Problem A: Analysis of the ion thruster to Saturn

Group : 241

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Abstract

This paper demonstrates thruster trajectory from the Earth to Saturn. Our team aims to minimize the fuel consumption by optimizing trajectory. Given the circumstance that spacecraft has fixed mass, less fuel consumption indicates more room for other detecting devices on the spacecraft, which is beneficial for research purpose. In this stimulation, the trajectory was controlled by change in velocity. In supplement to fuel, gravitational force from several planets in solar system are involved in accelerating and decelerating process.

An attempt to explain different stages of detailed flight path will be mentioned. Analysis of the impact of several gravitational field was involved in trajectory design. Additionally, mass reduction caused by continuously consumed fuels will be taken into consideration.

Contents

1 Introduction

1.1 Background

A spacecraft called Cassini was launched into space from the earth at 1997 by NASA. The final destination of Cassini is Saturn. However, the spacecraft was designed to follow a winding path. In April 1998 and June 1996, Cassini accept two gravity assists from Venus to help acceleration. The third time is by the earth in August 1999, followed by the fourth gravitational assist by Jupiter in December 2000. Finally, the spacecraft arrived at Saturn in July 1, 2004. This example inspired us to construct model to solve the topic.

1.2 Gravitational cancellation point

The spacecraft is continuously attracted by the earth and the sun while orbiting the earth. When the spacecraft is in its initial orbit, it receives much less gravitational force from the sun than it receives from the earth. According to the gravitational formula, the gravitational force of the earth on the spacecraft will decrease as the distance increases, and eventually decrease to the same level as the gravitational force of the sun at some specific points, which are gravitational cancellation points. Different from the Lagrange point, the motion state of the spacecraft at a gravitational cancellation point can be relatively moving. This point only cancels the two gravitational forces in an instant, making it easier for the aircraft to leave the orbit or the earth. Therefore, there should be enough gravitational cancellation points around the earth.

1.3 Slingshot effect

Slingshot effect, or gravity assist, it is used to accelerate or decelerate the spacecraft by utilizing the gravity of the planet to save fuel [1][2]. The process by which the speed of the spacecraft is changed by the gravity of planet is generally called elastic collision, although they do not actually touch each other. According to the formula, the speed of the spacecraft increases from the original v to v+2U after being accelerated by the slingshot effect of planet, and the whole process follows the conservation of momentum. One limitation of slingshot effect is that to complete this goal, Suitable actual is needed to be waited for, which means we need to wait for the target planet to move to the right position.

2 Assumptions and Notations

2.1 Assumptions

In the stimulation we can neglect the impact of the following factors:

1. Assume launching process to be mild, neglect the disturbance of sudden thrust in the direction of orbit.
2. Neglect the energy consumption when the spacecraft adjusts its angel in circular motion.
3. Assume the spacecraft is undisturbed by other space craft in space.
4. Neglect the gravitational force of the moon. Neglect the impact of solar tide wave.
5. Neglect the impact of solar pressure results in accelerating.
6. Neglect atmospheric friction when the spacecraft is in low earth orbit.

2.2 Notations

Variables	Explanations	Initial value
$\mathrm{M}_s, M_e, M_m, M_j, M_{ss}$	Mass of the sun, the earth, Mars, Jupiter, Saturn	
$\mathrm{R}_e, R_m, R_j, R_{ss}$	Radius of revolution cycles of the earth, Mars, Jupiter, Saturn	
$\mathrm{v}_e, v_m, v_j, v_{ss}$	Linear velocity of the earth, Mars, Jupiter, Saturn	
$\mathrm{T}_e, T_m, T_j, T_{ss}$	Revolution period of the earth, Mars, Jupiter, Saturn	
$\mathrm{r}_e, r_m, r_j, r_{ss}$	Radius of the earth, Mars, Jupiter, Saturn	
G	Gravitational constant	6.67259 $\times 10^{11} N \times m^2/kg^2$
Q	Propulsion force of the spacecraft	0.4 N
I_{sp}	Specific impulse	4000 s
ti	Time unit	1 s
H	Mass of the spacecraft	5000 kg
v_n, v_t	Normal and tangential velocity (relative to the sun)	
θ	Polar angle	0
r	Distance from the central	

3 Model

In the model of flight stimulation, we can divide it into three models. In model one, the probe accelerates by fuel. In the second model, the probe will be fast enough to escape circular orbit bounded by gravity. It will start to fly in elliptical circular motion and then the spacecraft will be accelerated by slingshot effect. The third model is the analysis of trajectory in space when spacecraft motion was not bound into circular or oval circular orbit.

3.1 Acceleration by fuel

After the probe was launched into low earth orbit and reach a state of plateau, we assume the potential energy at this point as initial potential energy E_{p0}. E_{p0} could be expressed as follows:

$$E_p = \frac{1}{2}mv^2 - \frac{GMm}{r_0} \tag{3.1}$$

As we know, v is the velocity of circular motion whose centripetal force is the gravitational force of the earth. The velocity v can be expressed as follows:

$$\frac{GMm}{r^2} = m\frac{v^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}} \tag{3.2}$$

We substitute v in equation (3.1) by the expression of v in equation (3.2), we can get the initial potential voltage as follows:

$$E_{p0} = \frac{1}{2}mv^2 - \frac{GM_eM}{r_0} = -\frac{GM_eM}{2r_0} \tag{3.3}$$

As the engine of the probe provides thrust to help acceleration. The thrust does positive work to the probe, which indicates the potential energy increase as the probe orbit. We assume the increased energy in every single orbit as ΔE, which could be expressed as:

$$\Delta E = F \cdot 2\pi r \quad E = E_{p0} = -\frac{GM_em}{2r_0} = -\frac{mv^2}{4} \tag{3.4}$$

We substitute E in Equation(3.4) with the expression of E from Equation(3.3). We will get:

$$F \cdot 2\pi r = \Delta(-\frac{GMm}{2r}) = -\frac{GMm}{2}(\frac{1}{r_2} - \frac{1}{r_1}) = \frac{GMm}{2} \cdot \frac{r_2 - r_1}{r_1 r_2} \tag{3.5}$$

We assume Δr to be small, which made both r_1 and r_2 similar to r when calculating.

$$\Delta r \to 0 \qquad \Delta r = r_2 - r_1 \tag{3.6}$$

$$r_1 = r_2 = r \tag{3.7}$$

Substitute r_1, r_2 in Equation (3.5) with r, we get the expression of Δr:

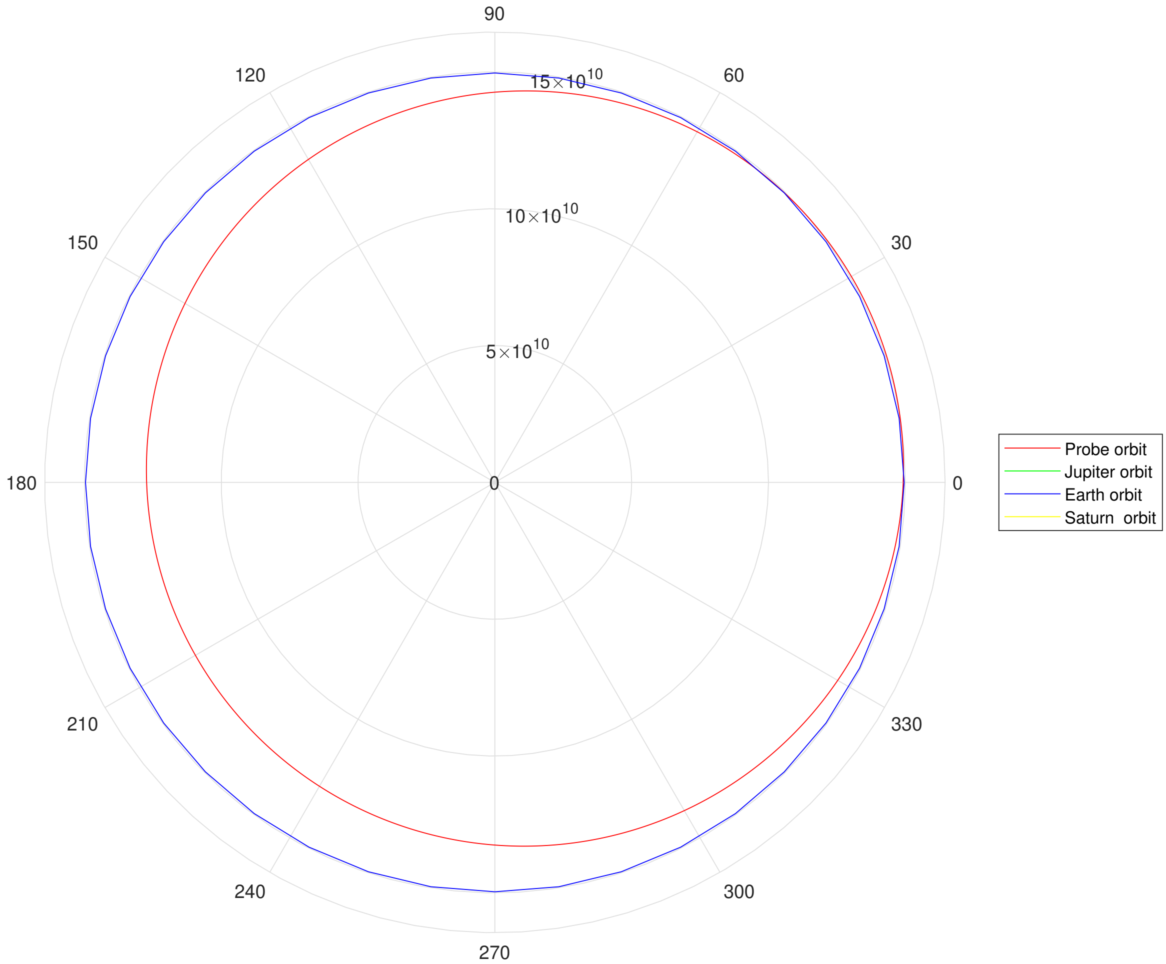
$$F \cdot 2\pi r^3 = \frac{GMm}{2} \cdot \Delta r \qquad \frac{F \cdot 4\pi r^3}{GMm} = \Delta r \tag{3.8}$$

Δr is the increase in orbit radius, which could be used in expression of Δr.
After the probe orbits multiple circles around the earth, there will be a point that the probe attained enough energy to escape the gravitation field of earth. The radius of the last orbit will be much larger than the penultimate radius. We use the following equation to examine after each circle to see whether the probe will escape earth's gravitational field.

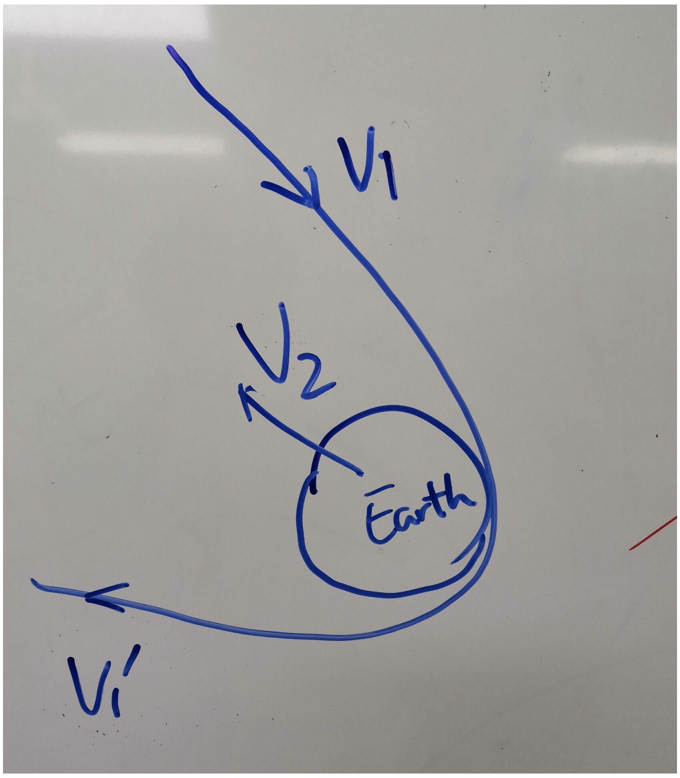
$$T_n = \sqrt{\frac{4\pi r_n^3}{GM}} \qquad v_n = \frac{2\pi r_n}{T_n} \tag{3.9}$$

3.2 Acceleration by the gravity

After the probe gain enough speed in stage 1, the probe will to do elliptical circular motion (as shown in Picture 1) until the probe encounter the earth again.



The spacecraft can be accelerated or decelerated by gravity applying slingshot effect. The effect of slingshot is similar to fully elastic collision when we are considering velocity change.



Define the velocity of the earth as v_2. Define the velocity of the spacecraft when encountering with the earth as v_1. Define the velocity of spacecraft after slingshot effect as $v_1^{`}$. Define the mass of spacecraft to be m_1. Define the mass of the earth to be m_2. Take the earth as reference frame.
Accordingly, we can express v_1:

$$v_1^{`} = \frac{v_1(m_1 - m_2) + 2m_2v_2}{m_1 + m_2} \tag{3.10}$$

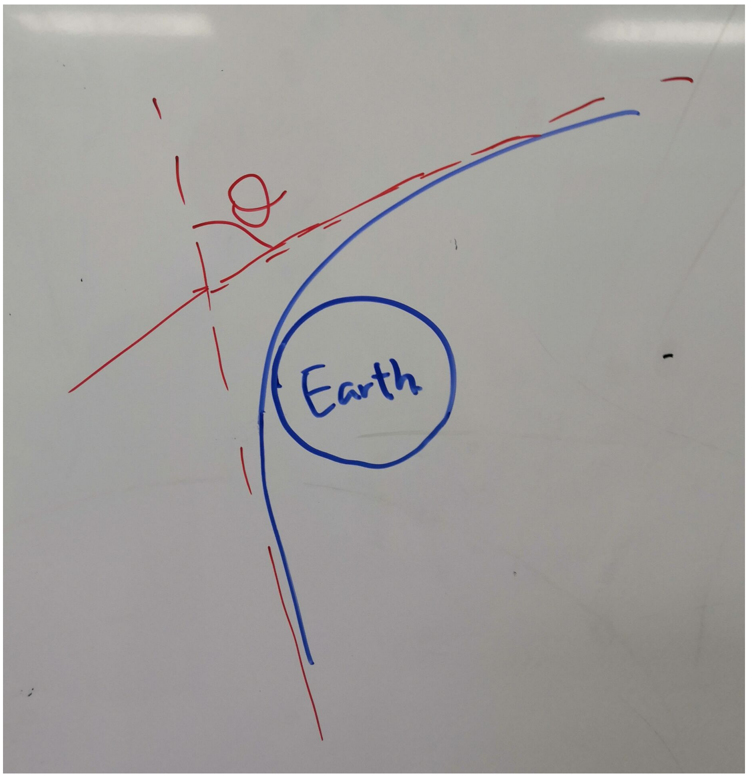
As we now, the mass of the earth is much larger than the mass of the space craft.

$$m_2 >> m_1 \tag{3.11}$$

Substitute the expression in (3.11) into (3.10):

$$v_1^{`} = \frac{-v_1m_2 + 2v_2m_2}{m_2} = -v_1 + 2v_2 \tag{3.12}$$

We can see that the magnitude of velocity is changed.
After slingshot, velocity also changes in direction.



We divide the velocity into normal velocity(v_n) and tangent velocity(v_t).
In normal direction, the spacecraft could be in either centripetal movement or in Eccentric movement depending on which of the following two inequality holds:

$$\frac{GMm}{r^2} > m\frac{v^2}{r} \quad (centripetal) \qquad \frac{GMm}{r^2} < m\frac{v^2}{r} \quad (centrifugal) \tag{3.13}$$

The normal vector v_n was obtained by the following equation:

$$v_n = \int (\frac{GM}{r^2} - \frac{v^2}{r})dt \tag{3.14}$$

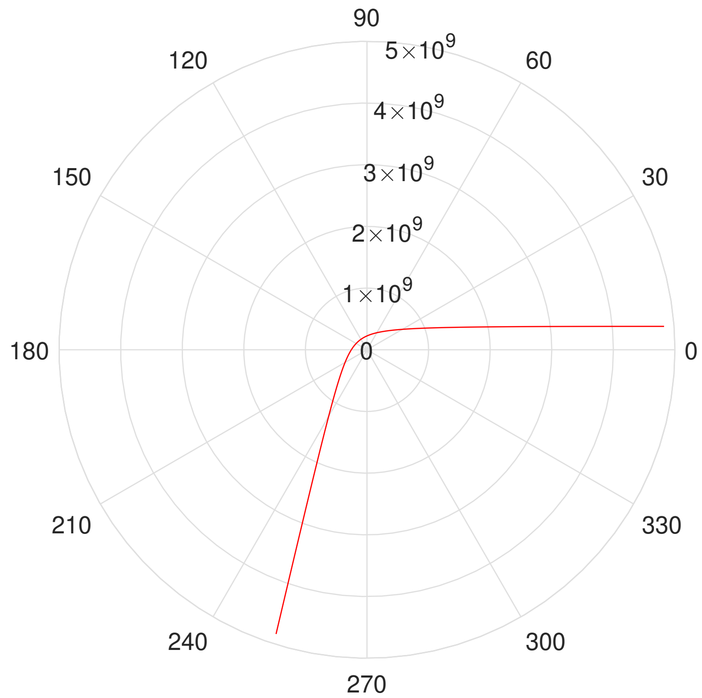
We assume r to be the distance from the position of spacecraft to the earth.

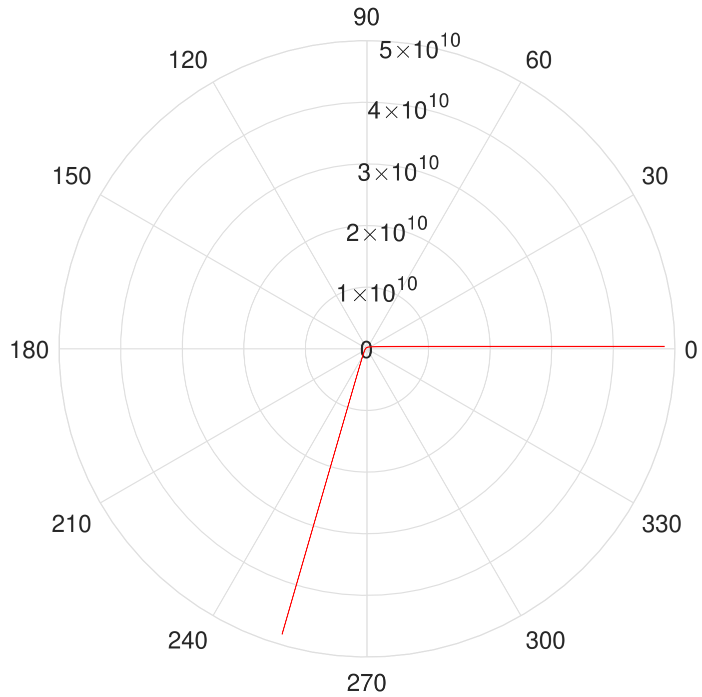
$$r_n = r_0 - \Delta r \qquad \Delta r = \int v_n dt \tag{3.15}$$

According to the conservation of angular momentum:

$$r^2 w_1 = r^2 w_2 \tag{3.16}$$

In this way, we can get the real-time location of the spacecraft.


90
120
60
150
30
180
0
210
330
240
300
270
5×10^9
4×10^9
3×10^9
2×10^9
1×10^9
0


90
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180
0
210
330
240
300
270
5×10^10
4×10^10
3×10^10
2×10^10
1×10^10
0

3.3 Analysis of spacecraft motion after acceleration

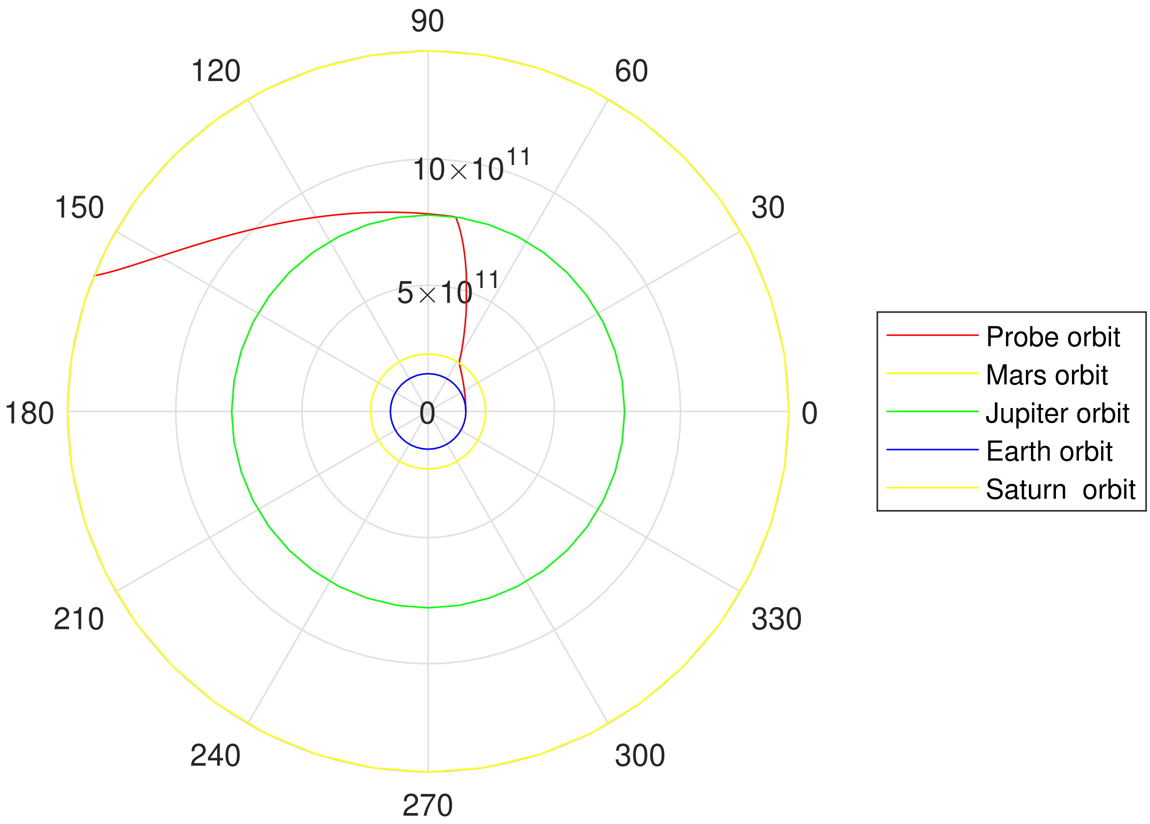
After accelerated by slingshot effect, the spacecraft will not be bounded into circular motion when travelling in space. However, the spacecraft will not travel in straight line either. We can obtain the track of spacecraft by analysing the force on the spacecraft. We define the positive direction of the spacecraft to be the planetary revolution direction of the solar system. We introduce Coriolis force when analyzing:

$$F_{col} = -2mw \times \vec{v} = -2m(\frac{\vec{v_t}}{r}) \cdot \vec{v}cos\theta \tag{3.17}$$

$$cos\theta = 1 \qquad \theta = \frac{\pi}{2} \tag{3.18}$$

$$F_{col} = -2m \cdot \frac{v_t}{r} \cdot v \tag{3.19}$$

In this way, we can explain the trajectory of spacecraft when travelling in space.



4 Results and Discussions

4.1 Results according to Model 1

$$R = \sqrt{\frac{GM_e}{0.0059}} = 2.5973 \times 10^8 \tag{4.1}$$

With G, M_s, R_e all known values, the gravitational force of sun is 0.0059 N.
R is the distance from spacecraft to the earth where the spacecraft began oval circuit, which is calculated to be 0.0059×10^8 meters.
Calculated by stimulation program, the spacecraft need to go through 4311 circuits to attain enough speed into oval circular orbit.

$$R(n) = 2.99 \times 10^8 > R \tag{4.2}$$

After the 4311^{th} orbit, R(4311) is larger than R.

$$R(n-1) = 2.51 \times 10^8 < R \tag{4.3}$$

After the 4310^{th} orbit, R(4310) is smaller than R.
We can conclude that in the 4311^{th} orbit we need to stop thrust supply at certain point in the orbit.
Fuel consumption:

$$H(n) = 4.2195 \times 10^3 \qquad H(1) - H(n) = 0.781 \times 10^3 \tag{4.4}$$

Time consumption:

$$R = \sqrt{\frac{GM_e}{0.0059}} = 2.5973 \times 10^8 \tag{4.5}$$

4.2 Acceleration by slingshot effect

The speed of spacecraft relative to earth is:

$$v_e = 2.8616 \times 10^4 \quad m/s \tag{4.6}$$

After accelerated using slingshot effect by the earth

$$v_t = 5.9938 \times 10^4 \quad m/s \tag{4.7}$$

$$v_n = 1.1348 \times 10^3 \quad m/s \tag{4.8}$$

The directional change (θ_1) in velocity after slingshot effect is

$$\theta_1 = 252^\circ \tag{4.9}$$

Then we keep the engine shut down.
When the spacecraft reaches the gravitational field of mars, the spacecraft was decelerated by the slingshot effect of the Mars. The directional change in velocity(θ_2):

$$\theta_2 = 135^\circ \tag{4.10}$$

After space travel, the spacecraft will reach the gravitational field of Jupiter and be accelerated by slingshot effect. The directional change in velocity is:

$$\theta_3 = 90^\circ \tag{4.11}$$

Once the spacecraft is ejected from Jupiter, turn the engine of spacecraft on. The velocity of the spacecraft when it reaches the gravitational field of Saturn is:

$$\sqrt{v_t^2 + v_n^2} = 6.039 \times 10^3 \tag{4.12}$$

Saturn speed is:

$$v_{ss} = 9.5962 \times 10^3 \quad m/s \tag{4.13}$$

By comparing the results of expression (4.12) and (4.13), we see that the speed of spacecraft is smaller than the Saturn speed. This means the spacecraft could be captured by Saturn.
Remaining fuel:

$$H = 1.6928 \times 10^3 \quad kg \tag{4.14}$$

The total time cost is 9 years for space travel and 4300 days for leaving earth orbit.

5 Evaluation

5.1 Advantages

1. Our team successfully build a model that works with high efficiency. The trajectory was clearly shown in the model.

2. Our team explain the theory of slingshot effect in detail.

3. Our team use MATLAB instead of Java to code, which improves efficiency and accuracy.

4. Our team minimize fuel consumption, which allows more space for other devices on space craft.

5.2 Disadvantages

1. In our model, it takes long time for the spacecraft to travel from Earth to Saturn.

2. We made too many assumptions and neglect the impact of too many factors. Our model might be impractical when it comes to application.

3. We neglect fuel consumption the space craft used to adjust. The magnitude of actual fuel consumption might be much larger than the stimulated vale.

4. After the spacecraft was captured by Saturn, we simplify our analysis. Analysis after being captured should be presented in detail in practical stimulation.

6 Conclusion

In the stimulation, it took the spacecraft 7585 days to reach the Saturn from the Earth. The first 9 years were spent to wait for the spacecraft in the oval circular circuit to encounter with the earth so that slingshot effect could happen again. The total energy consumption was 3307kg. Turn on the engine of the spacecraft to accelerate by fuel to escape gravitational circular orbit. Then turn off the engine and let the spacecraft to do elliptical motion until it encounters the Earth for slingshot. Then the spacecraft was decelerated by Mars and accelerated by Jupiter. After the spacecraft was captured by Saturn , turn on the engine until the spacecraft enters the orbit track with a cycle time of 40 hours.

7 Reference

[1] D. John, C. Robert, G. Asim, "Gravitational slingshot," American Journal of Physics, vol. 72, pp. 619-621, 2004.

[2] N. Arwen and F. Duncan, "Slingshot Dynamics for Self Replicating Probes and the Effect on Exploration Timescales," 2013.

8 Appendix

```
% code in matlab

% t1

Re = 1.49597 *10^11;        % distance from earth to sun
Rm = 2.2804e+11;            % distance of Mars
Rv = 1.08208930*10^11;      % Venus distance to sun
Ms = 1.9891*10^30;          % mass of Sun
G = 6.67259*10^-11;         % G
Q = 0.4;                    % push force
Isp = 4000;                 % fuel using

Me = 5.965*10^24;
Re1 = Re/100;
Rem = 3.84*10^8;
Lem = 6.5*10^7;
Lse = 1.5*10^8;
Ve = 2.98*10^4;


ti = 60;               % time unit (minute)
Tmax = 500000*ti;              % time max



a = zeros(1,Tmax/ti+1);        % acceleration
w = zeros(1,Tmax/ti+1);        % bu zhi dao
H = 5000*zeros(1,Tmax/ti+1);   % craft mass
Vn = zeros(1,Tmax/ti+1);       % normal speed
Vt = zeros(1,Tmax/ti+1);       % tangent speed
angle = zeros(1,Tmax/ti+1);    % polar coordinates
R = zeros(1,Tmax/ti+1);        % radis initial

H(1) = 5000;                % first craft mass
Vn(1) = 0;                  % first Vn
Vt(1) = (2*pi/(90*60))*(G*Me/(2*pi/(90*60))^2)^(1/3);     % first Vt
angle(1) = 0;               % first angel
```

```
R(1) = (G*Me/(2*pi/(90*60))^2)^(1/3);                  % first R = Re

n = 1;                      % init t (day count)
ti = zeros(1,500000);

Fe = zeros(1,500000);
Ve = 2*pi*Re/(365*24*3600);

Fs = G*Ms/Re^2;
Fe(1) = G*Me/R(1)^2;
Fc = Vt(1)^2/(Re);
F = 0;
 while H(n) > 0 && Fe(n) > 0.0059 && n<500000

% while H(n) > 0 && R(n)<Lse && n<500000
%while n < 314
    ti(n) = (4*pi^2*R(n)^3/(G*Me))^(1/2);
    R(n+1) = R(n) + Q*4*pi*R(n)^3/(G*Me*H(n));

    H(n+1) = H(n)-Q*ti(n)/(Isp*9.8);
    Vt(n) = 2*pi*R(n)/ti(n);
    Fe(n+1)=G*Me/(R(n))^2;
    n = n+1;
end % end while
n-1
Hn = H(n-1)
sum(ti)-ti(n-1)
R(n-1)
R(n-2)
sqrt(((Ve-Vt(n-1))^2/2-G*Me/R(n))*2)
```

```
% t2

% Mass of planets
Ms = 1.9891*10^30;            % mass of Sum
Me = 5.965*10^24;             % mass of Earth
Tm = 686.98*24*3600;
Tj = 11.8618*365.25*24*3600;
Tss = 10832.327*24*3600;
Mm = 6.4219*10^23
% Radius
Re = 1.49597 *10^11;
Rj = 7.7833 * 10^11;
Rm = 2.2794 * 10^11;
Rss = 1.4294* 10 ^12;
r_e =6.4871*10^6;
r_m =3.398*10^6;
r_j = 7.1398*10^7;
r_ss = 6.033*10^7;

% Radius
Re = 1.49597 *10^11;
Rj = 7.7833 * 10^11;
Rm = 2.2794 * 10^11;
Rss = 1.4294* 10 ^12;

% Gravitational constant
G = 6.67259*10^-11;

% Craft
Q = 0.4;                      % push force
Isp = 4000;

% Velocity
Ve = 2.98*10^4;

% Time unit
ti = 60;                       % 1 time unit(1 for 1 second)
T = ti * 500000;              % max compute time
V1 = Ve-2.8616e+04;

% Initial value
a = zeros(1,T/ti+1);          % acceleration
w = zeros(1,T/ti+1);          % bu zhi dao
H = 5000*zeros(1,T/ti+1);     % craft mass
```

```
Vn = zeros(1,T/ti+1);        % normal speed
Vt = zeros(1,T/ti+1);        % tangent speed
angle = zeros(1,T/ti+1);     % polar coordinates
R = zeros(1,T/ti+1);         % radius initial


% Input values
H(1) = 4.2028e+03;                 % first craft mass
Vn(1) = 0;                   % first Vn
angle(1) = pi/40;
Vt(1) =V1*sin(angle(1));       % first Vt
Vn(1) = -V1*cos(angle(1));
              % first angel
R(1) = 3.796e+08/sin(angle(1));

t = 1;                       % loop count

while R(t)<R(1)+1 && t<60000000 % if not reach Venus and not reach time max
    a1 = G*Me/(R(t)^2);      % gravity acc
    a2 = (Vt(t)^2)/R(t);     % xiang xin li
    a(t) = a2-a1;           % acc against Sun
    %Vn(t+1)=Vn(t)+a(t)*ti-(Q)*ti/H(t);
    Vn(t+1)=Vn(t)+a(t)*ti;
    R(t+1) = R(t)+Vn(t)*ti;
%     Vt(t+1) =  Vt(t)+(Vt(t)/R(t))*Vn(t)*ti;
    Vt(t+1) =  Vt(t)*R(t)/R(t+1);
    angle(t+1) = angle(t) + Vt(t)*ti/R(t);
%     H(t+1) = H(t)-Q*ti/(Isp*9.8);
    t = t+1;
end
t
polarplot(angle(1:t-1),R(1:t-1),'r');
angle(t)
angle(t)/pi*180
```

```
% t3

% Mass of planets
clear
Ms = 1.9891*10^30;          % mass of Sum
Me = 5.965*10^24;           % mass of Earth
Mss = 5.68*10^26;
Tm = 686.98*24*3600;
Tj = 11.8618*365.25*24*3600;
Tss = 10832.327*24*3600;

% Radius
Re = 1.49597 *10^11;
Rj = 7.7833 * 10^11;
Rm = 2.2794 * 10^11;
Rss = 1.4294* 10 ^12;
r_e =6.4871*10^6;
r_m =3.398*10^6;
r_j = 7.1398*10^7;
r_ss = 6.033*10^7;

% Gravitational constant
G = 6.67259*10^-11;

% Craft
Q = 0.4;                    % push force
Isp = 4000;

% Velocity
Ve = 2.98*10^4;
Vss = 2*pi*Rss/Tss;
% Time unit
ti = 300;                     % 1 time unit(1 for 1 second)
T = ti * 1000000;            % max compute time

% Initial value
a = zeros(1,T/ti+1);        % acceleration
w = zeros(1,T/ti+1);
H = 4.2195*10^03*ones(1,T/ti+1);   % craft mass
Vn = zeros(1,T/ti+1);       % normal speed
Vt = zeros(1,T/ti+1);       % tangent speed
angle = zeros(1,T/ti+1);    % polar coordinates
R = zeros(1,T/ti+1);        % radius initial
```

```
% Input values
Vn(1) = -(Ve-2.8616e+04)*sin(4.4230);                  % first Vn
Vt(1) =2*Ve-(Ve-2.8616e+04)*cos(4.4230);      % first Vt
angle(1) = 0;              % first angel
R(1) = Re-3.7968e+08;

t = 1;                     % loop count

while R(t)<Rm  % if not reach Venus and not reach time max
    a1 = G*Ms/(R(t)^2);     % gravity acc
    a2 = (Vt(t)^2)/R(t);
    a(t) = a2-a1;          % acc against Sun
    %Vn(t+1)=Vn(t)+a(t)*ti-(Q)*ti/H(t);
    Vn(t+1)=Vn(t)+a(t)*ti;
    R(t+1) = R(t)+Vn(t)*ti;
    Vt(t+1) =  Vt(t)-(Vt(t)/R(t))*Vn(t)*ti;
    angle(t+1) = angle(t) + Vt(t)*ti/R(t);
%     H(t+1) = H(t)-Q*ti/(Isp*9.8);
    t = t+1;
end
 t
%  Vt(t)
sqrt(Vt(t)^2+Vn(t)^2)
%  Vt(t) =2*2*pi*Re/(11.86*365*24*3600)-sqrt(Vt(t)^2+Vn(t)^2);
  Vt(t) =2*2*pi*Rm/Tm-cos(45*pi/180)*sqrt(Vt(t)^2+Vn(t)^2);
 Vt(t)
 Vn(t)=sin(45*pi/180)*sqrt(Vt(t)^2+Vn(t)^2);

while   R(t)<Rj && t<300000  % if not reach Venus and not reach time max
    a1 = G*Ms/(R(t)^2);     % gravity acc
    a2 = (Vt(t)^2)/R(t);
    a(t) = a2-a1;          % acc against Sun
    Vn(t+1)=Vn(t)+a(t)*ti;
    Vn(t+1)=Vn(t)+a(t)*ti;
    R(t+1) = R(t)+Vn(t)*ti;
    Vt(t+1) =  Vt(t)-(Vt(t)/R(t))*Vn(t)*ti;
    angle(t+1) = angle(t) + Vt(t)*ti/R(t);

    t = t+1;
end
t
Vt(t) =2*2*pi*Rj/Tj-sin(90 *pi/180)*sqrt(Vt(t)^2+Vn(t)^2);
% Vt(t)
```

```
 Vn(t)=cos(90 *pi/180)*sqrt(Vt(t)^2+Vn(t)^2);
H(t)
while R(t)<Rss && t<1000000 && H(t) > 10 % if not reach Venus and not reach time max
    a1 = G*Ms/(R(t)^2);     % gravity acc
    a2 = (Vt(t)^2)/R(t);
    a(t) = a2-a1;          % acc against Sun
    Vn(t+1)=Vn(t)+a(t)*ti;
    Vn(t+1)=Vn(t)+a(t)*ti;
    R(t+1) = R(t)+Vn(t)*ti;
    Vt(t+1) =  Vt(t)-(Vt(t)/R(t))*Vn(t)*ti-(Q)*ti/H(t);
    angle(t+1) = angle(t) + Vt(t)*ti/R(t);
    H(t+1) = H(t)-Q*ti/(Isp*9.8);
    t = t+1;
end
t
Vn(t)
sqrt((Vn(t)-Vss)^2+Vt(t)^2)

polarplot(angle(1:t-1),R(1:t-1),'r');
hold on
polarplot((0:pi/20:2*pi),Rm*ones(1,41),'y')
hold on
polarplot((0:pi/20:2*pi),Rj*ones(1,41),'g')
hold on
polarplot((0:pi/20:2*pi),Re*ones(1,41),'b')
% hold on
polarplot((0:pi/20:2*pi),Rss*ones(1,41),'y')
hold on
% polarplot(angle(1:t-1),R(1:t-1));
rlim([0 1.3*Rss])
legend('Probe orbit','Mars orbit','Jupiter orbit','Earth orbit','Saturn  orbit')
```