A Fuzzy Association Rule-Based Classification Model for High-Dimensional Problems With Genetic Rule Selection and Lateral Tuning

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Overview

1. INTRODUCTION
2. PRELIMINARIES
3. FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
4. EXPERIMENTAL SETUP
5. EXPERIMENTAL RESULTS
6. CONCLUDING REMARKS

Outline for section 1

1. INTRODUCTION
2. PRELIMINARIES
   - A.Fuzzy Rule-Based Classification Systems
   - B.Fuzzy Association Rules
   - C.Fuzzy Association Rules for Classification
3. FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
   - Stage 1. Fuzzy Association Rule Extraction for Classification
   - Stage 2. Candidate Rule Prescreening
   - Stage 3. Rule Selection and Lateral Tuning
   - Flowchart
4. EXPERIMENTAL SETUP
   - A.Datasetst
   - B.Methods Considered for Comparison
   - C.Parameters of the Methods
   - D.Statistical Analysis
5. EXPERIMENTAL RESULTS
   - A.Comparison With Other Genetic Fuzzy Systems
   - B.Comparison With Other Fuzzy Associative Classifiers
   - C.Comparison With Classical Approaches
   - D.Analysis of the Influence of Depthmax and the Number of Evaluations
   - E.AnalysisofScalability
6. CONCLUDING REMARKS

INTRODUCTION:

- FUZZY rule-based classification systems (FRBCSs) are useful and well-known tools in the machine learning framework, since they can provide an interpretable model for the end user .
- Association discovery is one of the most common data mining techniques that are used to extract interesting knowledge from large datasets.
- Much effort has been made to use its advantages for classification under the name of associative classification.

INTRODUCTION:

- Both association discovery and classification rules mining are essential in practical data mining applications , and their integration could result in greater savings and convenience for the user.
- A typical associative classification system is constructed in two stages:

1) discovering the association rules inherent in a database;
2) selecting a small set of relevant association rules to construct a classifier.

INTRODUCTION:

- In this paper, we present a fuzzy association rule-based classification method for high-dimensional problems (FARC-HD) to obtain an accurate and compact fuzzy rule-based classifier with a low computational cost.
- This method is based on the following three stages:

1) Fuzzy association rule extraction for classification.
2) Candidate rule prescreening.
3) Genetic rule selection and lateral tuning

- In order to assess the performance of the proposed approach, we have used 26 real-world datasets with a number of variables ranging from 4 to 90 and a number of patterns ranging from 150 to 19,020.

Outline for section 2

1 INTRODUCTION

2 PRELIMINARIES
- A.Fuzzy Rule-Based Classification Systems
- B.Fuzzy Association Rules
- C.Fuzzy Association Rules for Classification

3 FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
- Stage 1. Fuzzy Association Rule Extraction for Classification
- Stage 2. Candidate Rule Prescreening
- Stage 3. Rule Selection and Lateral Tuning
- Flowchart

4 EXPERIMENTAL SETUP
- A.Datasetst
- B.Methods Considered for Comparison
- C.Parameters of the Methods
- D.Statistical Analysis

5 EXPERIMENTAL RESULTS
- A.Comparison With Other Genetic Fuzzy Systems
- B.Comparison With Other Fuzzy Associative Classifiers
- C.Comparison With Classical Approaches
- D.Analysis of the Influence of Depthmax and the Number of Evaluations
- E.AnalysisofScalability

6 CONCLUDING REMARKS

PRELIMINARIES: A.Fuzzy Rule-Based Classification Systems

- Any classification problem consists of N training patterns, i.e., $x_p = (x_{p1},...,x_{pm})$, $p = 1,2,...,$N, from S classes, where x_{pi} is the ith attribute value ($i = 1,2,...,m$) of the pth training pattern.
- In this paper, we use fuzzy rules of the following form for our classifier:

Rule R_j**:IF** A_{j1} **and** $\cdots$**and** x_m **is** A_{jm}
THEN Class $= C_j$ **with** RW_j

PRELIMINARIES: A.Fuzzy Rule-Based Classification Systems

(1)

$$RW_j = CF_j = \frac{\sum_{x_p \in ClassC_j} \mu A_j(x_p)}{\sum_{p=1}^{N} \mu A_j(x_p)}$$

(2)

$$V_{Class_h}(x_p) = \sum_{R_j \in RB; R_j = h} \mu A_j(x_p) \bullet CF_j$$

$$h = 1, 2, ..., S, \qquad R_j \in RB.$$

PRELIMINARIES: B.Fuzzy Association Rules

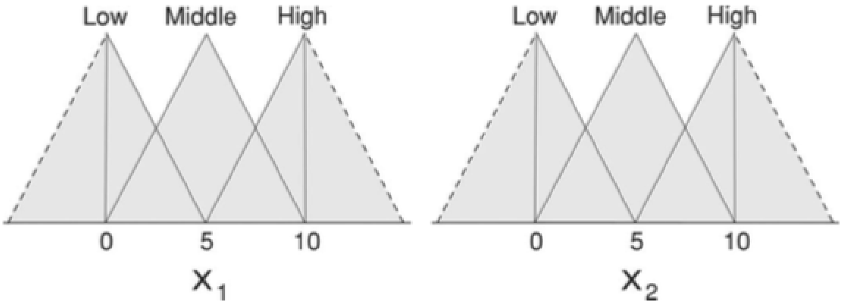


Fig. 1. Attributes and linguistic terms for the attributes X_1 and X_2.

PRELIMINARIES: B.Fuzzy Association Rules

(3)

$$Support(A \longrightarrow B) = \frac{\sum_{x_p \in T} \mu AB(x_p)}{|N|}$$

(4)

$$Confidence(A \longrightarrow B) = \frac{\sum_{x_p \in T} \mu AB(x_p)}{\sum_{x_p \in T} \mu A(x_p)}$$

PRELIMINARIES: C.Fuzzy Association Rules for Classification

(5)

$$Support(A \longrightarrow C_j) = \frac{\sum_{x_p \in ClassC_j} \mu A(x_p)}{|N|}$$

(6)

$$Confidence(A \longrightarrow C_j) = \frac{\sum_{x_p \in ClassC_j} \mu A(x_p)}{\sum_{x_p \in T} \mu A(x_p)}$$

Outline for section 3

1. INTRODUCTION
2. PRELIMINARIES
   - A.Fuzzy Rule-Based Classification Systems
   - B.Fuzzy Association Rules
   - C.Fuzzy Association Rules for Classification
3. FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
   - Stage 1. Fuzzy Association Rule Extraction for Classification
   - Stage 2. Candidate Rule Prescreening
   - Stage 3. Rule Selection and Lateral Tuning
   - Flowchart
4. EXPERIMENTAL SETUP
   - A.Datasetst
   - B.Methods Considered for Comparison
   - C.Parameters of the Methods
   - D.Statistical Analysis
5. EXPERIMENTAL RESULTS
   - A.Comparison With Other Genetic Fuzzy Systems
   - B.Comparison With Other Fuzzy Associative Classifiers
   - C.Comparison With Classical Approaches
   - D.Analysis of the Influence of Depthmax and the Number of Evaluations
   - E.AnalysisofScalability
6. CONCLUDING REMARKS

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 1. Fuzzy Association Rule Extraction for Classification

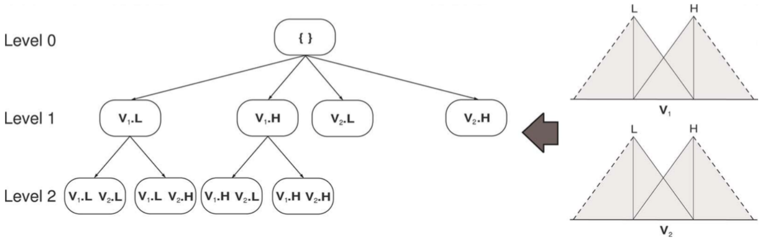


Fig. 2. Search tree for two quantitative attributes V_1 and V_2 with two linguistic terms L and H.

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 1. Fuzzy Association Rule Extraction for Classification

(7)

$$Support(A) = \frac{\sum_{x_p \in T} \mu A(x_p)}{|N|}$$

(8)

$$MinimumSupport_{C_j} = minSup * f_{C_j}$$

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 2. Candidate Rule Prescreening

(9)

$$wWRAcc'(A \longrightarrow C_j) = \frac{n'(A)}{N'} \bullet \left(\frac{n'(A \bullet C_j)}{n'(A)} - \frac{n(C_j)}{N}\right)$$

- N´ is the sum of the weights of all patterns.
- n´ (A) is the sum of the weights of all covered patterns .
- n´ (A $\cdot C_j$) is the sum of the weights of all correctly covered patterns.
- n(C_j) is the number of patterns of class C_j.
- N is the number of all patterns.

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 2. Candidate Rule Prescreening

TABLE I
FIVE PATTERNS IN THIS EXAMPLE

ID	X_1	X_2	$Class$	$Weight$
ID1	0.0	10.0	C_1	1.0
ID2	2.5	4.0	C_2	1.0
ID3	3.2	1.0	C_2	0.0
ID4	9.0	5.0	C_2	1.0
ID5	2.5	10	C_1	0.5

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 2. Candidate Rule Prescreening

i.e

$$R = \mathit{IF}\ X_1\ \mathit{is}\ [0.0, 5.0[\ \mathit{and}\ X_2\ \mathit{is}\ [5.0, 10.0] \rightarrow C_1$$

$$wWRAcc'(R) = \frac{1.0 + 0.5}{1.0 + 1.0 + 0.0 + 1.0 + 0.5} \bullet \left(\frac{1.0 + 0.5}{1.0 + 0.5} - \frac{2}{5}\right)$$

$$= 0.257.$$

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 2. Candidate Rule Prescreening

(10)

$$wWRAcc''(A \longrightarrow C_j) = \frac{n''(A \bullet C_j)}{n'(C_j)} \bullet (\frac{n''(A \bullet C_j)}{n''(A)} - \frac{n(C_j)}{N})$$

- n"(A) is the sum of the products of the weights of all covered patterns by their matching degrees with the antecedent part of the rule.
- n"(A $\cdot C_j$) is the sum of the products of the weights of all correctly covered patterns by their matching degrees with the antecedent part of the rules.
- n´ (C_j) is the sum of the weights of patterns of class C_j.
- Moreover, the first term in the definition of wWRAcc´ has been replaced by $\frac{n''(A \bullet C_j)}{n'(C_j)}$ to reward rules that cover uneliminated patterns of class C_j .

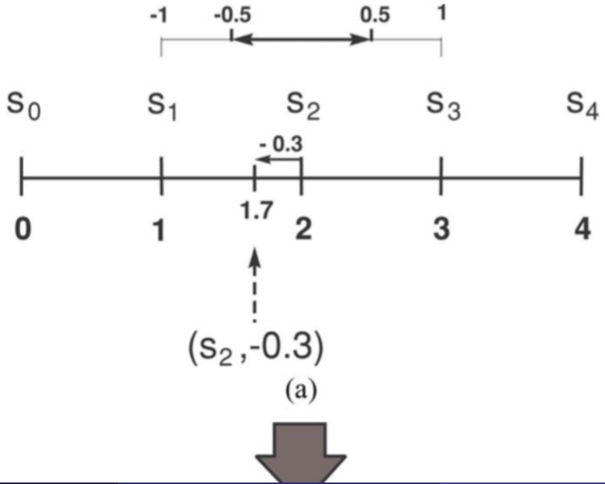
FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 2. Candidate Rule Prescreening

i.e

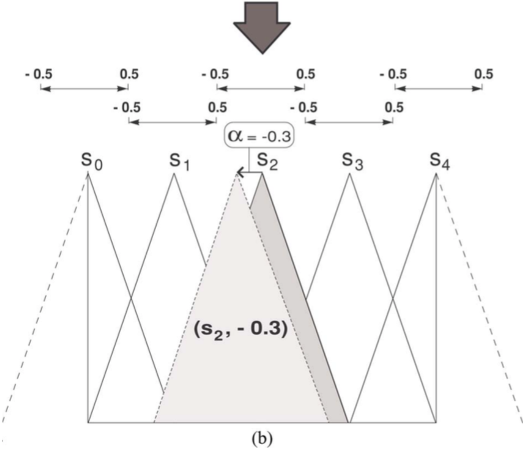
$$wWRAcc''(R) = \frac{1.0 * 1.0 + 0.5 * 0.5}{1.0 + 0.5} \bullet \left(\frac{1.0 * 1.0 + 0.5 * 0.5}{1.0 * 1.0 + 0.5 * 0.5} - \frac{2}{5}\right)$$
$$= 0.5.$$

- This measure can obtain positive or negative values in the interval [- 1.0, 1.0].
- A rule with a wWRAcc" value near to 1 may be more useful for the classification.

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 3. Rule Selection and Lateral Tuning



FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 3. Rule Selection and Lateral Tuning



(b)

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 3. Rule Selection and Lateral Tuning

- Fig. 3. of the involved MF. (a) Simbolic translation of a linguistic term. (b) Lateral displacement of an MF.

Classic Rule:

$$\textit{IF } X_1 \textit{ is Low and } X_2 \textit{ is Middle}$$
$$\textit{THEN Class is } C_1$$

Two-Tuple Representation:

$$\textit{IF } X_1 \textit{ is } (\textit{Low}, 0.1) \textit{ and } X_2 \textit{ is } (\textit{Middle}, -0.3)$$
$$\textit{THEN Class is } C_1.$$

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 3. Rule Selection and Lateral Tuning

the main characteristics of the genetic approach that combines rule selection and lateral tuning are presented:

- genetic model
- codification
- initial gene pool
- chromosome evaluation
- crossover operator
- restarting approach

1) CHC Genetic Model
2) Codification and Initial Gene Pool
3) Chromosome Evaluation
4) Crossover Operator

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Stage 3. Rule Selection and Lateral Tuning

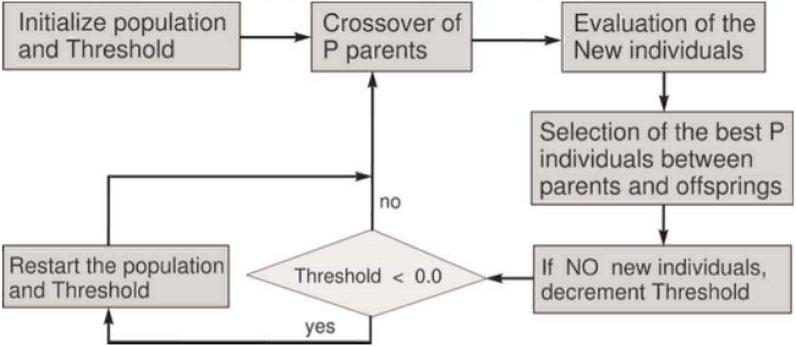


Fig. 4. Scheme of the CHC algorithm.

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Flowchart

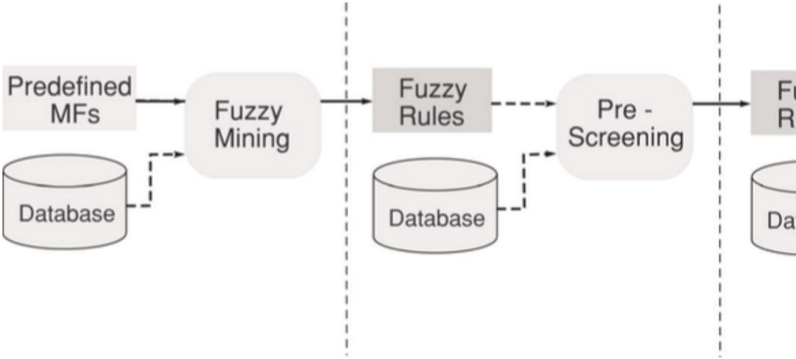
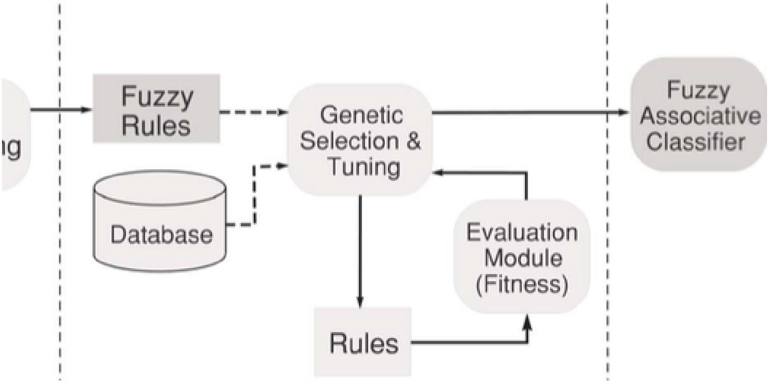


Fig. 5. Scheme of the FARC-HD method.

FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS: Flowchart



Outline for section 4

1. INTRODUCTION
2. PRELIMINARIES
   - A.Fuzzy Rule-Based Classification Systems
   - B.Fuzzy Association Rules
   - C.Fuzzy Association Rules for Classification
3. FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
   - Stage 1. Fuzzy Association Rule Extraction for Classification
   - Stage 2. Candidate Rule Prescreening
   - Stage 3. Rule Selection and Lateral Tuning
   - Flowchart
4. EXPERIMENTAL SETUP
   - A.Datasetst
   - B.Methods Considered for Comparison
   - C.Parameters of the Methods
   - D.Statistical Analysis
5. EXPERIMENTAL RESULTS
   - A.Comparison With Other Genetic Fuzzy Systems
   - B.Comparison With Other Fuzzy Associative Classifiers
   - C.Comparison With Classical Approaches
   - D.Analysis of the Influence of Depthmax and the Number of Evaluations
   - E.AnalysisofScalability
6. CONCLUDING REMARKS

EXPERIMENTAL SETUP:

Several experiments have been carried out in this paper to evaluate the usefulness of our proposal.

- we describe the real-world databases that are used in these experiments.
- we introduce a brief description of the methods considered for comparison.
- we show the configuration of the methods (determining all the parameters used).
- we describe the statistical analysis that is adopted in this study.

EXPERIMENTAL SETUP: A.Datasetst

TABLE II
DATASETS CONSIDERED FOR THE EXPERIMENTAL STUDY

Name	Attributes (R/I/N)	Patterns	Classes	Name	Attributes (R/I/N)	Patterns	Classes
Iris	4 (4/0/0)	150	3	Vowel	13 (10/3/0)	990	11
Phoneme	5 (5/0/0)	5404	2	Crx	15 (3/3/9)	653(690)	2
Monks	6 (0/6/0)	432	2	Pen-based	16 (0/16/0)	10992	10
Appendicitis	7 (7/0/0)	106	2	German	20 (0/7/13)	1000	2
Ecoli	7 (7/0/0)	336	8	Twonorm	20 (20/0/0)	7400	2
Pima	8 (8/0/0)	768	2	Ringnorm	20 (20/0/0)	7400	2
Yeast	8 (8/0/0)	1484	10	Wdbc	30 (30/0/0)	569	2
Glass	9 (9/0/0)	214	7	SatImage	36 (0/36/0)	6435	6
Page-blocks	10 (4/6/0)	5472	5	Texture	40 (40/0/0)	5500	11
Magic	10 (10/0/0)	19020	2	Spectfheart	44 (0/44/0)	267	2
Wine	13 (13/0/0)	178	3	Spambase	57 (57/0/0)	4597	2
Heart	13 (1/12/0)	270	2	Sonar	60 (60/0/0)	208	2
Cleveland	13 (13/0/0)	297(303)	5	Movementlibras	90 (90/0/0)	360	15

Available at `http://sci2s.ugr.es/keel/datasets.php`

EXPERIMENTAL SETUP: B.Methods Considered for Comparison

In these experiments, we compare the proposed approach with other ten methods, which are available in the Knowledge Extraction based on Evolutionary Learning (KEEL) software tool [61].

- C4.5 [39]
- Classification based on associations (CBA) [12]
- CBA2 [13]
- Classification based on multiple association rules (CMAR) [14]
- Structural learning algorithm on vague environment (2SLAVE) [64]
- Learning algorithm to discover fuzzy association rules for classification (LAFAR) [24]
- Classification based on predictive association rules (CPAR) [15]
- Fuzzy hybrid genetic-based machine learning algo- rithm (FH-GBML) [66]
- Steady-state GA for extracting fuzzy classification rules from data (SGERD) [67]
- Classification with fuzzy association rules (CFAR) [27]

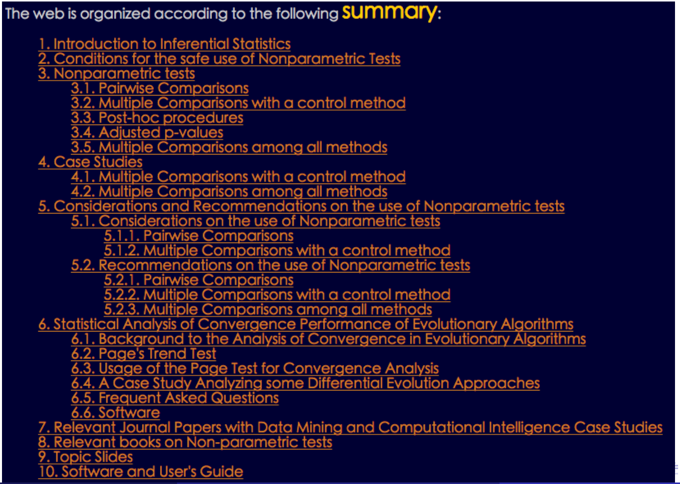
EXPERIMENTAL SETUP: C.Parameters of the Methods

TABLE III
PARAMETERS CONSIDERED FOR COMPARISON

Method	Parameters
C4.5	$Pruned = yes$, $Confidence = 0.25$, $InstancesPerLeaf = 2$
CBA	$Minsup = 0.01$, $Minconf = 0.5$, $Pruned = yes$, $RuleLimit = 80,000$
CBA2	$Minsup = 0.01$, $Minconf = 0.5$, $Pruned = yes$, $RuleLimit = 80,000$
CMAR	$Minsup = 0.01$, $Minconf = 0.5$, $\delta = 4$, $DifferenceThreshold = 20\%$
2SLAVE	$Pop = 100$, $Iter_{change} = 500$, $P_c = 0.6$, $P_m = 0.01$, $\kappa_1 = 0$, $\kappa_2 = 1$, $Op_{or} = 0.1$, $Op_{and} = 0.1$, $P_{gen} = 0.1$, $P_{rot} = 0.1$, $\alpha = 0.15$
LAFAR	$t_{max} = 50$, $N_{pop} = 30$, $\mid s \mid = \mid c \mid = 10$, $W_{CAR} = 10$, $W_V = 1$, $P_c = 1.0$, $P_m = 0.01$, $J_{max} = 100$, $\eta_1 = 0.001$, $\eta_2 = 0.1$
CPAR	$\delta = 0.05$, $min\_gain = 0.7$, $\alpha = 0.66$, $k = 5$
FH-GBML	$N_{rules} = 20$, $F_{sets} = 200$, $Gen = 1000$, $P_c = 0.9$, $P_{dcare} = \{0.5, 0.8, 0.95\}$, $P_{michigan} = 0.5$
SGERD	$Q = heuristics$
CFAR	$Minpsup = \{0.05, 0.1\}$, $Minpconf = 0.85$, $MS = 0.15$
FARC-HD	$Minsup = 0.05$, $Maxconf = 0.8$, $Depth_{max} = 3$, $k_t = 2$, $Pop = 50$, $Evaluations = 15,000$, $BITSGENE = 30$, $\delta = 0.2$

EXPERIMENTAL SETUP: D.Statistical Analysis

- We use $\alpha = 0.05$ as the level of confidence in all cases.
- A wider description of these tests, together with software for their use, can also be found at: http://sci2s.ugr.es/sicidm/


The web is organized according to the following **summary**:

1. Introduction to Inferential Statistics
2. Conditions for the safe use of Nonparametric Tests
3. Nonparametric tests
    3.1. Pairwise Comparisons
    3.2. Multiple Comparisons with a control method
    3.3. Post-hoc procedures
    3.4. Adjusted p-values
    3.5. Multiple Comparisons among all methods
4. Case Studies
    4.1. Multiple Comparisons with a control method
    4.2. Multiple Comparisons among all methods
5. Considerations and Recommendations on the use of Nonparametric tests
    5.1. Considerations on the use of Nonparametric tests
        5.1.1. Pairwise Comparisons
        5.1.2. Multiple Comparisons with a control method
    5.2. Recommendations on the use of Nonparametric tests
        5.2.1. Pairwise Comparisons
        5.2.2. Multiple Comparisons with a control method
        5.2.3. Multiple Comparisons among all methods
6. Statistical Analysis of Convergence Performance of Evolutionary Algorithms
    6.1. Background to the Analysis of Convergence in Evolutionary Algorithms
    6.2. Page's Trend Test
    6.3. Usage of the Page Test for Convergence Analysis
    6.4. A Case Study Analyzing some Differential Evolution Approaches
    6.5. Frequent Asked Questions
    6.6. Software
7. Relevant Journal Papers with Data Mining and Computational Intelligence Case Studies
8. Relevant books on Non-parametric tests
9. Topic Slides
10. Software and User's Guide

Outline for section 5

1 INTRODUCTION
2 PRELIMINARIES
- A.Fuzzy Rule-Based Classification Systems
- B.Fuzzy Association Rules
- C.Fuzzy Association Rules for Classification

3 FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
- Stage 1. Fuzzy Association Rule Extraction for Classification
- Stage 2. Candidate Rule Prescreening
- Stage 3. Rule Selection and Lateral Tuning
- Flowchart

4 EXPERIMENTAL SETUP
- A.Datasetst
- B.Methods Considered for Comparison
- C.Parameters of the Methods
- D.Statistical Analysis

5 EXPERIMENTAL RESULTS
- A.Comparison With Other Genetic Fuzzy Systems
- B.Comparison With Other Fuzzy Associative Classifiers
- C.Comparison With Classical Approaches
- D.Analysis of the Influence of Depthmax and the Number of Evaluations
- E.AnalysisofScalability

6 CONCLUDING REMARKS

EXPERIMENTAL RESULTS: A.Comparison With Other Genetic Fuzzy Systems

TABLE IV
RESULTS OBTAINED BY THE ANALYZED METHODS

	2SLAVE				FH-GBML				SGERD				FARC-HD			
Dataset	**#R**	**#C**	**Tra**	**Tst**	**#R**	**#C**	**Tra**	**Tst**	**#R**	**#C**	**Tra**	**Tst**	**#R**	**#C**	**Tra**	**Tst**
Iris	4.0	3.2	94.32	94.44	14.9	3.3	**98.89**	94.00	**3.4**	2.0	95.14	94.89	4.0	**1.1**	98.59	**96.00**
Phoneme	11.5	24.2	77.52	76.41	17.4	4.5	79.57	79.66	**3.6**	**1.9**	75.74	75.55	17.8	2.2	**83.52**	**82.14**
Monks	3.0	**1.3**	97.22	97.26	14.7	2.1	98.36	98.18	**2.2**	1.4	80.56	80.65	14.2	2.0	**99.92**	**99.77**
Appendicitis	4.4	7.5	91.20	82.91	13.8	7.0	93.19	**86.00**	**2.5**	2.0	87.88	84.48	6.8	**1.8**	**93.82**	84.18
Ecoli	12.6	9.6	89.51	**84.53**	10.3	4.2	75.83	69.38	**9.4**	**1.6**	76.53	74.05	33.8	2.4	**92.33**	82.19
Pima	7.8	8.8	76.35	73.71	10.6	6.0	77.18	75.26	**3.1**	**2.0**	74.01	73.37	22.7	2.4	**82.90**	**75.66**
Yeast	23.6	9.8	55.54	51.27	**7.5**	5.9	52.31	51.42	11.3	**1.5**	39.83	38.77	35.2	2.6	**63.81**	**58.50**
Glass	15.1	9.3	74.25	58.05	9.4	5.0	64.85	57.99	**6.9**	**2.0**	61.31	58.49	22.7	2.5	**81.10**	**70.24**
Page-blocks	7.5	10.3	91.39	91.39	7.4	8.1	94.37	94.21	**6.5**	**2.0**	90.83	90.72	19.1	2.3	**95.62**	**95.01**
Magic	4.1	10.5	73.97	73.96	9.9	8.2	81.23	81.30	**3.1**	**2.0**	72.17	72.06	43.3	2.5	**85.36**	**84.51**
Wine	5.5	10.3	92.52	89.47	9.2	4.7	95.51	92.61	**4.2**	2.0	93.67	91.88	8.7	**1.6**	**99.94**	**94.35**
Heart	4.3	10.7	75.35	71.36	12.7	3.2	84.65	75.93	**2.7**	**1.9**	74.83	73.21	27.0	2.6	**93.91**	**84.44**
Cleveland	11.9	12.8	54.24	48.82	6.9	4.5	58.29	53.51	**6.4**	**2.0**	56.55	51.59	61.3	2.9	**88.18**	**55.24**
Vowel	63.1	15.6	**82.06**	71.11	**9.2**	13.0	67.41	67.07	18.0	**1.9**	72.99	65.83	72.3	2.9	80.48	**71.82**
Crx	2.4	6.7	74.36	74.06	11.6	6.2	86.32	**86.60**	**2.1**	**1.9**	85.04	85.03	25.4	2.6	**91.17**	86.03
Pen-based	40.0	18.9	81.32	81.16	18.4	8.0	50.69	50.45	**15.9**	**2.0**	68.17	67.93	152.8	2.8	**97.04**	**96.04**
German	6.5	8.4	72.44	70.53	5.1	4.0	**87.11**	**87.01**	**3.4**	**2.0**	68.54	67.97	85.7	2.8	86.81	72.80
Twonorm	26.5	15.5	87.45	86.99	12.0	7.6	86.26	85.97	**3.1**	**2.0**	74.49	73.98	60.9	2.6	**96.64**	**95.28**
Ringnorm	**4.6**	23.7	80.12	79.63	6.9	11.3	87.34	86.92	6.8	2.0	73.21	72.63	24.0	**1.9**	**95.13**	**94.08**
Wdbc	5.2	8.1	92.43	92.33	7.2	4.9	95.12	92.26	**3.7**	2.0	91.79	90.68	10.4	**1.7**	**98.57**	**95.25**
SatImage	57.9	25.1	84.03	81.69	16.5	36.0	74.90	74.72	**12.2**	**2.0**	77.15	77.10	76.1	2.7	**88.68**	**87.32**
Texture	34.9	23.9	82.87	81.57	**14.6**	40.0	69.91	70.15	18.6	**2.0**	72.12	71.66	54.5	2.7	**93.71**	**92.89**
Spectfheart	6.1	21.7	80.71	79.17	10.8	44.0	79.28	72.36	**2.1**	1.9	79.05	78.16	12.9	**1.8**	**91.43**	**79.83**
Spambase	7.9	11.4	69.87	70.14	3.9	18.5	77.86	77.22	**3.7**	**2.0**	72.90	72.98	29.8	2.4	**92.37**	**91.93**
Sonar	9.6	17.5	77.92	71.42	10.3	4.7	80.56	68.24	**3.2**	**2.0**	74.22	71.90	18.0	2.3	**98.77**	**80.19**
Movementlibras	47.4	26.5	90.13	67.04	**12.1**	90.0	77.87	68.89	22.9	**2.0**	72.37	68.09	83.1	2.9	**95.52**	**76.67**
Mean	16.4	13.5	80.73	76.94	10.9	13.6	79.80	76.82	**6.9**	**1.9**	75.43	73.99	39.3	2.3	**90.97**	**83.94**

EXPERIMENTAL RESULTS: A.Comparison With Other Genetic Fuzzy Systems

TABLE V
RESULTS OF THE FRIEDMAN AND IMAN–DAVENPORT TESTS ($\alpha = 0.05$)

Friedman Test		
Statistic ($\mathcal{X}_F^2$)	**Critical Value**	**p value**
40.823	7.814	<0.0001
ImanDavenport Test		
Statistic ($\mathcal{F}_F$)	**Critical Value**	**p value**
27.452	2.7386	<0.0001

EXPERIMENTAL RESULTS: A.Comparison With Other Genetic Fuzzy Systems

TABLE VI
AVERAGE RANKINGS OF THE METHODS

Method	Ranking
SGERD	3.2884
2SLAVE	2.9808
FH-GBML	2.5577
FARC-HD	**1.1731**

EXPERIMENTAL RESULTS: A.Comparison With Other Genetic Fuzzy Systems

TABLE VII
HOLM'S TABLE FOR THE SELECTION METHODS WITH $\alpha = 0.05$

i	**Method**	z	p	α/i	**Hypothesis**
3	SGERD	5.01	3.46E-9	0.0166	**Rejected**
2	2SLAVE	5.05	4.45E-7	0.0125	**Rejected**
1	FH-GBML	3.87	1.10E-4	0.05	**Rejected**

EXPERIMENTAL RESULTS: B.Comparison With Other Fuzzy Associative Classifiers

TABLE VIII
RESULTS OBTAINED BY THE ANALYZED METHODS

	LAFAR				CFAR				FARC-HD			
Dataset	**#R**	**#C**	**Tra**	**Tst**	**#R**	**#C**	**Tra**	**Tst**	**#R**	**#C**	**Tra**	**Tst**
Iris	17.4	**1.0**	**98.74**	92.67	9.1	1.5	91.33	90.67	**4.0**	1.1	98.59	**96.00**
Phoneme	16.9	1.1	78.49	77.87	**2.0**	**1.0**	70.65	70.65	17.8	2.2	**83.52**	**82.14**
Monks	126.4	3.9	94.64	93.36	**12.1**	**1.8**	**100.00**	99.55	14.2	2.0	99.92	**99.77**
Appendicitis	39.1	**1.1**	**95.91**	86.00	14.8	1.4	88.47	**87.82**	**6.8**	1.8	93.82	84.18
Ecoli	68.4	**2.1**	81.58	72.36	**4.0**	3.1	48.45	47.61	33.8	2.4	**92.33**	**82.19**
Pima	32.3	**1.1**	77.36	75.40	**2.0**	1.9	65.10	65.11	22.7	2.4	**82.90**	**75.66**
Yeast	40.2	2.0	54.53	50.90	**3.0**	**1.0**	36.44	36.44	35.2	2.6	**63.81**	**58.50**
Glass	36.0	2.5	68.59	52.17	**3.4**	**2.1**	44.19	44.66	22.7	2.5	**81.10**	**70.24**
Page-blocks					**2.0**	**1.0**	89.78	89.78	19.1	2.3	**95.62**	**95.01**
Magic					**2.0**	**1.5**	64.84	64.84	43.3	2.5	**85.36**	**84.51**
Wine					115.7	3.0	99.63	93.24	**8.7**	**1.6**	**99.94**	**94.35**
Heart					45.3	3.3	86.01	82.22	**27.0**	**2.6**	**93.91**	**84.44**
Cleveland					**2.0**	**2.1**	53.87	53.88	61.3	2.9	**88.18**	**55.24**
Crx					89.9	5.1	89.45	**86.79**	**25.4**	**2.6**	**91.17**	86.03
Pen-based					**6.9**	2.9	36.54	36.43	152.8	**2.8**	**97.04**	**96.04**
German					**2.0**	**2.7**	70.00	70.00	85.7	2.8	**86.81**	**72.80**
Twonorm					498.7	3.7	91.96	91.66	**60.9**	**2.6**	**96.64**	**95.28**
Sonar					37.3	3.0	83.01	72.48	**18.0**	**2.3**	**98.77**	**80.19**
Mean	47.1	1.8	81.23	75.09	47.3	**2.3**	72.76	71.32	**36.6**	**2.3**	**90.52**	**82.92**

EXPERIMENTAL RESULTS: B.Comparison With Other Fuzzy Associative Classifiers

- The results obtained by these methods are shown in Table VIII.
- Notice that we show less datasets; this is due to scalability problems in the LAFAR and CFAR algorithms, which cannot run in all datasets.
- On the other hand, the results presented in Table VIII show that our approach obtains an average number of rules lower than the LAFAR and CFAR algorithms.
- However, the CFAR algorithm obtains less rules than our approach in 11 of the 18 datasets.

EXPERIMENTAL RESULTS: B.Comparison With Other Fuzzy Associative Classifiers

TABLE IX
WILCOXON'S TEST ($\alpha = 0.05$)

Comparison	R^+	R^-	Hypothesis	p-value
FARC-HD vs. CFAR	161	10	**Rejected**	0.001
FARC-HD vs. LAFAR	34	2	**Rejected**	0.025
LAFAR vs. CFAR	32	4	**Rejected**	0.05

In order to compare the two algorithms, we use a Wilcoxon test, which is shown in Table IX.

EXPERIMENTAL RESULTS: C.Comparison With Classical Approaches

TABLE X
RESULTS OBTAINED BY THE ANALYZED METHODS

	CBA			CBA2			CMAR			CPAR			C45			FARC-HD		
Data	**#R**	**Tra**	**Tst**	**#R**	**Tra**	**Tst**	**#R**	**Tra**	**Tst**	**#R**	**Tra**	**Tst**	**#R**	**Tra**	**Tst**	**#R**	**Tra**	**Tst**
Iris	**4**	96.7	93.3	**4**	96.7	93.3	41	96.4	94.0	34	96.4	**96.0**	5	98.0	**96.0**	**4**	**98.6**	**96.0**
Phoneme	81	81.4	80.5	116	82.2	81.2	358	79.3	78.6	1109	84.0	81.9	129	**91.9**	**86.8**	**18**	83.5	82.1
Monks	11	**100.0**	**100.0**	11	**100.0**	**100.0**	126	**100.0**	**100.0**	79	**100.0**	**100.0**	**5**	**100.0**	**100.0**	14	99.9	99.8
Appendicitis	6	91.5	89.6	6	91.5	89.6	42	90.8	**89.7**	25	89.5	87.8	**3**	91.0	83.3	7	**93.8**	84.2
Ecoli	29	87.3	78.0	35	89.2	77.1	178	82.5	77.7	158	82.6	76.2	**20**	91.7	79.5	34	**92.3**	**82.2**
Pima	42	79.6	72.7	44	79.9	72.5	237	79.4	75.1	166	79.1	74.5	**18**	**83.2**	74.0	23	82.9	**75.7**
Yeast	48	59.4	54.7	67	61.8	55.9	262	57.2	53.6	427	59.4	56.3	169	**81.5**	55.6	**35**	63.8	**58.5**
Glass	25	82.3	70.8	28	83.1	**71.3**	174	78.9	70.3	121	77.2	68.9	26	**93.7**	67.4	**23**	81.1	70.2
Page-blocks	172	94.6	94.0	256	98.2	95.9	1348	94.7	94.4	464	97.4	96.1	43	**98.5**	**97.1**	**19**	95.6	95.0
Magic	728	82.7	81.5	1015	84.8	83.7	4200	79.3	78.8	5422	87.9	84.9	321	**90.9**	**85.1**	**43**	85.4	84.5
Wine	9	**99.9**	93.8	9	**99.9**	93.8	73	**99.9**	**96.7**	44	99.7	95.6	**5**	98.9	93.3	9	**99.9**	94.3
Heart	40	93.8	83.0	40	**94.2**	81.5	306	91.5	82.2	93	90.1	80.7	**17**	91.7	78.5	27	93.9	**84.4**
Cleveland	**30**	64.3	**56.9**	47	70.0	54.9	230	54.1	53.9	107	61.8	54.9	40	83.1	54.5	61	**88.2**	55.2
Vowel	130	77.8	63.6	215	92.6	74.9	945	71.2	60.4	888	76.1	63.0	95	**97.1**	**81.5**	**72**	80.5	71.8
Crx	97	**95.2**	83.6	97	94.9	85.0	728	92.0	85.0	204	91.1	**87.3**	**21**	90.8	85.3	25	91.2	86.0
Pen-based	653	85.3	83.1	592	90.3	88.1	4328	72.0	71.7	2536	97.9	93.1	191	**99.3**	**96.5**	**153**	97.0	96.0
German	177	**94.5**	**75.3**	227	92.9	73.7	1984	84.4	71.9	666	88.1	73.3	**83**	84.9	72.5	86	86.8	72.8
Twonorm	708	97.7	91.6	724	97.0	91.7	2163	97.7	**95.5**	1646	**99.0**	89.4	286	98.7	84.5	**61**	96.6	95.3
Ringnorm	295	96.3	**94.1**	184	97.2	93.7	2788	87.2	83.7	1209	**99.1**	92.2	203	98.7	90.2	**24**	95.1	94.1
Wdbc	50	**100.0**	94.7	51	99.9	95.1	309	99.9	94.9	66	99.0	95.1	12	99.1	95.2	**10**	98.6	**95.3**
SatImage	722	88.5	85.2	587	87.6	83.6	7083	86.6	84.9	2032	97.3	85.8	287	**97.7**	85.8	**76**	88.7	**87.3**
Texture	377	88.3	84.5	**50**	53.5	52.5	5616	76.3	73.8	1492	98.5	90.7	152	**99.0**	92.6	55	93.7	**92.9**
Spectfheart	31	92.0	79.8	33	91.8	79.8	305	85.5	79.4	87	87.3	78.3	19	**98.3**	76.5	**13**	91.4	**79.8**
Spambase	278	95.0	93.2	302	95.2	93.0	2317	92.9	92.0	686	**97.9**	**93.5**	109	97.4	92.7	**30**	92.4	91.9
Sonar	27	92.3	75.4	39	97.7	77.9	234	98.3	78.8	92	95.8	75.0	**14**	97.7	70.5	18	**98.8**	**80.2**
Movementlibras	41	54.8	36.1	**4**	11.6	7.2	510	51.7	39.2	636	85.3	63.6	47	94.4	69.4	83	**95.5**	**76.7**
Mean	185	87.4	80.3	184	85.9	78.7	1419	83.8	79.1	788	89.1	82.1	89	**94.1**	82.5	**39**	91.0	**83.9**

EXPERIMENTAL RESULTS: C.Comparison With Classical Approaches

TABLE XI
RESULTS OF THE FRIEDMAN AND IMAN–DAVENPORT TESTS ($\alpha = 0.05$)

Friedman Test		
Statistic ($\mathcal{X}_F^2$)	**Critical Value**	**p value**
15.84	11.0705	<0.01
ImanDavenport Test		
Statistic ($\mathcal{F}_F$)	**Critical Value**	**p value**
3.4703	2.2885	<0.01

EXPERIMENTAL RESULTS: C.Comparison With Classical Approaches

TABLE XII
AVERAGE RANKINGS OF THE METHODS

Method	Ranking
CBA	3.807
CMAR	3.775
CBA2	3.730
C45	3.576
CPAR	3.346
FARC-HD	**2.307**

EXPERIMENTAL RESULTS: C.Comparison With Classical Approaches

TABLE XIII
HOLM'S TABLE FOR THE SELECTION METHODS WITH $\alpha = 0.05$

i	**Method**	z	p	α/i	**Hypothesis**
5	CMAR	3.71	2.1035E-4	0.01	**Rejected**
4	CBA	2.89	0.0038	0.0125	**Rejected**
3	CBA2	2.74	0.0060	0.0166	**Rejected**
2	C45	2.45	0.0144	0.025	**Rejected**
1	CPAR	2.01	0.0453	0.05	**Rejected**

EXPERIMENTAL RESULTS: D.Analysis of the Influence of Depthmax and the Number of Evaluations

ANALYSIS OF THE PERFORMANCE DEPENDING ON $Depth_{max}$

Data	$Depth_{max} = 2$					
	#R1	**#R2**	**#R3**	**Tra**	**Tst**	**Time**
Yeast	96.9	38.9	27.9	62.2	57.7	0:00:51
Vowel	211.9	48.3	35.2	60.4	52.3	0:00:58
Ringnorm	1460.4	36.7	25.9	94.8	93.8	0:05:44
Spectfheart	2870.0	37.6	13.8	91.3	79.1	0:00:19

Data	$Depth_{max} = 3$					
	#R1	**#R2**	**#R3**	**Tra**	**Tst**	**Time**
Yeast	373.2	56.3	35.2	63.8	58.5	0:01:00
Vowel	1482.3	97.5	72.3	80.4	71.8	0:02:16
Ringnorm	9927.7	41.4	24.0	95.1	94.1	0:07:28
Spectfheart	45508.1	34.7	12.9	91.4	79.8	0:01:59

Data	$Depth_{max} = 4$					
	#R1	**#R2**	**#R3**	**Tra**	**Tst**	**Time**
Yeast	845.5	68.1	43.7	64.6	57.6	0:02:03
Vowel	6423	111.1	90.5	89.8	78.4	0:07:10
Ringnorm	35201.1	45.4	28.0	95.9	94.6	0:19:42
Spectfheart	389983	32.8	15.4	92.6	77.5	1:10:21

EXPERIMENTAL RESULTS: D.Analysis of the Influence of Depthmax and the Number of Evaluations

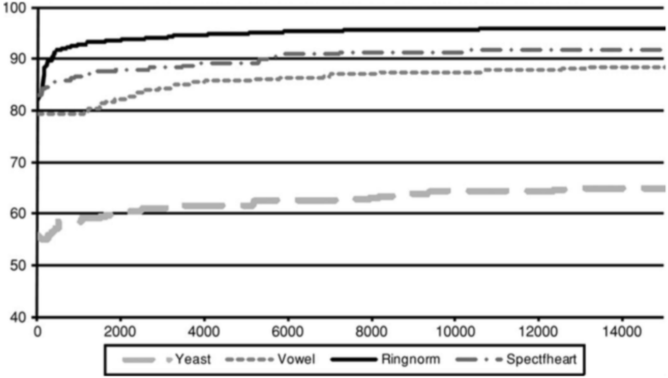


Fig. 6. Accuracy obtained over the training data with different numbers of evaluations in the genetic process with $\text{Depth}_{max} = 4$.

EXPERIMENTAL RESULTS: E.AnalysisofScalability

TABLE XV
AVERAGE RUNTIME OF THE ANALYZED METHODS (HH:MM:SS)

Dataset	Var	Patt	2SLAVE	FH-GBML	SGERD	CFAR	LAFAR	CBA	CBA2	CMAR	CPAR	C4.5	FARC-HD
Iris	4	150	00:00:05	00:12:57	**00:00:00**	**00:00:00**	00:09:39	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:01
Phoneme	5	5404	00:03:01	01:39:48	00:00:01	00:00:05	01:08:27	00:00:01	00:00:01	00:00:01	00:00:05	**00:00:00**	00:01:44
Monks	6	432	00:00:07	00:10:57	**00:00:00**	00:00:01	07:39:18	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:09
Appendicitis	7	106	00:00:03	00:02:05	**00:00:00**	00:00:05	01:00:44	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:02
Ecoli	7	336	00:00:50	00:11:05	**00:00:00**	00:00:01	01:20:10	**00:00:00**	**00:00:00**	**00:00:00**	00:00:01	**00:00:00**	00:00:15
Pima	8	768	00:00:41	00:23:31	**00:00:00**	00:00:06	04:18:18	**00:00:00**	**00:00:00**	**00:00:00**	00:00:01	**00:00:00**	00:00:22
Yeast	8	1484	00:07:04	00:48:51	**00:00:00**	00:00:01	15:27:38	**00:00:00**	**00:00:00**	**00:00:00**	00:00:01	00:00:01	00:01:00
Glass	9	214	00:00:38	00:08:05	**00:00:00**	00:00:04	07:58:01	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:18
Page-blocks	10	5472	00:09:59	03:09:10	**00:00:02**	00:01:08	-	00:00:06	00:01:09	00:00:10	00:00:04	00:00:07	00:02:15
Magic	10	19020	01:03:25	13:04:02	**00:00:21**	00:16:30	-	00:00:28	00:00:50	00:01:18	00:01:20	00:00:52	00:17:52
Wine	13	178	00:00:12	00:10:16	**00:00:00**	00:00:45	-	00:04:45	00:43:04	00:15:58	**00:00:00**	**00:00:00**	00:00:09
Heart	13	270	00:00:15	00:12:51	**00:00:00**	00:00:12	-	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:14
Cleveland	13	303	00:01:29	00:14:36	**00:00:00**	00:00:09	-	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	**00:00:00**	00:00:37
Vowel	13	990	00:13:20	00:13:40	**00:00:00**	00:00:10	-	00:01:12	00:02:13	00:00:05	00:00:03	**00:00:00**	00:02:16
Crx	15	690	00:00:35	00:35:37	**00:00:00**	00:02:02	-	00:02:06	00:00:18	00:06:19	00:00:01	**00:00:00**	00:00:27
Pen-based	16	10992	02:32:58	10:29:28	00:00:13	**00:00:07**	-	00:00:29	00:00:52	00:01:24	00:00:36	00:00:27	00:45:08
German	20	1000	00:02:26	01:03:06	**00:00:00**	00:02:17	-	00:01:16	00:00:15	00:04:46	00:00:03	**00:00:00**	00:02:20
Twonorm	20	7400	00:39:37	09:29:46	**00:00:07**	05:03:33	-	00:00:49	00:00:39	00:00:13	00:00:14	00:00:10	00:15:46
Ringnorm	20	7400	00:08:04	10:05:13	**00:00:07**	-	-	00:02:12	00:01:14	00:13:33	00:00:16	00:00:31	00:07:28
Wdbc	30	569	00:00:30	00:50:01	**00:00:00**	-	-	00:00:09	00:00:05	01:01:00	00:00:01	00:00:01	00:00:51
SatImage	36	6435	01:57:23	04:07:42	**00:00:13**	-	-	00:00:43	00:00:24	00:04:50	00:00:54	00:00:18	00:35:08
Texture	40	5500	01:29:43	03:40:14	**00:00:12**	-	-	00:00:28	00:00:15	00:03:50	00:00:40	00:00:15	00:36:42
Spectfheart	44	267	00:00:31	00:14:30	**00:00:00**	-	-	00:00:09	00:00:05	00:02:39	**00:00:00**	00:00:10	00:01:59
Spambase	57	4597	00:42:58	13:01:47	**00:00:10**	-	-	00:00:15	00:00:12	00:24:55	00:00:13	00:00:27	00:14:36
Sonar	60	208	00:01:04	00:21:02	00:00:01	00:00:20	-	00:00:03	00:00:08	00:10:26	00:00:01	**00:00:00**	00:17:27
Movementlibras	90	360	00:10:45	05:17:58	**00:00:01**	-	-	**00:00:01**	**00:00:01**	**00:00:01**	00:00:06	00:00:27	01:20:10

EXPERIMENTAL RESULTS: E.AnalysisofScalability

By the analysis of the results presented in Table XV, we can draw the following conclusions.

- The SGERD algorithm presents a very low average runtime in all datasets.
- The2SLAVE,FH-GBML,LAFAR,andCFARalgorithms expend a large amount of time .
  (Notice that the CFAR and LAFAR cannot run in 7 and 18 of the 26 datasets, respectively.)
- our proposal obtains the best ranking in Fried- man's test .
- NoticethattheCBAandCBA2algorithmspresentsimilar runtimes to the CMAR and CPAR algorithms.
- The FARC-HD approach presents a good computational cost in all datasets.

Outline for section 6

1 INTRODUCTION
2 PRELIMINARIES
- A.Fuzzy Rule-Based Classification Systems
- B.Fuzzy Association Rules
- C.Fuzzy Association Rules for Classification

3 FUZZY ASSOCIATION RULE-BASED CLASSIFIER FOR HIGH-DIMENSIONAL PROBLEMS
- Stage 1. Fuzzy Association Rule Extraction for Classification
- Stage 2. Candidate Rule Prescreening
- Stage 3. Rule Selection and Lateral Tuning
- Flowchart

4 EXPERIMENTAL SETUP
- A.Datasetst
- B.Methods Considered for Comparison
- C.Parameters of the Methods
- D.Statistical Analysis

5 EXPERIMENTAL RESULTS
- A.Comparison With Other Genetic Fuzzy Systems
- B.Comparison With Other Fuzzy Associative Classifiers
- C.Comparison With Classical Approaches
- D.Analysis of the Influence of Depthmax and the Number of Evaluations
- E.AnalysisofScalability

6 CONCLUDING REMARKS

CONCLUDING REMARKS:

- In this paper, we have proposed a new fuzzy associative classification method for high-dimensional datasets, named FARC- HD.
- Our aim was to obtain accurate and compact fuzzy asso- ciative classifiers with a low computational cost.
- Finally, the limit in the depth of the trees, along with candidate rule prescreening using the fuzzy measure wWRACC", allows us to reduce the search space considerably.

Thank you for your attention