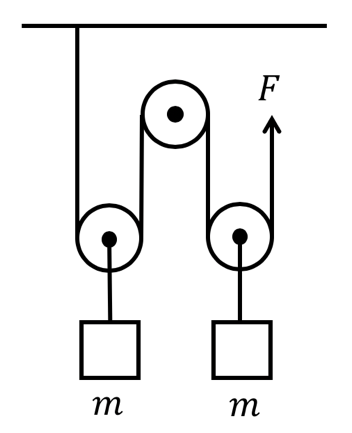
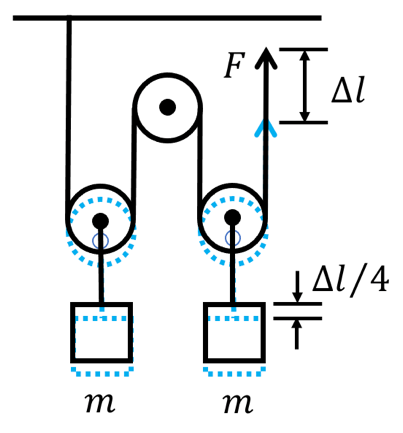
PHYS1110D – Engineering Physics: Mechanics and Thermodynamics

Tutorial Problems for Week 4: Newton’s Laws of Motion

**Problem 1 – Pulleys**

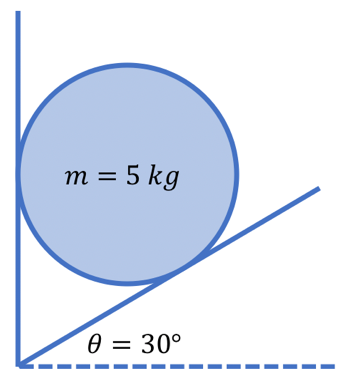
Two blocks with the same mass are hanging on two pulleys (see the figure). The pulley in the middle is fixed on a wall, while the other two can move freely. A force (upward) is applied to the rope so that both blocks are accelerating upwards with constant acceleration . All segments of the rope are in vertical direction (except when they are winding around the pulleys). Ignoring all frictions and the mass of the pulleys and the rope:

1. If the end of the rope moves upward by a distance of , how will the two blocks move?
2. What is the magnitude of this force?



**Solution:**

1. The two blocks will move upward by (You can just feel it from the geometry – we don’t want to say too much here)
2. For each block, we obtain



**Problem 2 – Normal Force**

A solid ball of mass is placed in a wedge formed by two planes as shown in the figure. Assuming no friction between the ball and the walls, find the magnitude and direction of both forces. ()

**Solution**:

The normal force is *always perpendicular to the contacting plane*.

Diagram

Description automatically generatedLet the magnitude of the two forces be . Draw the forces on the ball, and write down the equilibrium condition in the vertical and horizontal directions:

Therefore

*Remark*: You can benchmark your results by setting (then and ). In this case, the force exerted by the two walls should be very large. Such checking of answers prevents you from writing down things like (this will happen for some confused students).

**Problem 3 - Friction**

Chart, waterfall chart

Description automatically generatedA block of mass is placed on a rough plane. The static and kinetic friction coefficient are both . An external force is applied to the block at an angle . Find the magnitude of the friction on the block when the magnitude of is (taking ):

Diagram, schematic

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**Solution**:

The question is designed to remind you that the static friction can be smaller than , as long as the object is not slipping.

Let us first draw all forces acting on the block (shown on the right). We first need to determine *whether the static friction can be large enough to balance* . Thus, let us first determine :

The corresponding maximum static friction forces in these two cases are

The horizontal component of is

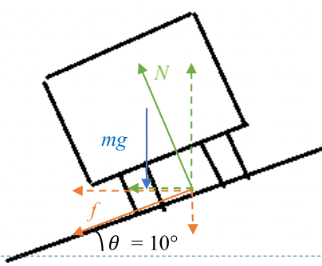
Therefore, when , the static friction can be large enough to balance :

But when , the block will start to slip:

A close up of a logo

Description automatically generated**Problem 4 – Circular Motion on Banked Road**

A car is moving on a circular path on a highway. The distance between the car and the center of the circular path is 60 m (*ignoring the size of the car*). The road is banked at an angle of 10°. Given that the coefficient of static friction between the tires and the road surface is 1, find the range of the speed of the car such that it can move without slipping.

**Solution**:

The equation of motion along the radial and the vertical directions are (here we assume that the friction is pointing inwards; when it is in fact outwards, will be negative)

We first write solve for and as functions of :

(*You can check whether these results are reasonable (“benchmarking”) by setting : then*

*These are as expected.*)

When the car does not slip, the range of should be

* Let us first check :

We find that always holds for *all* values of.

* Next, check :

Solving this inequality and notice that , we obtain

**Appendix – The Terminal Speed**

In the lecture, you learned that for a body falling in the air, the **drag force (air resistance)** is given by the formula

The minus sign means that the force is in the opposite direction of . Suppose that the body starts falling from stationary, you may wonder if we can find an explicit expression for the velocity (as shown in the figure below).

A close up of a map

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*Can you guess which function matches the actual velocity-time graph?*

It turns out that we can. First, let us write down the equation of motion for the falling body:

To solve this differential equation, we use the routine of **separation of variables**: the equation can be rewritten as

Then we *integrate over both sides* from the initial to the final state ( is the initial velocity):

On the right-hand side, since starts from 0, we shall assume that will not exceed 1. We shall check later if this is indeed the case. Making the substitution

We obtain (omitting the integration limits here)

Therefore

Finally, we obtain the explicit expression of :

Since always holds, we indeed have , as assumed earlier. Note that

This is just the expression of the **terminal speed**.