Yue Pan
yuepan@student.ethz.ch

Assignment 0

Exercise 1

Question: A minor outbreak of cooties has occurred in a city. There are a million healthy inhabitants and one hundred infected ones. There are no visible symptoms for the infected citizens. A test is available that produces a positive result with 97.5% probability if the tested person is infected, and with 2.5% probability if the tested person is healthy. If a person tests positive, how high is the probability that he/she is infected? Try to identify the **prior** probability, the **likelihood**, and the **posterior** probability in your calculation.

Answer:

According to the question, we can get equation 1,2 and 3.

$$prior = p\left(infected\right) = 100/\left(100 + 1000000\right) = 9.999 \times 10^{-5} \tag{1}$$

$$likelihood = p\left(positive \mid infected\right) = 97.5\% = 0.975 \tag{2}$$

$$p\left(positive \mid uninfected\right) = 2.5\% = 0.025 \tag{3}$$

What we'd like to solve here is the posterior. According to the Bayes Rule, we can draw equation 4.

$$posterior = p\left(infected \mid positive\right) = \frac{p\left(positive \mid infected\right) p\left(infected\right)}{p\left(positive\right)} \tag{4}$$

Then we can expand $p\left(positive\right)$ to equation 5.

$$p\left(positive\right) = p\left(positive \mid infected\right) p\left(infected\right) + p\left(positive \mid uninfected\right) p\left(uninfected\right) \tag{5}$$

And we know that

$$p\left(uninfected\right) = 1 - p\left(infected\right) \tag{6}$$

Therefore, we can solve the posterior probability as equation 7.

$$posterior = p\left(infected \mid positive\right) = \frac{0.975 \times 9.999 \times 10^{-5}}{0.975 \times 9.999 \times 10^{-5} + 0.025 \times \left(1 - 9.999 \times 10^{-5}\right)} = 0.0039 = 0.39\% \tag{7}$$

Exercise 2

Question: A merchant has two bags of coins. The first bag contains 30 silver and 20 golden coins, while the second bag contains 10 silver and 40 golden coins. He chooses a bag randomly and picks a random coin inside of it. If he pulls a golden coin out, how high is the probability that he chose the first bag?

Answer:

According to the question, we can get equation 8,9 and 10.

$$p\left(golden \mid first\right) = 20/\left(20+30\right) = 0.4 \tag{8}$$

$$p\left(golden \mid second\right) = 40/\left(10+40\right) = 0.8 \tag{9}$$

$$p\left(first\right) = p\left(second\right) = 0.5 \tag{10}$$

According to the Bayes Rule, we can get the posterior using the likelihood and prior and draw equation 11.

$$\begin{aligned} p\left(first \mid golden\right) &= \frac{p\left(golden \mid first\right)p\left(first\right)}{p\left(golden\right)} \\ &= \frac{p\left(golden \mid first\right)p\left(first\right)}{p\left(golden \mid first\right)p\left(first\right)+p\left(golden \mid second\right)p\left(second\right)} \\ &= \frac{0.4\times0.5}{0.4\times0.5+0.8\times0.5} = 1/3 = 0.3333 \end{aligned} \tag{11}$$