PHYS1110D – Engineering Physics: Mechanics and Thermodynamics

Week 12: Ideal Fluids

Diagram

Description automatically generated**Problem 1 – Bernoulli’s Equation**

A pitot tube can be used to determine the velocity of air. If the fluid in the tube is mercury and , find the speed of the air flow. Assume that the air is stagnant at point A and the height between point A and point B is negligible. Take and

**Solution:**

Apply the Bernoulli’s equation to points A and B for the flowing air:

The tube tells us that the pressure difference between A and B is

Therefore

Chart, line chart

Description automatically generated**Problem 2 – Buoyancy and Equilibrium**

A uniform thin rod (its cross-section area can be neglected) is hung at one end and is partially submerged in water. If the density of the rod is 5/9 that of water, find the fraction of the length of the rod below water.

**Solution:**

We first analyze all the forces on the rod. Since its density is uniform, the buoyancy on the rod acts as if on the mid-point of the fraction of the rod in the water.

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Let be the fraction of the rod length submerged in water. The buoyancy is

The gravity is

For convenience, we choose the right end as the origin to avoid considering the tension in the string. The total torque should be zero:

Thus, we obtain

A picture containing shape

Description automatically generated**Problem 3 – Continuity of Fluids**

Water flows steadily in a fork shape water pipe horizontally from left to right as shown in the figure. In the left section of the pipe, the water flows at a speed . The cross-section area of the left section is . In the right section, the water flows into three separated channels with a smaller speed. The cross-section area of one channel is and that of other channels are /2.

1. Find the speed of water in the right section of the pipe.
2. Find the pressure difference between two ends of the middle pipe.

**Solution:**

1. Note that the following two configurations are equivalent.

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Then, we can apply the continuity equation to determine the speed of water in the right section

1. Applying Bernoulli’s Equation, we can relate the pressure at the two ends of the pipe by

Then, the pressure difference is given by

*Remark*: As pressure difference exists inside the pipe, there is a pressure gradient force acting on the water. Note that the pressure gradient force is pointing from higher pressure to lower pressure, so the force is in opposite direction to the flow in this case. It explains why the flow speed is decreased.

**Problem 4 – Torricelli’s law**

Diagram

Description automatically generatedThe water in a water pipe with a small hole is flowing out with a varying rate. The cross-section areas of the pipe and the hole are respectively.

1. Let the volume of water remaining in the pipe be . Show the volume outflow rate is given by

Here is the height of water above the hole.

1. Suppose that the initial height . When the water level reaches the hole, the flow stops. Show the total time of flowing is

**Solution:**

1. Obviously

Here is the flowing velocity through the hole, and is the velocity at the top of water. They are related by Bernoulli’s Equation:

Then

1. The equation for is a differential equation of : it is not difficult to see that

Therefore

Now we follow the standard steps: we rewrite it as

Integrate both sides from to :

Therefore