Q6.

(1) Computational Cost of Communicating One Sample

When $\beta = 1$, the proposal distribution follows Laplace distribution.

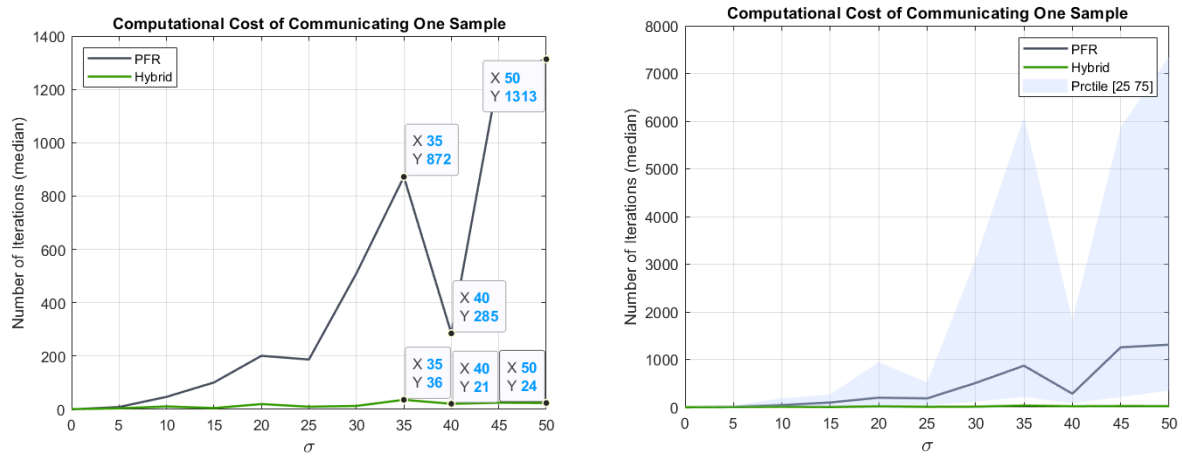

Figure 1. Computational Cost of Communication one sample: (a) median value of n chosen by PFC and Hybrid coding respectively; (b) Region of 25th and 75th percentile of n* chosen by PFC is shaded.*

The generating distribution $\hat{p}$ also follows Laplacian distribution, with $\mu = x$ and $var = \sigma^2 + I$. In our experiment, x acting as the mean is a standard Gaussian random variable $\sim \mathcal{N}(0,1)$.

The target distribution $\hat{q}$ is generated analogous to Appendix H. in [1]. It is a truncated Laplacian distribution.

As can observed from the figures above, hybrid coding outperforms PFC for large σ.

```
function [q_z, M] = q(x,z,sigma)
    theta = 10e-4; mu1=0;s1=1;
    a = lap_invcdf(theta/2,mu1,s1);
    b = lap_invcdf(1-theta/2,mu1,s1);

    % Candidate Distribution
    mu2=0; s2 = sqrt(sigma^2 + 1);
    a_phi = lap_cdf(a,mu2,s2);
    b_phi = lap_cdf(b,mu2,s2);
    M = floor(1/(b_phi - a_phi));

    % Generate a random sample from Canadidate Distribution
    mu3=0; s3=sqrt(sigma^2+1);
    z_hat = lap_invcdf(z,mu3,s3);
    z_pdf = lap_pdf(z_hat,mu3,s3);
    % Transformed Target Distribution
    mu4=x; s4 = 1;
    qhat_pdf = lap_pdf(z_hat,mu4,s4);

    % Compute pdf of z in target distribution
    q_z = (1/M)*(qhat_pdf/z_pdf);

end
```

Figure 2. MATLAB code for generating target distribution for Hybrid Coding

(2) Performance Comparison between Different Sampling Methods

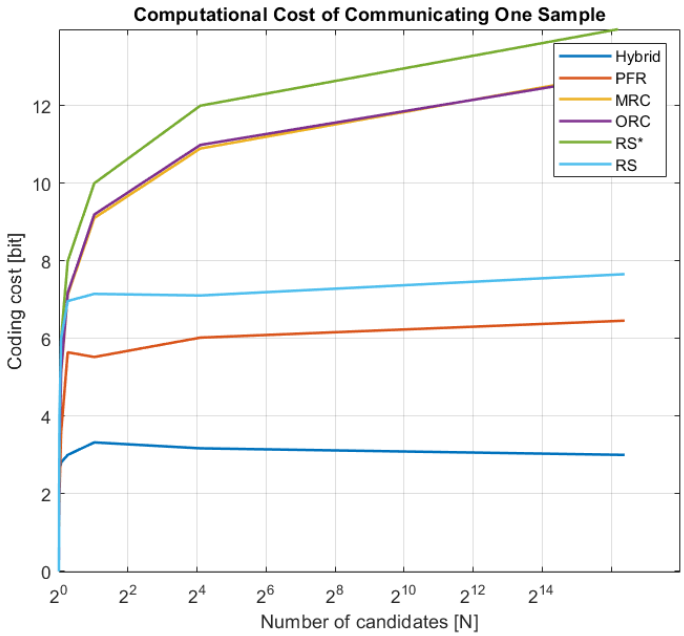

Figure 3. Coding cost v.s. Number of max candidates. The coding cost as a function of

a. When generating $\boldsymbol{n}^*$ for MRC and ORC, we inevitably modify the presumed target distribution $\hat{q}$ as a *3rd-order* generalized exponential distribution, otherwise the importance associated with each sample is relatively small. Thus, intuitively, MRC and ORC required more parameters to describe the target distribution.
b. As can be seen from Fig. 3, hybrid coding outperforms all the other approaches, and saturates quickly as the number of candidates increases. However, *RS* (RS-greedy)* and MRC/ORC tend to grow unboundedly.

```
function fx = q_MRC(y,mu,sigma)
    beta = 3;
    fx = beta/2*gamma(beta)*exp(- (sqrt(2)*abs(y-mu)/sigma)^beta );
end
```

Figure 4. MATLAB code for generating target distribution for MRC (Minimal Random Coding)

Reference

[1] L. Theis and N. Y. Ahmed, 'Algorithms for the Communication of Samples', in *Proceedings of the 39th International Conference on Machine Learning*, 17--23 Jul 2022, vol. 162, pp. 21308–21328.