n

$2n$

$n = 1, 2, 3, \ldots$

$1 \leq n \leq 6$

$2^3 + 2^2 + 2^1 + 2^0$

1^2

$$(2^0)^2 = 2^0$$

$(2^1 + 2^0)^2$

$$(2^1)^2 + 2 \cdot 2^1 \cdot 2^0 + \underline{(2^0)^2}$$

$(2^2 + 2^2) + \underline{2^0}$ by (1)

$2^3 + 2^0$

11^2

$(2^2 + 2^1 + 2^0)^2$

$$(2^2)^2 + 2 \cdot 2^2(2^1 + 2^0) + \underline{(2^1 + 2^0)^2}$$

$2^4 + 2^3(2^1 + 2^0) + \underline{2^3 + 2^0}$ by (2)

$$(2^4 + 2^4) + (2^3 + 2^3) + 2^0$$

$2^5 + 2^4 + 2^0$

111^2

$(2^3 + 2^2 + 2^1 + 2^0)^2$

$$(2^3)^2 + 2 \cdot 2^3(2^2 + 2^1 + 2^0) + \underline{(2^2 + 2^1 + 2^0)^2}$$

$2^6 + 2^4(2^2 + 2^1 + 2^0) + \underline{2^5 + 2^4 + 2^0}$ by (3)

$$(2^6 + 2^6) + (2^5 + 2^4) + (2^5 + 2^4) + 2^0$$

$2^7 + 2^6 + 2^5 + 2^0$

1111^2

$$(2^4 + 2^3 + 2^2 + 2^1 + 2^0)^2$$

$$(2^4)^2 + 2 \cdot 2^4(2^3 + 2^2 + 2^1 + 2^0) + \underline{(2^3 + 2^2 + 2^1 + 2^0)^2}$$

$2^8 + 2^5(2^3 + 2^2 + 2^1 + 2^0) + \underline{2^7 + 2^6 + 2^5 + 2^0}$ by (4)

$$(2^8 + 2^8) + (2^7 + 2^6 + 2^5) + (2^7 + 2^6 + 2^5) + 2^0$$

$2^9 + 2^8 + 2^7 + 2^6 + 2^0$

11111^2

$$(2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)^2$$

$$(2^5)^2 + 2 \cdot 2^5(2^4 + 2^3 + 2^2 + 2^1 + 2^0) + \underline{(2^4 + 2^3 + 2^2 + 2^1 + 2^0)^2}$$

$2^{10} + 2^6(2^4 + 2^3 + 2^2 + 2^1 + 2^0) + \underline{2^9 + 2^8 + 2^7 + 2^6 + 2^0}$ by (5)

$$(2^{10} + 2^{10}) + (2^9 + 2^8 + 2^7 + 2^6) + (2^9 + 2^8 + 2^7 + 2^6) + 2^0$$

$2^{11} + 2^{10} + 2^9 + 2^8 + 2^7 + 2^0$

111111^2

$n = 0, 1, 2, 3, \ldots$

$$(2^n + 2^{n-1} + \cdots + 2^0)^2 = 2^{2n+1} + 2^{2n} + \cdots + 2^{n+2} + 2^0.$$

2^0

$n = 0$

$2^3 + 2^0$

$n = 1$

$2^5 + 2^4 + 2^0$

$n = 2$

$(n+1)$

$(2^{n+1} + 2^n + \cdots + 2^0)^2$

$$(2^{n+1})^2 + 2 \cdot 2^{n+1}(2^n + 2^{n-1} + \cdots + 2^0) + \underline{(2^n + 2^{n-1} + \cdots + 2^0)^2}$$

$2^{2(n+1)} + 2^{n+2}(2^n + 2^{n-1} + \cdots + 2^0) + \underline{2^{2n+1} + 2^{2n} \cdots + 2^{n+2} + 2^0}$ by (7)

$$\left(2^{2(n+1)}+2^{2(n+1)}\right)+(2^{2n+1}+\cdots+2^{n+2})+(2^{2n+1}+\cdots+2^{n+2})+2^{0}$$

$$2^{2(n+1)+1} + 2^{2(n+1)} + \cdots + 2^{(n+1)+2} + 2^0$$