Hub Discovery from Hidden Networks in Knowledge Graphs [Technical Report]

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ABSTRACT

How can we efficiently detect important nodes, or *hubs*, in a highly heterogeneous knowledge graph (KG), based on implicit connections in a *hidden network* induced by a meta-path? Unfortunately, materializing a large hidden network raises a prohibitive cost. In this paper, we propose a novel *sketch propagation* framework that discovers hubs without explicitly materializing a hidden network. We develop this framework for the *degree centrality* measure and extend it to that of *h-index*, i.e., the largest value h such that a node q has h neighbors each of degree at least h. Further, we devise efficient pruning techniques to accelerate *personalized* queries which ask whether a node q is a hub, under both measures. Through extensive experimentation, we confirm the efficacy and efficiency of our solutions, reaching orders of magnitude speedups compared to exact methods while consistently achieving accuracy beyond 90%.

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1 INTRODUCTION

Knowledge graphs (KGs) are networks that organize diverse, complex, and heterogeneous information. In such networks, a *meta-path* [12, 29, 41] is a generic path pattern that captures implicit connections between nodes. Such implicit connections induce a *hidden network* within a KG, which reveals valuable insights. For instance, the meta-path "(*author*) → (*paper*) ← (*author*)" in an academic knowledge graph such as DBLP[1] and MAKG[2] reveals a hidden academic co-authorship network that connects any two authors who have collaborated on at least one paper. Hidden networks drive the application of KGs in similarity measurement [26, 29, 33, 36], community search [12, 18, 28, 38], and recommendation [16, 27, 39, 40].

A *hub* is a network node that is central by some measure [8, 19]; hubs are useful in evaluating network robustness [22, 24, 30, 35] and information dissemination [7, 9, 21]. For example, hubs in a hidden academic co-authorship network represent central authors with high connectivity to others who can act as influential knowledge brokers in the academic community. It is therefore purposeful to discover hubs in a *hidden network* induced by a *meta-path*. Still, to our knowledge, the extensive literature on meta-path-based KG analytics has yet to address this fundamental problem.

In this paper, we investigate the problem of *hub discovery* (HUD) in hidden networks induced by meta-paths on knowledge graphs. We aim to find the set of nodes in the top-λ quantile of the hidden network induced by a given meta-path $\mathcal{M}$ in a KG $\mathcal{G}$, based on the measure of *degree centrality* or *h-index*. A naive solution to this problem would enumerate all instances of $\mathcal{M}$ in $\mathcal{G}$ to construct the hidden network H, compute the degrees or h-index values of all nodes in H, and sort them to answer HUD queries. However, such explicit hidden-network construction can be very time- and memory-intensive, especially for large KGs [37]. One alternative would be to traverse meta-path instances in the KG and find a node's hidden-network neighbors without explicitly materializing the hidden networks. Nevertheless, to find hubs exactly, we should access the neighborhood information of all nodes in the hidden graph and potentially traverse any edge involved in any meta-path instance up to $O(|V_{\mathcal{M}}|)$ times, where $|V_{\mathcal{M}}|$ is the number of hidden network nodes. Thus, this approach also falters when the hidden network or KG is large. Besides, as the number of meta-paths mined from a heterogeneous KG using existing methods can be huge (up to several thousand in our experiments), it is inefficient to identify hubs exactly for each meta-path one by one.

Our Contributions. We find that, in real-world scenarios, enumerating all hubs exactly is often unnecessary, and it suffices to find an approximate set of hubs accurately yet efficiently. Inspired by cardinality sketches [1, 10, 11, 14, 17], we propose a novel *sketch propagation* framework for approximate HUD queries. We construct a *neighborhood cardinality* sketch for each node to estimate its degree in the hidden network by iteratively propagating sketches along edges in matching instances of the given meta-path $\mathcal{M}$. This way, we jointly estimate the degrees of all nodes in the hidden network in a *single* propagation process, thereby significantly enhancing the efficiency of hub discovery by degree centrality.

We extend this sketch propagation framework to approximate HUD queries by the *h-index* measure [19]. A key challenge is that the h-index of a node is determined by not only its own degree, but also by the degrees of its neighbors, while cardinality sketches only provide information on a node's own degree. We propose a *pivot-based* algorithm built upon the sketch propagation framework to fill this gap. Rather than directly estimating a node's h-index, it recursively finds nodes with h-index value no less than that of a random pivot node based on the sketches, akin to quick-sort. Thereby, it effectively skips nodes that cannot be hubs and quickly answers the HUD queries.

Furthermore, we provide efficient methods to answer *personalized* HUD queries, that is, to determine whether a query node is a hidden-network hub. We maintain a lower bound on the number of nodes with higher hidden-network centrality score than the

[1] https://dblp.org
[2] https://makg.org

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query node, and terminate sketch propagation early when the lower bound has exceeded the desired quantile. This early termination mechanism applies to personalized HUD queries based on both the degree centrality score and h-index score.

On the theoretical side, we prove that the sketch propagation framework provides unbiased and efficient approximations for HUD queries based on both measures. Our experiments demonstrate that the estimations converge much faster than the worst-case bound in terms of the number of propagations, achieving speedups of several orders of magnitude compared to exact methods.

Our total contributions are summarized as follows:

- We introduce the novel problem of *hub discovery* (HUD) in hidden networks induced by meta-paths on KGs under degree centrality and h-index. (Section 2)
- We propose a *sketch propagation* framework for approximate degree-based HUD queries and extend it to personalized HUD queries with early termination. (Section 3)
- We further devise a *pivot-based* algorithm, with early termination, for approximate (personalized) HUD queries by the h-index measure. (Section 4)
- Through extensive experimentation, we demonstrate that our proposals scale well to large hidden networks where exact methods fail to finish in a reasonable time and achieve speedups of orders of magnitude over exact methods in most cases while yielding accuracy beyond 90% with default parameter settings. (Section 5)

2 PRELIMINARIES

In this section, we first introduce the basic notations and the formulations of *hub discovery* (HUD) problems on hidden networks induced from KGs by meta-paths. Next, we present exact algorithms for HUD query processing.

2.1 Notations and Problem Formulations

A knowledge graph (KG) is denoted as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where $\mathcal{V}$ and $\mathcal{E}$ represent the sets of node and edge instances. An edge instance $e \in \mathcal{E}$ connects two node instances $u, v \in \mathcal{V}$. The function $\mathcal{L}$ maps each node or edge instance v or e to its type $\mathcal{L}(v)$ or $\mathcal{L}(e)$. We use $\mathcal{G}$ to refer to the KG for ease of presentation.

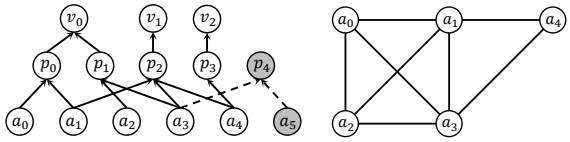
Meta-paths are commonly used to extract hidden networks from KGs [12, 38]. We provide the formal definitions of *meta-path* and its *matching instances* on the KG as the following:

Definition 2.1 (Meta-path). An L-hop meta-path is a sequence of node and edge types denoted as $\mathcal{M}(x_0, y_0, x_1, \ldots, x_{L-1}, y_{L-1}, x_L)$, where x_i is the type of the i-th node and y_i is the type of the i-th edge. The reverse of $\mathcal{M}$ is defined as $\mathcal{M}^{-1}$ $(x_L, y_{L-1}^{-1}, x_{L-1}, \ldots, x_1, y_0^{-1}, x_0)$, where y_i^{-1} is the reverse type of y_i.

Definition 2.2 (Matching Instance). A matching instance M to a meta-path $\mathcal{M}$ is a sequence of node and edge instances in $\mathcal{G}$ denoted by $M(v_0, e_0, v_1, \ldots, v_{L-1}, e_{L-1}, v_L) \triangleright \mathcal{M}$ satisfying that:

(1) $\forall i \in \{0, \ldots, L\}$, $\mathcal{L}(v_i) = x_i$.
(2) $\forall i \in \{0, \ldots, L-1\}$, $e_i = (v_i, v_{i+1}) \in \mathcal{E}$ and $\mathcal{L}(e_i) = y_i$.

When the context is clear, we use $M(v_0, v_1, \ldots, v_L)$ or simply M to denote the matching instance and use $M(x_i)$ to denote the node v_i in M. Our methods support any meta-path $\mathcal{M}$, but we follow established literature [12, 18, 38] by concatenating $\mathcal{M}$ and its inverse $\mathcal{M}^{-1}$ to create a homogeneous hidden network $H_{\mathcal{M}}$.



(a) Exemplary knowledge graph $\mathcal{G}$ (b) Hidden network H

Figure 1: An exemplary knowledge graph $\mathcal{G}$ and a hidden network H induced from $\mathcal{G}$ by a meta-path $\mathcal{M}(A,$ 'write', $P,$ 'publish', $V)$. The unshaded nodes and solid edges in (a) denote the matching graph of $\mathcal{M}$.

Definition 2.3 (Hidden Network). Given a KG $\mathcal{G}$ and a meta-path $\mathcal{M}$, $H_{\mathcal{M}} = (V_{\mathcal{M}}, E_{\mathcal{M}})$ is a hidden network induced from $\mathcal{G}$ by $\mathcal{M}$ if its node set $V_{\mathcal{M}}$ contains all the nodes in $\mathcal{V}$ that match x_0 in any instance of $\mathcal{M}$ and there is an edge $e = (u, v)$ in its edge set $E_{\mathcal{M}}$ for any two nodes $u, v \in V_{\mathcal{M}}$ such that there are $M \triangleright \mathcal{M}$ and $M' \triangleright \mathcal{M}^{-1}$ in $\mathcal{G}$ with (1) $M(x_0) = u$, (2) $M'(x_0) = v$, and (3) $M(x_L) = M'(x_L)$.

We use $\mathcal{N}(u)$ to denote the set of neighbors of u in $H_{\mathcal{M}}$ and $N(u) = |\mathcal{N}(u)|$ to denote the *degree* of u in $H_{\mathcal{M}}$. Moreover, we use $n = \max_{u \in H} N(u)$ to denote the maximum degree in $H_{\mathcal{M}}$. When the context is clear, we drop $\mathcal{M}$ and use $H = (V, E)$ to represent a hidden network for ease of presentation.

Example 2.4. Figure 1 presents an exemplary academic KG with three node types A ('author'), P ('paper'), and V ('venue') and two edge types $A \rightarrow P$ ('write') and $P \rightarrow V$ ('publish'). A sequence $(A,$ 'write', $P,$ 'publish', $V)$ denotes a meta-path $\mathcal{M}$, which induces the hidden network H in Figure 1(b). For instance, two nodes a_0 and a_2 are neighbors in H because there exist an instance $M = (a_0, p_0, v_0)$ of $\mathcal{M}$ and another instance $M' = (v_0, p_1, a_2)$ of the reverse $\mathcal{M}^{-1}$ of $\mathcal{M}$. More intuitively, two authors a_0 and a_2 are connected since they published papers in the same venue v_0.

Problem Statement. In this paper, we study the HUb Discovery (HUD) problem in hidden networks. Hubs are defined as nodes that rank higher than most other nodes in the network according to a centrality function $c : V \rightarrow \mathbb{R}_{\geq 0}$ that measures node importance. Specifically, we focus on the degree centrality [8], i.e., the number of nodes connected to a node, and h-index [19], i.e., the largest integer h such that a node has h neighbors each of degree at least h, as measures for hub discovery. The HUD query we consider is formalized as the following:

Problem 1 (HUD Query). *Given a hidden network H and $\lambda \in (0, 1)$, find every node $u \in V$ such that $c(u) > c(v^{\lambda})$, where v^{λ} is the $(1-\lambda)$-quantile node under a centrality measure $c(\cdot)$.*

We also consider *personalized* queries, i.e., to determine whether a query node q is a hub of H.

Problem 2 (Personalized HUD Query). *Given a hidden network H, a query node q, and $\lambda \in (0, 1)$, determine if $c(q) > c(v^{\lambda})$.*

Processing HUD queries on large-scale hidden networks is computationally expensive. In this work, we consider approximate HUD

queries, which can be answered efficiently by randomized algorithms based on the notion of ϵ-separable sets [15].

Definition 2.5 (ϵ-separable set). Given any node $u \in V$ and $\epsilon \in (0, 1)$, the two ϵ-separable sets of u, denoted as $S^+_\epsilon(u)$ and $S^-_\epsilon(u)$, are the sets of nodes in H with centrality larger and smaller than $c(u)$ by a relative ratio of ϵ, respectively, i.e., $S^+_\epsilon(u) = \{v \in V | c(v) > (1+\epsilon) \cdot c(u)\}$ and $S^-_\epsilon(u) = \{v \in V | c(v) < (1-\epsilon) \cdot c(u)\}$.

Accordingly, we define the approximate versions of HUD and personalized HUD queries as follows:

Problem 3 (ϵ-Approximate HUD Query). *Given a hidden network H and $\epsilon, \lambda \in (0, 1)$, return a set S of nodes in H such that $S^+_\epsilon(v^\lambda) \subseteq S$ and $S \cap S^-_\epsilon(v^\lambda) = \emptyset$.*

Problem 4 (ϵ-Approximate Personalized HUD Query). *Given a hidden network H, a query node q, and $\epsilon, \lambda \in (0, 1)$, return* TRUE *if $q \in S^+_\epsilon(v^\lambda)$ or* FALSE *if $q \in S^-_\epsilon(v^\lambda)$.*

2.2 The Exact Algorithms

Constructing large hidden networks is often memory-prohibitive [37]. To address this issue, we propose a memory-efficient baseline solution to exact HUD queries, which computes the exact centrality of each node in H using any generic worst-case optimal join algorithm [2, 13, 23, 32] on the *matching graph* of $\mathcal{M}$ over $\mathcal{G}$. We first introduce the notion of *matching graph* of a meta-path as follows:

Definition 2.6 (Matching Graph). Given a KG $\mathcal{G}$ and a meta-path $\mathcal{M}$, the matching graph of $\mathcal{M}$ over $\mathcal{G}$ is a subgraph $\mathcal{G}^*$ of $\mathcal{G}$ with a node set $\mathcal{V}^* = \bigcup_{0 \le i \le L} \mathcal{V}_i$ and an edge set $\mathcal{E}^* = \bigcup_{0 \le i < L} \mathcal{E}_i$, where $\mathcal{V}_i$ is the set of all node instances of type x_i contained in any matching instance of $\mathcal{M}$ and $\mathcal{E}_i$ consists of all edges of type y_i that connects two nodes between $\mathcal{V}_i$ and $\mathcal{V}_{i+1}$.

For each node $u \in \mathcal{V}_i$, we use $\mathcal{V}^+(u)$ to denote the set of nodes in $\mathcal{V}_{i+1}$ connected with u through edge type y_i, i.e., the successors of u in the matching graph; and use $\mathcal{V}^-(u)$ to denote the set of nodes in $\mathcal{V}_{i-1}$ connected with u through edge type y_{i-1}, i.e., the predecessors of u in the matching graph. The matching graph can be extracted from $\mathcal{G}$ efficiently by a *forward* and *backward* breadth-first search (BFS). At each level $i \in (0, \ldots, L-1)$, the forward BFS iteratively visits all neighbors of each candidate matching node of x_i via edge type y_i as the candidates for x_{i+1}. Subsequently, the backward BFS visits all neighbors of the candidates for x_{i+1} via edge type y_i as the candidates for x_i at each level $i \in (L-1, \ldots, 0)$. Only the candidate nodes and their adjacent edges visited by both forward and backward BFS are identified as the matching nodes and edges. The time and space complexities for matching graph extraction are $O(|\mathcal{V}|+|\mathcal{E}|)$ since only two rounds of BFS are needed.

To avoid generating excessive intermediate results, a generic worst-case optimal join is employed on the matching graph to compute the neighbors of each node $u \in V$ in the hidden graph. This process starts with a depth-first search (DFS) from each node u and traverses the matching instances of $\mathcal{M}$ and $\mathcal{M}^{-1}$. Specifically, DFS$(u, u, +)$ (Procedure DFS) is executed for each node $u \in V$. Starting from u, it recursively traverses the successors (Lines 6–8) of the current node. Once it visits a node in $\mathcal{V}_L$, the direction of DFS will be switched from traversing the instances of $\mathcal{M}$ to those of $\mathcal{M}^{-1}$ (Lines 2 and 3). Once the traversal reaches $\mathcal{V}_0$ via any instance

Procedure DFS$(u, v, \circ)$

Input: The matching graph $\mathcal{G}^*$ and the direction $\circ \in \{+, -\}$

1 Mark $(v, \circ)$ as visited;
2 **if** $v \in \mathcal{V}_L$ *and* $\circ = +$ **then**
3 DFS$(u, v, -)$;
4 **if** $v \in \mathcal{V}_0$ *and* $\circ = -$ **then**
5 Add v to $\mathcal{N}(u)$;
6 **for** $w \in \mathcal{V}^\circ(v)$ **do**
7 **if** $(w, \circ)$ *is not visited* **then**
8 DFS$(u, w, \circ)$;

of $\mathcal{M}^{-1}$, the node v will be added to $\mathcal{N}(u)$ (Lines 4 and 5). Note that after performing a DFS for each node, the set $\mathcal{N}(u)$ is discarded, and only the degree $|\mathcal{N}(u)|$ is kept to ensure a small memory footprint. To compute the h-index values of all nodes without *materializing* the hidden network, the worst-case optimal join is performed in two rounds. The first round also computes and stores the degree of each node $v \in V$. Then, the second round computes the h-index value of each node v by combining its neighborhood $\mathcal{N}(v)$ with the degrees kept in the first round.

Although memory consumption is bounded by $O(|\mathcal{V}| + |\mathcal{E}|)$ and thus not a bottleneck, the exact algorithms are still very inefficient due to repeated edge traversal. Each edge in $\mathcal{E}^*$ is visited up to $O(|V|)$ times to compute the neighborhood of each node in V. Consequently, the time complexity of the exact algorithms is $O(|V| \cdot |\mathcal{E}^*|)$, which can be prohibitively expensive when dealing with large hidden networks.

3 SKETCH PROPAGATION FRAMEWORK

In this section, we introduce a novel *sketch propagation* framework to efficiently estimate the degree of each node in H by constructing cardinality sketches for their neighbors. As a result, it effectively answers HUD queries based on degree centrality. Additionally, we propose an early termination technique to improve the efficiency of personalized HUD query processing.

3.1 Sketch Propagation for Approximate HUD

Our basic idea is to approximate the degree of each node in H by constructing KMV sketches for the neighborhood $\mathcal{N}(v)$ of each node $v \in V$ without materializing H. The KMV sketch was initially proposed for distinct-value estimation [5] and defined as follows.

Definition 3.1 (KMV Sketch). Given a collection C of items and an integer $k > 0$, the KMV (k-th Minimum Value) sketch $\mathcal{K}(C)$ is built by independently drawing a number uniformly at random from $(0, 1)$ for each item in C and maintaining the k smallest random numbers. The set of random numbers generated for each item in C is called the *basis* for the KMV sketch. The cardinality of C is estimated by the estimator $|\widetilde{C}| = \mathbb{E}[(k-1)/\zeta]$, where ζ is a random variable by taking the largest random number in $\mathcal{K}(C)$.

There are two key challenges for applying KMV sketches and their corresponding estimators to HUD queries. First, the KMV sketch and its analogs are primarily designed for stream processing [1, 10, 11, 14, 17], where a one-pass scan over the collection C is required. However, for HUD queries, scanning the neighborhood

of each node is an expensive operation, as discussed for exact algorithms. Second, prior works only provide asymptotic error bounds for the estimation when $|C| \to \infty$. Although assuming that a large number of elements will appear is reasonable for stream processing, the neighborhood size of each node in H can be relatively small. Thus, the original estimator for the KMV sketch does not provide meaningful error bounds for degree estimations in HUD queries.

To address the above two challenges, we first propose the *sketch propagation* framework that constructs a KMV sketch for $\mathcal{N}(v)$ of each $v \in H$ without explicitly generating and scanning $\mathcal{N}(v)$ and then offers a different estimator with a tight error bound when $\mathcal{N}(v)$ is relatively small. Our framework is based on the notion of *images*.

Definition 3.2 (Images). The forward image of a matching node $u \in \mathcal{V}_i$, denoted as $\mathcal{I}_f(u)$, contains a node $v \in \mathcal{V}_0$ if there exists an instance M of $\mathcal{M}$ including both u and v, i.e.,

$$\mathcal{I}_f(u) = \{v \in \mathcal{V}_0 \mid \exists M \triangleright \mathcal{M}, M(x_0) = v \wedge M(x_i) = u\}.$$

The backward image of $u \in \mathcal{V}_i$, denoted as $\mathcal{I}_b(u)$, contains a node $v \in \mathcal{V}_0$ if there exists a pair of concatenated instances M and M' of $\mathcal{M}$ and $\mathcal{M}^{-1}$ that includes u and v, respectively, i.e.,

$$\mathcal{I}_b(u) = \{v \in \mathcal{V}_0 \mid \exists M \triangleright \mathcal{M}, M' \triangleright \mathcal{M}^{-1}, M(x_i) = u \wedge M(x_L) = M'(x_L) \wedge M'(x_0) = v\}.$$

Due to the definition of hidden network, it is easy to see that $\mathcal{N}(u) = \mathcal{I}_b(u)$ for each node $u \in V$ in the hidden network.

Forward and Backward Propagation. We build a KMV sketch for $\mathcal{N}(u)$ of each $u \in V$ by iteratively maintaining the sketches for the forward and backward images of nodes from $\mathcal{V}_0$ to $\mathcal{V}_L$. First, a random number $r(u)$ is generated for each node $u \in \mathcal{V}_0$. Since $\mathcal{I}_f(u) = \{u\}$, the sketch is initialized as $\mathcal{K}(\mathcal{I}_f(u)) = \{r(u)\}$. The sketches for the forward images of nodes in $\mathcal{V}_i$ are then constructed by propagating the sketches of nodes in $\mathcal{V}_{i-1}$ iteratively. Specifically, each node in $\mathcal{V}_{i-1}$ will propagate its sketch to all its successor nodes in $\mathcal{V}_i$ and a node in $\mathcal{V}_i$ will build its sketch by retaining only the k smallest random numbers among all the sketches it receives. After building the sketches for the forward images of nodes in $\mathcal{V}_L$ (which equals to the sketches w.r.t. their backward images), the algorithm propagates the sketches back from the nodes in $\mathcal{V}_L$ to the nodes in $\mathcal{V}_0$ in the same way as for forward propagation so as to build the sketches for the backward images of nodes in $\mathcal{V}_0$ for estimating the degree of each node in H.

The following lemma ensures the correctness of the *sketch propagation* framework.

Lemma 3.3. *Given a meta-path $\mathcal{M}$, for each node $u \in \mathcal{V}_i$, we have*

$$\mathcal{K}(\mathcal{I}_f(u)) = \oplus_{v \in \mathcal{V}^-(u)} \mathcal{K}(\mathcal{I}_f(v)) \text{ and} \quad (1)$$

$$\mathcal{K}(\mathcal{I}_b(u)) = \oplus_{v \in \mathcal{V}^+(u)} \mathcal{K}(\mathcal{I}_b(v)), \quad (2)$$

where all KMV sketches are constructed from the same basis on $\mathcal{V}_0$, and the operator $\oplus$ extracts the k smallest random numbers from the union of the KMV sketches.

Proof. According to [5, Thm. 5], $\mathcal{K}(A) \oplus \mathcal{K}(B)$ is a KMV sketch for the collection $A \cup B$ for any two collections A and B. We thus only need to prove $\mathcal{I}_f(u) = \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$ and $\mathcal{I}_b(u) = \bigcup_{v \in \mathcal{V}^+} \mathcal{I}_b(v)$ for each node $u \in \mathcal{V}_i$.

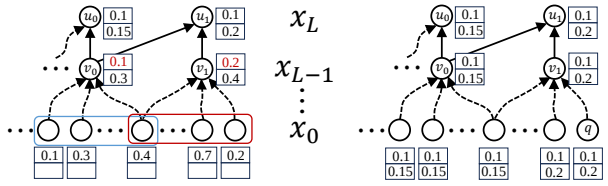
Algorithm 1: Sketch Propagation

Input: Knowledge graph $\mathcal{G}$, meta-path $\mathcal{M}$, percentile λ, number of propagations θ, KMV sketch size k
Output: A set of nodes S for (approximate) HUD query

1 **forall** $u \in \mathcal{V}^*$ **do**
2 **for** $t = 1$ **to** θ **do**
3 $\mathcal{K}_f[u][t] \leftarrow \emptyset$ and $\mathcal{K}_b[u][t] \leftarrow \emptyset$;
4 **if** $u \in \mathcal{V}_0$ **then** add $r(u) = \text{Rand}(0, 1)$ to $\mathcal{K}_f[u][t]$;
5 $\widetilde{N} \leftarrow$ Propagate($\mathcal{K}_f, \mathcal{K}_b$);
6 $S \leftarrow$ top-$(\lambda \cdot |V|)$ nodes with the highest degrees in H w.r.t. $\widetilde{N}$;
7 **return** S;
8 **Function** Propagate($\mathcal{K}_f, \mathcal{K}_b$):
9 **for** $i = 1$ **to** L **do**
10 **forall** $u \in \mathcal{V}_i$ **do** Receive($u, \mathcal{K}_f, -$);
11 **forall** $u \in \mathcal{V}_L$ **do** $\mathcal{K}_b[u] \leftarrow \mathcal{K}_f[u]$;
12 **for** $i = L - 1$ **to** 0 **do**
13 **forall** $u \in \mathcal{V}_i$ **do** Receive($u, \mathcal{K}_b, +$);
14 **forall** $u \in \mathcal{V}_0$ **do**
15 $\widetilde{\mu} \leftarrow 0$;
16 **for** $t = 1$ **to** θ **do**
17 **if** $|\mathcal{K}_b[u][t]| = k$ **then**
18 $\widetilde{\mu} \leftarrow \widetilde{\mu} + \frac{\max(\mathcal{K}_b[u][t])}{\theta}$;
19 **else**
20 $\widetilde{\mu} \leftarrow \widetilde{\mu} + \frac{k}{(|\mathcal{K}_b[u][t]|+1)\cdot\theta}$;
21 $\widetilde{N}[u] \leftarrow k/\widetilde{\mu} - 1$;
22 **return** $\widetilde{N}$;
23 **Function** Receive($u, \mathcal{K}, \circ$):
24 **for** $t = 1$ **to** θ **do**
25 $\mathcal{K}[u][t] \leftarrow \oplus_{v \in \mathcal{V}^\circ(u)} \mathcal{K}[v][t]$;

On the one hand, for any node $u' \in \mathcal{I}_f(u)$, there exists $M \triangleright \mathcal{M}$ such that $M(x_i) = u$ and $M(x_0) = u'$. Suppose that $v = M(x_{i-1})$, we have $u' \in \mathcal{I}_f(v)$ and $v \in \mathcal{V}^-(u)$. Thus, $u' \in \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$, i.e., $\mathcal{I}_f(u) \subseteq \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$. On the other hand, for any node $u' \in \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$, there exists a node $v \in \mathcal{V}^-(u)$ such that $u' \in \mathcal{I}_f(v)$, i.e., there exists an instance $M' \triangleright \mathcal{M}$ such that $M'(x_0) = u'$ and $M'(x_{i-1}) = v$. Moreover, since $v \in \mathcal{V}^-(u)$, there exists an instance $M'' \triangleright \mathcal{M}$ such that $M''(x_{i-1}) = v$ and $M''(x_i) = u$. By concatenating $M'(x_0) = u'$ to $M'(x_{i-1}) = v$ with $M''(x_{i-1}) = v$ to $M''(x_L)$, we construct an instance $M \triangleright \mathcal{M}$ such that $M(x_0) = u'$ and $M(x_i) = u$. Thus, $u' \in \mathcal{I}_f(u)$, i.e., $\bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v) \subseteq \mathcal{I}_f(u)$. Therefore, we have $\mathcal{I}_f(u) = \bigcup_{v \in \mathcal{V}^-(u)} \mathcal{I}_f(v)$ for each node $u \in \mathcal{V}_i$. By symmetry, we can also prove that $\mathcal{I}_b(u) = \bigcup_{v \in \mathcal{V}^+} \mathcal{I}_b(v)$. □

Algorithm 1 describes the procedure of the *sketch propagation* framework. It receives a knowledge graph $\mathcal{G}$, a meta-path $\mathcal{M}$, a ratio $\lambda \in (0, 1)$, the number of propagations $\theta \in \mathbb{Z}^+$, and the sketch size $k \in \mathbb{Z}^+$ as input, and returns a set of nodes S for the (approximate) HUD query. It first initializes two arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ to store θ KMV sketches for the respective forward and backward images of each node $u \in \mathcal{V}^*$ (Lines 1–4). Next, it calls the function Propagate, which constructs KMV sketches within $\mathcal{K}_f$ and $\mathcal{K}_b$ (Lines 9–13) and estimates the degree of each node u in H using $\mathcal{K}_b$ (Lines 14–21).



(a) Forward propagation (b) Backward propagation

Figure 2: Illustration of forward and backward propagation with $k = 2$ and $\theta = 1$. Nodes in the blue and red boxes represent the forward images $\mathcal{I}_f(v_0)$ and $\mathcal{I}_f(v_1)$, respectively.

Specifically, the function Receive depicts the step for KMV sketch construction when each node receives the sketches propagated from other nodes. Given a node u, the array $\mathcal{K}$ to store sketches, and a parameter $\circ$ indicating the propagation direction, it scans the random numbers in the sketches of nodes in either $\mathcal{V}^+(u)$ or $\mathcal{V}^-(u)$ and maintains the k-smallest numbers in $\mathcal{K}[u][t]$ with a min-heap of size k (Lines 24 and 25). Finally, it returns $\lambda \cdot |V|$ nodes with the largest estimated degrees as S (Lines 6 and 7). The tied values are broken arbitrarily.

Example 3.4. Figure 2 illustrates the sketch propagation framework with $k = 2$ and $\theta = 1$. The forward image $\mathcal{I}_f(u_1)$ is the union of forward images of nodes in its predecessors $\mathcal{V}^-(u_1) = \{v_0, v_1\}$, i.e., the union of nodes in the blue and red boxes at layer x_0. The sketch $\mathcal{K}(\mathcal{I}_f(u_1))$ is constructed by taking the smallest two numbers from the union of sketches of forward images of nodes in $\mathcal{V}^-(u_1)$. Thus, $\mathcal{K}(\mathcal{I}_f(u_1)) = \{0.1, 0.3\} \oplus \{0.2, 0.4\} = \{0.1, 0.2\}$. During the backward propagation, the sketch $\mathcal{K}(\mathcal{I}_b(v_0))$ is constructed similarly but in the reverse direction by taking the smallest two numbers from the union of sketches of backward images of nodes in $\mathcal{V}^+(v_0)$. Thus, $\mathcal{K}(\mathcal{I}_b(v_0)) = \{0.1, 0.15\} \oplus \{0.1, 0.2\} = \{0.1, 0.15\}$.

Theoretical Analysis. Next, we will prove that the *sketch propagation* framework accurately approximates HUD queries with high probability (w.h.p.) (cf. Theorem 3.8). For the proof, we first establish the following two lemmas: (1) Lemma 3.5 indicates the unbiasedness of sketch propagation in estimating the image sizes for each node in the matching graph; (2) Lemma 3.7 demonstrates the concentration properties that guarantee tight approximations around the true values.

For any node $u \in \mathcal{V}^*$, we use $I_f(u)$ and $I_b(u)$ to denote the sizes of $\mathcal{I}_f(u)$ and $\mathcal{I}_b(u)$. Moreover, we use ζ to denote the random variable that takes the maximum random number in the sketch of either u's forward or backward image and μ to denote the expectation of ζ. When the context is clear, we abbreviate $I_b(u)$ and $I_f(u)$ as I. We further use $\widetilde{I}$ and $\widetilde{\mu}$ to denote the estimation of I and μ given by the sketch propagation framework.

Lemma 3.5. *For any $u \in \mathcal{V}^*$, $I = k/\mu - 1$ if $I \geq k$.*

Proof. According to [5], if $I \geq k$, the probability density function of ζ will be $f_\zeta(t) = \frac{t^{k-1}(1-t)^{I-k}}{B(k,I-k+1)}$, where $B(a, b)$ denotes the beta function. Thus, we have

$$\mu = \int_0^1 t f_\zeta(t)\, dt = \int_0^1 \frac{t^k(1-t)^{I-k}}{B(k, I-k+1)}\, dt = \frac{B(k+1, I-k+1)}{B(k, I-k+1)} = \frac{k \cdot \binom{I}{k}}{(k+1) \cdot \binom{I+1}{k+1}} = \frac{k}{I+1}, \tag{3}$$

which indicates that $I = k/\mu - 1$. □

Remark. When $I < k$, a KMV sketch directly provides the true value of I with the number of random numbers in the sketch. Further, the original estimator $\mathbb{E}[(k-1)/\zeta]$ in Definition 3.1 is unbiased but with only asymptotic error bounds. In contrast, our proposed estimator in Lemma 3.5 utilizes Bernstein's inequality to establish a tighter confidence bound for arbitrary image sizes.

Theorem 3.6 (Bernstein Inequality [3]). *Let $X_1, X_2, \ldots, X_\theta$ be real-valued i.i.d. random variables such that $X_i \in [0, r]$, $\mathbb{E}[X_i] = \mu$, and $\mathrm{Var}[X_i] = \sigma^2$. Let $\overline{X}_\theta = \frac{1}{\theta}\sum_{i=1}^{\theta} X_i$ denote the empirical mean. With probability at least $1 - p$, it holds that*

$$|\overline{X} - \mu| \leq \sqrt{\frac{\sigma^2 \log(2/p)}{\theta}} + \frac{2r\log(1/p)}{\theta}.$$

Lemma 3.7. *For any $\delta, p \in (0, 1)$ and $u \in \mathcal{V}^*$, if $\theta = O(\frac{n}{\delta k}\log\frac{1}{p})$, $(1-\delta)\cdot I \leq \widetilde{I} \leq (1+\delta)\cdot I$ will hold with probability at least $1 - p$.*

Proof. When $I < k$, the exact value of I will always be obtained, i.e. $\widetilde{I} = I$, and the lemma holds trivially. When $I \geq k$, let $\Delta\mu = |\widetilde{\mu} - \mu|$ and $\Delta I = |\widetilde{I} - I|$. According to Lemma 3.5, $I = \frac{k}{\mu} - 1$ and thus

$$\Delta I = \begin{cases} \frac{k}{\mu} - \frac{k}{\mu + \Delta\mu}, & \text{if } \mu < \widetilde{\mu}; \\ \frac{k}{\mu - \Delta\mu} - \frac{k}{\mu}, & \text{if } \mu \geq \widetilde{\mu}. \end{cases}$$

For any $\delta \in (0, 1)$, it follows that $\Delta I \leq \delta I$ if $\Delta\mu \leq \frac{\delta I k}{(I+1)(I+1+\delta I)}$. Furthermore, the variance of ζ is

$$\sigma^2 = \mathbb{E}[\zeta^2] - \mu^2 = \frac{k\cdot(k+1)}{(I+1)(I+2)} - \left(\frac{k}{I+1}\right)^2 = \frac{k(I-k+1)}{(I+1)^2(I+2)}. \tag{4}$$

According to Bernstein's inequality, for any $p \in (0, 1)$, we have

$$\Delta\mu \leq \sqrt{\frac{\sigma^2\log(2/p)}{\theta}} + \frac{2\log(1/p)}{\theta}$$

with probability at least $1 - p$. Thus, we ensure $\Delta I \leq \delta I$ with probability at least $1 - p$ by setting θ such that

$$\sqrt{\frac{\sigma^2\log(2/p)}{\theta}} + \frac{2\log(1/p)}{\theta} \leq \frac{\delta I k}{(I+1)(I+1+\delta I)}.$$

By solving the above inequality as a quadratic inequality in terms of $\sqrt{\theta}$, we have $\Delta I \leq \delta I$ with probability at least $1 - p$ if

$$\theta \geq \left(\sqrt{\tfrac{I-k+1}{I+2}\log\tfrac{2}{p}} + \sqrt{\tfrac{I-k+1}{I+2}\log\tfrac{2}{p} + 8\tfrac{\delta I(I+1)}{I+1+\delta I}\log\tfrac{1}{p}}\right)^2 \cdot \tfrac{(I+1+\delta I)^2}{4\delta^2 I^2 k}.$$

By simplifying the above inequality, we obtain $\theta = O(\frac{I}{\delta k}\log\frac{1}{p})$. Since $I \leq n$ for any image, by setting $\theta = O(\frac{n}{\delta k}\log\frac{1}{p})$, $\Delta I \leq \delta \cdot I$ holds with probability at least $1 - p$. □

Theorem 3.8. *Let $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$ for any $\epsilon \in (0, 1)$, the sketch propagation framework answers an ϵ-approximate HUD query under degree centrality correctly with probability at least $1 - p$.*

Proof. Let $\widetilde{N}(u)$ denote the estimated degree of node v through sketch propagation. Given any nodes $u \in S_0^+(v^\lambda)$ and $v \in S_\epsilon^-(v^\lambda)$,

$$\widetilde{N}(v) \le (1+\delta)\cdot N(v) \le (1+\delta)(1-\epsilon)\cdot N(v^\lambda)$$
$$\le (1+\delta)(1-\epsilon)\cdot N(u) \le \frac{(1+\delta)(1-\epsilon)}{1-\delta}\cdot \widetilde{N}(u)$$

hold due to Lemma 3.7 and the definitions of $S_\epsilon^-(v^\lambda)$ and $S_0^+(v^\lambda)$. By setting $\delta < \frac{\epsilon}{2-\epsilon}$, we have $\widetilde{N}(v) < \widetilde{N}(u)$, i.e., the sketches rank v lower than u, with probability at least $1-p$. Taking the union bound over all nodes in $S_0^+(v^\lambda)$ and setting $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$, the sketches rank all nodes in $S_0^+(v^\lambda)$ higher than $v \in S_\epsilon^-(v^\lambda)$. Since there are at least $\lambda|V|$ nodes in $S_0^+(v^\lambda)$, v will not be included in the result S. Similarly, given any nodes $u \in S_0^-(v^\lambda)$ and $v \in S_\epsilon^+(v^\lambda)$, we have $\widetilde{N}(v) \ge (1-\delta)\cdot N(v) \ge (1-\delta)(1+\epsilon)\cdot N(v^\lambda) \ge (1-\delta)(1+\epsilon)\cdot N(u) \ge \frac{(1-\delta)(1+\epsilon)}{1+\delta}\cdot \widetilde{N}(u)$. By setting $\delta < \frac{\epsilon}{2+\epsilon}$, it holds that $\widetilde{N}(v) > \widetilde{N}(u)$, i.e. the sketches rank v higher than u, with probability at least $1-p$. Also taking the union bound over all nodes in $S_0^-(v^\lambda)$ and setting $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$, the sketches rank all nodes in $S_0^-(v^\lambda)$ lower than $v \in S_\epsilon^+(v^\lambda)$. Since there are at least $(1-\lambda)|V|$ nodes in $S_0^-(v^\lambda)$, v will be included in S. Finally, by taking the union bound over all nodes in $S_\epsilon^+(v^\lambda)$ and $S_\epsilon^-(v^\lambda)$ and setting $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|^2}{p}) = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$, all nodes in $S_\epsilon^+(v^\lambda)$ are included in S, whereas all nodes in $S_\epsilon^-(v^\lambda)$ are excluded from S, with probability at least $1-p$. □

Finally, the time complexity of the *sketch propagation* framework is $O(\frac{n|\mathcal{E}^*|}{\epsilon}\log\frac{|V|}{p})$. The bottleneck of the *sketch propagation* framework lies in constructing the KMV sketches, which propagate θ KMV sketches of size k over the matching graph with $|\mathcal{E}^*|$ edges.

3.2 Early Termination for Approximate Personalized HUD

Given a query node q, a personalized HUD query can be answered directly using the estimated node degrees through *sketch propagation*. Specifically, if q is ranked higher than the $(1-\lambda)$-quantile node in terms of estimated degree, it returns TRUE for the personalized query, and FALSE otherwise. The following corollary is a direct extension of Theorem 3.8 for personalized queries.

Corollary 3.9. *Let $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$ for any $\epsilon \in (0,1)$. Sketch propagation answers personalized ϵ-approximate HUD queries under degree centrality correctly with probability at least $1-p$.*

Nevertheless, we can further optimize personalized HUD query processing by terminating the *sketch propagation* early once we have determined that q is not a hub with high confidence. We first give the following lemma to inspire the design of the early termination optimization.

Lemma 3.10. *Given a query node q, if there exists a node $u \in \mathcal{V}^*$ such that $I_f(u) \ge \lambda|V|$ and $I_b(u) \ge N(q)$, then the answer to the personalized HUD query is* FALSE.

Proof. For any node $u \in \mathcal{V}^*$, it is easy to see that any $v_1 \in \mathcal{I}_f(u)$ and $v_2 \in \mathcal{I}_b(u)$ must be neighbors in H. Hence, each node in $\mathcal{I}_f(u)$ has at least $I_b(u)$ neighbors. Since $I_b(u) \ge N(q)$ and $I_f(u) \ge \lambda|V|$, there are at least $\lambda|V|$ nodes with higher degrees than $N(q)$. □

Based on Lemma 3.10, we can terminate *sketch propagation* when the estimations $\widetilde{I}_f(u)$ and $\widetilde{I}_b(u)$ of $I_f(u)$ and $I_b(u)$ for a node $u \in \mathcal{V}^*$ by KMV sketches satisfy that $\widetilde{I}_f(u) \ge \lambda|V|$ and $\widetilde{I}_b(u) \ge N(q)$. However, this may cause a false negative result once $I_f(u)$ and $I_b(u)$ are overestimated. To avoid such errors, we only terminate sketch propagation when $\widetilde{I}_f(u)$ and $\widetilde{I}_b(u)$ are significantly larger than $\lambda|V|$ and $N(q)$, respectively. Specifically, the early termination is performed only when a node u satisfies $\widetilde{I}_f(u) \ge (1+\beta)\cdot\lambda|V|$ and $\widetilde{I}_b(u) \ge (1+\beta)\cdot N(q)$. By selecting an appropriate value of β, we can ensure that the probability of a false negative is small enough.

Algorithm 2 presents the procedure of OptimizedReceive, an optimized version of Receive at Line 23 in Algorithm 1 with early termination of sketch propagation. Specifically, OptimizedReceive returns whether sketch propagation should be terminated at node u. It first calls Receive at Line 23 of Algorithm 1 to construct the KMV sketch for the backward image of u (Line 1) and then estimates $I_f(u)$ and $I_b(u)$ accordingly (Lines 2–12). Finally, it returns whether the estimated $\widetilde{I}_f(u)$ and $\widetilde{I}_b(u)$ have satisfied the early termination conditions (Lines 13–15).

Algorithm 2: OptimizedReceive

Input: Node u, arrays of KMV sketches $\mathcal{K}_f, \mathcal{K}_b$
Output: If sketch propagation can be terminated

1 Receive$(u, \mathcal{K}_b, +)$;
2 $\widetilde{I}_f \leftarrow 0$ and $\widetilde{I}_b \leftarrow 0$;
3 **for** $t = 1$ **to** θ **do**
4 **if** $|\mathcal{K}_f[u][t]| = k$ **then**
5 $\widetilde{I}_f \leftarrow \widetilde{I}_f + \frac{\max(\mathcal{K}_f[u][t])}{\theta}$;
6 **else**
7 $\widetilde{I}_f \leftarrow \widetilde{I}_f + \frac{k}{(|\mathcal{K}_f[u][t]|+1)\cdot\theta}$;
8 **if** $|\mathcal{K}_b[u][t]| = k$ **then**
9 $\widetilde{I}_b \leftarrow \widetilde{I}_b + \frac{\max(\mathcal{K}_b[u][t])}{\theta}$;
10 **else**
11 $\widetilde{I}_b \leftarrow \widetilde{I}_b + \frac{k}{(|\mathcal{K}_b[u][t]|+1)\cdot\theta}$;
12 $\widetilde{I}_f \leftarrow k/\widetilde{I}_f - 1$ and $\widetilde{I}_b \leftarrow k/\widetilde{I}_b - 1$;
13 **if** $\widetilde{I}_f \ge (1+\beta)\cdot\lambda|V|$ and $\widetilde{I}_b \ge (1+\beta)\cdot N(q)$ **then**
14 **return** TRUE;
15 **return** FALSE;

Example 3.11. Continuing with Example 3.4 where $\mathcal{K}(\mathcal{I}_b(v_0)) = \{0.1, 0.15\}$, the algorithm estimates $\widetilde{I}_b(v_0) = \frac{2}{0.15} - 1 \approx 12.3$. The size $\widetilde{I}_f(v_0) = \frac{2}{0.3} - 1 \approx 5.7$ is estimated from the sketch $\mathcal{K}(\mathcal{I}_f(v_0))$ in Figure 2(a). Assuming $\lambda = 0.01$, $|V| = 500$, $N(q) = 9$ and setting $\beta = 0$, we have $\widetilde{I}_f(u_1) > 0.01\cdot 500$ and $\widetilde{I}_b(u_1) > 9$, and thus sketch propagation can be early terminated.

Choosing Appropriate β. A key problem in optimized query processing is deciding the value of β for early termination. The following theorem provides an upper bound for β so that the probability that early termination yields a false negative result is at most p_e for any $p_e \in (0,1)$.

Theorem 3.12. *If* $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$ *and* $\beta = \frac{\log p_e - \log|\mathcal{V}^*|}{\log p - \log|V|}\cdot\epsilon$, *the probability that a personalized HUD query whose answer is* TRUE *is incorrectly answered with* FALSE *is at most* p_e.

Proof. Early termination occurs only when there exists a node $u \in \mathcal{V}^*$ such that $\widetilde{I}_f(u) \geq (1+\beta)\cdot\lambda|V|$ and $\widetilde{I}_b(u) \geq (1+\beta)\cdot N(q)$. Thus, sketch propagation will not be early terminated incorrectly if

$$\widetilde{I}_f(u) < (1+\beta)\cdot\lambda|V| \text{ or } \widetilde{I}_b(u) < (1+\beta)\cdot N(q),\ \forall u \in \mathcal{V}^*. \quad (5)$$

For any given $\delta' > 0$, we have

$$O(\frac{n}{\epsilon k}\log\frac{|V|}{p}) = O(\frac{n}{(\frac{\log\delta' p}{\log(p/|V|)})\epsilon k}\log\frac{1}{\delta' p}) \quad (6)$$

Combining Theorem 3.7 and Eqn. 6, if $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$, we have

$$\widetilde{I}_f(u) \leq (1+\frac{\log\delta' p}{\log(p/|V|)}\cdot\epsilon)\cdot I_f(u);\ \widetilde{I}_b(u) \leq (1+\frac{\log\delta' p}{\log(p/|V|)}\cdot\epsilon)\cdot I_b(u).$$

Each holds with probability at least $1-\delta' p$ for a given node u. Let $\delta' = \frac{p_e}{p|\mathcal{V}^*|}$ and $\beta = \frac{\log p_e - \log|\mathcal{V}^*|}{\log p - \log|V|}\cdot\epsilon$. Taking the union bound,

$$\widetilde{I}_f(u) \leq (1+\beta)\cdot I_f(u) \text{ or } \widetilde{I}_b(u) \leq (1+\beta)\cdot I_b(u),\ \forall u \in \mathcal{V}^* \quad (7)$$

holds with probability at least $1-p_e$.

According to Lemma 3.10, since the answer to a personalized HUD query is TRUE, $I_f(u) < \lambda|V|$ or $I_b(u) < N(q), \forall u \in \mathcal{V}^*$. Combined with Eqn. 7, Eqn. 5 holds with probability $1-p_e$. □

4 EXTENSION TO H-INDEX

In this section, we extend the sketch propagation framework to process approximate HUD queries based on the h-index measure. To compute the h-indexes of the nodes in H, we need to compute the degrees of their neighbors. However, the KMV sketches can only estimate the neighborhood sizes of each node but fail to provide the degree estimations of their neighbors. Therefore, we propose a novel pivot-based algorithm that can estimate the set of hubs in terms of h-index without explicitly calculating the h-index of every node in H. Before diving into the algorithm, we first review the definition of *h-index* [19].

Definition 4.1 (H-index). Given a node $u \in H$, the h-index of u is the largest value $h(u)$ such that u has no fewer than $h(u)$ neighbors of degrees at least $h(u)$.

4.1 Pivot-based Method for Approximate HUD

The basic idea of the pivot-based algorithm is to choose a random pivot node $u \in V$ and iteratively partition the nodes in H into two disjoint sets, Q_u^+ and Q_u^-, based on their h-index values in comparison with $h(u)$, i.e., $Q_u^+ = \{v \in V \mid h(v) \geq h(u)\}$ and $Q_u^- = \{v \in V \mid h(v) < h(u)\}$. As such, we can decide that $Q_u^+ \subseteq S_0^+(v_\lambda)$ if $u \in S_0^+(v_\lambda)$ since all nodes in Q_u^+ have h-index values greater than or equal to $h(u)$ and thus are ranked higher than u, or $Q_u^- \cap S_0^+(v_\lambda) = \emptyset$ if $u \in S_0^-(v_\lambda)$ similarly. Such a partitioning strategy allows for efficient bisection when identifying hubs, i.e., $S_0^+(v^\lambda)$. If $u \in S_0^+(v_\lambda)$, we can add all nodes in Q_u^+ to the result set and exclude them from consideration in the subsequent iteration. On the contrary, if $u \in S_0^-(v^\lambda)$, the nodes in Q_u^+ may contain those in both $S_0^+(v^\lambda)$ and $S_0^-(v^\lambda)$, which require further partitioning in next iterations. Meanwhile, all nodes in Q_u^- must not be in $S_0^+(v^\lambda)$ and are excluded from consideration. The above iterative process proceeds until all nodes have been decided. Then, all the nodes in $S_0^+(v^\lambda)$ have been added to the result set.

Pivot-based Partitioning Method. The key challenge lies in partitioning the nodes in H without explicitly computing the h-index of each node. In fact, we can compare the h-indexes of two nodes using only one of them. By the definition of h-index, for any two nodes $u, v \in V$, we have $h(v) \geq h(u)$ if and only if v has at least $h(u)$ neighbors in H with degrees larger than or equal to $h(u)$. Based on the above observation, we introduce a two-stage pivotal partitioning method for our pivot-based algorithm.

Stage (I): Estimating $h(u)$. We first employ *sketch propagation* on the matching graph to estimate the degrees of all nodes in H. Subsequently, at each iteration, we scan through the set of neighbors $\mathcal{N}(u)$ of a randomly chosen node u (obtained by a single DFS on the matching graph starting from u). Based on the degree estimations, we compute an approximate h-index $\widetilde{h}(u)$ of node u.

Stage (II): Estimating Q_u^+ *and* Q_u^-. Once we have the approximate h-index $\widetilde{h}(u)$, we proceed to eliminate nodes in H with degrees smaller than $\widetilde{h}(u)$. Next, we perform *sketch propagation* on the subgraph of the matching graph that only includes the remaining nodes. The resulting KMV sketches estimate the degrees of nodes in the subgraph of H induced by the set of nodes with degrees greater than or equal to $\widetilde{h}(u)$ in the original graph H, denoted as $\widetilde{H}(u)$. Finally, we obtain Q_u^+ as the set of nodes with estimated degrees greater than $\widetilde{h}(u)$ in the induced subgraph of $\widetilde{H}(u)$, while the remaining nodes are added to Q_u^-.

Algorithm 3 depicts the procedure of our pivot-based algorithm. It also receives a knowledge graph $\mathcal{G}$, a meta-path $\mathcal{M}$, the percentile λ, the number of propagations θ, and the KMV sketch size k as input, and returns a set of nodes S to answer the (approximate) HUD query based on h-index. It first calls the same Propagate function as Algorithm 1 over the arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ of KMV sketches to estimate the degree of each node in H (Lines 1–5). Note that the estimated degrees will be reused across all iterations for partitioning. The iterative process proceeds until Q, which contains the nodes to be partitioned in each iteration, is empty (Lines 7–26). In each iteration, it first randomly chooses a pivot node u from Q and estimates its h-index $\widetilde{h}(u)$ (Lines 8-12). Then, it re-initializes arrays $\mathcal{K}_f$ and $\mathcal{K}_b$ of KMV sketches for the nodes with degrees greater than $\widetilde{h}(u)$ (Lines 13–17). After that, the Propagate function is performed over $\mathcal{K}_f$ and $\mathcal{K}_b$ to estimate the number of neighbors with degrees larger than $\widetilde{h}(u)$ for each node in Q (Line 18). Accordingly, it partitions Q into Q_u^+ and Q_u^- according to $\widetilde{h}$ (Lines 19–21) and updates the result set S and the candidate node set Q based on Q_u^+ and Q_u^- (Lines 22–26). Finally, when Q is empty, the result set S is returned (Line 27). The tied values are broken arbitrarily.

Theoretical Analysis. Next, we prove that the pivot-based algorithm returns the correct answer to an approximate HUD query based on h-index w.h.p. We first give the following lemma, which states that Algorithm 3 provides a concentrated estimation of $h(u)$ for the randomly chosen pivot node u in each iteration.

Algorithm 3: Pivot-based Algorithm

Input: Knowledge graph $\mathcal{G}$, meta-path $\mathcal{M}$, percentile λ, number of propagations θ, KMV sketch size k
Output: The set of nodes S for (approximate) HUD query

forall $u \in \mathcal{V}^*$ **do**
  for $t = 1$ **to** θ **do**
    $\mathcal{K}_f[u][t] \leftarrow \emptyset$ and $\mathcal{K}_b[u][t] \leftarrow \emptyset$;
    if $u \in \mathcal{V}_0$ **do** add $r(u) = \text{Rand}(0,1)$ to $\mathcal{K}_f[u][t]$;
$\widetilde{N} \leftarrow \text{Propagate}(\mathcal{K}_f, \mathcal{K}_b)$;
$Q \leftarrow V$ and $S \leftarrow \emptyset$;
while $|Q| > 0$ **do**
  Sample a random node from Q as the pivot node u;
  Sort the neighbors $\mathcal{N}(u)$ of u in descending order by $\widetilde{N}$;
  Initialize $\widetilde{h}(u) \leftarrow 0$;
  forall $v \in \mathcal{N}(u)$ **do**
    if $\widetilde{N}(v) \geq \widetilde{h}(u)$ **then** $\widetilde{h}(u) \leftarrow \widetilde{h}(u) + 1$;
  forall $v \in \mathcal{V}^*$ **do**
    for $t = 1$ **to** θ **do**
      $\mathcal{K}_f[v][t] \leftarrow \emptyset$ and $\mathcal{K}_b[v][t] \leftarrow \emptyset$;
      if $v \in \mathcal{V}_0$ *and* $\widetilde{N}(v) \geq \widetilde{h}(u)$ **then**
        Add $r(v) = \text{Rand}(0,1)$ to $\mathcal{K}_f[v][t]$;
  $\widetilde{h} \leftarrow \text{Propagate}(\mathcal{K}_f, \mathcal{K}_b)$;
  Initialize $Q_u^+ \leftarrow \emptyset$ and $Q_u^- \leftarrow \emptyset$;
  forall $v \in Q$ **do**
    if $\widetilde{h}(v) \geq \widetilde{h}(u)$ **then** add v to Q_u^+ **else** add v to Q_u^-;
  if $|Q_u^+| + |S| \leq \lambda|V|$ **then**
    $Q \leftarrow Q_u^-$ and $S \leftarrow S \cup Q_u^+$;
    if $|S| \leq \lambda|V|$ **then** $S \leftarrow S \cup \{u\}$;
  else
    $Q \leftarrow Q_u^+$;
return S;

Lemma 4.2. *For any $\delta, p \in (0,1)$, if $\theta = O(\frac{n}{\delta k}\log\frac{|V|}{p})$, we have $(1-\delta)\cdot h(u) \leq \widetilde{h}(u) \leq (1+\delta)\cdot h(u)$ with probability at least $1-p$ for the pivot node u in each iteration.*

Proof. Let us order the nodes in $\mathcal{N}(u)$ according to their degrees in H descendingly and denote v_i as the i-th neighbor of u. Then, we have $N(v_i) \geq h(u)$ for $0 \leq i \leq h(u)-1$ and $N(v_i) \leq h(u)$ for $h(u) \leq i \leq N(u)-1$.

By combining Lemma 3.7 with the union bound over V, when $\theta = O(\frac{n}{\delta k}\log\frac{|V|}{p})$ in the first stage, we have $\widetilde{N}(v_i) \geq (1-\delta)N(v_i) \geq (1-\delta)h(u)$ for $0 \leq i \leq h(u)-1$ with probability at least $1-p$. Thus, node u has $h(u) \geq (1-\delta)h(u)$ neighbors with estimated degrees larger than or equal to $(1-\delta)h(u)$. Thus, $\widetilde{h}(u) \geq (1-\delta)h(u)$ with probability at least $1-p$. Similarly, we have $\widetilde{N}(v_i) \leq (1+\delta)N(v_i) \leq (1+\delta)h(u)$ for $i \geq h(u)$ with probability at least $1-p$. Thus, node u has no more than $h(u)$ neighbors with estimated degrees larger than $(1+\delta)h(u)$, and we have $\widetilde{h}(u) \leq (1+\delta)h(u)$ with probability at least $1-p$. □

Then, the following lemma indicates that the pivot-based partitioning method can divide candidate nodes into ϵ-separable sets.

Lemma 4.3. *For any $\delta' \in (0,1)$ and pivot node u, by setting $\theta = O(\frac{n}{\delta' k}\log\frac{|V|}{p})$, we have $S_{\delta'}^-(u) \subseteq Q_u^-$ and $S_{\delta'}^+(u) \subseteq Q_u^+$ hold with probability at least $1-p$.*

Proof. For any node $v \in S_{\delta'}^-(u)$, let $\mathcal{N}_u(v)$ denote the neighbors of v in $\widetilde{H}(u)$ and $N_u(v) = |\mathcal{N}_u(v)|$. Since $h(v) < (1-\delta')h(u)$, v has at most $(1-\delta')h(u)$ neighbors with degrees larger than or equal to $(1-\delta')h(u)$. For each neighbor v' of v with $N(v') < (1-\delta')h(u)$, when $\theta = O(\frac{n}{\delta k}\log\frac{|V|}{p})$, $\widetilde{N}(v') < (1+\delta)N(v') < (1+\delta)(1-\delta')h(u) < \frac{(1+\delta)(1-\delta')}{1-\delta}\widetilde{h}(u)$. By setting $\delta < \frac{\delta'}{2-\delta'}$, we have $\widetilde{N}(v') < \widetilde{h}(u)$, i.e., v' is filtered out in *Stage (I)*, with probability at least $1-p$. Thus, we have $N_u(v) < (1-\delta')h(u)$ with probability at least $1-p$. Let $\widetilde{N}_u(v)$ denote the estimation of $N_u(v)$ in *Stage (II)* of pivot-based partitioning. We have $\widetilde{N}_u(v) \leq (1+\delta)N_u(u)$ with probability at least $1-p$, and thus $\widetilde{N}_u(v) \leq (1+\delta)N_u(v) < (1+\delta)(1-\delta')h(u) \leq \frac{(1+\delta)(1-\delta')}{1-\delta}\widetilde{h}(u) \leq \widetilde{h}(u)$, i.e., v is added into Q_u^- with probability at least $1-p$. Thus, by setting $\theta = O(\frac{n}{\frac{\delta'}{2-\delta'}k}\log\frac{|V|}{p}) = O(\frac{n}{\delta' k}\log\frac{|V|}{p})$, we have $S_{\delta'}^-(u) \subseteq Q_u^-$. Similarly, we also get $S_{\delta'}^+(u) \subseteq Q_u^+$ with probability at least $1-p$ when $\theta = O(\frac{n}{\delta' k}\log\frac{|V|}{p})$. □

Subsequently, we can formally analyze the correctness of the pivot-based algorithm for processing ϵ-approximate HUD queries based on the h-index in the following theorem.

Theorem 4.4. *Let $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$ for any $\epsilon \in (0,1)$, the pivot-based algorithm returns the answer to an ϵ-approximate HUD query based on the h-index correctly with probability at least $1-p$.*

Proof. We prove the theorem by examining three different cases of the ranges of the h-index $h(u)$ for the pivot node u. For each case, we show the correctness of the first iteration of the pivot-based algorithm, while subsequent iterations are proven by induction. By selecting $\delta' < \frac{\epsilon}{2}$ as Lemma 4.3, we establish that:

- **Case 1 ($h(u) > (1+\frac{\epsilon}{2})h(v^\lambda)$):** Each v with $h(v) \leq h(v^\lambda)$ will be added to Q_u^-. Thus, $|Q_u^-| \geq (1-\lambda)|V|$. Therefore, Q_u^- will be used as the candidate set Q in the next iteration, and Q^+, which does not contain any node in $S_\epsilon^-(v^\lambda)$, will be added to S.
- **Case 2 ($h(u) < (1-\frac{\epsilon}{2})h(v^\lambda)$):** Each v with $h(v) \geq h(v^\lambda)$ will be added to Q_u^+. Thus, $|Q_u^+| \geq \lambda|V|$. Therefore, Q^+, which contains all nodes in $S_\epsilon^+(v^\lambda)$, will be used as the candidate set Q in the next iteration.
- **Case 3 ($(1-\frac{\epsilon}{2})h(v^\lambda) < h(u) < (1+\frac{\epsilon}{2})h(v_\lambda)$):** Any v with $h(v) \geq (1+\epsilon)h(v^\lambda)$ will be added to Q_u^+. And any v' with $h(v') \leq (1-\epsilon)h(v^\lambda)$ will be added to Q_u^-. Thus, Q_u^+, which contains all nodes in $S_\epsilon^+(v^\lambda)$, will be either added to S or used as the candidate set Q in the next iteration; whereas Q_u^-, which contains all nodes in $S_\epsilon^-(v^\lambda)$, will be either eliminated or used as the candidate set Q for the next iteration.

Considering all the above cases, it follows that, for any randomly chosen pivot u, none of the nodes in $S_\epsilon^-(v^\lambda)$ is added to the result set S. In contrast, all nodes in $S_\epsilon^+(v^\lambda)$ have a probability of at least $1-p$ to be either added to S or retained for the subsequent iteration. Consequently, we prove the theorem by applying the union bound over all iterations. □

Finally, the amortized time complexity of the pivot-based algorithm is $O(\frac{n|\mathcal{E}^*|}{\epsilon}\log^2\frac{|V|}{p})$ since it invokes the sketch propagation framework $O(\log|V|)$ times in amortization.

4.2 Early Termination for Approximate Personalized HUD

Given a query node q, the personalized HUD query based on the h-index can also be answered by directly using the query node q as the pivot node and performing the pivot-based partitioning method to obtain $\mathcal{Q}_u^+$. Then, it simply returns FALSE if $|\mathcal{Q}_u^+| > \lambda|V|$ or TRUE otherwise. Following Lemma 4.3, the following corollary indicates that the above algorithm provides correct answers for approximate personalized HUD queries w.h.p.

Corollary 4.5. *For any $\epsilon \in (0,1)$, by setting $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$, the pivot-based algorithm answers an ϵ-approximate personalized HUD query based on h-index with probability at least $1-p$.*

Early termination. An early termination optimization can also accelerate personalized HUD queries based on h-index.

Lemma 4.6. *Given a query node q and percentile λ, if there exists a node $u \in \mathcal{V}^*$ such that $I_f(u) \geq h(q)$ and $I_b(u) \geq \max(h(q), \lambda|V|)$, then the answer to the personalized HUD query must be* FALSE.

Proof. Note that each pair of nodes in $\mathcal{I}_b(u)$ and $\mathcal{I}_f(u)$ are a neighbor of each other in H according to the proof of Lemma 3.10. Thus, each node $v_1 \in \mathcal{I}_b(u)$ has at least $I_f(u)$ neighbors with degrees larger than or equal to $I_b(u)$. Since $I_f(u) \geq h(q)$ and $I_b(u) \geq \max(h(q), \lambda \cdot |V|)$, there exists at least $I_b(u) \geq \lambda|V|$ nodes with h-indexes larger than or equal to $h(q)$. Therefore, q is not a hub in terms of h-index. □

Based on Lemma 4.6, sketch propagation can be early terminated during *Stage (I)* of the pivot-based partitioning method. Specifically, we use the KMV sketches to estimate $I_f(u)$ and $I_b(u)$ for each $u \in \mathcal{V}^*$. If there exists a node $u \in \mathcal{V}^*$ such that $\widetilde{I}_f(u) \geq (1+\beta)N(q)$ and $\widetilde{I}_b(u) \geq (1+\beta)\max(N(q), \lambda|V|)$, the algorithm can be early terminated since $N(q)$ is an upper bound of $h(q)$.

Once we obtain $\widetilde{h}(q)$ after *Stage (I)*, the algorithm can also be terminated without entering *Stage (II)* if there exists a node $u \in \mathcal{V}^*$ such that $\widetilde{I}_f(u) \geq (1+\beta)\widetilde{h}(q)$ and $\widetilde{I}_b(u) \geq (1+\beta)\max(\widetilde{h}(q), \lambda|V|)$. The following theorem provides an upper bound of β that assures the probability of false negatives caused by early termination is at most $p_e \in (0,1)$.

Theorem 4.7. *If $\theta = O(\frac{n}{\epsilon k}\log\frac{|V|}{p})$ and $\beta = \frac{\log p_e - \log|\mathcal{V}^*|}{\log p - \log|V|}\cdot\epsilon$, the pivot-based algorithm with early termination answers a personalized HUD query with probability at least $1-p_e$.*

The proof is similar to that of Theorem 3.12, and we leave it to the technical report [4] due to space limits.

5 EXPERIMENTS

5.1 Experimental Setup

Datasets. We conduct our experiments on six real-world KGs. The statistics of those KGs are reported in Table 1. ACM and DBLP are two academic KGs denoting the co-authorships between researchers through papers and publishing venues. FreeBase is extracted from a general-purpose KG developed by Google[3]. IMDB is a KG about actors and directors of movies. PubMed is a biomedical KG representing the relationships between genes, chemicals, and diseases of different species. Yelp is a business KG recording the user ratings and comments on businesses at different locations.

Table 1: Statistics of KGs used in the experiments, where $|\mathcal{L}(\mathcal{V})|$ and $|\mathcal{L}(\mathcal{E})|$ denote the numbers of node and edge types, and $|\mathbb{M}|$ is the number of evaluated meta-paths.

Dataset	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	$\lvert\mathcal{L}(\mathcal{V})\rvert$	$\lvert\mathcal{L}(\mathcal{E})\rvert$	$\lvert\mathbb{M}\rvert$
ACM	10.9K	548K	4	8	20
DBLP	26.1K	239.6K	4	6	48
FreeBase	180K	1057.7K	8	36	151
IMDB	21.4K	86.6K	4	6	679
PubMed	63.1K	236.5K	4	10	172
Yelp	82.5K	29.9M	4	4	2230

Query Generation. For each dataset, we use AnyBURL [20] to discover a set of candidate meta-paths $\mathbb{M}$. We set the confidence threshold to 0.1 for AnyBURL and take the snapshot at ten seconds. Note that AnyBURL discovers meta-paths with extra node constraints, resulting in a large number of potential meta-paths for validation. This makes the efficient processing of HUD queries even more required. $|\mathbb{M}|$ in Table 1 shows the number of meta-paths found on each dataset. For personalized HUD queries, we randomly sample 10 nodes from $\mathcal{V}_0$ as the query nodes for each meta-path.

Compared Methods. To the best of our knowledge, there has not yet been any prior study on the HUD problem. Therefore, we only compare our sketch propagation-based algorithms with the exact algorithm based on the generic worst-case optimal join.

- ExactD and ExactH use the exact algorithms in Section 2.2 to compute the degrees and h-indexes of all nodes in H, thereby identifying the hubs by sorting the nodes in descending order of degree and h-index.
- GloD and PerD apply sketch propagation in Section 3.1 to estimate the degrees of all nodes in H. GloD returns the set of hubs based on estimated degrees, whereas PerD decides if a query node q is a hub.
- PerD$^+$ optimizes PerD with early termination in Section 3.2.
- GloH and PerH apply the pivot-based algorithm in Section 4.1 to find the hubs based on estimated h-indexes. GloH recursively partitions the nodes until all hubs are found, whereas PerH uses a query node q as the pivot to check whether it is a hub.
- PerH$^+$ optimizes PerH with early termination in Section 4.2.

Parameter Settings. By default, we set $\lambda = 0.05$ to find the hubs as the top 5% of nodes with the highest degrees or h-indexes in hidden networks. Our results in Section 5.3 show that h-index-based HUD queries are well approximated using smaller values of θ and k compared to degree-based HUD queries. Therefore, for algorithms targeting degree-based HUD queries (GloD, PerD, PerD$^+$), we set $\theta = 8$ and $k = 32$ by default, while for algorithms targeting h-index-based HUD queries (GloH, PerH, PerD$^+$), we set $\theta = 8$ and $k = 4$ by default. We use $\beta = 0$ as the default setting since it

[3] https://developers.google.com/freebase

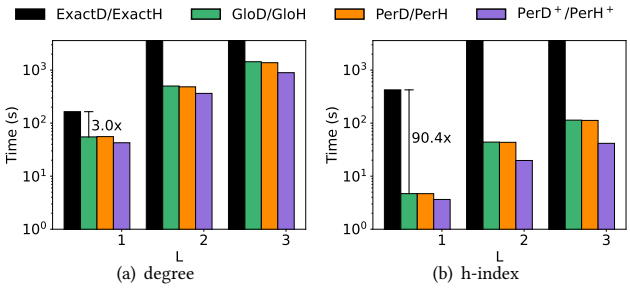


Figure 3: Running times of different algorithms on `Yelp` for meta-paths with varying length L.

produces correct early termination for the personalized queries across different datasets, as shown in Section 5.3.

Evaluation Metrics. We evaluate the quality of hubs each algorithm returns for global HUD queries (GloD and GloH) with the F1-score. Our algorithms return a node set S containing $\lambda \cdot |V|$ nodes. However, there might be less than $\lambda \cdot |V|$ nodes satisfying the criteria of Problem 1 due to tied values with v^λ. As such, we define *precision* as $|S \cap S^+|/|S|$ and *recall* as $|S \cap S^*|/|S^*|$ for F1-score computation. Here, S^+ represents the set of nodes with centralities at least the same as that of v^λ, and S^* represents the set of nodes with strictly higher centralities than v^λ. In effect, we include the nodes with tied centralities to v^λ in the precision calculation. To assess the effectiveness of PerD, PerD$^+$, PerH, and PerH$^+$ for personalized HUD queries, we record their accuracy in determining whether a query node q has equal or greater centrality compared to v^λ. We report the average F1 and accuracy scores across all meta-paths in Table 1. For efficiency evaluation, we report the average running time of each algorithm to process one HUD query.

Environment. All experiments were conducted on a Linux Server with AMD EPYC 7643 CPU and 256 GB memory running Ubuntu 20.04. All algorithms were implemented in C++ with `-O3` optimization and executed in a single thread.

5.2 Efficiency Evaluation

In Table 2, we present the execution time per query for each method, along with its speedup ratios compared to the corresponding exact method across five datasets. We also report the average time to construct the matching graph, which is nearly negligible compared with the running time of each algorithm. Based on these results, we make the following observations.

First, our sketch propagation algorithms are consistently more efficient than the exact baselines across all datasets except `IMDB` and provide more than 10x speedups for degree-based global queries (on `FreeBase`, GloD achieves a 29x speedup over ExactD) and upto two-orders of magnitude speedups for h-index-based global queries (on `PubMed`, GloH achieves a 135.7x speedup over ExactH). The speedup over degree-based global queries is smaller than h-index-based ones since h-index-based queries are approximated accurately with smaller k and θ as discussed in Section 5.3. GloD and GloH are slightly slower than ExactD and ExactH for degree-based queries on `IMDB` since `IMDB` is a very small dataset, where the worst-case optimal join can be performed quickly over the matching graph, whereas our methods require additional costs for sketch construction and degree estimation. Nevertheless, GloD answers HUD queries on `IMDB` in 0.0013s.

Second, the algorithms for personalized queries, i.e., PerD$^+$ and PerH$^+$, benefit from early termination optimization and achieve more than 2x additional speedups over PerD and PerH. For degree-based personalized queries, backward propagation, which can be early terminated, is more time-consuming than forward propagation since forward sketches usually contain fewer random numbers than backward sketches.

The experimental results for memory consumption are omitted here and left to the technical report [4] due to space limits.

Scalability. On the largest dataset `Yelp` (with 29.9M edges and more than 2000 candidate meta-paths), ExactD and ExactH cannot be finished within 24 hours. To evaluate the efficiencies of different algorithms, we sampled ten meta-paths of different lengths ($L = 1, 2, 3$) and ran each method within a one-hour limit per meta-path. Figure 3 shows that our proposed methods process all queries within the time limit, while the exact algorithms exceed the time limit for meta-paths of length $L = 2, 3$.

5.3 Effectiveness Evaluation

Results for Degree-based HUD Queries. Figures 4(a)–4(e) illustrate the effectiveness of GloD, PerD, and PerD$^+$ on each dataset with varying the parameter λ. We first observe that GloD achieves F1-scores of more than 0.85 across all datasets for global HUD queries, and PerD and PerD$^+$ consistently achieve at least 0.98 accuracy for personalized HUD queries. Moreover, GloD provides more accurate answers with increasing λ and achieves over 0.9 F1-score when $\lambda = 0.05$. The reason is that, for a larger λ, the HUD queries apply less strict requirements to nodes in the top-λ percentile, which are more easily approximated.

Figures 4(k)–4(o) show the effectiveness of GloD, PerD, and PerD$^+$ on each dataset with varying the parameter k. First, GloD, PerD, and PerD$^+$ all provide better results with increasing k. Larger KMV sketches allow the sketch propagation-based methods to approximate HUD queries with higher F1-scores or accuracy owing to more accurate degree estimations. Second, personalized HUD queries are accurately approximated by PerD and PerD$^+$, even with relatively smaller values of k. This is because personalized HUD queries only require comparing the degrees of q and v^λ. For the default setting of a relatively small $\lambda = 0.05$, most nodes in G have significantly different degrees from the degree of v^λ, facilitating the comparison. In the technical report [4], we also experiment with greater values of λ ranging from 0.1 to 0.5. In those experiments, PerD and PerD$^+$ still consistently achieve high accuracy for personalized HUD queries. Third, the gap between the accuracy of PerD$^+$ and PerD is marginal, even when $k = 2$ (with a maximum gap of 0.044 on `DBLP`), and narrows further with increasing k. This is because a larger k leads to more accurate estimations of image sizes, thus reducing the chance of false early termination.

Figures 4(p)–4(t) demonstrate the effectiveness of GloD, PerD, and PerD$^+$ on each dataset with varying parameter θ. First, we still observe that GloD returns monotonically better results with increasing θ since HUD queries are better approximated with more KMV sketches. We also observe that GloD is less sensitive to θ than

Table 2: Overall results for the efficiencies of different algorithms. For ExactD and ExactH, the running time on each dataset is reported. For GloD,PerD$^+$,GloH, and PerH$^+$, the running time, as well as the speedup ratios over ExactD and ExactH, are reported. The average time to construct the matching graph $\mathcal{G}^*$ is reported as $T(\mathcal{G}^*)$.

Centrality	Degree							H-index							$T(\mathcal{G}^*)$
Method	ExactD	GloD		PerD		PerD$^+$		ExactH	GloH		PerH		PerH$^+$		
IMDB	0.0009s	0.0013s	0.66x	0.0016s	0.55x	0.0017s	0.53x	0.0013s	0.001s	1.23x	0.0015s	0.82x	0.0015s	0.81x	0.88ms
DBLP	7.95s	0.63s	12.59x	0.67s	11.79x	0.27s	29x	14.59s	0.23s	64.32x	0.1s	148x	0.049s	298x	3.58ms
PubMed	5.96s	0.45s	13.24x	0.35s	17.05x	0.13s	45x	12.11s	0.09s	135.7x	0.056s	215x	0.032s	381x	8ms
ACM	4.77s	1.77s	2.7x	2.04s	2.34x	0.81s	5.91x	8.01s	0.19s	41.7x	0.25s	32x	0.076s	105x	4.18ms
FreeBase	25.15s	0.87s	29x	0.92s	27.4x	0.41s	60.9x	50.5s	0.26s	191x	0.19s	268x	0.13s	379x	26ms

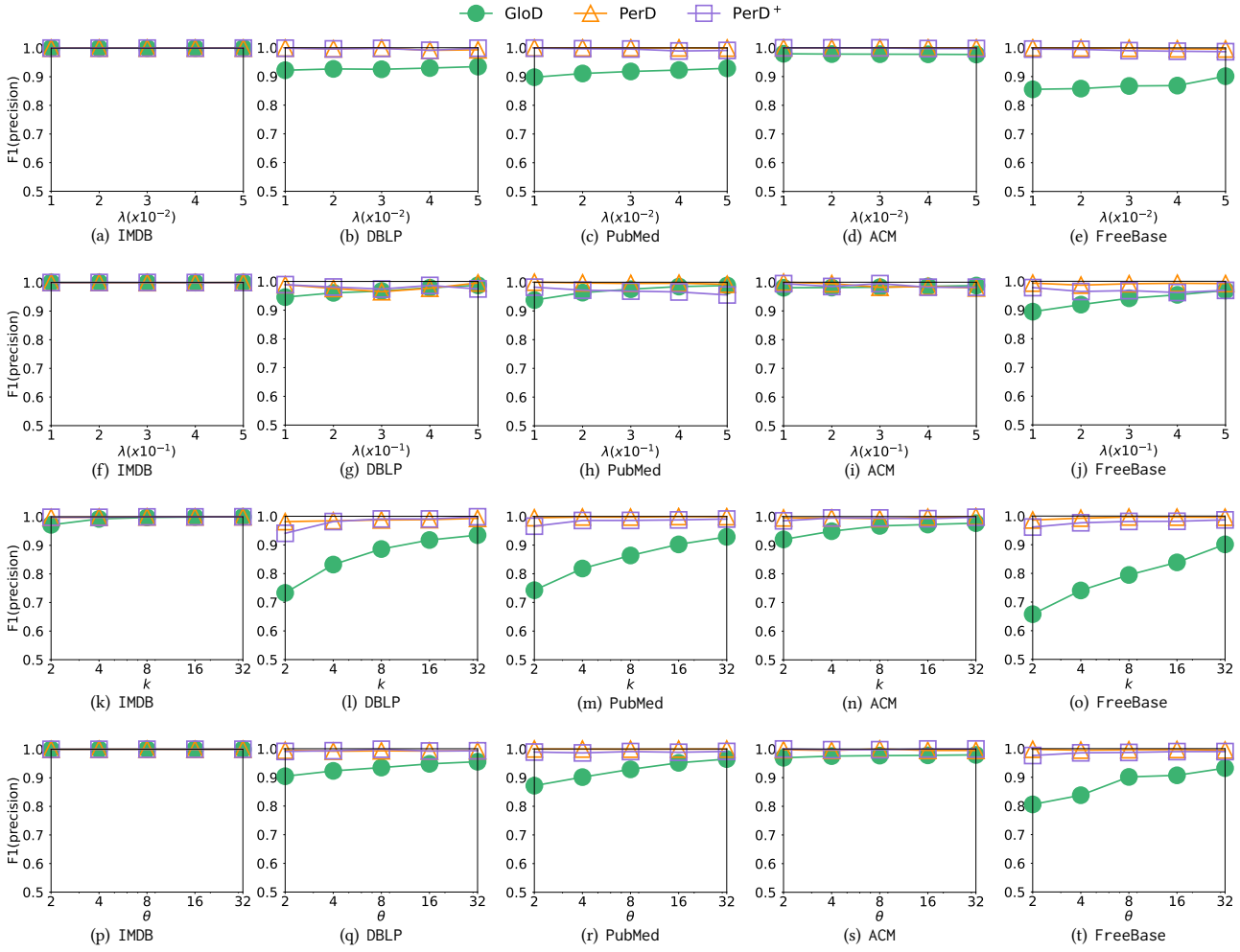


Figure 4: Effectiveness of GloD, PerD, and PerD$^+$ for HUD queries based on degree centrality with varying parameters. Subfigures (a)-(e): varying parameter λ. Subfigures (f)-(j): varying parameter k. Subfigures (k)-(o): varying parameter θ.

k. This is attributed to the default setting of $k = 32$, which already provides a reasonably accurate approximation. For personalized HUD queries, PerD and PerD$^+$ consistently achieve an accuracy of at least 0.95 across all datasets, regardless of the changes in θ.

Results for HUD Queries based on H-index. The effectiveness results of GloH, PerH, and PerH$^+$ are presented in Figure 5. Similar observations can be made for the methods for h-index-based HUD queries as those for degree-based HUD queries. Furthermore, we note that GloH achieves a higher F1-score for h-index-based HUD queries compared to GloD under the same parameter settings. This is because the range of h-index values is much smaller than that of degrees, resulting in a larger number of tied h-index values. By excluding the nodes with tied values to v^λ from the recall evaluation, h-index-based HUD queries become much easier to approximate.

Summary. GloD, PerD, and PerD$^+$ achieve F1-scores or accuracy of above 0.9 across all datasets when $k = 32$ and $\theta = 8$ for degree-based HUD queries with $\lambda = 0.05$. Similarly, for h-index-based HUD queries, GloH, PerH, and PerH$^+$ achieve F1 scores or accuracy of above 0.9 across all datasets when $k = 4$ and $\theta = 8$. All these results establish the effectiveness of our proposed methods in the default parameter settings.

6 RELATED WORK

Hubs in Hidden Graphs. Previous work has extensively explored the problem of discovering hubs in hidden graphs, where the existence of any edge can only be decided via *probe operations*. Tao et al. [31] proposed two instance-optimal exact algorithms over two classes of hidden networks. Wang et al. [34] extended this problem to include group testing, whereby probes can verify edge connections between multiple nodes. Sheng et al. [25] proposed a sampling algorithm for hub discovery, which Cao et al. [6] subsequently improved. Strouthopoulos and Papadopoulos [28] studied the discovery of k-cores in hidden networks. However, all these algorithms face two notable challenges when applied to HUD queries. First, they focus on bipartite graphs and potentially take $O(|V|^2)$ probe operations when handling multi-partite graphs induced by meta-paths. Second, their probe operations are time-consuming for our problem because they involve many breadth-first search (BFS) operations on the matching graph.

Xirogiannopoulos et al. [37] construct compact representations of hidden networks extracted by *path joins* in relational databases, which are analogous to meta-paths in KGs. However, this work is orthogonal to ours, while our sketch propagation framework is also applicable to the compact structure of [37]. Further, we employ exact algorithms that leverage worst-case optimal joins to compute neighbors efficiently. This aligns with the future direction suggested by [37] for improving the efficiencies of neighbor computations.

Meta-paths in KGs. Meta-paths have emerged as a prominent approach for extracting information from knowledge graphs (KGs). They serve as a fundamental concept for designing node similarity measures in KGs. Notable examples include Pathsim [29], Joinsim [36], RelSim [33], and Hetesim [26], among others. These measures leverage meta-paths to quantify the node similarity and play a crucial role in various KG analysis tasks. For instance, Pathsim evaluates the similarity between nodes in heterogeneous networks with the number of matching instances connecting them. Joinsim, on the other hand, is a variant of Pathsim that satisfies the triangle inequality, facilitating the efficient discovery of top-k similarity node pairs. Meta-paths have also found applications in community search within KGs. Fang et al. [12] and Yang et al. [38] proposed the $(k, \mathcal{M})$-core and $(k, \mathcal{M})$-Btruss models, respectively, for community search in KGs. These models are analogous to the well-known k-cores and k-trusses in the hidden network induced by $\mathcal{M}$. Additionally, meta-paths have been applied in recommender systems based on KGs [16, 27, 39, 40].

Even though hidden graphs induced by meta-paths have been widely used in various scenarios, the fundamental problem of hub discovery has received little attention. To the best of our knowledge, this is the first systematic investigation of efficient hub discovery from hidden graphs using meta-paths on KGs.

7 CONCLUSION

We introduced and investigated the problem of hub discovery (HUD) over hidden networks induced by meta-paths on KGs, based on degree or h-index scores. We proposed a sketch propagation framework for approximate degree-based HUD queries and a pivot-based algorithm that extends sketch propagation to h-index-based HUD queries. We studied personalized HUD queries based on both degree and h-index, and enhanced the corresponding algorithms using early termination. On the theoretical side, we showd that our sketch propagation and pivot-based algorithms answer approximate HUD queries correctly with high probability. Through extensive experimentation, we verified the effectiveness and efficiency of our proposals on real-world KGs.

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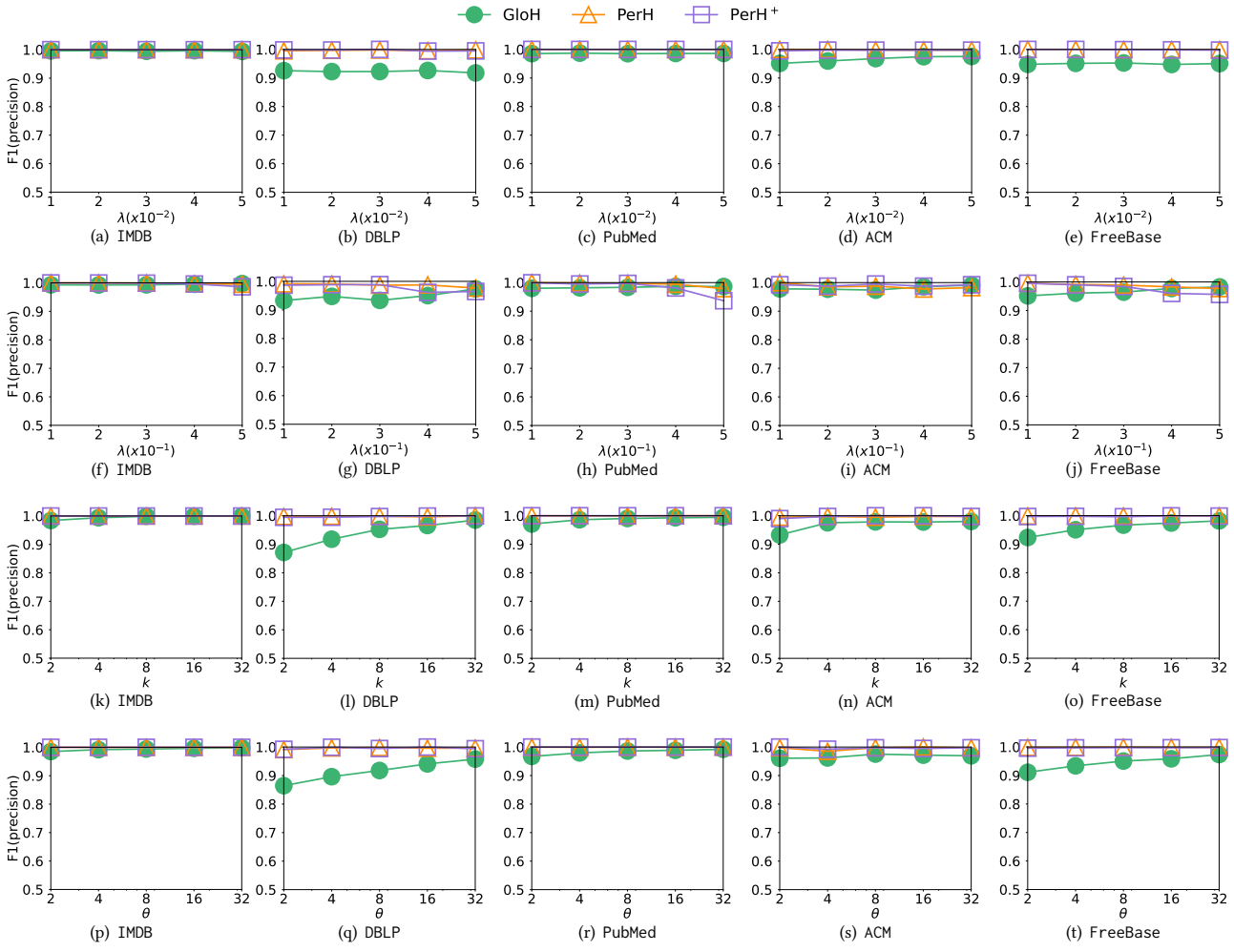

Figure 5: Effectiveness of GloH, PerH, and PerH$^+$ for HUD queries based on h-index centrality with varying parameters. Subfigures (a)-(e): varying parameter λ. Subfigures (f)-(j): varying parameter k. Subfigures (k)-(o): varying parameter θ.

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