PHYS1110D – Engineering Physics: Mechanics and Thermodynamics

Tutorial Problems for Week 3: Integration

**Formulae You Should Memorize:**

**Problem 1 – Linear Motion with Changing Acceleration**

1. A body is moving on the axis. At time , it is at the origin and is moving at velocity 10 m/s2 towards the direction. Find the position for if its acceleration is given by:

a) ; b) (and in m/s2 and seconds respectively.)

1. A body is moving on the axis. Find the position and acceleration for if the velocity of the body is given by

a) ; b) (and in m/s and seconds respectively)

**Solution:**

Solving this problem is in principle simple:

However, we remind you an important detail: whenever you are going to integrate, don’t forget the *initial condition*! They correspond to the *integration constants* when calculating indefinite integrals.

* 1. (*Again, don’t forget !*)

*Remark*: You can try applying the rule for differentiating the product of two functions

when finding the acceleration. Check if the results agree with each other.

**Problem 2 – Average Value**

The average velocity with respect to time over is given by

If an object moves with velocity on the axis ( are positive constants with appropriate units):

1. Is this object moving with constant acceleration?
2. Find the average velocity in the time interval (. Is it equal to or ?

**Solution:**

1. Obviously not, since
2. By definition

Meanwhile

They are of course not equal to .

*Remark: Do Dimensional Analysis for Your Answer*

The last question is designed to emphasize the importance of *units* of physical quantities. In the past, some really confused students thought that the average velocity is given by

But we can *immediately* say that it must be wrong, since it is *not even a velocity*. To see this, we check the unit of this expression

It corresponds to the unit of the *acceleration*. However, if you thought

We cannot easily tell whether it is true or not, since this wrong expression is indeed some kind of velocity. Nevertheless, such checking of the unit (called **dimensional analysis**) can quickly help you eliminate obviously unreasonable results.

**Problem 3 – Integration by Parts**

Integrating both sides of the rule

We obtain

Rearranging the terms, we obtain the formula of **integration by parts:**

Or, in terms of indefinite integrals

Using this theorem, calculate the following integrals ( is a constant number):

**Solution**:

1. The traditional solution is to use integration by parts:

But there exists a clever trick: although is a constant in the integration, we can still formally calculate the derivative of :

Recall the definition of integration:

That is, *the differentiation with respect to can be exchanged with integration over* . This is true except for some weird examples constructed by the mathematicians.

It can be easily calculated that

Then

1. The clever trick shown above can also be applied to this question with slight modifications, but here we only show the standard approach by integration by parts:

**Problem 4 – Introduction to Fourier Series**

Review: Euler’s formula

1. Calculate the following integral:

Here is the imaginary unit; are arbitrary integers; is a positive real number; is an arbitrary real number.

1. Directly, or make use of the previous result, to calculate the following integrals:

Here are *non-negative* integers; is a positive real number; is an arbitrary real number.

*Remark*: These formulas are important for constructing the **Fourier series** of a periodic function.

**Solution**:

1. Using the property , we obtain
   1. When : (LHS = Left-Hand Side)

But we know that is an integer, thus

And .

* 1. When :

To summarize:

Here we used the notation of **Kronecker delta**:

1. Using Euler’s formula, we obtain
   1. Integral 1
   2. Integral 2

Since are both nonnegative, if , then we must have , which further leads to . Therefore

* 1. Integral 3