--- Page 1 ---

## Chapter 3 Energy Release Rate

### 3.1 Griffith Postulation

\* Inglis found a solution for stress concentration effect in an elliptical hole. \[\sigma\_{22}^{\text{ max}}=\hat{\sigma}\_{22}=\sigma\_{0}(1+\frac{2a}{b})\]

\* According to Inglis solution, for a sharp crack where \(a>>b\), \(\sigma\_{22}^{\text{ max}}\) will be much greater than the strength of the material, which is contrary to actual observation.

--- Page 2 ---

\* \*\*Griffth Postulation\*\* - A crack would not grow unless certain amount of energy is released while cracking to overcome the energy required to form two new surfaces. - From this postulation, a crack growth condition is stated: (Total energy released when cracking (Released Energy) \(\geq\) The energy required to form two new crack surfaces (Surface Energy))

#### A simple model

--- Page 3 ---

\(\bullet\) \*\*Released Energy\*\*:

\[E\_{\rm R}(\mbox{Released Energy}) =U\_{\rm 0}\times\mbox{Volume of }\mbox{ the Triangles}\] \[=\frac{\sigma^{2}}{2E}\times 2[\frac{1}{2}(2a)(\lambda 2a)B]\] \[=\frac{2\lambda a^{2}B\sigma^{2}}{E}=\frac{2\biggl{(}\frac{\pi}{2 }\biggr{)}a^{2}B\sigma^{2}}{E}=\frac{\pi a^{2}B\sigma^{2}}{E}\]

- It has been shown that \(\lambda=\frac{\pi}{2}\) for thin plates.

Thus,

\[E\_{R}=\frac{\pi a^{2}B\sigma^{2}}{E}\] (1)

\(\bullet\) \*\*Surface Energy\*\*:

- To grow a crack, new surfaces must be created by breaking the bonds that hold atoms together. The bond strength is the result of the attractive forces between atoms.

- The energy required to break the bonds along the growing crack surface is called "Surface Energy".

\(\gamma\_{\rm s}\) = Surface Energy per unit area (J/m\({}^{2}\))

\(\bigtimes\) \(\gamma\_{\rm s}\) is a material property

- The surface energy required in the plate is

\[E\_{\rm s}=2(2a)B\gamma\_{\rm s}=4aB\gamma\_{\rm s}\] (2)

\(\bigtimes\) two new surfaces are created.

--- Page 4 ---

- From (1) and (2), it can be observed that

\[\left(\begin{array}[]{c}E\_{\rm s}\propto a\\ E\_{\rm R}\propto a^{2}\end{array}\right)\]

\begin{table} \begin{tabular}{|c|c|} \hline Material & \(\gamma\_{\rm s}\left(J\,/\,m^{2}\right)\) \\ \hline Copper & 0.98 \\ Mild steel & 1.20 \\ Aluminum & 0.60 \\ NaCl & 1.35 \\ MgO & 3.30 \\ Glass Pane & 2.30 \\ Ice & 0.07 \\ Diamond & 5.50 \\ \hline \end{tabular} \end{table} Table 1: Surface energy for some materials

--- Page 5 ---

- At critical crack size, the rate of \(E\_{\rm s}\) and \(E\_{\rm R}\) against the incremental crack length (\(\Delta a\)) become the same:

\[\frac{dE\_{\rm R}}{da}=\frac{dE\_{\rm s}}{da}\]

- Beyond this point, the crack will grow by itself.

- Thus the condition for self-crack growth is

\[\frac{dE\_{\rm R}}{da}\geq\frac{dE\_{\rm s}}{da}\] (3.3)

which means

\[\frac{2\pi aB\sigma^{2}}{E}\geq 4B\gamma\_{\rm s}\] (3.4)

- The critical crack length \(a\_{\rm c}\) is

\[\begin{split}\frac{2\pi a\_{\rm c}B\sigma^{2}}{E}=4B\gamma\_{\rm s }\\ \rightarrow\ a\_{\rm c}=\frac{2E\gamma\_{\rm s}}{\pi\sigma^{2}} \end{split}\] (3.5)

- The critical stress \(\sigma\_{\rm c}\) :

\[\begin{split}\frac{2\pi aB\sigma\_{\rm c}^{2}}{E}=4B\gamma\_{\rm s }\\ \rightarrow\ \sigma\_{\rm c}=(\frac{2E\gamma\_{\rm s}}{\pi a})^{1/2} \end{split}\] (3.6)

\(\mathbb{X}\) For plane strain case

\[\sigma\_{\rm c}=(\frac{2E^{\prime}\gamma\_{\rm s}}{\pi a})^{1/2}=(\frac{2E \gamma\_{\rm s}}{\pi(1-\nu^{2})a})^{1/2}\]

--- Page 6 ---

### Energy release rate (Irwin, 1956)

\* Griffith provided the energy approach for cracking process. However, no useful parameter that can be used to determine crack expansion is suggested.

\* Irwin suggested Energy Release Rate as a useful parameter for that purpose.

\* Energy Release Rate is a characterization of the driving force for crack growth.

\* In general, when a crack grows in size: 1. Elastic Strain Energy is changed (\(\Delta U\)) 2. Stiffness of the component decreases 3. Energy is consumed to create two new surfaces. 4. External loads do work to the components (\(\Delta W\_{\text{ext}}\))

\* For an incremental increase in the crack area \(\Delta A\), the total potential energy change is : \[\Delta\pi=\Delta U-\Delta W\_{\text{ext}}\]

\* The Energy Release Rate is \[G=\lim\_{\Delta A\to 0}(-\frac{\Delta\pi}{\Delta A})=-\frac{d\pi}{dA}\] (11)

\* Energy is released from the system as the potential energy is decreased. With \(\Delta A=B\Delta a\) (\(B\) = thickness) \[G=-\frac{1}{B}\frac{d\pi}{da}\] (12)

--- Page 7 ---

If the crack moves rapidly, some energy is being consumed to impart kinetic energy to the crack portions. In this case

\[G=-\frac{d\pi}{dA}-\frac{d\Upsilon}{dA}\]

where T = kinetic energy

### Energy Release Rate in the Forms of Compliance change

\(u=CP\quad or\quad P=\frac{u}{C}=Ku\)

\(C=compliance\)

\(K=stiffness\)

\* Now consider a double cantilever beam (DCB)

--- Page 8 ---

1. DCB with a constant load \(P\) (load controlled) u: tip displacement of a cantilever \[U=\frac{1}{2}\,Pu\] \[W\_{\text{ext}}=Pu\] Thus, \[\Delta\pi=\Delta U-\Delta W\_{\text{ext}}=\frac{1}{2}\,P\Delta u-P\Delta u=- \frac{1}{2}\,P\Delta u\] \[\begin{array}[]{l}\includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ \includegraphics[width=142.26378pt]{figures/DCB.eps} \\ 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--- Page 9 ---

\*\*ii. DCB with a fixed grip (displacement controlled)\*\*

\(P\)\(a\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)\(d\)

--- Page 10 ---

new surfaces, the crack will grow.

\* Now, since \(\Delta W\_{\rm ext}=P\Delta u\)=0, \[G=-\frac{1}{B}\frac{d\pi}{da}=-\frac{1}{B}\frac{d}{da}(\frac{1}{2}Pu)=-\frac{u}{ 2B}\frac{dP}{da}\quad(u=const)\] \[with\ \ P=\frac{u}{C}\] \[G=-\frac{u}{2B}\frac{d}{da}(\frac{u}{C})=-\frac{u^{2}}{2B}\frac{ d}{da}(\frac{1}{C})=\frac{u^{2}}{2BC^{2}}\frac{dC}{da}\] \[=\frac{(CP)^{2}}{2BC^{2}}\frac{dC}{da}\] \[=\frac{P^{2}}{2B}\frac{dC}{da}\] which is exactly the same with (3.9).

\* Therefore, G is the same both for load-controlled and displacement-controlled.

\* G in terms of compliance change provides convenient way for tests.

\* \((\frac{dU}{da})\_{p}=-(\frac{dU}{da})\_{u}\ \ :\) will be shown in the next section

--- Page 11 ---

\* If both \(P\) and \(u\) change simultaneously, expression (3.9) is still valid. The continuous curve can be treated as the integration of small steps which are load-controlled (displacement controlled)

\* Finding G: \(\Delta u\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M}\)\(u\_{\rm M

--- Page 12 ---

### G in terms of Strain Energy Change

1. \*\*DCB with load-controlled (const. \(P\))\*\* \[G=-\frac{d\pi}{dA}=-\frac{d}{dA}(U-W\_{\rm ext})\] \[{\rm where}\,dW\_{\rm ext}=Pdu=2dU\] \[{\rm Thus,}\;\;G=\frac{dU}{dA}\] \(\rightarrow\) half of the external work is used to increase the strain energy.

2. \*\*DCB with displacement controlled (const. \(u\))\*\* \[\bullet\;\;{\rm In\;this\;case,}\;\;\Delta W\_{\rm ext}=0\] \(\bullet\;\;{\rm Thus,}\) \[G=-\frac{dU}{dA}\] \(\rightarrow\) already existing strain energy is decreased as the crack advances.

--- Page 13 ---

\* \*\*Example 3.1\*\* Determine \(G\) for an edge crack loaded as shown below: \(U=\int\frac{1}{2}M\kappa\ da\) for a beam under bending \(\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \

--- Page 14 ---

### Energy Release Rate of DCB specimen

\* DCB specimen is frequently used to determine \(G\) for a specific material.

\* The tip deflection of a cantilever with a free-end load \(P\): \[\delta=\frac{1}{3}\frac{Pl^{3}}{EI}\]

\* For a DCB, same \(P\) produces a displacement twice the deflection of one cantilever: \[u=\frac{2}{3}\frac{Pa^{3}}{EI}\] Then, \[C=\frac{u}{P}=\frac{2a^{3}}{3EI}=\frac{8a^{3}}{Ebh^{3}}\quad(I=\frac{Bh^{3}} {12})\] (a) Using (a) in (3.9) \[G=\frac{P^{2}}{2B}(\frac{dC}{da})=\frac{12}{E}\frac{a^{2}}{B^{2}}\frac{P^{2}}{ h^{3}}\] (3.10)

\* Energy Release Rate depends on the capacity of the body to store strain energy

\* In this context "\(h\)" plays dominant role. The next dominant is \(B\).

--- Page 15 ---

\* \*\*Example 3.2\*\* Determine \(G\) of a DCB specimen whose thickness is 30mm, depth of each cantilever 12mm, and crack length 50mm. The pulling load is 15,405 N. The specimen is made of hardened steel with E = 207 GPa. From (3.10) \[G=\frac{12}{E}\frac{a^{2}}{B^{2}}\frac{P^{2}}{h^{3}}=\frac{12\times(0.05m)^{2} \times(15405N)^{2}}{(207\times 10^{9}Pa)(0.03m)^{2}(0.012m)^{3}}=22.0KJ\ /\ m^{2}\]

### Inelastic Deformation Effect at Crack Tip

\* The eqn (3.6) is valid for brittle materials.

\* At the crack tip high stresses cause plastic deformation in most metals (ductile). A lot of energy is dissipated in the process of plastic deformation.

\* Irwin and Orowan modified Griffith expression to account for the dissipation.

\* The eqn (3.6) is modified to \[\sigma\_{\mathrm{c}}=[\frac{2E(\gamma\_{\mathrm{s}}+\gamma\_{\mathrm{p}})}{\pi a }]^{1/2}\] where \(\gamma\_{\mathrm{p}}\) is the plastic work per unit area of surface created.

\* For most structural metal \(\gamma\_{\mathrm{p}}\)\(\square\)\(\gamma\_{\mathrm{s}}\). For example, for a mild steel, \(\gamma\_{\mathrm{s}}=1.20\,J\ /\ m^{2}\) while \(\gamma\_{\mathrm{p}}=125,000\,J\ /\ m^{2}\)

\* For polymer, energy is dissipated for polymer chains near cracked surface to align themselves under the stresses. This energy is also several times higher than \(\gamma\_{\mathrm{s}}\).

--- Page 16 ---

\* To reflect these facts, (3.6) is rewritten as \[\sigma\_{c}=(\frac{2E\gamma}{\pi a})^{1/2}\] (3.11) where \(\gamma\) is the overall surface energy that may include plastic, viscoelastic or viscoplastic effects at the crack tip. The Griffith model is for linear elastic material. Therefore the global behavior of the structure must be elastic.

### Crack Resistance and Instability

\* A crack starts to grow when \(G=R\)=2\(\gamma\) ( From(3.1); \(\frac{dE\_{S}}{dA}=2\gamma,A=\) crack surface area). R is called CRACK RESISTANCE

i. \*\*Brittle materials\*\*

\* R is almost constant w.r.t the change in crack size.

\* For a center crack shown below \(G=-\frac{d\pi}{dA}=\frac{\pi\sigma^{2}a}{E}\) (from(3.2))

Figure 3.1: Driving Force (\(G\)) curves for different values of \(\sigma\)

--- Page 17 ---

(G is a function of the crack size)

\* When \(\sigma=\sigma\_{2}\), it becomes unstable as \(G\) exceeds \(R\) when crack starts to grow.

\*\*i. Ductile materials\*\*

\* As the crack grows in size, the plastic zone at the crack tip increases and the resistance to crack opening increases.

\* When \(\sigma=\sigma\_{1}\), the crack does not grow. (\(a>a\_{0}\), \(G<R\))

\* For \(\sigma=\sigma\_{2}\), the crack grows from \(a\_{0}\) to \(a\_{2}\) ( \(\Delta a=a\_{2}-a\_{0}\) ). However, beyond point A, \(G<R\) and it is stable.

\* For \(\sigma=\sigma\_{3}\), the crack starts to grow by itself. (beyond point B, \(G>R\))

\* The condition for stable crack growth: \[G = R,\] \[and\ \ \frac{dG}{da}\leq\frac{dR}{da}\] (3.12)

\* Unstable crack growth occurs when

--- Page 18 ---

\[G>R\]

and \[\frac{dG}{da}>\frac{dR}{da}\] (3.13)

### The Effect of Thickness of the Specimen

\begin{tabular}{c c} Mode I & In a thick plate, plane-stress state prevails near specimen surfaces. \\ & In the interior, plane-strain state exists. \\ & As the thickness increases the plane-strain state dominates. Beyond certain size of the thickness, the thickness effects are not felt anymore. \\ \end{tabular}

--- Page 19 ---

\* As B increases, \(G\_{c}\to G\_{\rm IC}\).

\* \(G\_{\rm IC}\) is called the Critical Energy Release Rate for mode I and is considered a material property whose values can be found in handbooks.

\* For a thin plate, the stress state is plane-stress state. Therefore, if the handbook values of \(G\_{\rm IC}\) are used, the designs will be conservative.

\* \*\*Plane-stress state in a thin plate\*\*

\* \*\*Plane-strain state in a thick plate\*\*

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--- Page 20 ---

\begin{table} \begin{tabular}{|c|c|} \hline Material & \(G\_{\rm IC}\) (\(J\,/\,m^{2}\)) \\ \hline Mild Steel & \(\approx\) 250,000 \\ Alloy Steel (\(\sigma^{\*}\_{\mbox{\tiny{\it sys}}}\) = 1070) & 30,000 \\ EN 24 (U.K.) & \\

4340 (U.S.A.) & \\

40Ni2Cr1Mo28 (I.S.) & \\ Aluminum 7075-T6 & 8,000 \\ Titanium Ti-6Al-4V & 29,000 \\ Perspex (PMMA) & 800 \\ PVC & 4,500 \\ \hline \multicolumn{2}{|l|}{\({}^{\*}\)Yield Stress in MPa} \\ \hline \end{tabular} \end{table} Table 2: Representative plane strain \(G\_{\rm IC}\) of some common materials

## Chapter 3

## Energy Release Rate

## 3.1 Griffith Postulation

Plate of thickness B

<!-- image -->

- ⚫ Inglis found a solution for stress concentration effect in an elliptical hole.

<!-- formula-not-decoded -->

- ⚫ According to Inglis solution, for a sharp crack where a b  , max 22  will be much greater than the strength of the material, which is contrary to actual observation.

## ⚫ Griffth Postulation

- - A crack would not grow unless certain amount of energy is released while cracking to overcome the energy required to form two new surfaces.

- - From this postulation, a crack growth condition is stated:

(Total energy released when cracking (Released Energy)  The energy required to form two new crack surfaces (Surface Energy))

## 3.1.1 A simple model

<!-- image -->

## ⚫ Released Energy :

<!-- image -->

<!-- formula-not-decoded -->

- It has been shown that 2   = for thin plates.

Thus,

<!-- formula-not-decoded -->

## ⚫ Surface Energy :

- - To grow a crack, new surfaces must be created by breaking the bonds that hold atoms together. The bond strength is the result of the attractive forces between atoms.

- - The energy required to break the bonds along the growing crack surface is called 'Surface Energy'.

- s  = Surface Energy per unit area (J/m 2 )

- ※ s  is a material property

- - The surface energy required in the plate is

<!-- formula-not-decoded -->

- ※ two new surfaces are created.

Table 3.1 Surface energy for some materials

| Material | 2 s ( / ) J m  |

|------------|-------------------|

| Copper | 0.98 |

| Mild steel | 1.2 |

| Aluminum | 0.6 |

| NaCl | 1.35 |

| MgO | 3.3 |

| Glass Pane | 2.3 |

| Ice | 0.07 |

| Diamond | 5.5 |

## - From (2.1) and (2.2), it can be observed that

<!-- formula-not-decoded -->

<!-- image -->

- - At critical crack size, the rate of s E and R E against the incremental crack length ( a  ) become the same:

<!-- formula-not-decoded -->

- - Beyond this point, the crack will grow by itself.

- - Thus the condition for self-crack growth is

<!-- formula-not-decoded -->

which means

<!-- formula-not-decoded -->

- - The critical crack length c a is

<!-- formula-not-decoded -->

- - The critical stress c  :

<!-- formula-not-decoded -->

## ※ For plane strain case

<!-- formula-not-decoded -->

## 3.2 Energy release rate (Irwin, 1956)

- ⚫ Griffith provided the energy approach for cracking process. However, no useful parameter that can be used to determine crack expansion is suggested.

- ⚫ Irwin suggested Energy Release Rate as a useful parameter for that purpose.

- ⚫ Energy Release Rate is a characterization of the driving force for crack growth.

- ⚫ In general, when a crack grows in size:

- 1. Elastic Strain Energy is changed ( U  )

- 2. Stiffness of the component decreases

- 3. Energy is consumed to create two new surfaces.

- 4. External loads do work to the components ( ext W  )

- ⚫ For an incremental increase in the crack area A  , the total potential energy change is :

<!-- formula-not-decoded -->

- ⚫ The Energy Release Rate is

<!-- formula-not-decoded -->

- ⚫ Energy is released from the system as the potential energy is decreased. With A B a  =  ( B = thickness)

<!-- formula-not-decoded -->

※ If the crack moves rapidly, some energy is being consumed to impart kinetic energy to the crack portions. In this case

<!-- formula-not-decoded -->

where T = kinetic energy

## 3.3 Energy Release Rate in the Forms of Compliance change

<!-- image -->

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

- ⚫ Now consider a double cantilever beam (DCB)

## i. DCB with a constant load P (load controlled)

- u: tip displacement of a cantilever

<!-- image -->

<!-- image -->

Now,

<!-- formula-not-decoded -->

- ⚫ G is determined by finding the rate of compliance change w.r.t a.

- ※ The external work done by the constant load P → i.) half goes to deform the beam to a higher curvature increasing the strain energy, and ii.) the other half is released for the crack growth.

<!-- image -->

<!-- formula-not-decoded -->

B P u = 

: if no crack expansion U A B = +

<!-- image -->

When crack expands

<!-- formula-not-decoded -->

is half of C B

Only half of is stored as strain energy B →

## ii. DCB with a fixed grip (displacement controlled)

<!-- image -->

- ⚫ Initial work stored:

<!-- formula-not-decoded -->

- ⚫ With the increase in the crack length, the cantilever acquires smaller curvature releasing strain energy. ( 2 : U curvature    = )

- ⚫ If the release is large enough to meet the demands of producing

new surfaces, the crack will grow.

- ⚫ Now, since 𝛥𝑊ext = 𝑃𝛥𝑢 =0,

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

which is exactly the same with (3.9).

- ⚫ Therefore, G is the same both for load-controlled and displacement-controlled.

- ⚫ G in terms of compliance change provides convenient way for tests.

- ⚫ ( ) ( ) p u dU dU da da = -: will be shown in the next section

- ⚫ If both P and u change simultaneously, expression (3.9) is still valid.

<!-- image -->

The continuous curve can be treated as the integration of small steps which are load-controlled (displacement controlled)

- ⚫ Finding G:

<!-- image -->

𝑇𝑜𝑡𝑎𝑙 𝐸𝑛𝑒𝑟𝑔𝑦 𝑅𝑒𝑙𝑒𝑎𝑠𝑒𝑑

= 𝐺 × 𝑆𝑢𝑟𝑓𝑎𝑐𝑒 𝐴𝑟𝑒𝑎 𝐼𝑛𝑐𝑟𝑒𝑚𝑒𝑛𝑡 = 𝐺 × 𝐵𝛥𝑎 = 𝑎𝑟𝑒𝑎 𝐒

<!-- formula-not-decoded -->

## 3.4 G in terms of Strain Energy Change

## i. DCB with load-controlled (const. P )

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

→ half of the external work is used to increase the strain energy.

## ii. DCB with displacement controlled (const. u )

- ⚫ In this case, 𝛥𝑊ext =0

- ⚫ Thus,

<!-- formula-not-decoded -->

→ already existing strain energy is decreased as the crack advances.

- ⚫ Example 3.1 Determine G for an edge crack loaded as shown below:

1 for a beam under bending 2 U M da  = 

<!-- image -->

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

## 3.5 Energy Release Rate of DCB specimen

- ⚫ DCB specimen is frequently used to determine G for a specific material.

- ⚫ The tip deflection of a cantilever with a free-end load P :

<!-- formula-not-decoded -->

- ⚫ For a DCB, same P produces a displacement twice the deflection of one cantilever:

<!-- image -->

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

Using (a) in (3.9)

<!-- formula-not-decoded -->

※ Energy Release Rate depends on the capacity of the body to store strain energy

※ In this context ' h ' plays dominant role. The next dominant is B.

- ⚫ Example 3.2 Determine G of a DCB specimen whose thickness is 30mm, depth of each cantilever 12mm, and crack length 50mm. The pulling load is 15,405 N. The specimen is made of hardened steel with E = 207 GPa. From (3.10)

<!-- formula-not-decoded -->

## 3.6 Inelastic Deformation Effect at Crack Tip

- ⚫ The eqn (3.6) is valid for brittle materials.

- ⚫ At the crack tip high stresses cause plastic deformation in most metals (ductile). A lot of energy is dissipated in the process of plastic deformation.

- ⚫ Irwin and Orowan modified Griffith expression to account for the dissipation.

- ⚫ The eqn (3.6) is modified to

<!-- formula-not-decoded -->

where p  is the plastic work per unit area of surface created.

※ For most structural metal p s   . For example, for a mild steel, 2 s 1.20 / J m  = while 2 p 125,000 / J m  =

※ For polymer, energy is dissipated for polymer chains near cracked surface to align themselves under the stresses. This energy is also several times higher than s  .

- ⚫ To reflect these facts, (3.6) is rewritten as

<!-- formula-not-decoded -->

where 𝛾 is the overall surface energy that may include plastic, viscoelastic or viscoplastic effects at the crack tip. The Griffith model is for linear elastic material. Therefore the global behavior of the structure must be elastic.

## 3.7 Crack Resistance and Instability

- ⚫ A crack starts to grow when G = R =2 γ ( From(3.1); ⅆEs ⅆA = 2γ, A = crack surface area ). R is called CRACK RESISTANCE

## i. Brittle materials

- ⚫ R is almost constant w.r.t the change in crack size.

- ⚫ For a center crack shown below G = ⅆπ ⅆA = 𝝅𝝈 𝟐 𝒂 𝑬 (from(3.2))

Figure 3.1 Driving Force ( G ) curves for different values of 

<!-- image -->

(G is a function of the crack size)

- ⚫ When 2  = , it becomes unstable as G exceeds R when crack starts to grow.

## ii. Ductile materials

- ⚫ As the crack grows in size, the plastic zone at the crack tip increases and the resistance to crack opening increases.

- ⚫ When 1  = , the crack does not grow. ( 0 a a  , G R  )

- ⚫ For 2  = , the crack grows from 0 a to 2 a ( 2 0 a a a  = -). However, beyond point A, G R  and it is stable.

- ⚫ For 3  = , the crack starts to grow by itself. (beyond point B, G R  )

- ⚫ The condition for stable crack growth:

<!-- image -->

<!-- formula-not-decoded -->

- ⚫ Unstable crack growth occurs when

<!-- formula-not-decoded -->

<!-- formula-not-decoded -->

## 3.8 The Effect of Thickness of the Specimen

<!-- image -->

- ⚫ In a thick plate, plane-stress state prevails near specimen surfaces.

- ⚫ In the interior, plane-strain state exists.

- ⚫ As the thickness increases the plane-strain state dominates. Beyond certain size of the thickness, the thickness effects are not felt anymore.

<!-- image -->

Crack length

- ⚫ As B increases, c IC G G → .

- ⚫ IC G is called the Critical Energy Release Rate for mode Ⅰ and is considered a material property whose values can be found in handbooks.

- ⚫ For a thin plate, the stress state is plane-stress state. Therefore, if the handbook values of IC G are used, the designs will be conservative.

## ⚫ Plane-stress state in a thin plate

Large max  → Yielding (energy dissipation)

<!-- image -->

## ⚫ Plane-strain state in a thick plate

Small max  → No yielding or Small yielding → easy fracture

<!-- image -->

Table 3.2 Representative plane strain IC G of some common materials

| Material | 2 IC ( / ) G J m |

|-------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------|

| Mild Steel Alloy Steel ( \* ys  = 1070) EN 24 (U.K.) 4340 (U.S.A.) 40Ni2Cr1Mo28 (I.S.) Aluminum 7075-T6 Titanium Ti-6Al-4V Perspex (PMMA) |  250,000 30,000 8,000 29,000 |

| | 800 |

| PVC | 4,500 |

| \*Yield Stress in MPa | |

## Plane stresslplane strain Specimen Thickness Effects

Thickness B

<!-- image -->

Image: figure-1-1.jpg

The image shows a diagram of a plate with an elliptical hole in the center. The plate is subjected to uniform stress, denoted as \( \sigma\_0 \), applied vertically on the top and bottom edges. The elliptical hole has a horizontal axis labeled \( 2a \) and a vertical axis labeled \( 2b \). The center of the ellipse is marked as point \( A \). The coordinate axes are labeled \( x\_1 \) (horizontal) and \( x\_2 \) (vertical). Below the diagram, the text reads "Plate of thickness \( B \)".

In summary, the image illustrates a stressed plate with an elliptical hole, showing dimensions and stress directions.

Image: figure-11-14.jpg

The image is a graph with axes labeled \( P \) (vertical) and \( u \) (horizontal). It features a curve that decreases as it moves from left to right. Superimposed on the curve are step-like segments. Two labeled measurements are shown: \(\Delta P\) (vertical distance) and \(\Delta u\) (horizontal distance). The graph likely represents a relationship between two variables, with the step segments indicating discrete changes or approximations.

Image: figure-11-15.jpg

The image consists of two diagrams related to mechanical deformation and load-displacement analysis.

1. \*\*Left Diagram:\*\*

- Shows a beam subjected to a load \( P \).

- The beam is deflected, and two points are marked: \( a\_A \) and \( a\_M \).

- The vertical axis is labeled \( u \).

2. \*\*Right Diagram:\*\*

- A graph with axes labeled \( P \) (vertical) and \( u \) (horizontal).

- Points \( O \), \( A \), \( B \), \( C \), and \( M \) are marked on the graph.

- The area under the curve is shaded and labeled \( S \).

- The horizontal distance between \( u\_A \) and \( u\_M \) is marked as \( \Delta u \).

\*\*Summary:\*\*

The image illustrates a mechanical system with a beam under load, showing deflection and corresponding load-displacement graph. The graph highlights the relationship between load \( P \) and displacement \( u \), with specific points and areas of interest marked.

Image: figure-11-16.jpg

The image shows a horizontal line with arrows at both ends. The left arrow is pointing left, and the right arrow is pointing right. There is no text content in the image.

Image: figure-11-17.jpg

The image appears to be a simple black comma on a white background. There is no additional text or content to summarize.

Image: figure-13-18.jpg

The image depicts a 3D diagram of a beam with a rectangular cross-section. The beam is subjected to a bending moment, denoted by "M," which is constant along its length. The dimensions of the beam are labeled as "B" for the width, "h" for the height (with two sections labeled "h"), and "a" for the length of a section. The beam is shown with a cutout section to illustrate the internal structure and the application of the moment. The text "M = const" indicates that the moment is constant.

Image: figure-14-19.jpg

The image is a diagram of a mechanical component with the following text labels:

- Thickness = \( B \)

- \( P \) (indicating force or pressure)

- \( \delta \) (indicating a vertical distance)

- \( u \) (indicating a vertical distance)

- \( h \) (indicating a vertical distance)

- \( a \) (indicating a horizontal distance)

The diagram shows a tapered structure with a thickness labeled \( B \). The structure appears to be under some form of pressure or force, indicated by arrows labeled \( P \). The dimensions \( \delta \), \( u \), \( h \), and \( a \) are marked to show various distances within the structure.

Image: figure-16-20.jpg

The image consists of two parts: a graph and a diagram.

\*\*Graph:\*\*

- Axes labeled as \( G, R \) (vertical) and \( a\_o \) (horizontal).

- A dashed horizontal line labeled \( G\_c \).

- Two lines intersecting at a point on the \( G\_c \) line, labeled \( G \) and \( R \).

- The graph shows regions labeled "Stable" and "Unstable."

- Two stress levels are indicated: \( \sigma = \sigma\_1 \) (stable) and \( \sigma = \sigma\_2 \) (unstable).

\*\*Diagram:\*\*

- A rectangular block with arrows indicating stress (\( \sigma \)) applied vertically.

- A crack or defect is shown in the middle, labeled with a width of \( 2a \).

- A horizontal arrow labeled \( B \).

\*\*Summary:\*\*

The image illustrates a stability analysis of a material under stress. The graph shows the relationship between growth rate (\( G \)) and resistance (\( R \)) at different stress levels, indicating stable and unstable regions. The diagram depicts a material block with a crack, showing how stress is applied.

Image: figure-17-21.jpg

The image is a graph with axes labeled \(G, R\) on the vertical axis and \(a\_0, a\_2, a\_c\) on the horizontal axis. The graph features two curves, one labeled \(G\) and the other \(R\). There are three lines intersecting the curves, labeled \(\sigma\_1\), \(\sigma\_2\), and \(\sigma\_3\), with arrows pointing in the direction of increasing values. Points \(A\) and \(B\) are marked on the curves, with \(B\) at a higher position than \(A\). The vertical line from point \(B\) intersects the horizontal axis at \(a\_c\), and a horizontal line from \(B\) intersects the vertical axis at \(G\_c\).

In summary, the graph illustrates the relationship between variables \(G\) and \(R\) with respect to parameters \(\sigma\) and \(a\), showing how these variables change along the curves and lines.

Image: figure-18-22.jpg

The image shows a diagram labeled "Mode I." It depicts a rectangular block with arrows pointing upwards and downwards, indicating stress or force, labeled as "σ." There is an elliptical shape in the center with a horizontal dimension labeled "2a." A line labeled "B" is shown at the bottom right, indicating a boundary or edge.

Summary: The diagram illustrates a block under Mode I loading, showing stress distribution with a central crack of length "2a" and a boundary labeled "B."

Image: figure-18-23.jpg

The image is a graph showing the relationship between crack length and energy release rate (G, R). The x-axis is labeled "Crack length" and the y-axis is labeled "G, R". There are several curves labeled \(B\_1\), \(B\_2\), and \(B\_3\), with the relationship \(B\_3 > B\_2 > B\_1\).

Horizontal dashed lines are labeled \(G\_{c}^{(1)}\), \(G\_{c}^{(2)}\), \(G\_{c}^{(3)}\), and \(G\_{Ic}\). The curves represent different conditions or materials, with \(B\_3\) having the highest energy release rate and \(B\_1\) the lowest. The graph illustrates how the energy release rate changes with crack length for different conditions.

Image: figure-19-24.jpg

The image consists of two diagrams related to stress analysis near a crack tip.

1. \*\*Left Diagram\*\*:

- Shows a 3D block with a crack.

- Axes labeled as \(x\_1\), \(x\_2\), and \(x\_3\).

- Arrows labeled with \(\sigma\) indicate stress applied on the block.

2. \*\*Right Diagram\*\*:

- A Mohr's circle representation.

- Axes labeled as \(\tau\) (shear stress) and \(\sigma\) (normal stress).

- Points labeled \(\sigma\_1\), \(\sigma\_2\), and \(\sigma\_3\).

- Text "Large \(\tau\_{\text{max}}\)" and "Near crack tip" are present.

The diagrams illustrate stress distribution and analysis near a crack tip, using a 3D model and Mohr's circle.

Image: figure-19-25.jpg

The image consists of two diagrams related to stress analysis near a crack tip.

1. \*\*Left Diagram\*\*:

- A 3D block with a crack is shown.

- Axes labeled \(x\_1\), \(x\_2\), and \(x\_3\).

- Arrows labeled \(\sigma\) indicate stress applied on the block.

2. \*\*Right Diagram\*\*:

- A 2D plot with axes labeled \(\tau\) (vertical) and \(\sigma\) (horizontal).

- A Mohr's circle is depicted with points labeled \(\sigma\_1\), \(\sigma\_2\), and \(\sigma\_3\).

- The text "Small \(\tau\_{\text{max}}\)" is near the top of the circle.

- The text "Near crack tip" is below the diagram.

\*\*Summary\*\*: The image illustrates stress distribution and analysis near a crack tip using a 3D model and a Mohr's circle, highlighting the stress components and maximum shear stress.

Image: figure-2-2.jpg

The image shows a series of diagrams illustrating a plate being stretched.

1. \*\*Left Diagram\*\*:

- Labeled "Plate of thickness B."

- Shows a solid plate with no visible alterations.

2. \*\*Middle Diagram\*\*:

- Labeled "Stretched."

- The plate is shown being stretched vertically with arrows indicating the direction of the force.

3. \*\*Right Diagram\*\*:

- Shows the same plate with a crack.

- Labeled "A crack cut by a knife."

- The crack is depicted as a diamond shape with dimensions labeled as \(2a\) horizontally and \(\lambda 2a\) vertically.

The diagrams illustrate the process of stretching a plate and introducing a crack to study its effects.

Image: figure-20-26.jpg

The image is a graph titled "Plane stress/plane strain Specimen Thickness Effects." It shows the relationship between fracture toughness and specimen thickness. The y-axis is labeled with \( K\_c \) and \( K\_{Ic} \), while the x-axis is labeled "Thickness B."

The graph is divided into three regions: "Plane Stress," "Mixed," and "Plane Strain." The transition from plane stress to plane strain occurs around a thickness of approximately \( 25 r\_p \), where \( r\_p \) is defined as the radius of the plastic zone at the crack tip.

The curve starts high on the y-axis in the plane stress region and decreases as it moves through the mixed region, leveling off in the plane strain region. This indicates that as thickness increases, the fracture toughness decreases until it stabilizes.

Image: figure-3-3.jpg

The image shows a geometric diagram with a diamond shape centered on a set of axes. The diamond is outlined with black lines, and there is a shaded elliptical area inside the diamond. The axes have arrows pointing outward, indicating direction. There is no text content in the image.

Image: figure-4-4.jpg

The image is a graph with two curves labeled \(E\_R\) and \(E\_S\), representing different energy components. The x-axis is labeled with \(a\), and specific points are marked as \(a\_0\) and \(a\_{\text{critical}}\). The y-axis is labeled \(E\_R, E\_S\).

There are vertical arrows indicating changes in energy, labeled \(\Delta E\_S\) and \(\Delta E\_R\). The graph shows the relationship between these energy components as a function of \(a\), with a critical point where the behavior of the curves changes. The area between \(a\_0\) and \(a\_{\text{critical}}\) is marked as \(\Delta a\).

Overall, the graph illustrates how the energies \(E\_R\) and \(E\_S\) vary with the parameter \(a\), highlighting a critical point and changes in energy.

Image: figure-7-5.jpg

The image shows a mechanical diagram with a shape that has a hole in the center. The hole is labeled with "2a" indicating its width. There is a force labeled "P" pointing upwards at the top of the shape. An arrow labeled "u" is also pointing upwards near the force "P". At the bottom of the shape, there is a symbol representing a support or a fixed point.

Summary: The diagram illustrates a structure with a central hole of width "2a", subjected to an upward force "P" and displacement "u", with a support at the bottom.

Image: figure-8-6.jpg

The image is a diagram showing a mechanical setup. It includes:

- A weight labeled "P" hanging from a rope.

- The rope is looped over two pulleys.

- The rope is connected to a structure that appears to be a beam or lever.

- The beam is split into two branches, with one end fixed.

- There are two labeled distances: "a" and "Δa" along the beam.

This setup likely illustrates a physics or engineering concept related to forces, tension, or mechanical advantage.

Image: figure-8-7.jpg

The image contains a diagram and some equations related to mechanics or physics.

\*\*Diagram:\*\*

- It shows a beam or rod with a force applied, supported by a structure.

- There are two points marked with distances "a" and "Δa."

\*\*Equations:\*\*

1. \( U = \frac{1}{2} Pu \)

2. \( W\_{\text{ext}} = Pu \)

3. Thus,

\[

\Delta \pi = \Delta U - \Delta W\_{\text{ext}} = \frac{1}{2} P \Delta u - P \Delta u =

\]

\*\*Summary:\*\*

The image illustrates a mechanical setup with a beam and applied forces, accompanied by equations calculating potential energy, external work, and changes in these quantities.

Image: figure-8-8.jpg

The image is a diagram with a graph that includes several labeled components. Here's a summary of the text and elements:

- Axes: The vertical axis is labeled "P" and the horizontal axis is labeled with "u" and "u + Δu".

- Lines and Areas:

- A line labeled "U" extends from the origin.

- A vertical line at "u" and another at "u + Δu".

- A diagonal line labeled "dU" connects these vertical lines.

- Areas labeled "A", "B", and "C" are formed between these lines.

- Text:

- "for a" and "for a + Δa" are written along the horizontal axis.

- "1/2 PΔu" is written above the top horizontal line.

- A box with the text "= PΔu" is present.

The diagram appears to represent a mathematical or physical concept involving changes in variables, possibly related to work or energy.

Image: figure-9-10.jpg

The image is a graph with axes labeled "P" (vertical) and an unlabeled horizontal axis. There is a parallelogram-like shape on the graph, with a slanted top and bottom. The shape is divided into smaller sections with diagonal lines. The letter "C" is inside the shape. The horizontal axis has two points marked: "u" and "u + Δu," with vertical dashed lines extending from these points to the shape.

Summary: The image depicts a graph with a geometric shape labeled "C" between two points "u" and "u + Δu" on the horizontal axis, likely representing a change or interval.

Image: figure-9-11.jpg

The image contains a diagram and text related to crack expansion in a material.

\*\*Diagram:\*\*

- A triangular area labeled "C" is shown between two vertical lines marked "u" and "u+Δu."

\*\*Text:\*\*

- "When crack expands"

- "C is lost (= \(\frac{1}{2} P \Delta u\)) → Outside"

- "C is half of B"

- "→ Only half of B is stored as strain energy."

\*\*Summary:\*\*

The image illustrates the concept of energy loss during crack expansion in a material. The area "C" represents energy that is lost to the outside, calculated as half of the product of force and displacement change. "C" is half of another area "B," indicating that only half of "B" is stored as strain energy.

Image: figure-9-12.jpg

The image is a diagram with a graph that has axes labeled "P" (vertical) and "u" (horizontal). It features a triangular shape with annotations. The text in the image includes:

- "P" (vertical axis)

- "u" (horizontal axis)

- "-dU" inside the triangle

- "-dP" with a bracket indicating a section of the triangle

- "for a" on the left side of the triangle

- "for a + Δa" on the right side of the triangle

The diagram appears to represent a change in variables, possibly in a thermodynamic or mechanical context, with the triangle indicating changes in "U" and "P" over an interval from "a" to "a + Δa".

Image: figure-9-13.jpg

The image shows a diagram of a beam or structure with two supports on the left side. The beam is split into two curved sections that converge into a single section on the right. The supports are represented by triangular symbols with lines indicating fixed points. Below the beam, there are two labeled distances: "a" and "Δa," indicating the length of the sections. The diagram likely represents a mechanical or structural concept, possibly related to deflection or stress analysis.

Image: figure-9-9.jpg

The image is a graph with axes labeled "P" (vertical) and "u" (horizontal). There are two regions marked on the graph:

1. Region A: A trapezoidal area on the left, filled with diagonal lines.

2. Region B: A rectangular area on the right.

The horizontal axis has two points labeled "u" and "u + Δu," indicating an interval on the axis. The graph likely represents a relationship between variables P and u, with areas A and B representing different segments or contributions within this interval.