

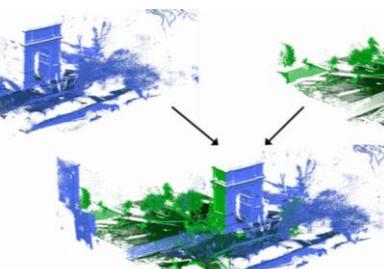
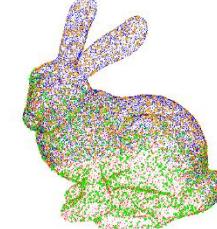
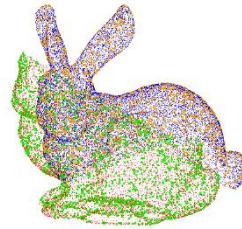


Target-less registration of point cloud: A Review

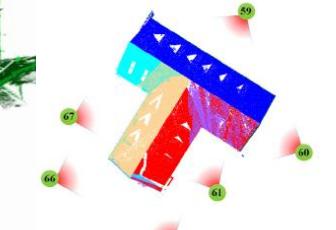
Yue Pan

Introduction

What is point cloud registration?

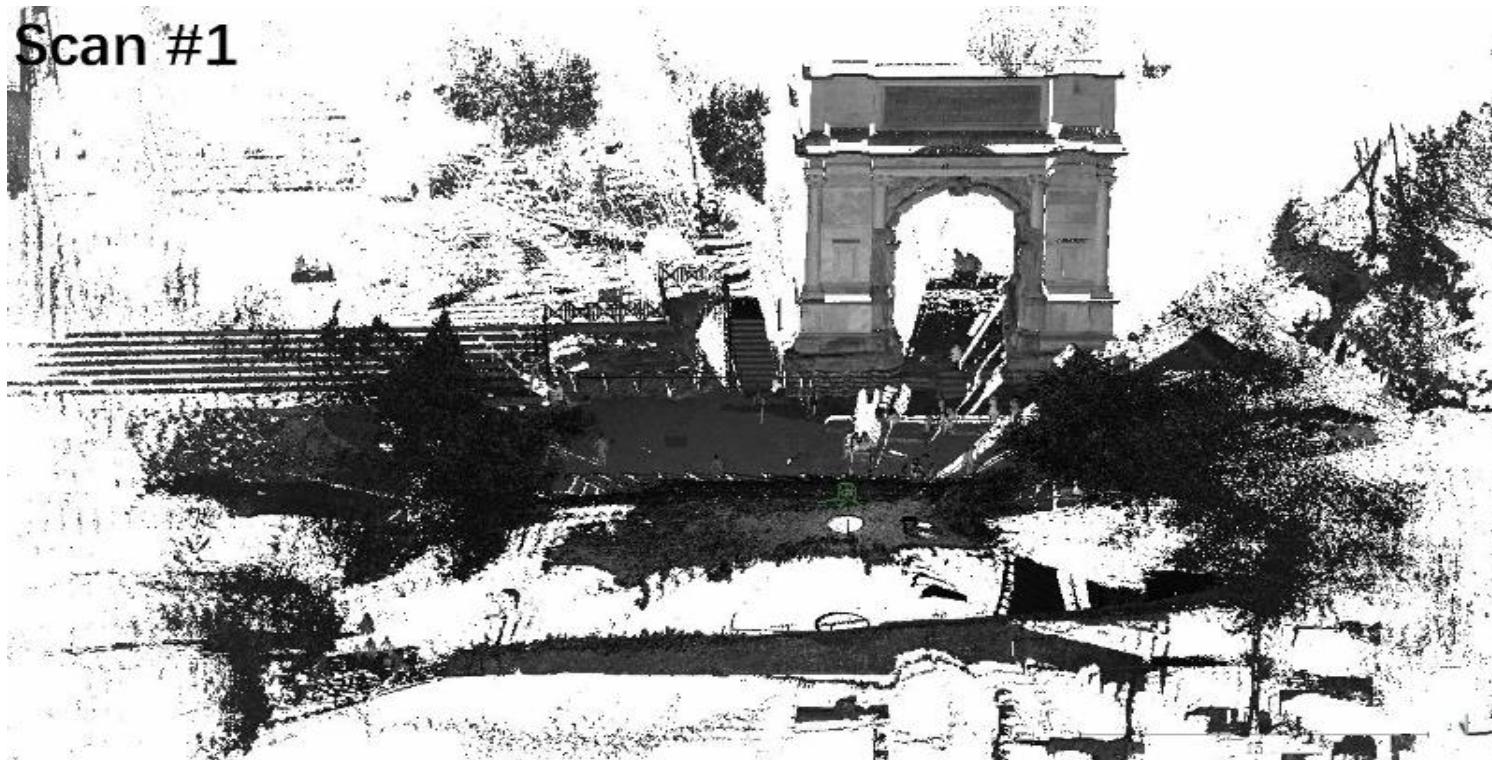


<https://prs.igp.ethz.ch/>



Dong et.al. (2018 ISPRSJ)

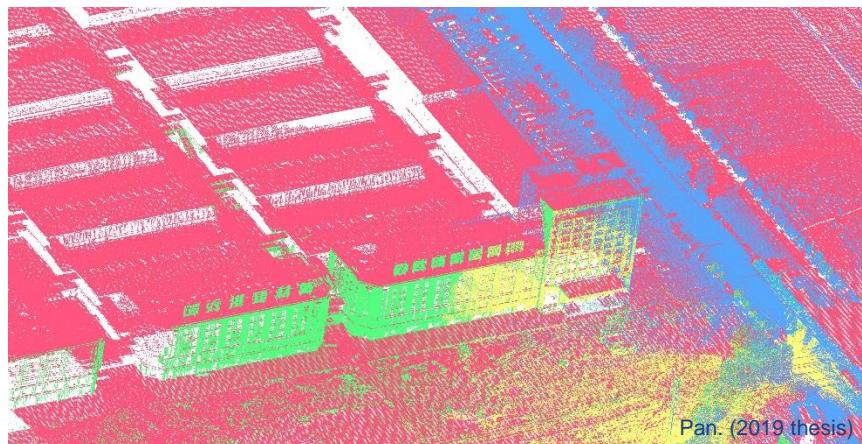
Scan #1



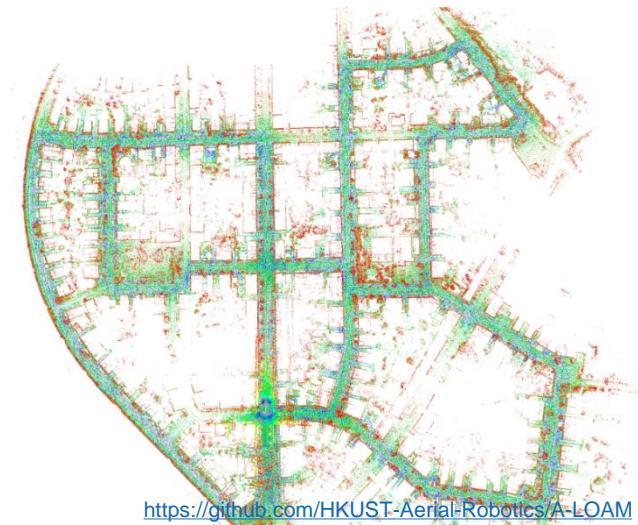
Application of registration

- Geomatics: Unify coordinate system of scans
- Simultaneous Localization and Mapping (SLAM)

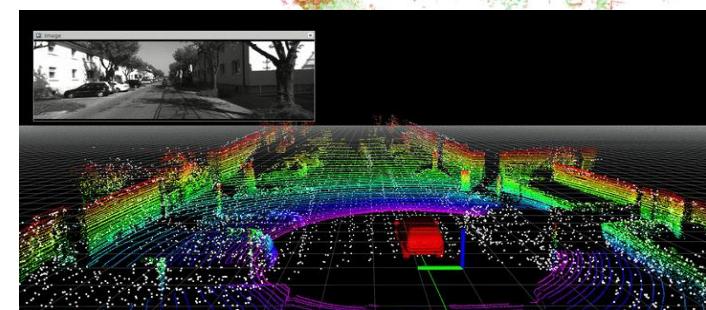
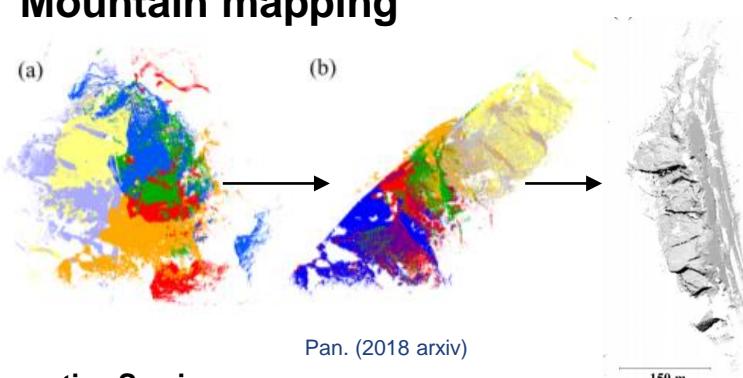
Multi-source point cloud fusion



Lidar Odometry and Mapping

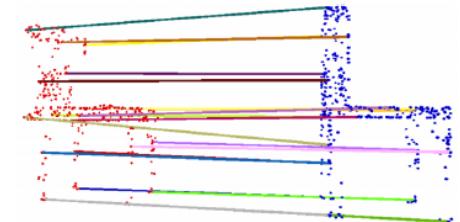


Mountain mapping



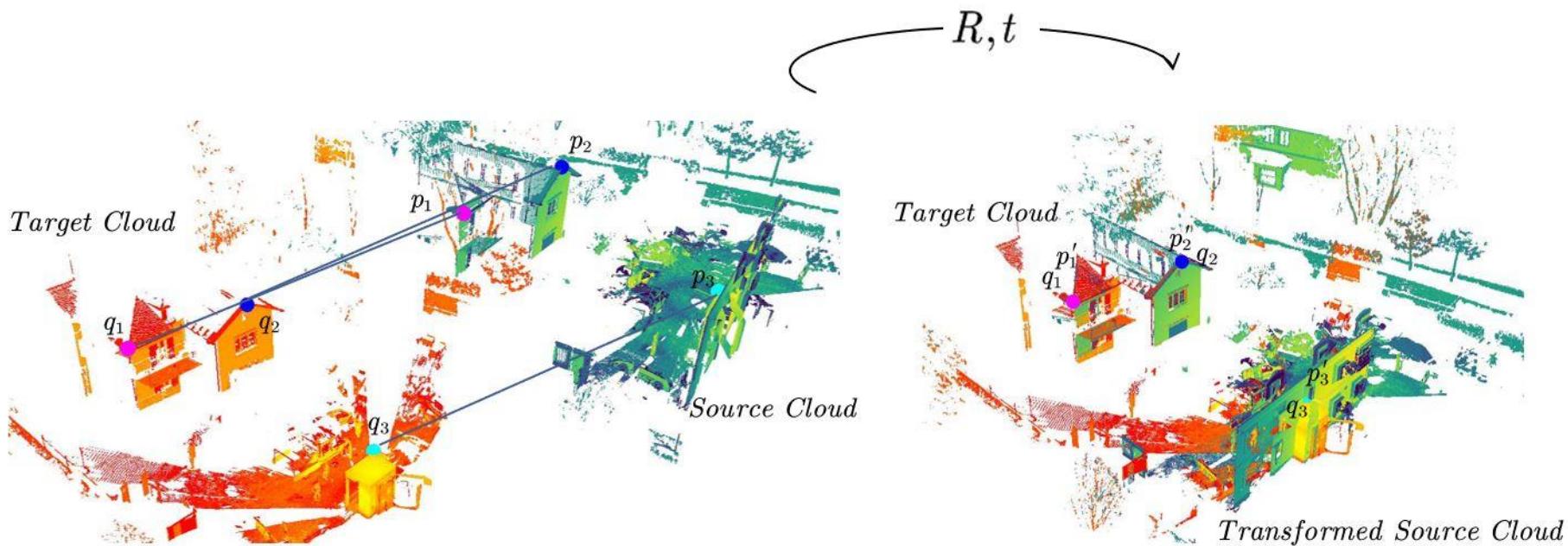
General Solution

- **First Step:** Determine Correspondences
 - Corresponding **points** / lines / **planes** / objects...
 - Prepare: Detect key points, Fit lines / planes, Recognize objects
- **Second Step:** Estimate Transformation
 - Define target function
 - Minimize (optimize) target function
 - By SVD, Linear least square, Non-linear optimization (LM, GN, GA...)
- If two steps are both solved accurately
- ——————> successful registration



Transformation Estimation for Point Correspondences

- Given point correspondences $\{p_i, q_i\}$
- Target function: $\{R^*, t^*\} = \operatorname{argmin}_{\{R, t\}} \left(\sum_i \|Rp_i + t - q_i\|^2 \right)$
- How to solve?
 - SVD, Linear Least Square, Non-linear Optimization



Transformation Estimation

SVD

Horn (1987 JOSA)

$$\{R^*, t^*\} = \underset{\{R, t\}}{\operatorname{argmin}} \left(\sum_i \|Rp_i + t - q_i\|^2 \right)$$

Correspondences known
 $\{p_i, q_i\}$

1. Calculate the centroids of two point clouds

$$\bar{p} = \frac{1}{N} \sum_{i=1}^N p_i, \bar{q} = \frac{1}{N} \sum_{i=1}^N q_i$$

2. Calculate the decentralized coordinate

$$p'_i = p_i - \bar{p}, \quad q'_i = q_i - \bar{q}$$

3. Apply Singular Value Decomposition (SVD)

$$U\Sigma V^T = \sum_{i=1}^N p'_i {q'}_i^T$$

4. Get rotation matrix R and translation vector t

$$R^* = VU^T, \quad t^* = \bar{q} - R^* \bar{p}$$

Deducing Details

Transformation Estimation

Linear least square

Correspondences known
 $\{p_i, q_i\}$

$$\{R^*, t^*\} = \underset{\{R, t\}}{\operatorname{argmin}} \left(\sum_i \|Rp_i + t - q_i\|^2 \right)$$

3. Parameter estimation by LS

1. Linear Approximation

$\sin(\alpha) \approx \alpha$ when $\alpha \rightarrow 0$

$$R \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_x + I$$

2. Observation function

$$v_i = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} x_i^{(p)} \\ y_i^{(p)} \\ z_i^{(p)} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \begin{bmatrix} x_i^{(q)} \\ y_i^{(q)} \\ z_i^{(q)} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} v_i^{(x)} \\ v_i^{(y)} \\ v_i^{(z)} \end{bmatrix}}_{v_i} = \underbrace{\begin{bmatrix} 0 & z_i^{(p)} & -y_i^{(p)} & 1 & 0 & 0 \\ -z_i^{(p)} & 0 & x_i^{(p)} & 0 & 1 & 0 \\ y_i^{(p)} & -x_i^{(p)} & 0 & 0 & 0 & 1 \end{bmatrix}}_{A_i} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ t_x \\ t_y \\ t_z \end{bmatrix}}_{l} - \underbrace{\begin{bmatrix} x_i^{(q)} - x_i^{(p)} \\ y_i^{(q)} - y_i^{(p)} \\ z_i^{(q)} - z_i^{(p)} \end{bmatrix}}_l$$

$$\underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}}_V = \underbrace{\begin{bmatrix} A_1 \\ \vdots \\ A_n \end{bmatrix}}_A x - \underbrace{\begin{bmatrix} l_1 \\ \vdots \\ l_n \end{bmatrix}}_l$$

$$x = [\alpha \ \beta \ \gamma \ \underbrace{t_x \ t_y \ t_z}_l]^T = (A^T A)^{-1} A^T l$$

get R^*, t^*

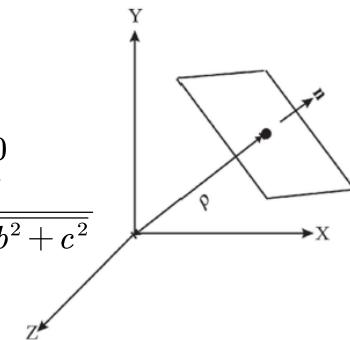
Transformation Estimation for Plane Correspondences

- Plane Correspondences
- Prepare: plane fitting by RANSAC

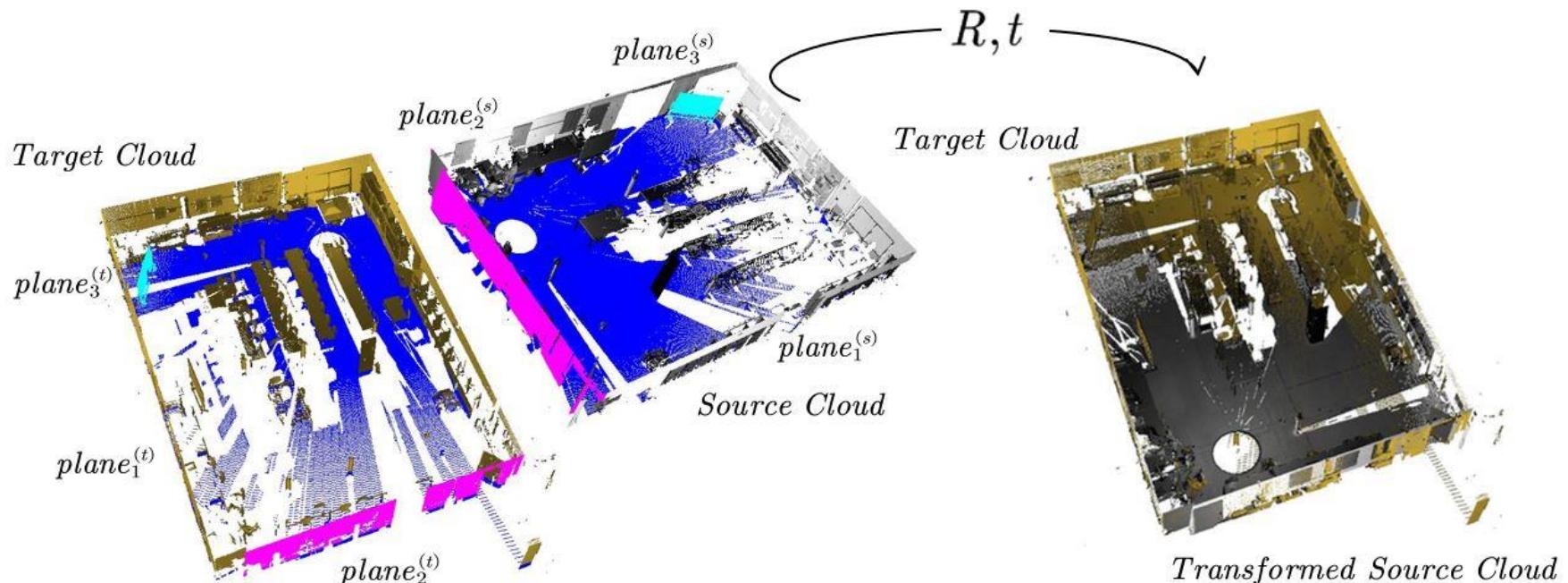
■ Target function: $\{R^*, t^*\} = \operatorname{argmin}_{\{R, t\}} \left(\sum_i \left\| \begin{bmatrix} Rn_i^{(s)} - n_i^{(t)} \\ \rho_i^{(s)} - \rho_i^{(t)} + (Rn_i^{(s)})^T t \end{bmatrix} \right\|^2 \right)$

$$ax + by + cz + d = 0$$

$$n = (a, b, c), \rho = \frac{d}{\sqrt{a^2 + b^2 + c^2}}$$

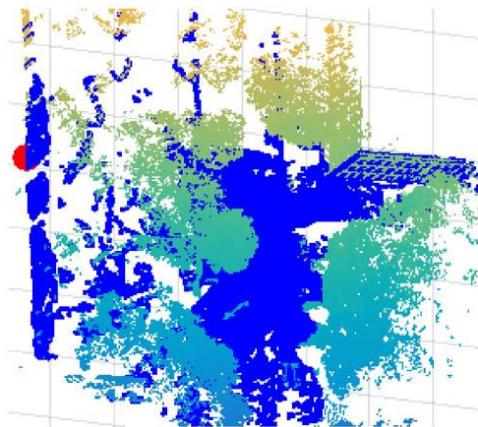
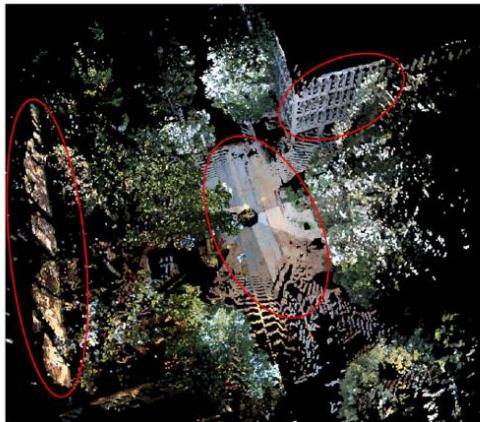


Previtali et.al. (2014 ISPRS)



Transformation Estimation for Plane Correspondences

- 3 planes + 1 intersection point fine registration Kim et.al. (2017 JCCE)



1. Calculate rotation matrix from normal vectors

$$R = \text{getRotation}(v_1, v_2) = I + [v_1 \times v_2]_{\times} + [v_1 \times v_2]_{\times}^2 \frac{1 - v_1 \cdot v_2}{\|v_1 \times v_2\|}$$

$$R_1 = \text{getRotation}(n_1^{(s)}, n_1^{(t)})$$

$$R_2 = \text{getRotation}(R_1 n_2^{(s)}, n_2^{(t)})$$

$$R_3 = \text{getRotation}(R_2 R_1 n_3^{(s)}, n_3^{(t)})$$

$$R^* = R_3 R_2 R_1$$

$$ax + by + cz + d = 0$$

$$n = (a, b, c), \rho = \frac{d}{\sqrt{a^2 + b^2 + c^2}}$$

2. Calculate translation vector from intersection points

$$x_{int}^{(t)} = \begin{bmatrix} a_1^{(t)} & b_1^{(t)} & c_1^{(t)} \\ a_2^{(t)} & b_2^{(t)} & c_2^{(t)} \\ a_3^{(t)} & b_3^{(t)} & c_3^{(t)} \end{bmatrix}^{-1} \begin{bmatrix} -d_1^{(t)} \\ -d_2^{(t)} \\ -d_3^{(t)} \end{bmatrix}$$

$$x_{int}^{(s)} = \begin{bmatrix} a_1^{(s)} & b_1^{(s)} & c_1^{(s)} \\ a_2^{(s)} & b_2^{(s)} & c_2^{(s)} \\ a_3^{(s)} & b_3^{(s)} & c_3^{(s)} \end{bmatrix}^{-1} \begin{bmatrix} -d_1^{(s)} \\ -d_2^{(s)} \\ -d_3^{(s)} \end{bmatrix}$$

$$t^* = x_{int}^{(t)} - x_{int}^{(s)}$$

Plane correspondences are set according to the closest normal vector

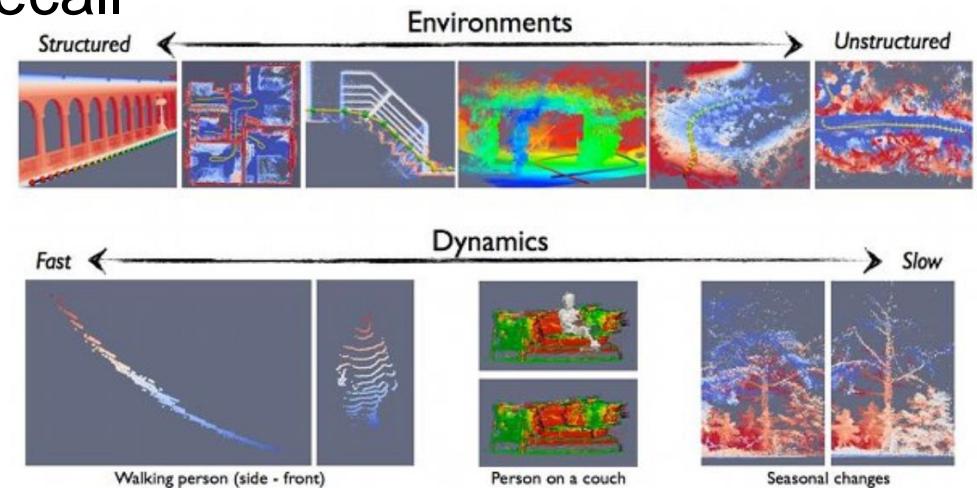
Determine Correspondences

- Manual: Target based
- Automatic:
 - Geometric closest : ICP
 - Feature matching
 - Random sample



Bustos et.al. (2017 PAMI)

- Goal: High precision & recall
- Challenges
 - Low overlapping rate
 - Occlusion
 - Noise
 - Repetitive structure



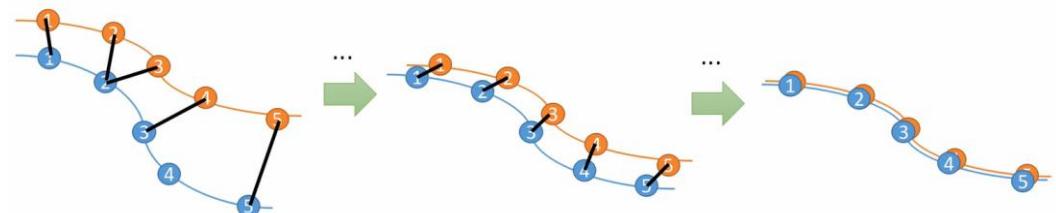
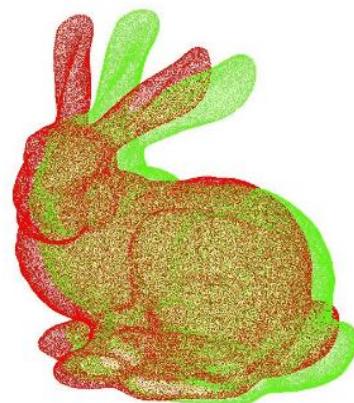
ETH ASL Registration Dataset

Iterative Closest Points (ICP) principle

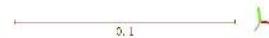
Besl et.al. (1992 PAMI)

- Iteratively
 - Assume **closest points** of source and target point cloud as correspondences $M = \{p_i, q_i\}$
 - Estimate transformation T_{temp} from M
 - Transform source cloud with T_{temp}
 - Converged to local optima

1



ICP Demo



ICP variants

Algorithm 1 Summary of ICP algorithm.

```

Require:  ${}^A\mathcal{P}$  Source Point Cloud
Require:  ${}^B\mathcal{Q}$  Target Point Cloud
Require:  $\mathcal{T}_{init}$  Transformation Initial Guess
 ${}^A\mathcal{P}' \leftarrow \text{datafilter}({}^A\mathcal{P})$ 
 ${}^B\mathcal{Q}' \leftarrow \text{datafilter}({}^B\mathcal{Q})$ 
 ${}^{i-1}\mathcal{T} \leftarrow \mathcal{T}_{init}$ 
repeat
     ${}^i\mathcal{P}' \leftarrow {}^{i-1}\mathcal{T}({}^{i-1}\mathcal{P}')$  Update Source Point Cloud
     $\mathcal{M}_i \leftarrow \text{match}({}^i\mathcal{P}', \mathcal{Q}')$  Determine correspondences
     $\mathcal{W}_i \leftarrow \text{outlier}(\mathcal{M}_i)$  Filter outliers
     ${}^{i+1}\mathcal{T} \leftarrow \arg \min_{\mathcal{T}} (\text{error}(\mathcal{T}({}^i\mathcal{P}'), \mathcal{Q}'))$  Estimate Transformation
    until convergence
Ensure:  ${}^A\hat{\mathcal{T}} = \left( \bigcirc_i {}^{i-1}\mathcal{T} \right) \circ \mathcal{T}_{init}$  Cumulate Final Transformation
  
```

François Pomerleau et.al. (2015 FTR)

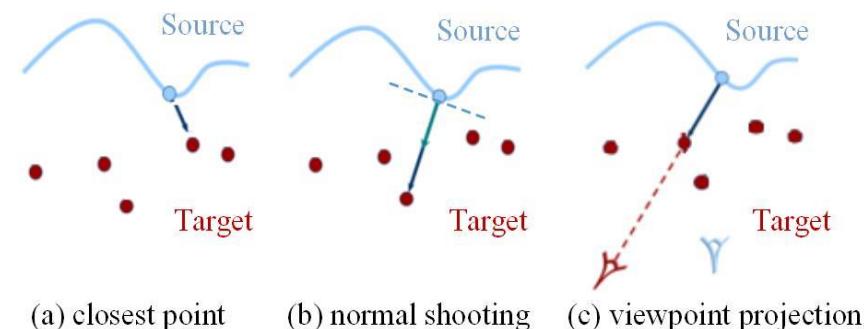
- **Determine correspondences:**
- Closest point (classic)
- Normal shooting (smooth structure)
- Viewpoint projection (efficiency)

Source Point Cloud
Target Point Cloud
Transformation Initial Guess

Filter Input Data



- **Filter input data:**
- Downsampling (efficiency)
- Extract keypoints (efficiency and saliency)



Wolfram et.al. Introduction to Mobile Robotics

ICP variants

- Filter outlier:
 - Correspondence distance threshold
 - Trimmed-ICP: estimate distance threshold using overlapping ratio Chetverkiov et.al. (2002 ICPR)
 - Normal compatibility, Unique...

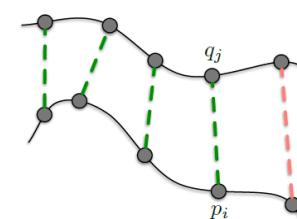
- Distance metrics for minimization:
 - Point-to-Point (classic)

$$\{R^*, t^*\} = \operatorname{argmin}_{\{R, t\}} \left(\sum_i \|Rp_i + t - q_i\|^2 \right)$$
 - Point-to-Plane (façade, ground) Chen et.al. (1992 IVC)

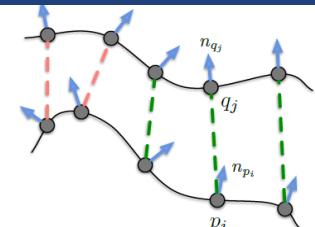
$$\{R^*, t^*\} = \operatorname{argmin}_{\{R, t\}} \left(\sum_i \|(Rp_i + t - q_i) \cdot n_i^{(q)}\|^2 \right)$$
 - Point-to-Line (trunk, pillar) Censi (2008 ICRA)

$$\{R^*, t^*\} = \operatorname{argmin}_{\{R, t\}} \left(\sum_i \|(Rp_i + t - q_i) \times (\widehat{n_i^{(q)} \times n_i^{(p)}})\|^2 \right)$$
 - General ICP (neighborhood covariance) Segal et.al. (2009 RSS)

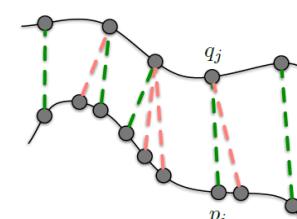
Holz et.al. (2015 RAM)



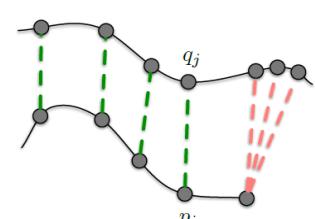
(a) Rejection based on the distance between the points.



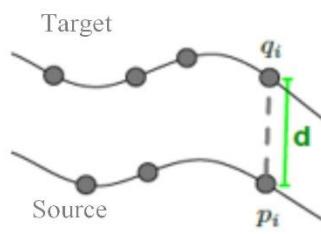
(b) Rejection based on normal compatibility.



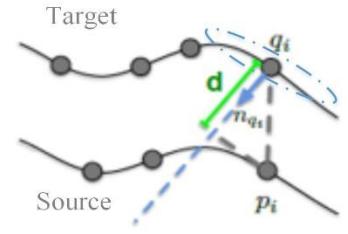
(c) Rejection of pairs with duplicate target matches.



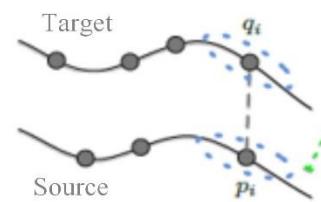
(d) Rejection of pairs that contain boundary points.



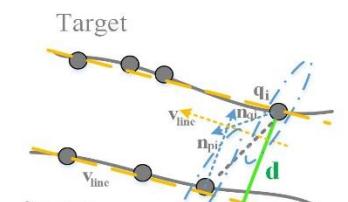
(a) Point to point error



(b) Point to plane error



(c) Generalized-ICP



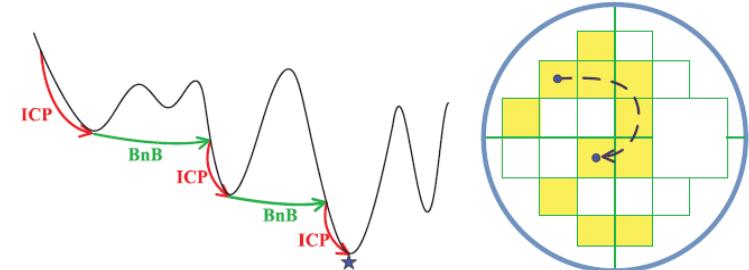
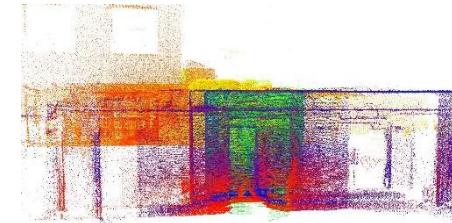
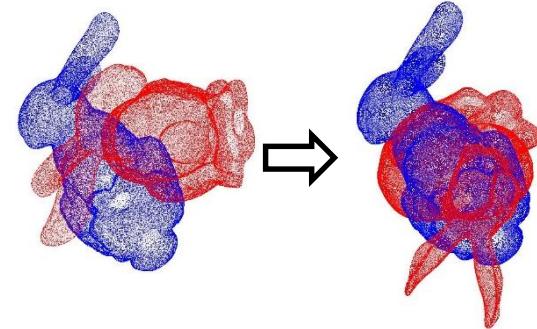
(d) Point to line error

ICP: Pros and Cons

- Pros:
 - Simple structure, easy to implement
 - Somewhat efficient
 - High accuracy when reaching global optima (**Fine registration**)
[popular in SLAM applications]

- Cons:
 - Converge to local optima when without good initial guess (**Local registration**)
 - Not robust to ...

- Improvement:
 - Go-ICP (arbitrary initial guess)
 - S-ICP (larger convergence domain)



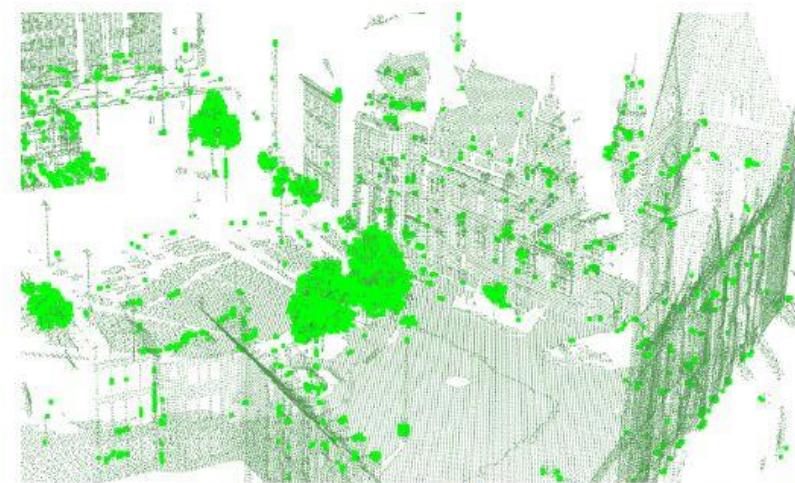
Yang et.al. (2017 PAMI)

Feature matching

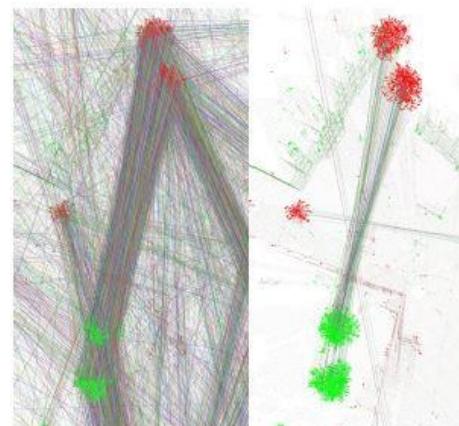
- Determine Correspondences:
Feature matching

- Workflow:
 - 1.Detect keypoints
 - 2.Extract feature
 - 3.Initial feature matching
 - 4.Filter outliers

- Keypoints detection:
 - local curvature extremum
(neighborhood eigen value calculation)
 - 3D Harris ...

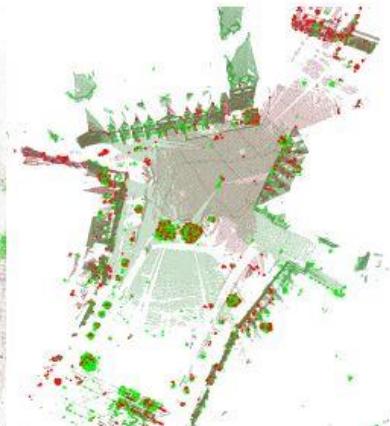


(a) Target point cloud with estimated keypoints (visualized with larger radii)



(b) Estimated correspondences

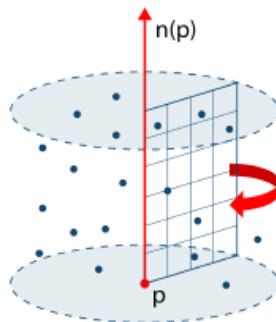
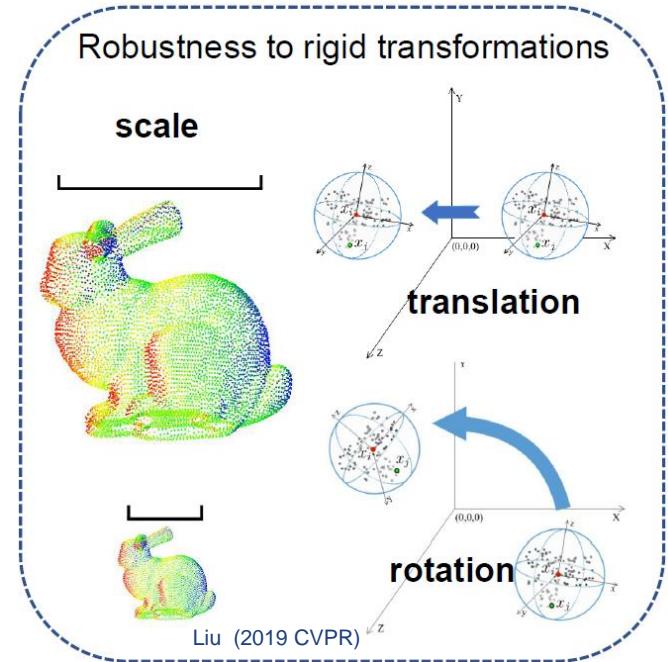
(c) Filtered correspondences



(d) Aligned point clouds

Holz et.al. (2015 RAM)

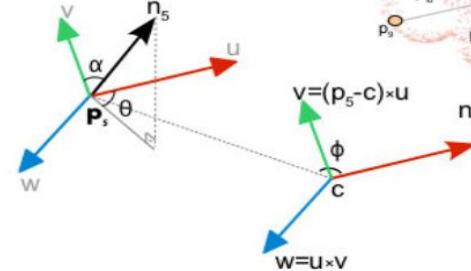
Handcrafted features



(a) Spin Images

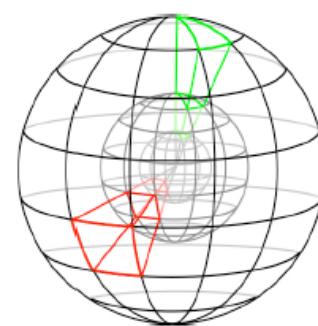
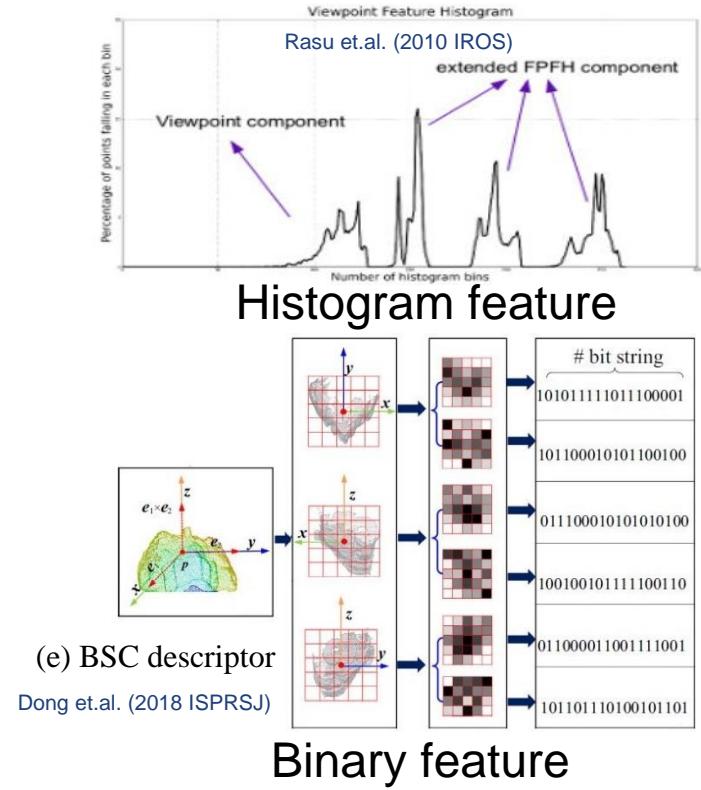
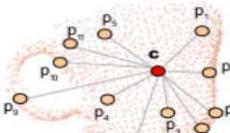
Andrew (1997 thesis)

Geomatics Seminar



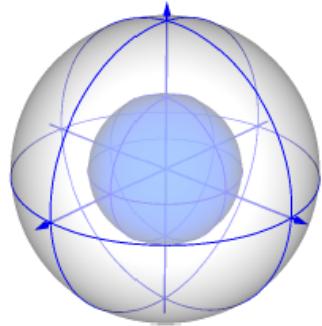
(b) FPFH Descriptor

Rusu et.al. (2009 ICRA)



(c) 3DSC Descriptor

Frome et.al. (2014 ECCV)



(d) SHOT Descriptor

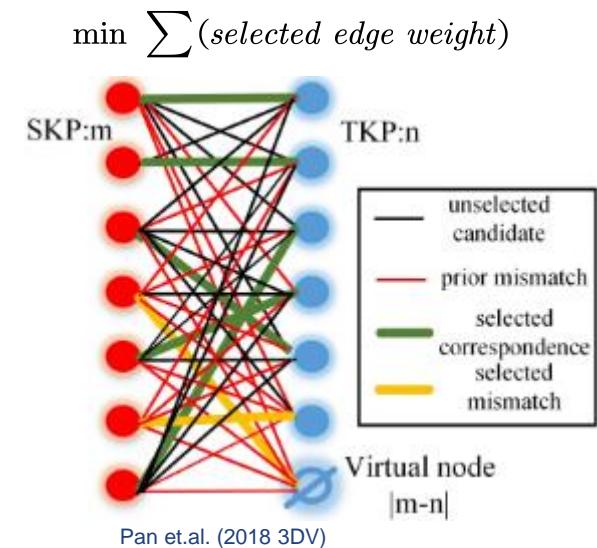
Tombari et.al. (2010 ECCV)

Matching strategy

- Similarity / Distance calculation
 - SAD (L1), SSD (L2), Hamming (Binary)
 - Cosine similarity, Correlation

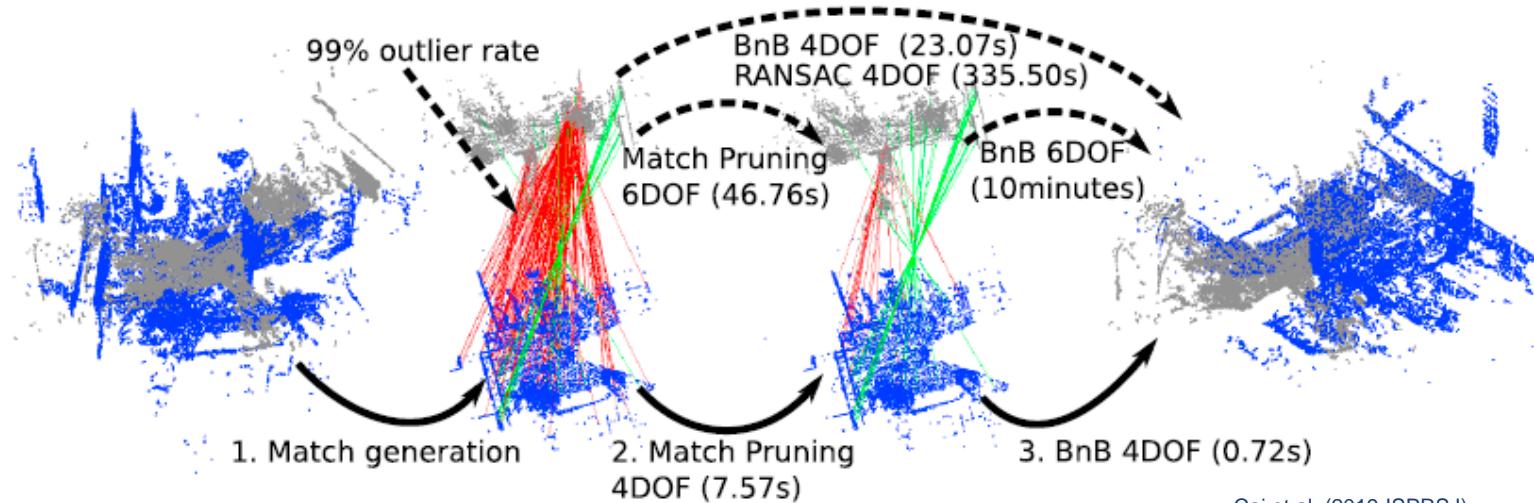
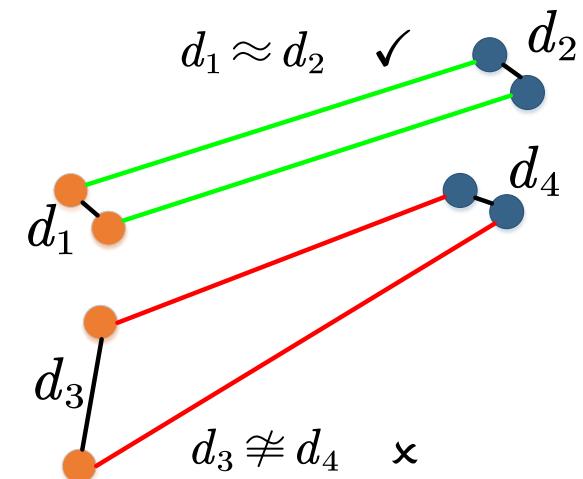
- Matching
 - One way / Reciprocal Nearest Neighbor
 - NN-SNN Ratio test
 - Bipartite graph

		TKP: n							$ m-n $
		11	19	4	30	10	30	30	
		17	10	16	30	17	30	30	
		20	30	5	28	11	29	30	
		30	21	30	24	30	30	30	
		18	26	6	7	12	30	30	
		23	30	27	30	30	30	30	
		22	24	7	21	13	30	30	



Robust outlier filtering

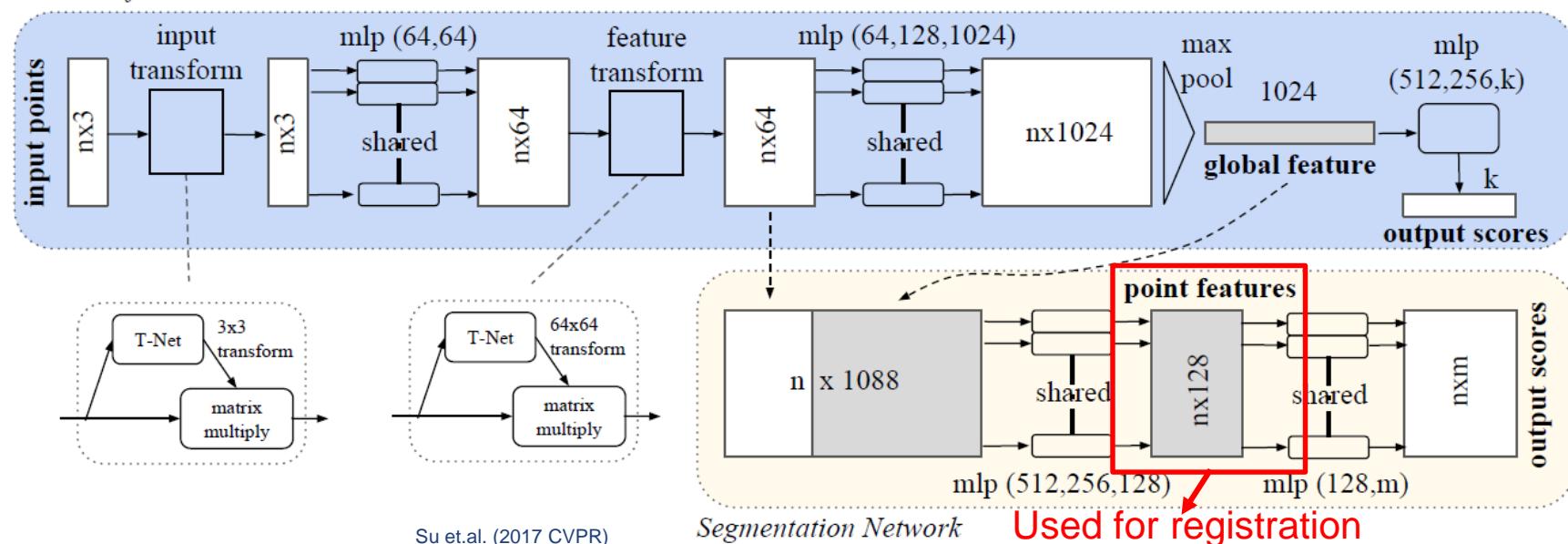
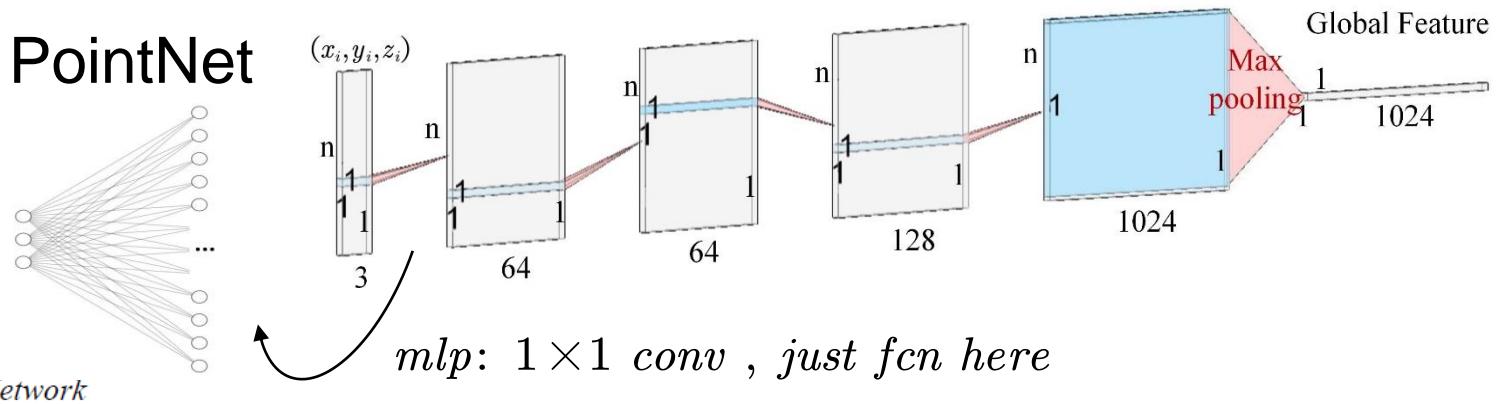
- RANSAC based on correspondence
 - Find 3 outlier-free correspondences
- Geometric consistency
- Guaranteed outlier removal (GORE)



Cai et.al. (2019 ISPRSJ)

Deep Learning based Registration

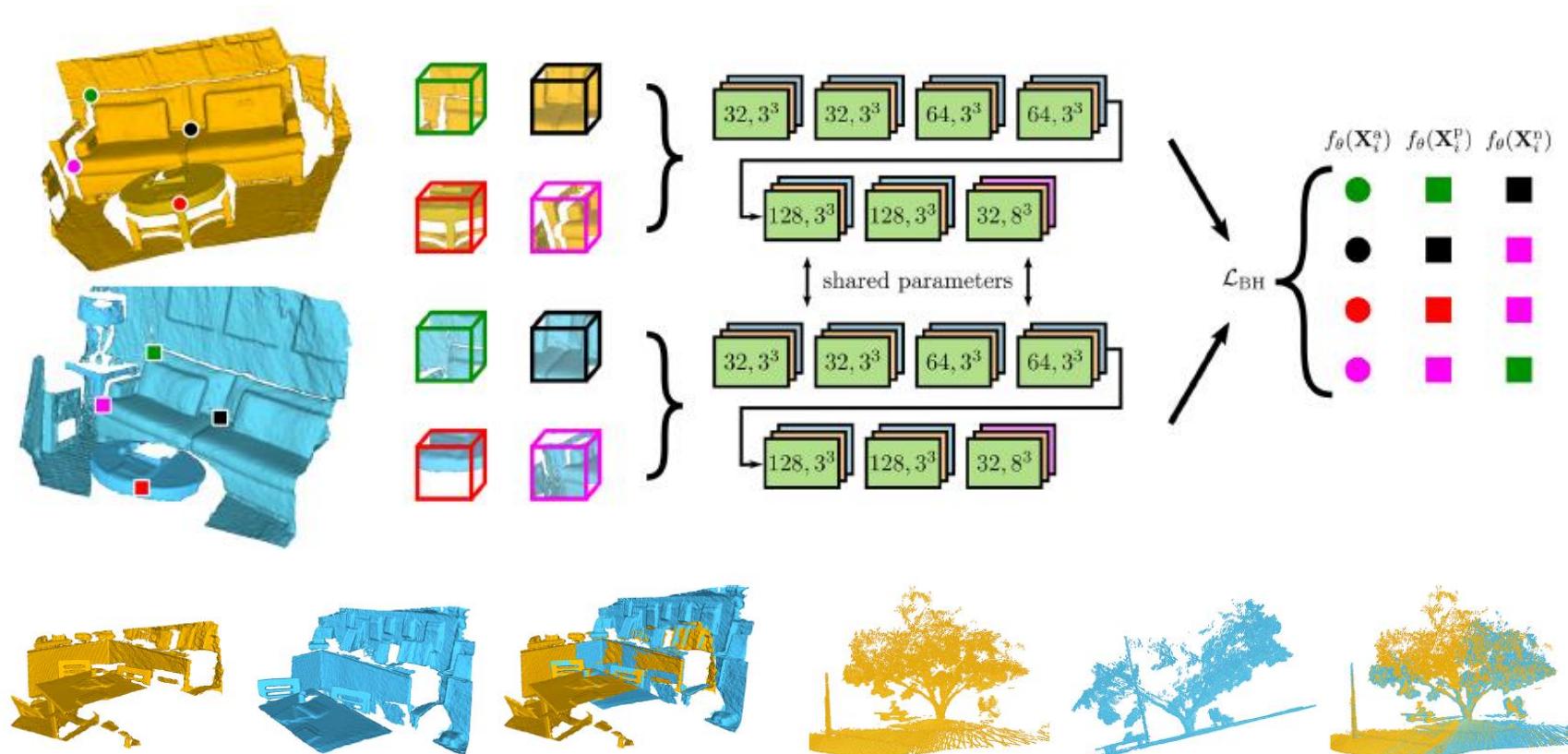
- Deep feature
- Basic: PointNet



Su et.al. (2017 CVPR)

Deep Learning based Registration

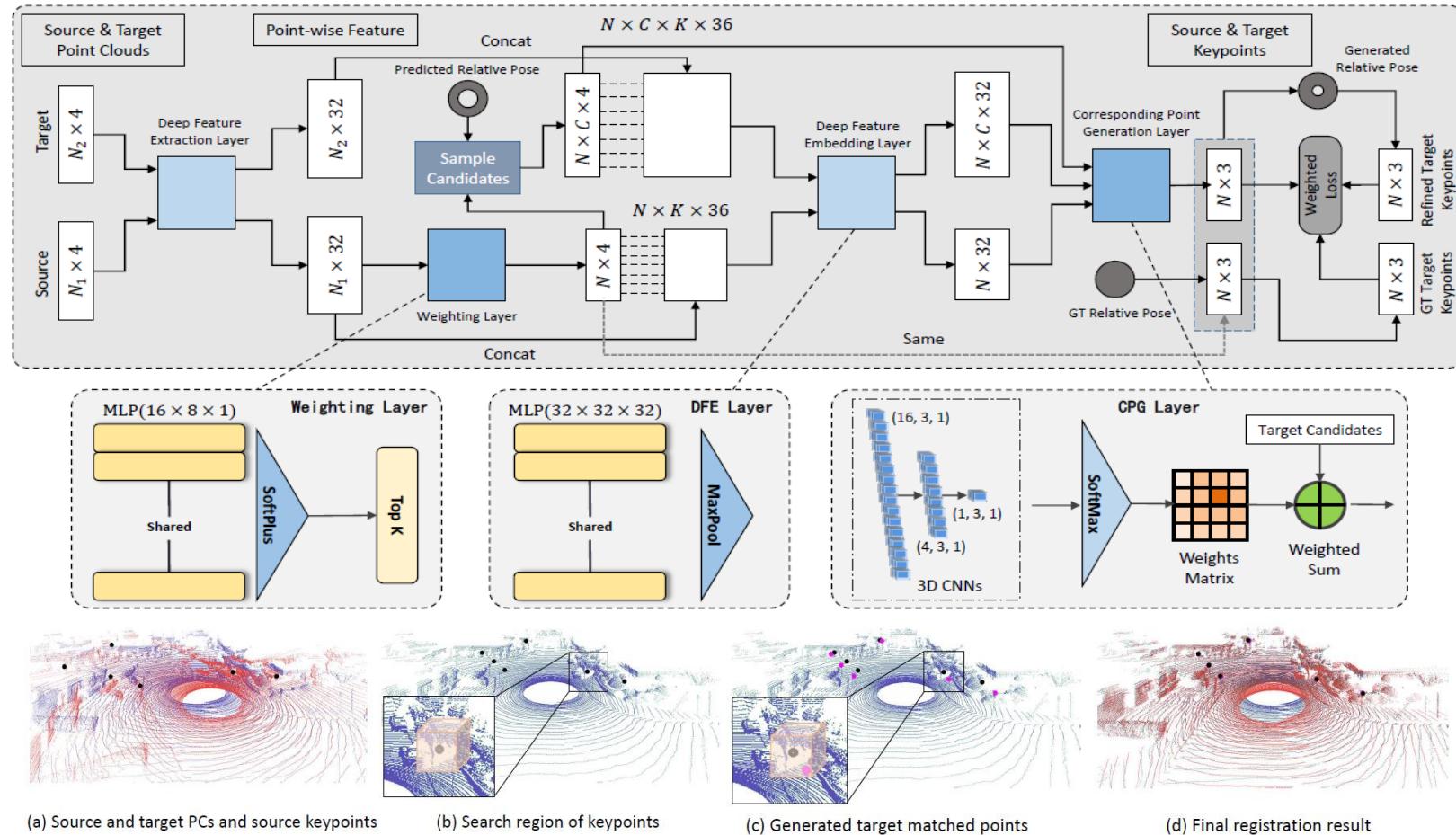
- End-to-end: features and correspondences
 - Example: 3D Smooth Net



Gojcic et.al. (2019 CVPR)

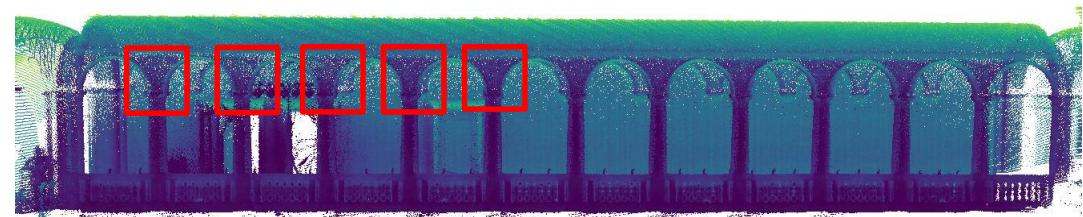
Deep Learning based Registration

- End-to-end: features, correspondence and transformation
- Example: DeepVCP



Feature based methods: Pros and Cons

- Pros:
 - No requirement of transformation initial guess (**Global registration**)
 - Robust (with proper matching strategy and outlier filter)
- Cons:
 - Not accurate enough (**Coarse registration**)
 - Often based on keypoints (the raw point cloud should be dense)
 - May be time-consuming
- Outlook:
 - Deep learning based



Repetitive structures

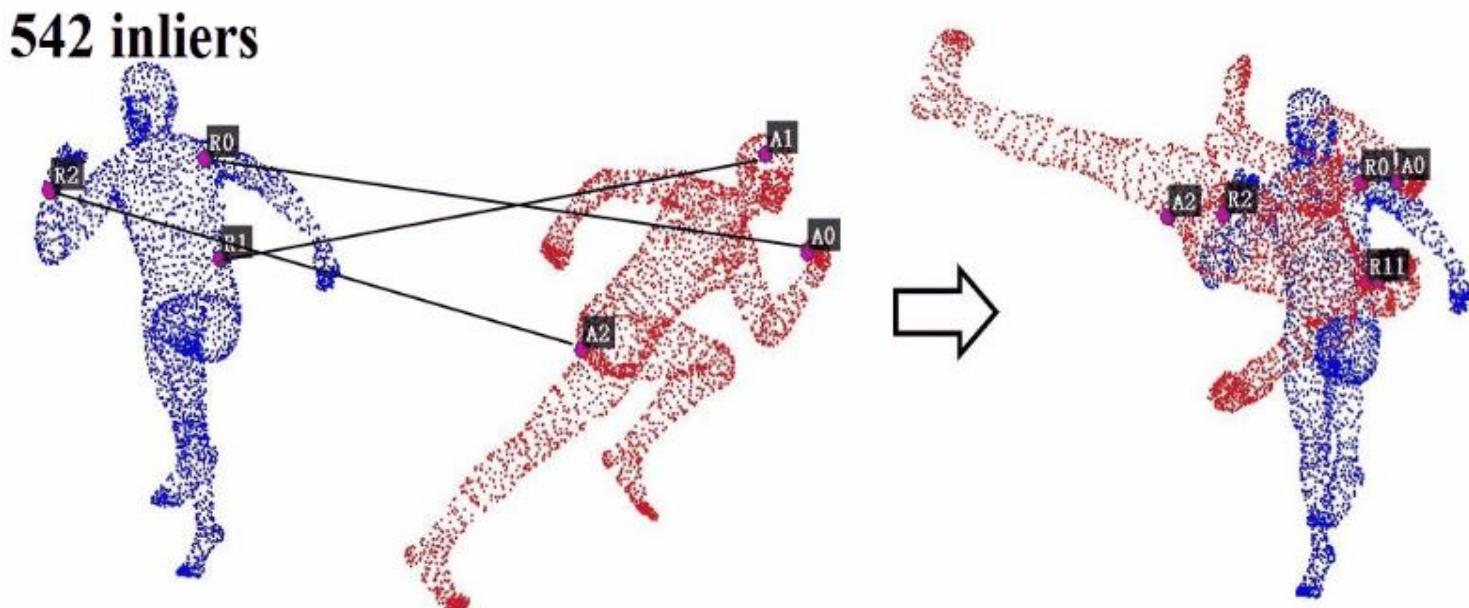
Randomized hypothesize-and-verify

- RANSAC without correspondences
 - Find two outlier-free 3-point subsets with correct correspondences
- Problem: efficiency (Total sampling number M)

Chen et.al. (1999 PAMI)

$$p = 1 - (1 - r^N)^M > 0.99 \quad r: \text{point inlier ratio}, \ N=3$$

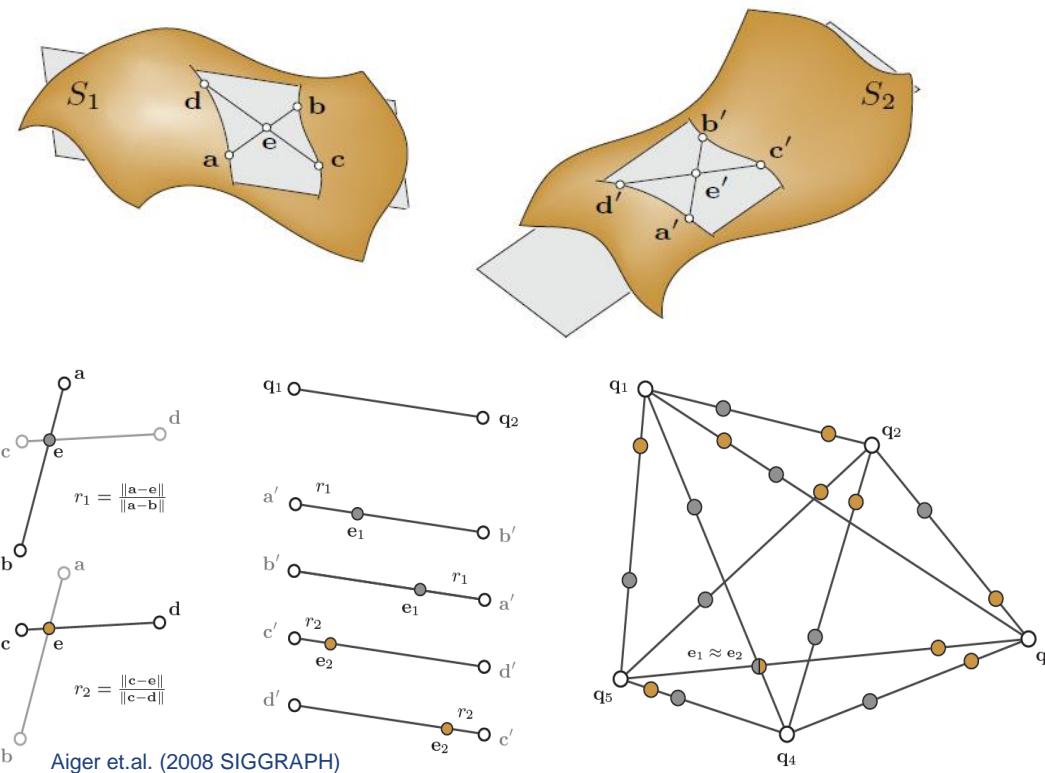
For each hypothesize to find the geometric consistent 3-point subset in the other point cloud $O(n^3)$



Efficiency improvement: 4PCS

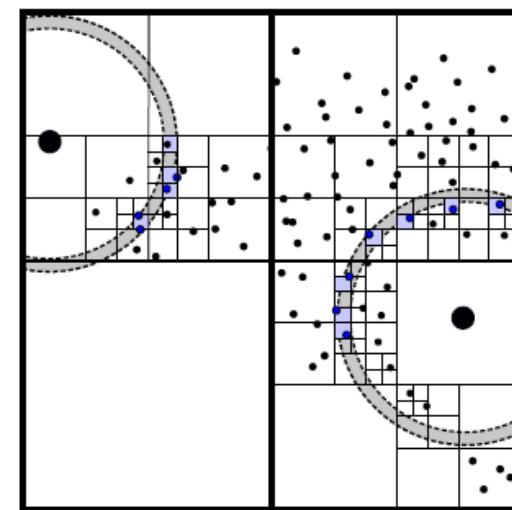
4 Points Congruent Set

- $O(n^2)$ searching of affine-invariant 4 coplanar point sets
- Invariant of intersection ratio



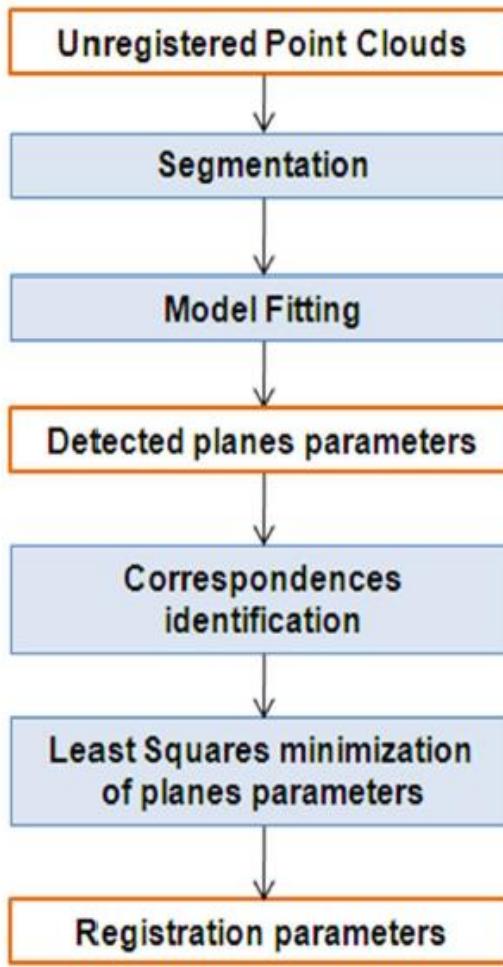
■ Variants:

- Super4PCS: Mellado et.al. (2014 SGP)
Smart-indexing
(efficiency) $O(n^2) \Rightarrow O(n)$



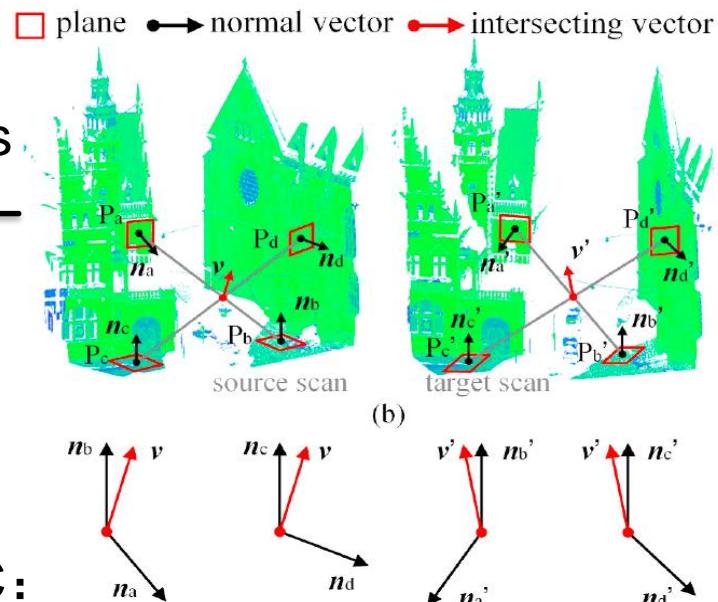
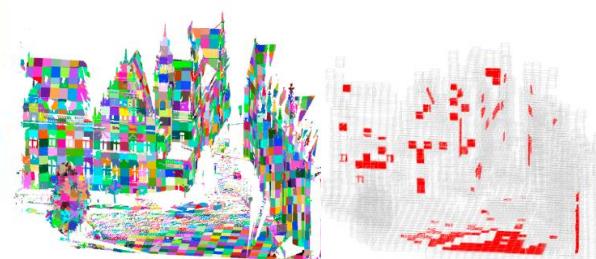
- K4PCS: Theiler et.al. (2017 ISPRSJ)
Base on keypoints
(efficiency)

Example: Plane Based RANSAC



Voxel-based
4-plane congruent sets
(V4PCS):
Improvement of
sampling strategies,
plane version of 4PCS

Plane based RANSAC:
→ Minimum 3-planes
correspondence



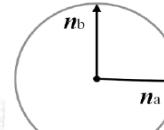
$$r_1 = \frac{\leq n_a, v \geq}{\leq n_a, n_b \geq}$$

$$r_2 = \frac{\leq n_c, v \geq}{\leq n_c, n_d \geq}$$

$$r_1' = \frac{\leq n_a', v' \geq}{\leq n_a', n_b' \geq}$$

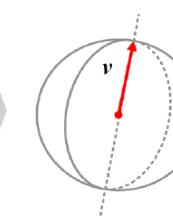
$$r_2' = \frac{\leq n_c', v' \geq}{\leq n_c', n_d' \geq}$$

○ plane formed by two
normal vectors

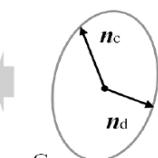


plane of pair I

→ intersecting vector



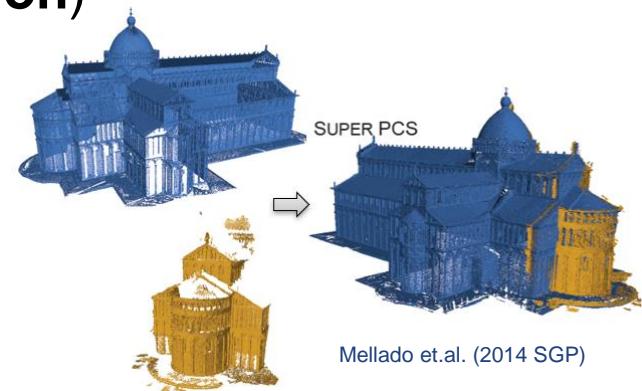
intersection of planes



plane of pair II

RANSAC based methods: Pros and Cons

- Pros:
 - No requirement of transformation initial guess (**Global registration**)
 - Robust to noise, outliers and similar / repetitive structures
- Cons:
 - With noise and outliers, the likelihood of picking outlier-free subsets degrades rapidly
 - Could be very time-consuming or you need to do downsampling
 - Not accurate enough (**Coarse registration**)
- Outlooks:
 - Better sampling strategies

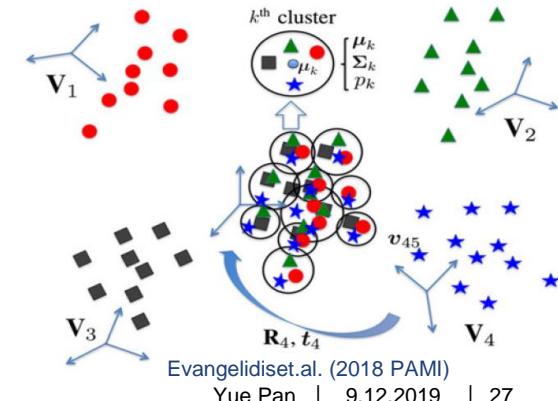
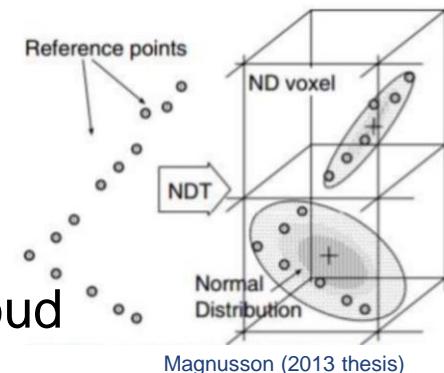


Another way: Probability a brief intro.

Magnusson et.al. (2009 ICRA)

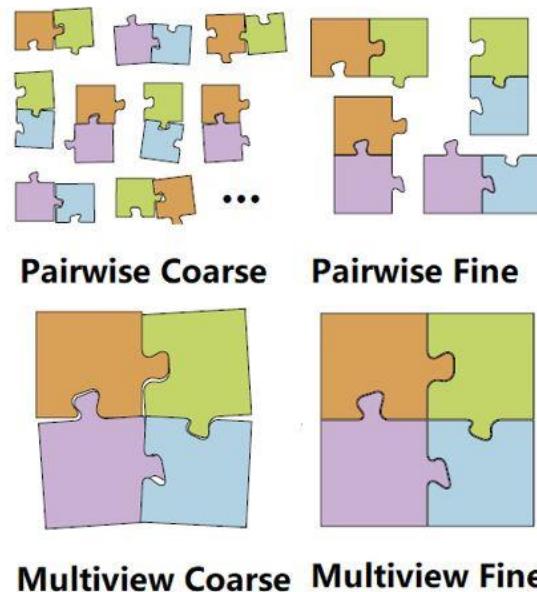
- Normal Distribution Transformation (NDT)
 - Fit Gaussian distribution $p_N^{(t)}$ in each voxel of target cloud

$$R^*, t^* = \arg \max_{R, t} \left(\prod_i p_N^{(t)}(Rx_i^{(s)} + t) \right)$$
 - Faster and has larger convergence domain than ICP, but not stable
 - Fine registration (Applications like SLAM, Localization)
- Coherent Point Drift (CPD) Myronenko et.al. (2010 PAMI)
 - Each point in Target cloud: one Gaussian kernel centroid
 - Points in Source cloud: Generate by the GMM of Target cloud
 - Solved by EM
- GMM-Registration Jian et.al. (2011 PAMI)
- GUGMA : GMMReg.+BnB Campbell et.al. (2016 CVPR)

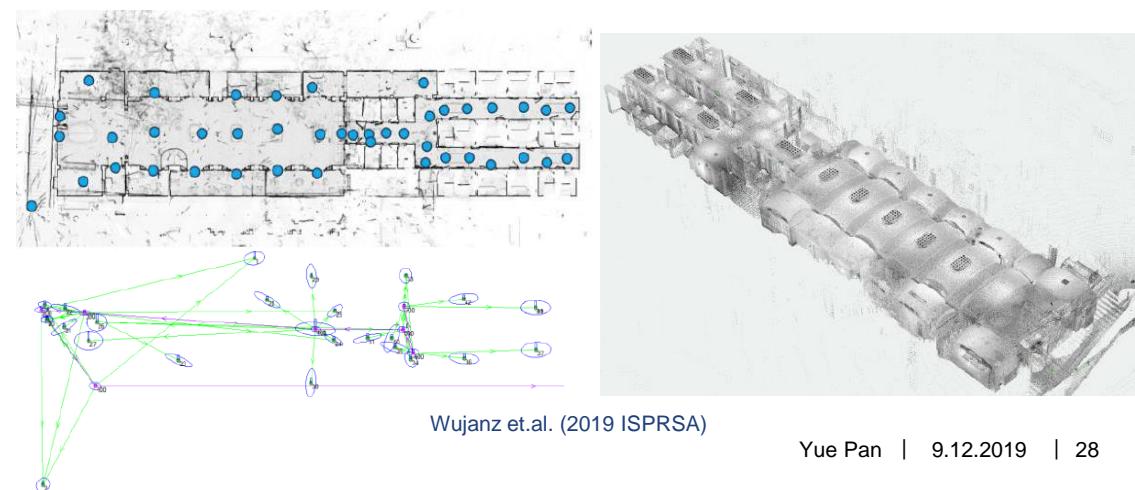
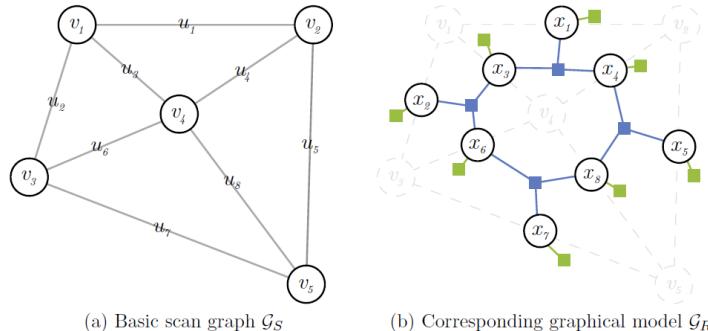


Combination

- Coarse to Fine
- Pairwise to Multiview
 - Joint optimization (adjustment)
 - Minimum Spanning Tree
 - Pose Graph Optimization



Theiler et.al. (2015 ISPRSJ)

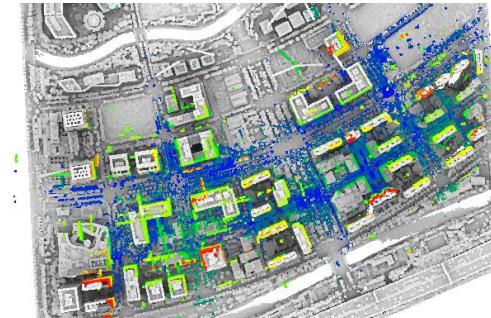


Conclusions

- Two steps
 - Determine correspondences (points, planes, etc.)
 - Estimate transformation according to the target function
- Methods
 - Feature based (Coarse)
 - RANSAC based (Coarse)
 - ICP (Fine)
 - Probability based
- Challenges
 - small overlap, occlusion, noise...
 - large variance of point density
 - consuming time (for real-time applications)

Open Questions

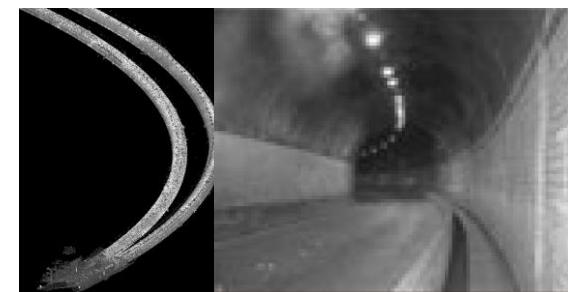
- Cross-platform Registration
 - ALS and TLS
- Real-time Lidar Odometry
 - KITTI Odometry top: V-LOAM
 - (ATE 0.55, ARE 0.13) still not accurate enough
 - Challenging case: Tunnel mapping
- Little-overlapping registration
 - Digital cultural relic restoration



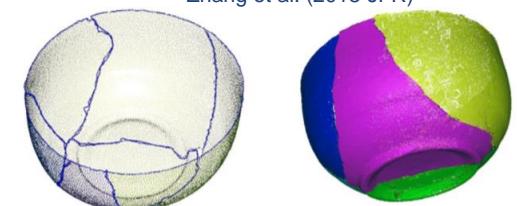
Liang et.al. (2019 CLC)



KITTI Dataset: Odometry



Zhang et.al. (2018 JFR)



Wang et.al. (2019 CLC)

Q & A:

- Questions for audiences:

- Thanks.

Transformation Estimation

SVD in detail

$$\bar{p} = \frac{1}{N} \sum_{i=1}^N p_i, \bar{q} = \frac{1}{N} \sum_{i=1}^N q_i \quad \text{Calculate centroids}$$

$$\begin{aligned} E(R, t) &= \sum_i \|Rp_i + t - q_i\|^2 \\ &= \sum_i \|R(p_i - \bar{p}) - (q_i - \bar{q}) + (R\bar{p} + t - \bar{q})\|^2 \\ &= \sum_i \|R(p_i - \bar{p}) - (q_i - \bar{q})\|^2 \quad R\bar{p} + t - \bar{q} = 0 \end{aligned}$$

$$\begin{aligned} R^* &= \arg \min_R \sum_i \|R(p_i - \bar{p}) - (q_i - \bar{q})\|^2 \\ &= \arg \min_R \sum_i \|Rp'_i - q'_i\|^2 \quad \text{decentralized coordinates} \\ &= \arg \min_R \sum_i (p'^T_i R^T Rp'_i + q'^T_i q'_i - 2q'^T_i Rp'_i) \\ &= \arg \max_R \sum_i (q'^T_i Rp'_i) \end{aligned}$$

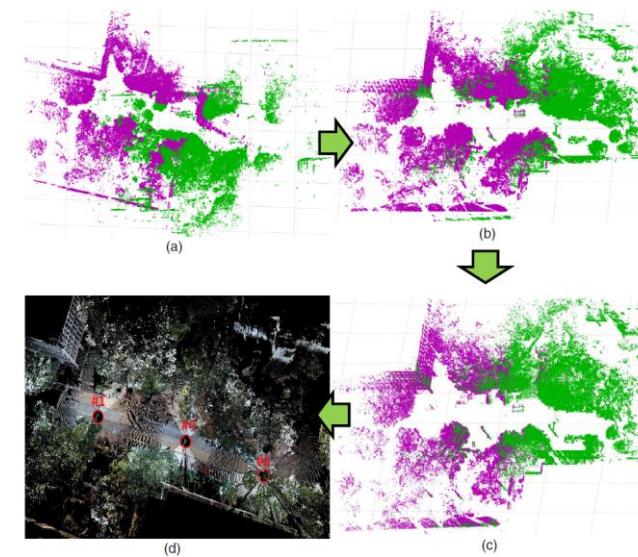
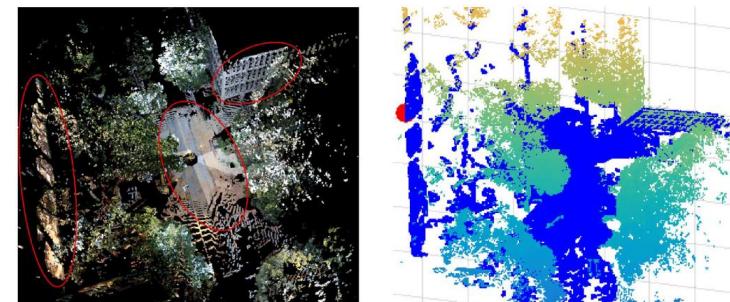
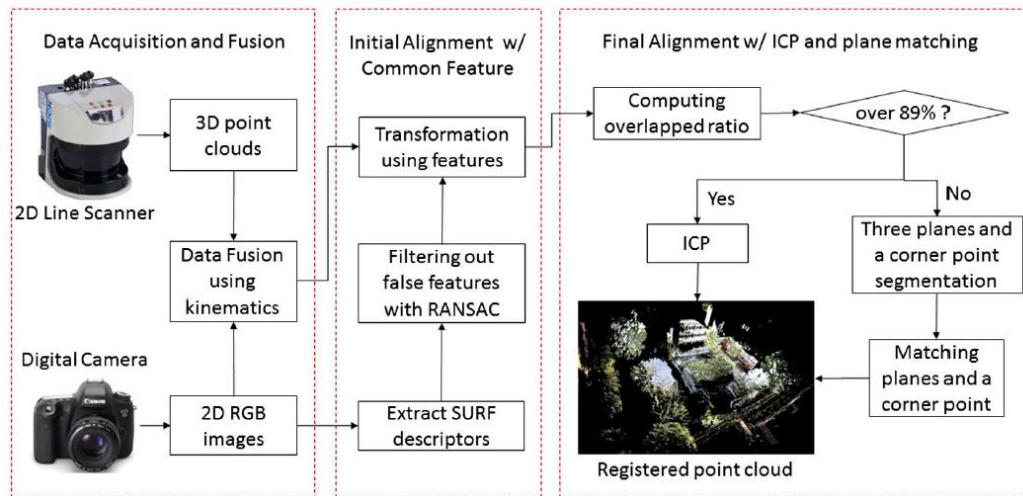
$$\begin{aligned} R^* &= \arg \max_R \sum_i (\text{trace}(Rp'_i q'^T_i)) \\ &= \arg \max_R (\text{trace}\left(R \sum_i p'_i q'^T_i\right)) \quad \text{the properties of trace} \\ &= \arg \max_R (\text{trace}(RH)) \quad \text{denote } \sum_i p'_i q'^T_i \text{ as } H \\ &= \arg \max_R (\text{trace}(RU\Sigma V^T)) \quad \text{SVD: } H = U\Sigma V^T \\ &= \arg \max_R (\text{trace}(V^T RU\Sigma)) \\ &= \arg \max_R (\text{trace}(R'\Sigma)) \quad \text{denote } V^T RU \text{ as } R' \\ &= V \left(\arg \max_{R'} (\text{trace}(R'\Sigma)) \right) U^T \quad \text{so } R = VR'U^T \\ &= V \left(\arg \max_{R'} (r'_{11}\sigma_1 + r'_{22}\sigma_2 + r'_{33}\sigma_3) \right) U^T \\ &= VU^T \quad \text{when } r'_{11} = r'_{22} = r'_{33} = 1 \Rightarrow R' = I \end{aligned}$$

$$t^* = \bar{q} - R^* \bar{p}$$

[Back](#)

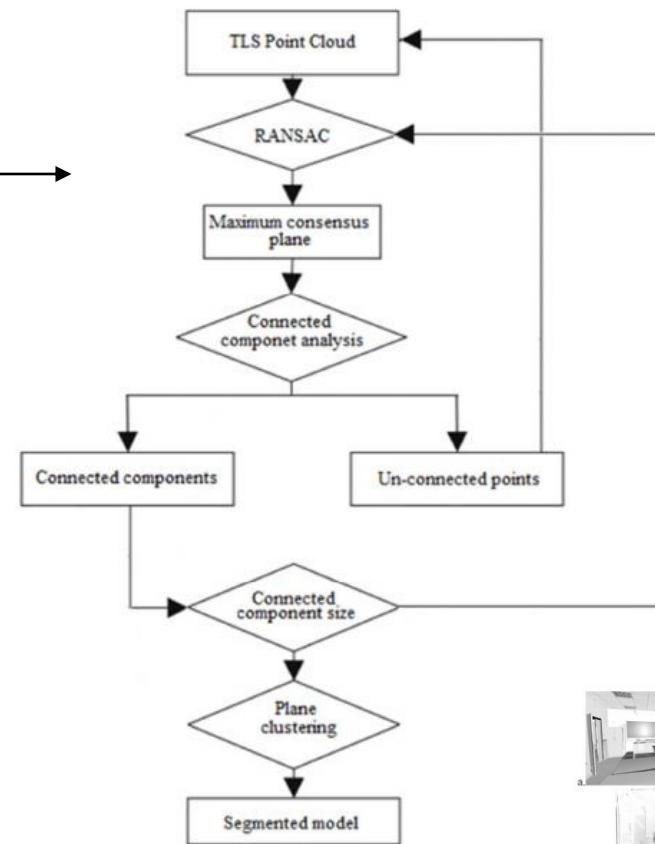
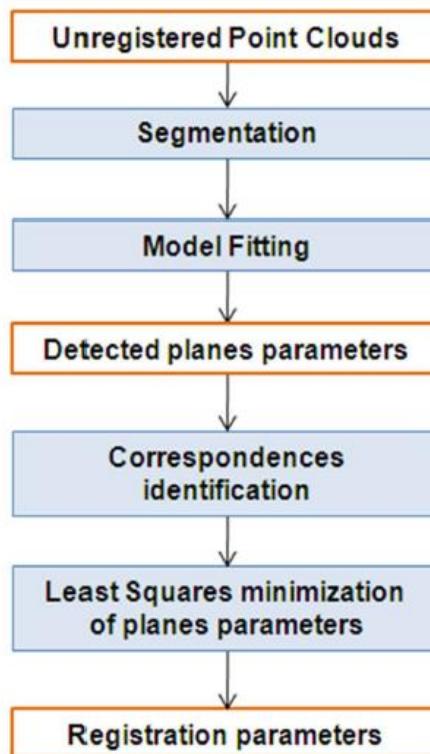
Paper Reading

Automated Point Cloud Registration Using Visual and Planar Features for Construction Environments

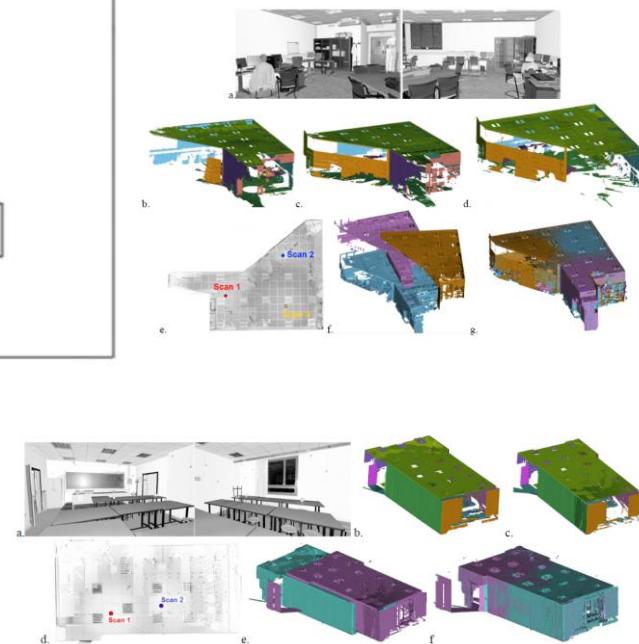


Paper Reading

Laser scan registration using planar features



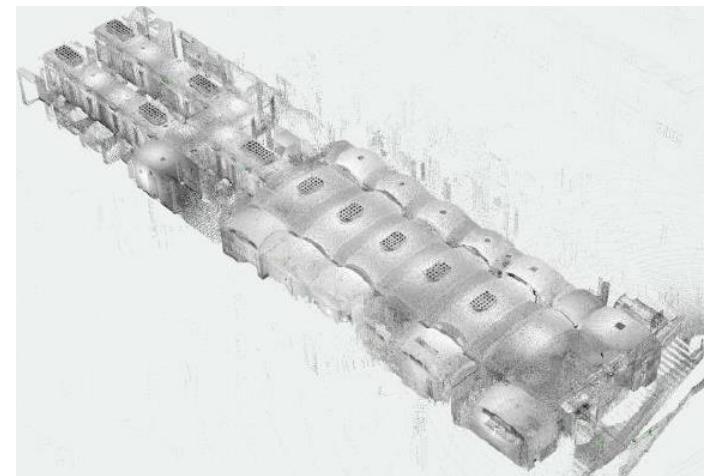
Examples



Paper Reading

A comparative study among three registration algorithms: performance, quality assurance and accuracy

- Compare target-based, surface-based and plane-based registration methods.
- Not SOTA enough



Plane-based vs target-based registration			
	DX (mm)	DY (mm)	DZ (mm)
Mean	-2.9	-0.7	-0.1
Std.dev.	2.3	1.5	6.3
Min	-8.5	-4.1	-10.5
Max	4.6	3.8	14.9

Contact information and credits

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