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Assignment 1

Task 1: Data normalization

Firstly, 9 (>= 6) corner points on the calibration chessboard are selected and clicked as shown in Fig.1. The 2D and corresponding 3D coordinates of these points are shown in Table 1.

After that, data normalization needs to be done to avoid potential ill-conditioned matrix. Then a similarity transform T is used to normalize the 2D image points, and a second similarity transform U is used to normalize the corresponding 3D space points.

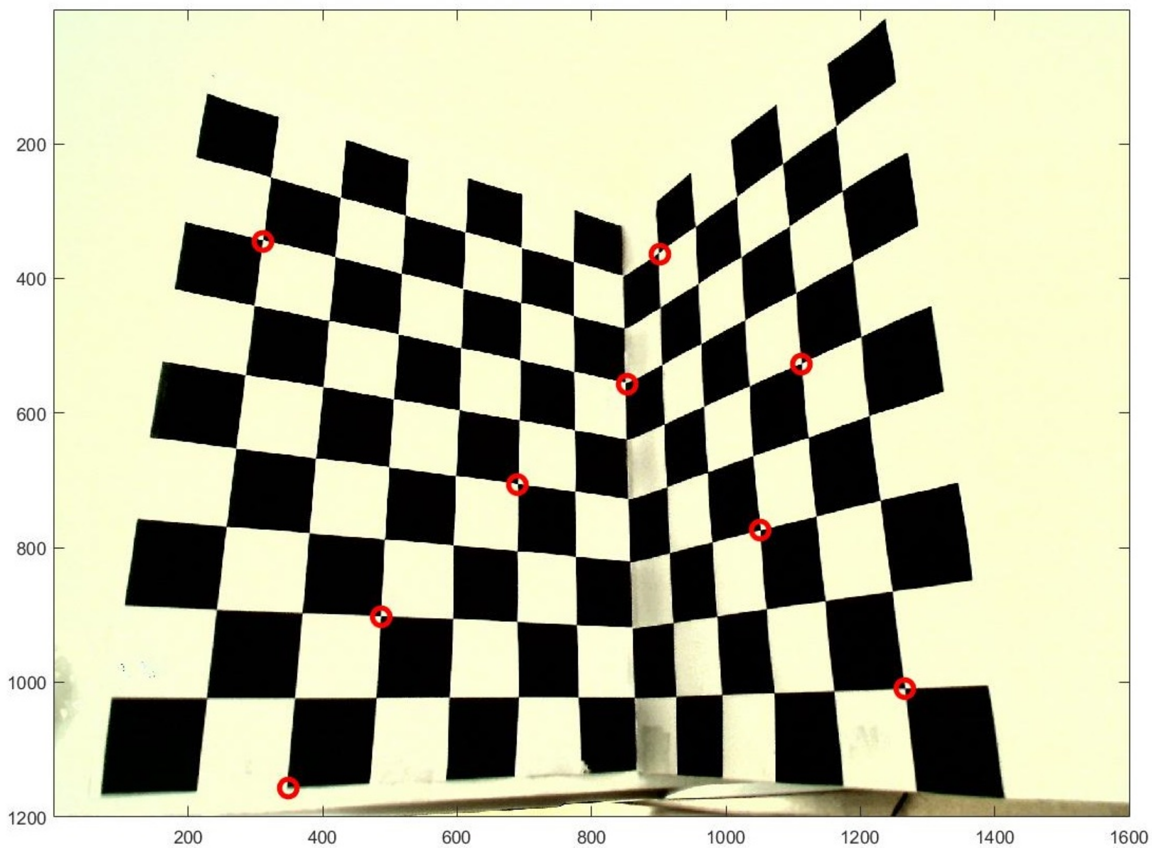

Figure 1: The clicked points on the chessboard

The normalization similarity transformation matrix T and U can then be calculated as Eq.1-8. Then we can get the normalized 2D and 3D points (Refer to *normalization.mat*)

$$\boldsymbol{t_{2D}} = \begin{bmatrix} t_x & t_y \end{bmatrix}^T = \begin{bmatrix} \sum_{i=1}^{n} -x_i & \sum_{i=1}^{n} -y_i \end{bmatrix}^T \tag{1}$$

$$\boldsymbol{t_{3D}} = \begin{bmatrix} t_X & t_Y & t_Z \end{bmatrix}^T = \begin{bmatrix} \sum_{i=1}^{n} -X_i & \sum_{i=1}^{n} -Y_i & \sum_{i=1}^{n} -Z_i \end{bmatrix}^T \tag{2}$$

Table 1: 2D and the corresponding 3D coordinates of the clicked point set

Point ID	x (pix)	y (pix)	X (m)	Y (m)	Z (m)
1	311.23	345.26	0.162	0.0	0.189
2	487.15	902.86	0.108	0.0	0.054
3	689.77	706.52	0.054	0.0	0.108
4	853.12	557.31	0.0	0.0	0.162
5	1266.21	1009.66	0.0	0.135	0.027
6	1051.02	774.06	0.0	0.081	0.081
7	901.81	364.11	0.0	0.027	0.216
8	1112.28	527.46	0.0	0.108	0.135
9	348.93	1157.31	0.135	0.0	0.0

$$s_{2D} = \frac{n\sqrt{2}}{\sum_{i=1}^{n}\sqrt{x_i^2 + y_i^2}} \tag{3}$$

$$s_{3D} = \frac{n\sqrt{3}}{\sum_{i=1}^{n}\sqrt{X_i^2 + Y_i^2 + Z_i^2}} \tag{4}$$

$$\begin{bmatrix} c_x & c_y \end{bmatrix}^T = s_{2D}\boldsymbol{t_{2D}} = s_{2D}\begin{bmatrix} t_x & t_y \end{bmatrix}^T \tag{5}$$

$$\begin{bmatrix} c_X & c_Y & c_Z \end{bmatrix}^T = s_{3D}\boldsymbol{t_{3D}} = s_{3D}\begin{bmatrix} t_X & t_Y & t_Z \end{bmatrix}^T \tag{6}$$

$$T = \begin{bmatrix} s_{2D} & 0 & c_x \\ 0 & s_{2D} & c_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0037 & 0 & -2.9066 \\ 0 & 0.0037 & -2.6264 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$U = \begin{bmatrix} s_{3D} & 0 & 0 & c_X \\ 0 & s_{3D} & 0 & c_Y \\ 0 & 0 & s_{3D} & c_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 17.0793 & 0 & 0 & -0.8710 \\ 0 & 17.0793 & 0 & -0.6661 \\ 0 & 0 & 17.0793 & -1.8446 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Task 2: Direct Linear Transform (DLT)

For this task, please refer to *runDLT.m* , *DLT.m* and *runDLTwithoutnormalization.m.*
Firstly, the matrix A in $\underset{2k*12}{A}\underset{12*1}{p} = \underset{2k*1}{0}$ is constructed. Then the Singular Value Decomposition (**SVD**) is used to estimate the projection parameters p which make up the projection matrix P. If the normalization is

done beforehand, the final projection matrix can be calculated from the normalized projection matrix P_{norm} and the similarity transformation matrix T and U as Eq.9. Then we get P_{DLT} as shown in Eq.10.

$$P = T^{-1} P_{norm} U \tag{9}$$

$$P_{\text{DLT}} = \begin{bmatrix} 2925.4 & -57.5 & -460.1 & -581.1 \\ 978.1 & 1615.1 & 2002.1 & -752.0 \\ 1.0 & 1.6 & -0.6 & -0.7 \end{bmatrix} \tag{10}$$

Next, according to *decompose.m*, the intrinsic and extrinsic part of the projection transformation (namely matrix K and matrix $[R|t]$) can be drawn from matrix P using **QR** decomposition and **SVD**. The result is shown as Eq.11 and Eq.12.

$$K_{\text{DLT}} = \begin{bmatrix} 1279.1 & -5.6 & 812.8 \\ 0 & 1272.2 & 603.7 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$[R|t]_{\text{DLT}} = \begin{bmatrix} -0.847 & 0.532 & -0.016 & 0.015 \\ -0.151 & -0.269 & -0.951 & 0.140 \\ -0.510 & -0.803 & 0.308 & 0.343 \end{bmatrix} \tag{12}$$

After that, by reprojecting the 3D space points using the estimated P and comparing them with their corresponding 2D image points, the evaluation for DLT is implemented. The results are shown in Fig.2 and Table 2.

Table 2: Reprojection error comparison, unit: pixel

Method	Pt1	Pt2	Pt3	Pt4	Pt5	Pt6	Pt7	Pt8	Pt9	Mean
DLT	0.326	0.747	3.717	0.774	1.430	0.565	2.737	2.151	2.313	**1.640**
Gold Standard	0.255	0.646	3.725	0.969	0.799	0.602	3.400	1.586	1.622	**1.512**

If the unnormalized points are used in DLT, the result (matrix P, K and $[R|t]$) is shown as Eq.13-15. For P, the last row is very small and approximately equal to zero. For homogeneous coordinates representation, the scale of projection matrix does not make sense. So we can see that there's a kind of propotional change between P with and without data normalization. There's also a bit minor difference for matrix K and $[R|t]$. According to the experiment result, average reprojection error for these two settings is almost the same (the one without normalization is even a bit smaller). The reason may be the inaccurate manual selection of corner points on the calibration board.

$$P_{\text{DLT\_withoutnorm}} = \begin{bmatrix} -0.704 & 0.013 & 0.111 & 0.140 \\ -0.236 & -0.389 & -0.482 & 0.181 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

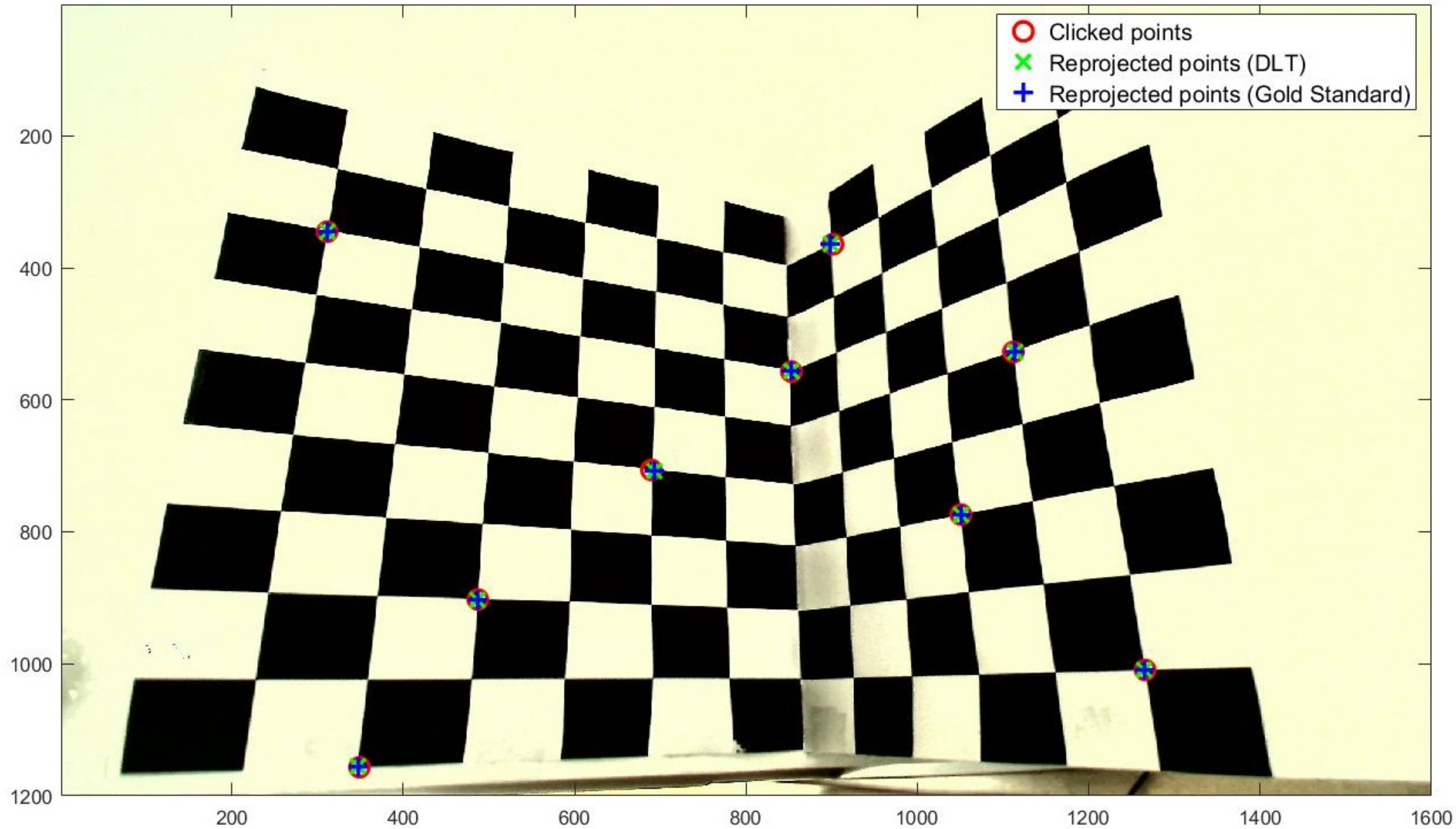

Figure 2: The clicked points and the reprojected 3D points using DLT and gold standard method

$$K_{\text{DLT\_withoutnorm}} = \begin{bmatrix} 1274.5 & -6.2 & 813.1 \\ 0 & 1267.8 & 606.1 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$[R|t]_{\text{DLT\_withoutnorm}} = \begin{bmatrix} -0.846 & 0.532 & -0.016 & 0.015 \\ -0.150 & -0.267 & -0.952 & 0.139 \\ -0.511 & -0.803 & 0.306 & 0.343 \end{bmatrix} \tag{15}$$

Task 3: Gold Standard algorithm

For this task, please refer to *runGoldStandard.m* and *fminGoldStandard.m*.

The gold standard algorithm is implemented taking the result of DLT as initial value. Then the geometric reprojection error is minimized using non-linear optimization while the projection parameters are refined through the process.

The estimated projection, intrinsic and extrinsic matrix are shown as Eq.16-18. After conduct the reprojection, the evaluation of gold standard algorithm is shown in Fig.2 and Table 2. A more detailed view of Fig.2 is Fig.3. According to Table 2, the mean reprojection error for all the nine points decreases from 1.640 pixel (DLT Alg.) to 1.512 pixel (Gold Standard Alg.), which proves the effect of gold standard algorithm.

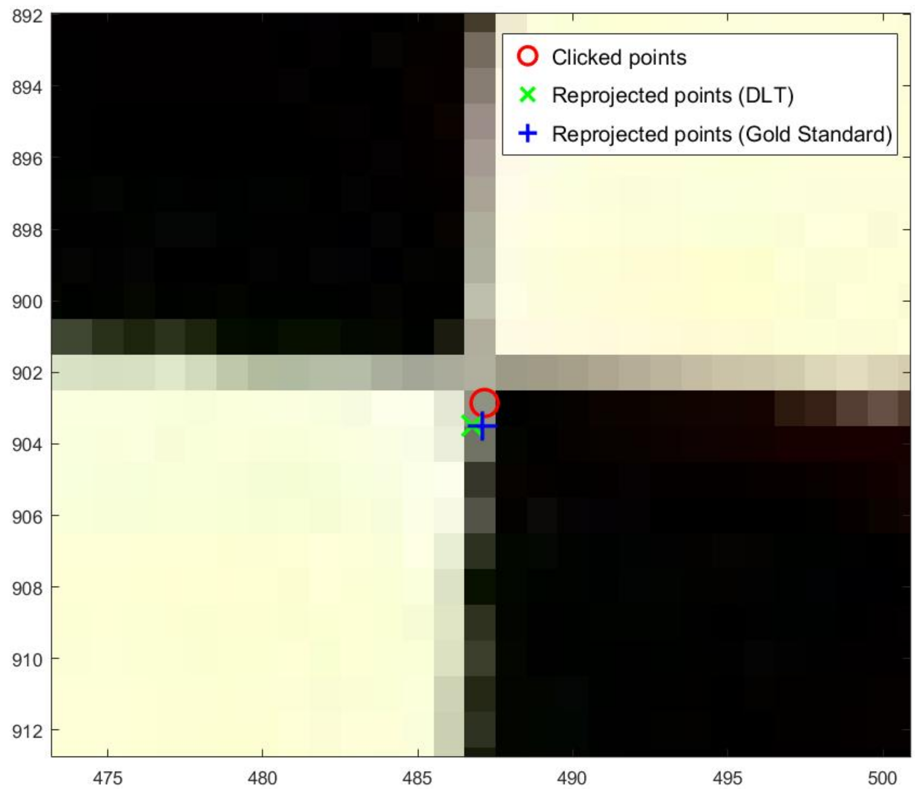

Figure 3: The clicked points and the reprojected 3D points using DLT and gold standard method (Zoom in)

$$P_{\text{Gold Standard}} = \begin{bmatrix} 2930.4 & -59.6 & -461.2 & -582.3 \\ 978.6 & 1615.6 & 2003.8 & -753.2 \\ 1.0 & 1.6 & -0.6 & -0.7 \end{bmatrix} \tag{16}$$

$$K_{\text{Gold Standard}} = \begin{bmatrix} 1281.1 & -2.9 & 812.9 \\ 0 & 1274.5 & 598.7 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

$$[R|t]_{\text{Gold Standard}} = \begin{bmatrix} -0.847 & 0.532 & -0.016 & 0.015 \\ -0.153 & -0.271 & -0.950 & 0.141 \\ -0.510 & -0.802 & 0.311 & 0.344 \end{bmatrix} \tag{18}$$

Task 4: Bouguet‘s Calibration Toolbox

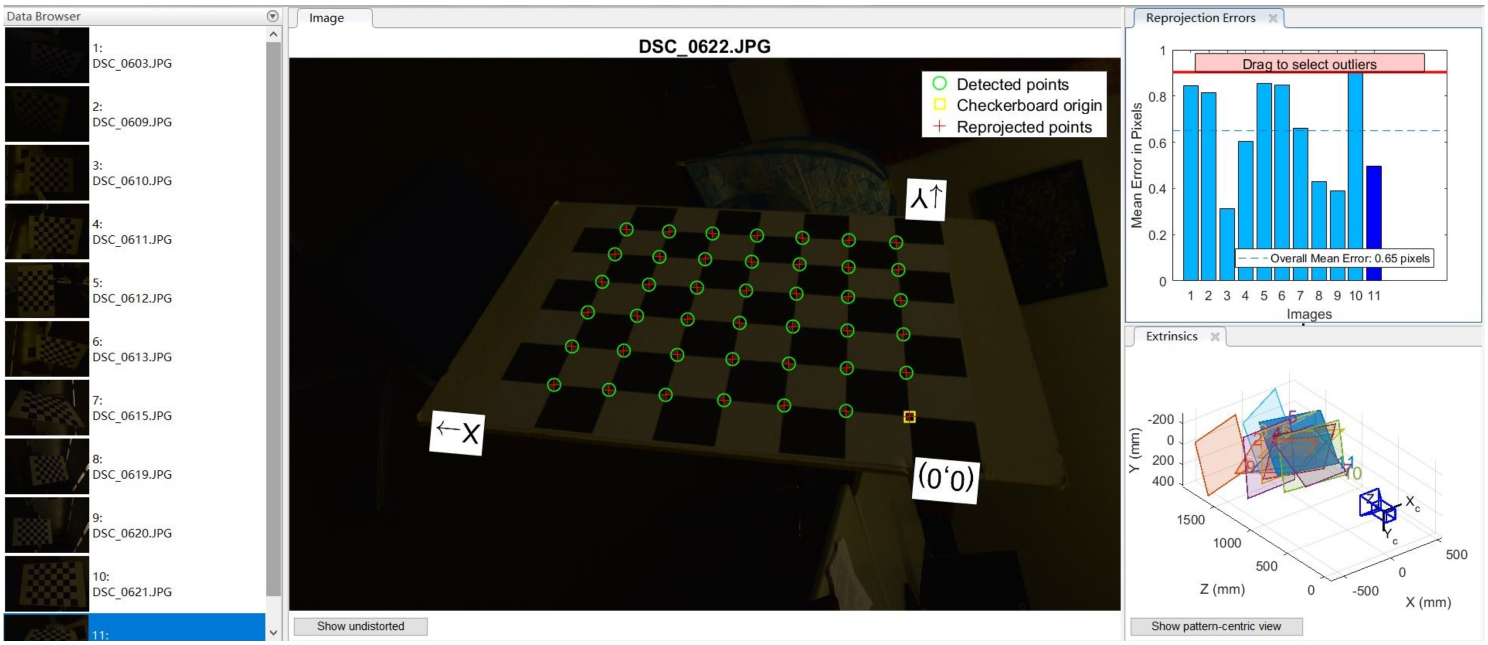

Figure 4: Calibration demo using Matlab’s calibration toolbox