation* when the estimations $\widetilde{I}_f(u)$ and $\widetilde{I}_b(u)$ of $I_f(u)$ and $I_b(u)$ for a node $u \in \mathcal{V}^*$ by KMV sketches satisfy $\widetilde{I}_f(u) \geq \lambda|V|$ and $\widetilde{I}_b(u) > N(q)$. Alg. 2 presents the procedure of `OptimizedReceive`, an optimized version of `Receive` at Line 23 in Alg. 1 with early termination. Specifically, `OptimizedReceive` returns whether the sketch propagation should end at node u. It first calls `Receive` in Line 23 of Alg. 1 to construct the KMV sketch for the backward image of u (Line 1) and then estimates $I_f(u)$ and $I_b(u)$ accordingly (Lines 2–8). Finally, it returns whether the estimated $\widetilde{I}_f(u)$ and $\widetilde{I}_b(u)$ have satisfied the early termination conditions (Lines 9–10).

Example 5. *Continuing with Example 4 where $\mathcal{K}(\mathcal{I}_b(v_0)) = \{0.1, 0.15\}$, the algorithm estimates $\widetilde{I}_b(v_0) = \frac{2}{0.15} - 1 \approx 12.3$. The size $\widetilde{I}_f(v_0) =$*

$\frac{2}{0.3} - 1 \approx 5.7$ is estimated from the sketch $\mathcal{K}(\mathcal{I}_f(v_0))$ in Fig. 2(a). Assuming $\lambda = 0.01$, $|V| = 500$, $N(q) = 9$, we have $\widetilde{I}_f(u_1) > 0.01 \cdot 500$ and $\widetilde{I}_b(u_1) > 9$, and thus the propagation can be terminated early.

4 EXTENSION TO H-INDEX

In this section, we extend the sketch propagation framework to process approximate HUB queries based on the h-index measure. To compute the h-index of each node, we should compute the degrees of its neighbors. However, KMV sketches can only estimate the neighborhood sizes of a node but fail to provide the degree estimations of its neighbors. Therefore, we propose a novel pivot-based algorithm to obtain the set of hubs in terms of h-index without explicitly calculating the h-index of every node in H. Before delving into the algorithm, we first review the definition of *h-index* [28].

Definition 9 (H-index). *Given a node $u \in H$, the h-index of u is the largest integer $h(u)$ such that u has no fewer than $h(u)$ neighbors of degrees at least $h(u)$.*

4.1 Pivot-based Method for HUB

The basic idea of the pivot-based algorithm is to choose a random pivot node $u \in V$ and iteratively partition the nodes in H into two disjoint sets, Q_u^+ and Q_u^-, based on their h-index values in comparison with $h(u)$, i.e., $Q_u^+ = \{v \in V \mid h(v) \geq h(u)\}$ and $Q_u^- = \{v \in V \mid h(v) < h(u)\}$. As such, we can decide that $Q_u^+ \subseteq S_0^+(v_\lambda)$ if $u \in S_0^+(v_\lambda)$ since all nodes in Q_u^+ have h-index values no less than $h(u)$ and thus are ranked higher than u, or $Q_u^- \cap S_0^+(v_\lambda) = \emptyset$ if $u \in S_0^-(v_\lambda)$ similarly. This partitioning strategy allows for efficient bisection when identifying hubs, i.e., $S_0^+(v^\lambda)$. If $u \in S_0^+(v_\lambda)$, we can add all nodes in Q_u^+ to the result set and exclude them from consideration in subsequent iterations. In contrast, if $u \in S_0^-(v^\lambda)$, the nodes in Q_u^+ may contain those in both $S_0^+(v^\lambda)$ and $S_0^-(v^\lambda)$, which require further partitioning in subsequent iterations. Meanwhile, all nodes in Q_u^- must not be in $S_0^+(v^\lambda)$ and are excluded from consideration. The iterative process above proceeds until all nodes have been decided. Then, all nodes in $S_0^+(v^\lambda)$ are added to the result.

Pivot-based Partitioning Method. The key challenge lies in partitioning the nodes in H without explicitly computing the h-index of each node. In fact, we can compare the h-indexes of two nodes using only one of them. By the definition of h-index, for any two nodes $u, v \in V$, we have $h(v) \geq h(u)$ if and only if v has at least $h(u)$ neighbors in H with degrees larger than or equal to $h(u)$. Based on the above observation, we introduce a two-stage pivotal partitioning method for our pivot-based algorithm.

Stage (I): Estimate $h(u)$. We first employ the *sketch propagation* on the matching graph to estimate the degrees of all nodes in H. Subsequently, at each iteration, we scan the set of neighbors $\mathcal{N}(u)$ of a randomly chosen node u (obtained by a single DFS on the matching graph starting from u). Based on the degree estimations, we compute an approximate h-index $\widetilde{h}(u)$ of node u.

Stage (II): Estimate Q_u^+ and Q_u^-. Once we have the approximate h-index $\widetilde{h}(u)$, we proceed to eliminate the nodes in H with degrees smaller than $\widetilde{h}(u)$. Next, we perform *sketch propagation* on the subgraph of the matching graph that only includes the remaining nodes. The resulting KMV sketches estimate the degrees of nodes in the subgraph of H induced by the set of nodes with degrees greater than or equal to $\widetilde{h}(u)$ in the original graph H, denoted as $\widetilde{H}(u)$. Finally, we obtain Q_u^+ as the set of nodes with estimated degrees greater than $\widetilde{h}(u)$ in the induced subgraph of $\widetilde{H}(u)$, while the remaining nodes are added to Q_u^-.

Algorithm 3: Pivot-based Algorithm

```
Input: KG $\mathcal{G}$, meta-path $\mathcal{M}$, quantile parameter $\lambda$, number of
       propagations $\theta$, KMV sketch size $k$
Output: The set of nodes $S$ for (approximate) HUB query
1  forall $u \in \mathcal{V}^*$ do
2      for $t = 1$ to $\theta$ do
3          $\mathcal{K}_f[u][t] \leftarrow \emptyset$ and $\mathcal{K}_b[u][t] \leftarrow \emptyset$;
4          if $u \in \mathcal{V}_0$ do add $r(u) = \mathsf{Rand}(0,1)$ to $\mathcal{K}_f[u][t]$;
5  $\widetilde{N} \leftarrow \mathtt{Propagate}(\mathcal{K}_f, \mathcal{K}_b)$;
6  $Q \leftarrow V$ and $S \leftarrow \emptyset$;
7  while $|Q| > 0$ do
8      Sample a random node from $Q$ as the pivot node $u$;
9      Sort the neighbors $\mathcal{N}(u)$ of $u$ in descending order by $\widetilde{N}$;
10     Initialize $\widetilde{h}(u) \leftarrow 0$;
11     forall $v \in \mathcal{N}(u)$ do
12         if $\widetilde{N}(v) \geq \widetilde{h}(u)$ then $\widetilde{h}(u) \leftarrow \widetilde{h}(u) + 1$;
13     forall $v \in \mathcal{V}^*$ do
14         for $t = 1$ to $\theta$ do
15             $\mathcal{K}_f[v][t] \leftarrow \emptyset$ and $\mathcal{K}_b[v][t] \leftarrow \emptyset$;
16             if $v \in \mathcal{V}_0$ and $\widetilde{N}(v) \geq \widetilde{h}(u)$ then
17                 Add $r(v) = \mathsf{Rand}(0,1)$ to $\mathcal{K}_f[v][t]$;
18     $\widetilde{h} \leftarrow \mathtt{Propagate}(\mathcal{K}_f, \mathcal{K}_b)$;
19     Initialize $Q_u^+ \leftarrow \emptyset$ and $Q_u^- \leftarrow \emptyset$;
20     forall $v \in Q$ do
21         if $\widetilde{h}(v) \geq \widetilde{h}(u)$ then add $v$ to $Q_u^+$ else add $v$ to $Q_u^-$;
22     if $|Q_u^+| + |S| \leq \lambda|V|$ then
23         $Q \leftarrow Q_u^-$ and $S \leftarrow S \cup Q_u^+$;
24         if $|S| \leq \lambda|V|$ then $S \leftarrow S \cup \{u\}$;
25     else
26         $Q \leftarrow Q_u^+$;
27 return $S$;
```

Algorithm 3 depicts the procedure of our pivot-based algorithm. It also receives a KG $\mathcal{G}$, a meta-path $\mathcal{M}$, the quantile parameter λ, the number of propagations θ, and the KMV sketch size k as input, and returns a set of nodes S to answer the (approximate) HUB query based on h-index. It first calls the same `Propagate` function as Algorithm 1 over the arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ of KMV sketches to estimate the degree of each node in H (Lines 1–5). Note that the estimated degrees will be reused across all iterations for partitioning. The iterative process proceeds until Q, which contains the nodes to be partitioned in each iteration, is empty (Lines 7–26). In each iteration, it first randomly chooses a pivot node u from Q and estimates its h-index $\widetilde{h}(u)$ (Lines 8-12). Then, it re-initializes arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ of KMV sketches for the nodes with degrees greater than $\widetilde{h}(u)$ (Lines 13–17). After that, the `Propagate` function is performed over $\mathcal{K}_f$ and $\mathcal{K}_b$ to estimate the number of neighbors with degrees greater than $\widetilde{h}(u)$ for each node in Q (Line 18). Accordingly, it partitions Q into Q_u^+ and Q_u^- according to $\widetilde{h}$ (Lines 19–21) and updates the result set S and the candidate node set Q based on Q_u^+ and Q_u^- (Lines 22–26). Finally, when Q is empty, the result set S is returned (Line 27). The ties are broken arbitrarily.

Theoretical Analysis. Next, we prove that Algorithm 3 returns the correct answer to an approximate HUB query based on h-index w.h.p. We first give the following lemma, which states that Algorithm 3 provides a concentrated estimation of $h(u)$ for the randomly chosen pivot node u in each iteration.

LEMMA 5. *For any $\delta, p \in (0,1)$, if $\theta = \Theta(\frac{n}{\delta k} \log \frac{|V|}{p})$, we have $(1-\delta) \cdot h(u) \le \widetilde{h}(u) \le (1+\delta) \cdot h(u)$ with probability at least $1-p$ for the pivot node u in each iteration.*

PROOF. Let us order the nodes in $\mathcal{N}(u)$ according to their degrees in H descendingly and denote v_i as the i-th neighbor of u. Then, we have $N(v_i) \ge h(u)$ for $0 \le i \le h(u)-1$ and $N(v_i) \le h(u)$ for $h(u) \le i \le N(u)-1$. By combining Lemma 3 with the union bound over V, when $\theta = \Theta(\frac{n}{\delta k} \log \frac{|V|}{p})$ in the first stage, we have $\widetilde{N}(v_i) \ge (1-\delta)N(v_i) \ge (1-\delta)h(u)$ for $0 \le i \le h(u)-1$ with probability at least $1-p$. Thus, node u has $h(u) \ge (1-\delta)h(u)$ neighbors with estimated degrees of at least $(1-\delta)h(u)$. Thus, $\widetilde{h}(u) \ge (1-\delta)h(u)$ with probability at least $1-p$. Similarly, we have $\widetilde{N}(v_i) \le (1+\delta)N(v_i) \le (1+\delta)h(u)$ for $i \ge h(u)$ with probability at least $1-p$. Thus, node u has no more than $h(u)$ neighbors with estimated degrees greater than $(1+\delta)h(u)$, and $\widetilde{h}(u) \le (1+\delta)h(u)$ with probability at least $1-p$. □

The following lemma indicates that the pivot-based partitioning method divides candidate nodes into ϵ-separable sets.

LEMMA 6. *For any $\delta' \in (0,1)$ and pivot node u, by setting $\theta = \Theta(\frac{n}{\delta' k} \log \frac{|V|}{p})$, we have $S_{\delta'}^-(u) \subseteq Q_u^-$ and $S_{\delta'}^+(u) \subseteq Q_u^+$ with probability at least $1-p$.*

PROOF. For any node $v \in S_{\delta'}^-(u)$, let $\mathcal{N}_u(v)$ denote the neighbors of v in $\widetilde{H}(u)$ and $N_u(v) = |\mathcal{N}_u(v)|$. Since $h(v) < (1-\delta')h(u)$, v has at most $(1-\delta')h(u)$ neighbors with degrees larger than or equal to $(1-\delta')h(u)$. For each neighbor v' of v with $N(v') < (1-\delta')h(u)$, when $\theta = \Theta(\frac{n}{\delta k} \log \frac{|V|}{p})$, $\widetilde{N}(v') < (1+\delta)N(v') < (1+\delta)(1-\delta')h(u) < \frac{(1+\delta)(1-\delta')}{1-\delta}\widetilde{h}(u)$. By setting $\delta < \frac{\delta'}{2-\delta'}$, we have $\widetilde{N}(v') < \widetilde{h}(u)$, i.e., v' is filtered out in *Stage (I)*, with probability at least $1-p$. Thus, we have $N_u(v) < (1-\delta')h(u)$ with probability at least $1-p$. Let $\widetilde{N}_u(v)$ denote the estimation of $N_u(v)$ in *Stage (II)* of pivot-based partitioning. We have $\widetilde{N}_u(v) \le (1+\delta)N_u(u)$ with probability at least $1-p$, and thus $\widetilde{N}_u(v) \le (1+\delta)N_u(v) < (1+\delta)(1-\delta')h(u) \le \frac{(1+\delta)(1-\delta')}{1-\delta}\widetilde{h}(u) \le \widetilde{h}(u)$, i.e., v is added to Q_u^- with probability at least $1-p$. Thus, by setting $\theta = \Theta(\frac{n}{\frac{\delta'}{2-\delta'}k} \log \frac{|V|}{p}) = \Theta(\frac{n}{\delta' k} \log \frac{|V|}{p})$, we have $S_{\delta'}^-(u) \subseteq Q_u^-$. Similarly, we also get $S_{\delta'}^+(u) \subseteq Q_u^+$ with probability at least $1-p$ when $\theta = \Theta(\frac{n}{\delta' k} \log \frac{|V|}{p})$. □

Subsequently, we can formally analyze the correctness of the pivot-based algorithm for processing ϵ-approximate HUB queries based on the h-index in the following theorem.

THEOREM 3. *Let $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$. Algorithm 3 returns the answer to an ϵ-approximate HUB query based on the h-index correctly with probability at least $1-p$.*

PROOF. We prove the theorem by examining three different cases for the ranges of the h-index $h(u)$ for the pivot node u. For each case, we show the correctness of the first iteration in Algorithm 3, while subsequent iterations are proven by induction. By selecting $\delta' < \frac{\epsilon}{2}$ as Lemma 6, we have:

- **Case 1 ($h(u) > (1+\frac{\epsilon}{2})h(v^\lambda)$):** Each v with $h(v) \le h(v^\lambda)$ will be added to Q_u^-. Thus, $|Q_u^-| \ge (1-\lambda)|V|$. Therefore, Q_u^- will be used as the candidate set Q in the next iteration, and Q^+, which does not contain any node in $S_\epsilon^-(v^\lambda)$, will be added to S.
- **Case 2 ($h(u) < (1-\frac{\epsilon}{2})h(v^\lambda)$):** Each v with $h(v) \ge h(v^\lambda)$ will be added to Q_u^+. Thus, $|Q_u^+| \ge \lambda|V|$. Therefore, Q^+, which contains all nodes in $S_\epsilon^+(v^\lambda)$, will be used as the candidate set Q in the next iteration.
- **Case 3 ($(1-\frac{\epsilon}{2})h(v^\lambda) < h(u) < (1+\frac{\epsilon}{2})h(v_\lambda)$):** Any v with $h(v) \ge (1+\epsilon)h(v^\lambda)$ will be added to Q_u^+. And any v' with $h(v') \le (1-\epsilon)h(v^\lambda)$ will be added to Q_u^-. Thus, Q_u^+, which contains all nodes in $S_\epsilon^+(v^\lambda)$, will be added to S or used as the candidate set Q in the next iteration; whereas Q_u^-, which contains all nodes in $S_\epsilon^-(v^\lambda)$, will be eliminated or used as the candidate set Q for the next iteration.

Considering all of the above cases, it follows that, for any randomly chosen pivot u, none of the nodes in $S_\epsilon^-(v^\lambda)$ is added to S. In contrast, all nodes in $S_\epsilon^+(v^\lambda)$ have a probability of at least $1-p$ to be either added to S or retained for the subsequent iteration. Thus, we prove the theorem by applying the union bound over all iterations. □

Finally, the amortized time complexity of the pivot-based algorithm is $O(\frac{n|\mathcal{E}^*|}{\epsilon} \log^2 \frac{|V|}{p})$ since it invokes the sketch propagation framework $O(\log|V|)$ times in amortization. The space complexity of the pivot-based algorithm is $O(k\theta|\mathcal{V}^*| + |\mathcal{E}^*|)$, which is equal to that of Algorithm 1.

4.2 Early Termination for Personalized HUB

Given a query node q, the personalized HUB query based on the h-index can also be answered by performing the pivot-based partitioning method with q as the pivot node to obtain Q_u^+ and returning FALSE if $|Q_u^+| > \lambda|V|$ or TRUE otherwise. Following Lemma 6, the following corollary indicates that the above algorithm provides correct answers for approximate personalized HUB queries w.h.p.

COROLLARY 2. *Let $\theta = \Theta(\frac{n}{\epsilon k} \log \frac{|V|}{p})$ for any $\epsilon \in (0,1)$. Algorithm 3 correctly answers an ϵ-approximate personalized HUB query based on h-index with probability at least $1-p$.*

Early Termination. An early termination optimization can also accelerate personalized HUB queries based on h-index.

LEMMA 7. *Given a query node q and the quantile parameter λ, if there exists a node $u \in \mathcal{V}^*$ such that $I_f(u) > h(q)$ and $I_b(u) \ge$*

Table 1: Statistics of KGs used in the experiments, where $|\mathcal{L}(\mathcal{V})|$ and $|\mathcal{L}(\mathcal{E})|$ are the numbers of node and edge types and $|\mathbb{M}|$ is the number of evaluated meta-paths.

Dataset	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{L}(\mathcal{V})\rvert$	$\lvert\mathcal{L}(\mathcal{E})\rvert$	$\lvert\mathbb{M}\rvert$
IMDB	21.4K	86.6K	4	6	679
ACM	10.9K	548K	4	8	20
DBLP	26.1K	239.6K	4	6	48
PubMed	63.1K	236.5K	4	10	172
FreeBase	180K	1.06M	8	36	151
Yelp	82.5K	29.9M	4	4	2230
BPM	1.2M~19.2M	120M~1.92B	2	1	1

$\max(h(q), \lambda|V|)$, *then the answer to the personalized HUB query must be* FALSE.

Proof. Each pair of nodes in $\mathcal{I}_b(u)$ and $\mathcal{I}_f(u)$ are neighbors of each other in H according to the proof of Lemma 4. Thus, each node $v_1 \in \mathcal{I}_b(u)$ has at least $I_f(u)$ neighbors with degrees larger than or equal to $I_b(u)$. Since $I_f(u) > h(q)$ and $I_b(u) \geq \max(h(q), \lambda \cdot |V|)$, there exist at least $I_b(u) \geq \lambda|V|$ nodes with h-indexes greater than or equal to $h(q)$. Therefore, q is not a hub based on the h-index. □

Based on Lemma 7, the sketch propagation can be terminated during *Stage (I)* of the pivot-based partitioning method. Specifically, we use the KMV sketches to estimate $I_f(u)$ and $I_b(u)$ for each $u \in \mathcal{V}^*$. If there exists a node $u \in \mathcal{V}^*$ such that $\widetilde{I_f}(u) > N(q)$ and $\widetilde{I_b}(u) \geq \max(N(q), \lambda|V|)$, the algorithm can be terminated early since $N(q)$ is an upper bound of $h(q)$. Once we obtain $\widetilde{h}(q)$ after *Stage (I)*, the algorithm can also be terminated without entering *Stage (II)* if there exists a node $u \in \mathcal{V}^*$ such that $\widetilde{I_f}(u) > \widetilde{h}(q)$ and $\widetilde{I_b}(u) \geq \max(\widetilde{h}(q), \lambda|V|)$.

5 EXPERIMENTS

5.1 Experimental Setup

Datasets. We conduct our experiments on six real-world KGs. The statistics of these KGs are reported in Table 1. IMDB is a KG about actors and directors of movies. ACM and DBLP are two academic KGs denoting the co-authorships between researchers through papers and publishing venues. PubMed is a biomedical KG representing the relationships between genes, chemicals, and diseases of different species. FreeBase is extracted from a general-purpose KG developed by Google[1]. Yelp is a KG built from user ratings and comments on businesses at different locations.

BPM represents a series of large synthetic datasets generated by Bipartite Preferential Model [19, 20, 51], which are used for scalability test. Specifically, Bipartite Preferential Model receives three parameters n, m, p and generates a bipartite graph with two types of nodes T_1, T_2. The first parameter n denotes the number of nodes of type T_1. For each node v of type T_1, the model connects v to m nodes of type T_2. When generating each edge of node v, the model will add a new node of T_2 and connect v to the newly added node with probability p; and with probability $1-p$, the model will connect v to an existing node of type T_2 by preferential attachment. In this experiments, we consider the query meta-path $T_1 \longrightarrow T_2$.

[1]https://developers.google.com/freebase

Query Generation. For each dataset, we use AnyBURL [29] to extract a set of candidate meta-paths $\mathbb{M}$. We set the confidence threshold to 0.1 for AnyBURL and take the snapshot at ten seconds. Note that AnyBURL can extract meta-paths with extra node constraints, resulting in a large number of potential meta-paths for validation. This makes efficient processing of HUB queries even more necessary. $|\mathbb{M}|$ in Table 1 shows the number of meta-paths found on each dataset. For personalized HUB queries, we randomly sample 10 nodes from $\mathcal{V}_0$ as query nodes for each meta-path.

Compared Methods. To the best of our knowledge, there has not yet been any prior study on the HUB problem. Therefore, we only compare our sketch propagation-based algorithms with the exact algorithms based on the generic worst-case optimal join.

- ExactD and ExactH are the exact algorithms (Sect. 2.2) to get hubs by computing the degrees and h-indexes of all nodes in H.
- GloD and PerD apply the sketch propagation framework (Sect. 3.1) to estimate the degrees of all nodes in H. GloD returns the set of hubs based on estimated degrees, whereas PerD decides whether a query node q is a hub.
- PerD$^+$ optimizes PerD with early termination (Sect. 3.2).
- GloH and PerH apply the pivot-based algorithm (Sect. 4.1) to find the hubs based on estimated h-indexes. GloH recursively partitions the nodes until all hubs are found, whereas PerH uses a query node q as the pivot to check whether it is a hub.
- PerH$^+$ optimizes PerH with early termination (Sect. 4.2).

Parameter Settings. By default, we set $\lambda = 0.05$ to find the hubs as the top 5% of the nodes with the highest degrees or h-indexes in a relational graph. Our results in Sect. 5.3 will show that h-index-based HUB queries are well approximated using smaller values of θ and k compared to degree-based HUB queries. Hence, for algorithms targeting degree-based HUB queries (GloD, PerD, PerD$^+$), we set $\theta = 8$ and $k = 32$ by default, while for algorithms targeting h-index-based HUB queries (GloH, PerH, PerD$^+$), we set $\theta = 8$ and $k = 4$. These settings also demonstrate that our proposed methods produce accurate results in practice with significantly less stringency than those outlined in the worst-case analysis.

Evaluation Metrics. We evaluate the quality of the hubs each algorithm returns for global HUB queries (GloD and GloH) with the F1-score. Our algorithms return a node set S containing $\lambda \cdot |V|$ nodes. However, there might be more than $\lambda \cdot |V|$ nodes satisfying the criteria of Problem 1 due to the tied values with v^λ. As such, we define the *precision* as $|S \cap S^*|/|S|$ and the *recall* as $|S \cap S^+|/|S^+|$ for the computation of F1-scores. Here, S^* represents the set of nodes with centralities at least the same as that of v^λ, and S^+ represents the set of nodes with strictly higher centralities than v^λ. In effect, we exclude nodes with centralities equal to v^λ in the recall calculation. To assess the effectiveness of PerD, PerD$^+$, PerH, and PerH$^+$ for personalized HUB queries, we record their accuracy in determining whether a query node q has equal or greater centrality compared to v^λ. We report the average F1 and accuracy scores across all meta-paths in Figs. 5 and 6. For efficiency evaluation, we report the average running time of each algorithm to process a HUB query.

Environment. All experiments were carried out on a Linux server with AMD EPYC 7643 CPU and 256 GB memory running Ubuntu

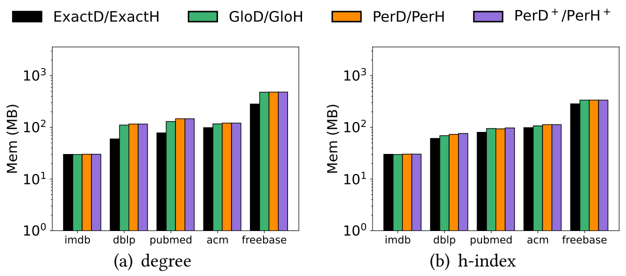


Figure 3: Memory consumption of different methods across datasets.

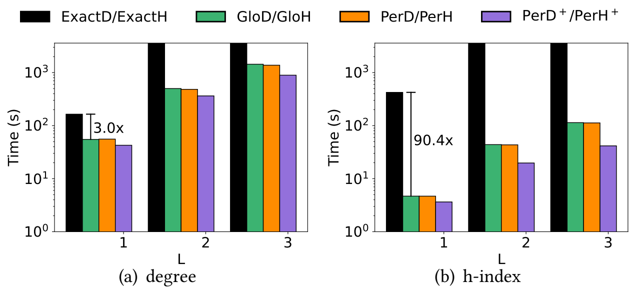


Figure 4: Running times of different algorithms on `Yelp` for meta-paths with varying length L.

20.04. All algorithms were implemented in C++ with `-O3` optimization and executed in a single thread. The source code and data are publicly available [2].

5.2 Efficiency Evaluation

In Table 2, we present the execution time per query for each method, along with its speedup ratios compared to the corresponding exact method across five datasets. We report the average time to construct the matching graph, which is nearly negligible compared to the running time of each algorithm. Based on these results, we make the following observations.

First, sketch propagation based algorithms are consistently more efficient than exact baselines across all datasets except `IMDB` and provide more than 10x speedups for degree-based global queries (on `FreeBase`, GloD is 29x faster than ExactD) and up to two orders of magnitude speedups for h-index-based global queries (on `PubMed`, GloH is 135.7x faster than ExactH). The speedup over degree-based global queries is smaller than h-index-based queries because h-index-based queries are accurately approximated with smaller k and θ as discussed in Sect. 5.3. GloD and GloH are slightly slower than ExactD and ExactH for degree-based queries on `IMDB` since `IMDB` is very small, where the worst-case optimal join is performed quickly over the matching graph, whereas our methods require additional costs for sketch construction and degree estimation. Nevertheless, GloD answers HUB queries on `IMDB` in 1.3ms.

Second, the algorithms for personalized queries, i.e., PerD$^+$ and PerH$^+$, benefit from early termination optimization and achieve more than 2x additional speedups over PerD and PerH. For degree-based personalized queries, backward propagation, which can be early terminated, is more time-consuming than forward propagation since forward sketches usually contain fewer random numbers than backward sketches.

We then report the memory consumption of different methods in Figure 3. We can observe from Figure 3(a) that GloD, PerD and PerD$^+$ take relatively more memory than the exact algorithm ExactD in order to maintain KMV sketches for images. Nevertheless, our methods take at most 1.93 times memory than ExactD (on `DBLP`). On the other hand, the gap between memory consumption for GloH, PerH, PerH$^+$ and the memory consumption of ExactH is marginal since h-index based queries require less KMV sketches for accurate estimation. Specifically, our methods take at most 1.21 times memory than ExactH (on `PubMed`).

Varying length L of meta-paths. Fig. 4 reports the time consumption of different methods on `Yelp` varying the length L of query meta-paths. We can observe that L has a significant impact over the efficiency of all methods. Specifically, exact methods ExactD and ExactH can only handle query meta-paths of length $L = 1$ while exceed the 1 hour limit per meta-path when $L = 2, 3$. On the other hand, our proposed methods process all queries within the limit.

5.3 Effectiveness Evaluation

Results for Degree-based HUB Queries. Figs. 5(a)–5(e) illustrate the effectiveness of GloD, PerD, and PerD$^+$ on each dataset by varying the parameter λ. We observe that GloD achieves F1-scores of more than 0.85 across all datasets for global HUB queries, and PerD and PerD$^+$ consistently achieve at least 0.98 accuracy for personalized HUB queries. Moreover, GloD provides more accurate answers with increasing λ and achieves an F1-score of over 0.9 when $\lambda = 0.05$. The reason is that for a larger λ, the $(1-\lambda)$-quantile node v^λ has a smaller degree, and the HUB queries apply less strict requirements to the hubs, which are more easily approximated.

Figs. 5(f)–5(j) show the effectiveness of GloD, PerD, and PerD$^+$ on each dataset with varying the parameter k. First, GloD, PerD, and PerD$^+$ all provide better results with increasing k. Larger KMV sketches allow the sketch propagation-based methods to approximate HUB queries with higher F1-scores or accuracy due to more accurate degree estimations. Second, personalized HUB queries are accurately approximated by PerD and PerD$^+$, even with relatively smaller values of k. This is because personalized HUB queries only require comparing the degrees of q and v^λ, which is relatively easy due to the significant difference between the degrees of q and v^λ. Third, the gap between the accuracy of PerD$^+$ and PerD is marginal, even when $k = 2$ (with a maximum gap of 0.044 on `DBLP`), and narrows further with increasing k. This is because a larger k leads to more accurate estimations of image sizes, thus reducing the chance of false early termination.

Figs. 5(k)–5(o) show the effectiveness of GloD, PerD, and PerD$^+$ on each dataset by varying the parameter θ. First, we still observe that GloD returns monotonically better results with increasing θ since HUB queries are better approximated with more KMV sketches. We also observe that GloD is less sensitive to θ than k. This is attributed to the default setting of $k = 32$, which already provides a reasonably accurate approximation. For personalized

[2] https://anonymous.4open.science/r/HUB-F5CF/readme.md

Table 2: Results for the efficiencies of different algorithms. For ExactD and ExactH, the running time on each dataset is reported. For GloD,PerD$^+$,GloH, and PerH$^+$, the running time, as well as the speedup ratios over ExactD and ExactH, are reported. The average time to construct the matching graph $\mathcal{G}^*$ is reported as $T(\mathcal{G}^*)$.

Centrality	**Degree**							**H-index**							$T(\mathcal{G}^*)$
Method	ExactD	GloD		PerD		PerD$^+$		ExactH	GloH		PerH		PerH$^+$		
IMDB	0.0009s	0.0013s	0.66x	0.0016s	0.55x	0.0017s	0.53x	0.0013s	0.001s	1.23x	0.0015s	0.82x	0.0015s	0.81x	0.88ms
ACM	4.77s	1.77s	2.7x	2.04s	2.34x	0.81s	5.91x	8.01s	0.19s	41.7x	0.25s	32x	0.076s	105x	4.18ms
DBLP	7.95s	0.63s	12.59x	0.67s	11.79x	0.27s	29x	14.59s	0.23s	64.32x	0.1s	148x	0.049s	298x	3.58ms
PubMed	5.96s	0.45s	13.24x	0.35s	17.05x	0.13s	45x	12.11s	0.09s	135.7x	0.056s	215x	0.032s	381x	8ms
FreeBase	25.15s	0.87s	29x	0.92s	27.4x	0.41s	60.9x	50.5s	0.26s	191x	0.19s	268x	0.13s	379x	26ms

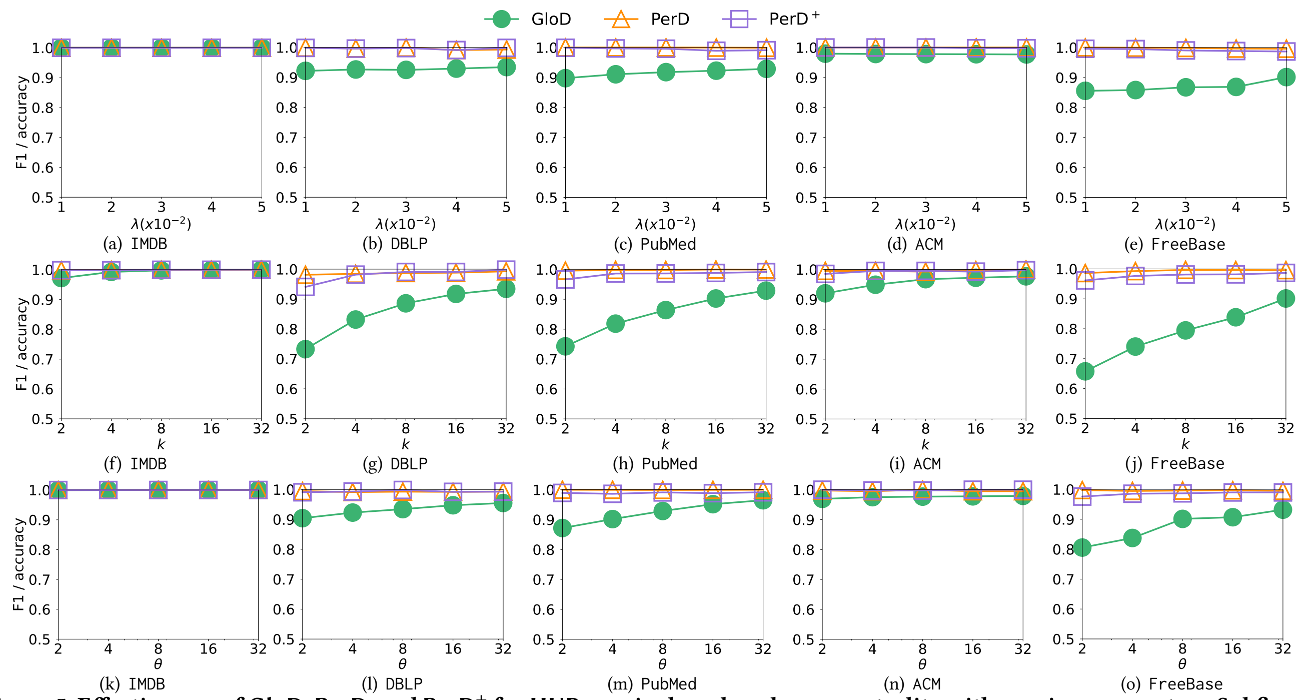


Figure 5: Effectiveness of GloD, PerD, and PerD$^+$ for HUB queries based on degree centrality with varying parameters. Subfigures (a)-(e): varying parameter λ. Subfigures (f)-(j): varying parameter k. Subfigures (k)-(o): varying parameter θ.

HUB queries, PerD and PerD$^+$ consistently achieve an accuracy of at least 0.95 across all datasets, regardless of changes in θ.

Results for HUB Queries based on H-index. The effectiveness results of GloH, PerH, and PerH$^+$ are presented in Fig. 6. Similar observations can be made for the methods for h-index-based HUB queries as those for degree-based HUB queries. Furthermore, we note that GloH achieves a higher F1-score for h-index-based HUB queries compared to GloD in the same parameter settings. This is because the range of h-index values is much smaller than that of degrees, resulting in a larger number of tied h-index values. By excluding nodes with tied values to v^λ from the recall evaluation, h-index-based HUB queries become much easier to approximate.

Summary. GloD, PerD, and PerD$^+$ achieve F1-scores or accuracy of above 0.9 across all datasets when $k = 32$ and $\theta = 8$ for degree-based HUB queries with $\lambda = 0.05$. Similarly, for h-index-based HUB queries, GloH, PerH, and PerH$^+$ achieve F1 scores or accuracy of above 0.9 across all datasets when $k = 4$ and $\theta = 8$. All of these results confirm the effectiveness of our proposed methods.

5.4 Influence Analysis

In this section, we further verify the effectiveness of HUB on influence maximization (IM) over relational graphs. We adopt the weighted cascade model for influence estimation [8, 17].

Fig. 7 reports the ratio between influences of hubs revealed by HUB methods (ExactD, GloD, ExactH and GloH) and influences achieved by the greedy framework [26]. First, we can observe that HUB methods (GloD and GloH) based on *sketch propagation* consistently achieve more than 85% of influences achieved by the greedy framework, which verifies the effectiveness of our methods in IM over relational graphs. Besides, GloD and GloH often achieve slight higher influences than ExactD and ExactH, which reveal hubs according to centralities exactly. The reason is that hubs discovered according to centralities exactly tends to cluster within large cliques and thus have severe overlaps in their influences.

5.5 Scalability

We further test the scalability of our methods over large synthetic datasets BPM. Fig. 8 reports the time and memory consumption of each method, except ExactD and ExactH which exceed the 1 hour limit even on the smallest BPM dataset. We can observe that all our methods based on *sketch propagation* scales linearly with the size

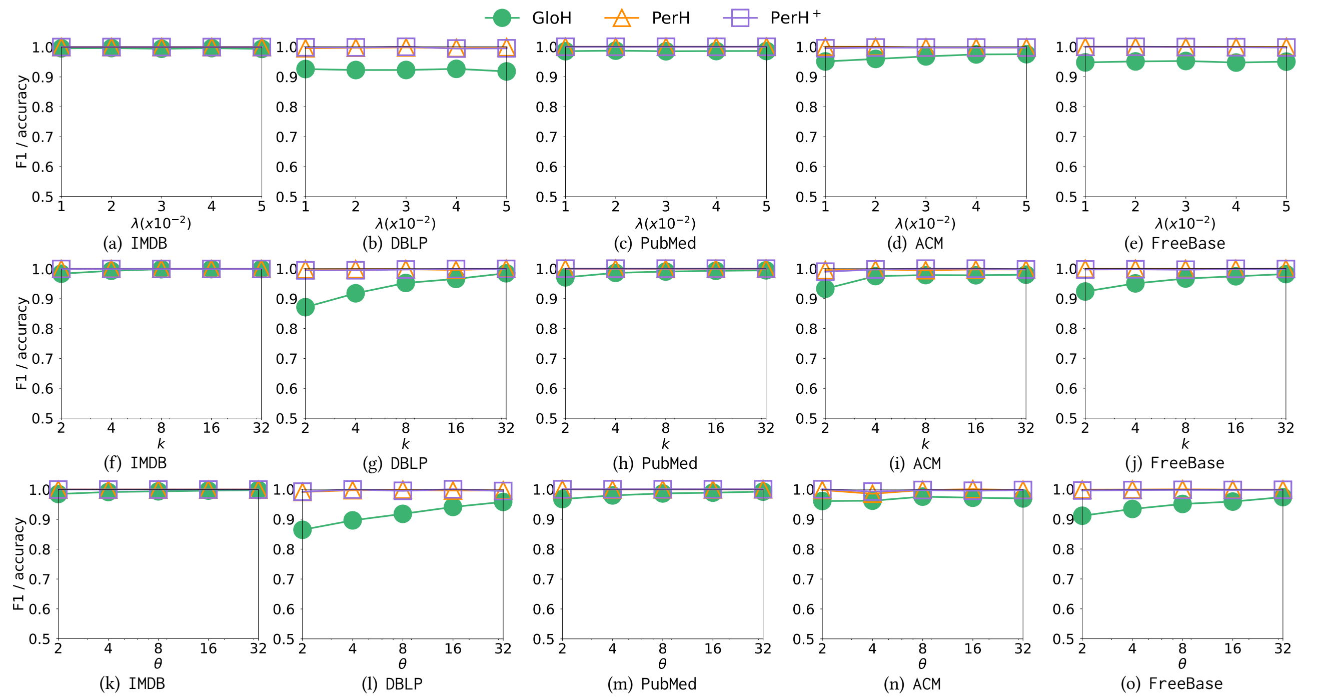


Figure 6: Effectiveness of GloH, PerH, and PerH$^+$ for HUB queries based on h-index with varying parameters. Subfigures (a)-(e): varying parameter λ. Subfigures (f)-(j): varying parameter k. Subfigures (k)-(o): varying parameter θ.

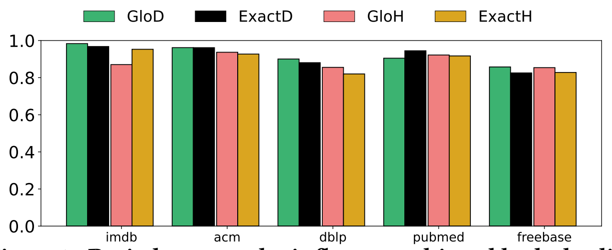


Figure 7: Ratio between the influence achieved by hubs discovered by HUB methods and the influence achieved by greedy framework.

of the datasets and can handle queries on BPM datasets with upto 192M nodes and 1.92B edges within 2552 seconds for degree based queries and 258 seconds for h-index based queries.

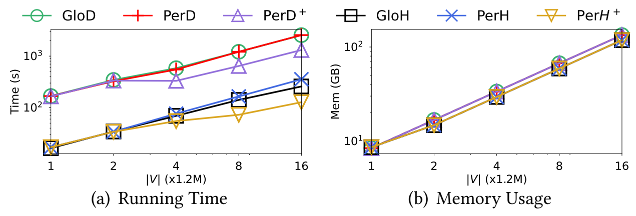


Figure 8: The running time and memory consumption of our methods on BPM.

6 RELATED WORK

Hubs in Hidden Networks. Previous work has extensively explored the problem of finding hubs in hidden networks, where the existence of any edge can only be decided via *probe operations*. Tao et al. [43] proposed two instance-optimal exact algorithms over two kinds of probing strategies of hidden networks. Wang et al. [46] extended this problem to include group testing, whereby probes can verify edge connections between multiple nodes. Sheng et al. [34] proposed a sampling algorithm for hub queries, which Cao et al. [7] subsequently improved. Strouthopoulos and Papadopoulos [39] studied the discovery of k-cores in hidden networks. However, all of these algorithms face two notable challenges when applied to HUB queries. First, they focus on bipartite networks and potentially take $O(|V|^2)$ probe operations when handling relational graphs. Second, the probe operations are time-consuming for our problem because they involve many breadth-first search (BFS) operations on the matching graph.

Xirogiannopoulos et al. [51] construct compact representations of hidden networks extracted by *path joins* in relational databases, which are analogous to meta-paths in KGs. However, this work is orthogonal to ours, while our sketch propagation framework is applicable to the compact structure of [51]. In addition, our exact algorithms that leverage worst-case optimal joins for efficient neighbor computation are aligned with the future direction for efficiency improvements suggested in [51].

Meta-paths in KGs. Meta-paths constitute a prominent approach to extracting information from KGs and lend a fundamental concept to the design of node similarity measures. Measures such as Pathsim [41], Joinsim [50], RelSim [45], and Hetesim [35], use meta-paths to quantify node similarity and play a crucial role in various KG analysis tasks. For instance, Pathsim evaluates the similarity between nodes in a heterogeneous network as the number of matching meta-path instances connecting them. Joinsim is a variant of Pathsim that satisfies the triangle inequality, facilitating efficient discovery of top-k similarity node pairs. Meta-paths also find applications in community search within KGs [14, 25, 52, 56]. Fang et al. [14] proposed the $(k, \mathcal{M})$-core model while Yang et al. [52] introduced the $(k, \mathcal{M})$-Btruss and $(k, \mathcal{M})$-Ctruss models for community search in KGs, which correspond to a k-core and a k-truss in the relational graph derived using $\mathcal{M}$, respectively. Additionally, meta-paths have been used in KG-based recommender systems [21, 37, 53, 54].

Nevertheless, even though relational graphs have been used in various scenarios, the fundamental problem of hub discovery has received little attention. To the best of our knowledge, ours is the first systematic investigation of efficient hub queries over relational graphs derived from KGs.

7 CONCLUSION

We introduced and investigated the problem of finding hubs over relational graphs (HUB), based on degree or h-index. We proposed a sketch propagation framework for approximate degree-based HUB queries and a pivot-based algorithm that extends the framework to h-index-based HUB queries. We also considered personalized HUB queries based on both centrality measures and enhanced the efficiencies of the corresponding algorithms with early termination. Theoretically, sketch propagation and pivot-based algorithms could correctly answer approximate HUB queries with high probability. Extensive experimentation verified the effectiveness and efficiency of our proposals on real-world heterogeneous data.

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