CS685 Homework 7

Yue Hao, yhao3@gmu.edu

1. The measurements has zero-mean Gaussian noise, and the range and the bearing measurements are independent of each other.

$$\begin{aligned} P(z|x,l) &= P(z_r|x,l)P(z_\theta|x,l) \\ &= \frac{1}{\sigma_r\sqrt{2\pi}}e^{-\frac{(z_r-||l-x||)^2}{2\sigma_r^2}} \cdot \frac{1}{\sigma_\theta\sqrt{2\pi}}e^{-\frac{(z_\theta-\theta)^2}{2\sigma_\theta^2}} \end{aligned}$$

where $||l-x||$ is the distance between x and l and θ is the angle between x and l.

2. (a) My friend is more likely to be on campus.
To calculate which place my friend is more like to be is to compare $P(p_0|d,x)$ and $P(p_1|d,x)$ which is larger. According to Baye's rule, we have

$$P(p|d,x) = \frac{P(d|p,x)P(p|x)}{P(d|x)}$$

where

$$\begin{aligned} P(d|p,x) &= P(d_0|p,x_1)P(d_1|p,x_0) \\ &= \frac{1}{\sigma_0\sqrt{2\pi}}e^{-\frac{(d_0-||p-x_0||)^2}{2\sigma_0^2}} \cdot \frac{1}{\sigma_1\sqrt{2\pi}}e^{-\frac{(d_1-||p-x_1||)^2}{2\sigma_1^2}} \\ &= \frac{1}{\sqrt{2\pi}}e^{-\frac{(3.9-||p-x_0||)^2}{2}} \cdot \frac{1}{\sqrt{3\pi}}e^{-\frac{(4.5-||p-x_1||)^2}{3}} \end{aligned}$$

because distance measurements are independent of each other, and disturbed by zero-mean Gaussian noise.
We have

$$||p_0-x_0|| = \sqrt{(10-12)^2+(8-4)^2} = \sqrt{20}$$
$$||p_0-x_1|| = \sqrt{(10-5)^2+(8-7)^2} = \sqrt{26}$$
$$||p_1-x_0|| = \sqrt{(6-12)^2+(3-4)^2} = \sqrt{37}$$
$$||p_1-x_1|| = \sqrt{(6-5)^2+(3-7)^2} = \sqrt{17}$$

So,

$$P(d|p_0,x) = 0.0979$$
$$P(d|p_1,x) = 0.0114$$

Also, because we have no prior knowledge.

$$P(p_0|x) = P(p_1|x) = 0.5$$

Subsequently,

$$P(p_0|d,x) = \alpha P(d|p_0,x)P(p_0|x) = 0.0489\alpha$$
$$P(p_1|d,x) = \alpha P(d|p_1,x)P(p_1|x) = 0.0057\alpha$$

(b) If we have prior knowledge that

$$P(p_0|x) = 0.3$$
$$P(p_1|x) = 0.7$$

Then

$$P(p_0|d, x) = \alpha P(d|p_0, x)P(p_0|x) = 0.0294\alpha$$
$$P(p_1|d, x) = \alpha P(d|p_1, x)P(p_1|x) = 0.0080\alpha$$

My friend is still more likely to be on campus.

3. Shown in Fig. 1.

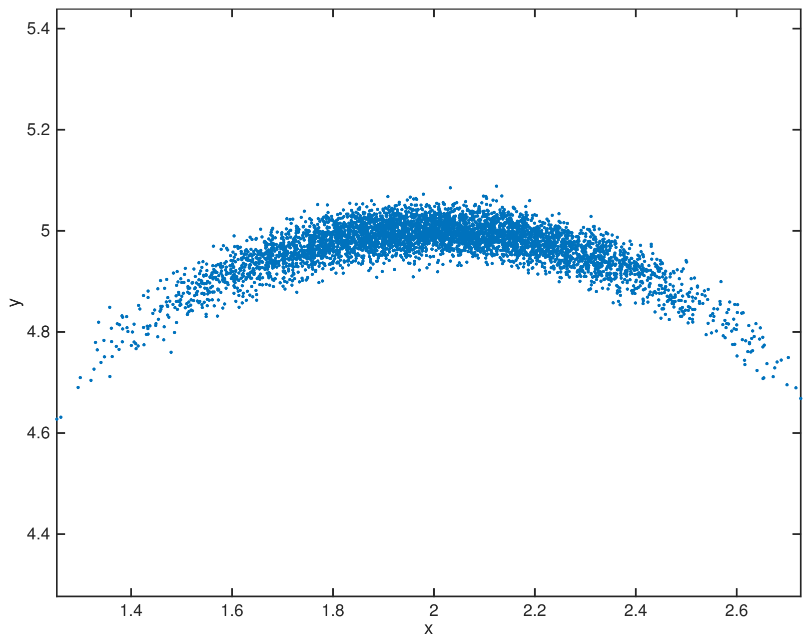

Figure 1: Motion model in 5000 runs.